Reliability Analysis of Aircraft Cannon Targeting System (ACTS)

1. Reliability Calculation for Mod 2 (Parallel Servo Controllers)

Unreliability of One Servo Controller (QD1) at 400 Hours:

The servo controller has a failure rate of 495.5 per million hours ($\lambda \approx 4.955\times10{-}4$ per hour).

$$Q_{D1}(400) = 1 - e^{-\lambda\cdot 400}.$$

$$\lambda \cdot 400 \approx 4.955 \times 10^{-4} \times 400 = 0.1982.$$

$$Q_{D1}(400) = 1 - e^{-0.1982} = 0.1797942044$$

$$R_{D1}(400) = 1 - Q_{D1}(400) = 0.8202$$

about an **18.0%** chance that a single servo controller fails or a reliability of 82% within 400 hours.

Reliability of the Parallel Servo Controller Subsystem (Rcomp) at 400 Hours: In Mod 2, two identical servo controllers operate in parallel (active redundancy). The subsystem works as long as **at least one** controller is functional.

$$R_{\text{comp}}(400) = 1 - [Q_{D1}(400)]^2.$$

Using $Q_{D1}(400) \approx 0.1800$, we get:

$$R_{\text{comp}}(400) = 1 - (0.1797942044)^2 = 1 - 0.03232606381 = 0.9677\ (to\ 4\ s.f)$$

So the dual-controller servo subsystem has **96.77% reliability at 400 hours**, a dramatic improvement over a single controller's 82%. In other words, the chance that *both* controllers fail by 400h is only ~3.24%, so the parallel servo setup is very robust.

2. Reliability of ACTS for Mod 1, Mod 2, and Mod 3

The given failure rates are in **failures per million flying hours** and the failure rate is given by the following equation

$$\lambda = \frac{\text{Failure Rate per million hours}}{10^6}$$

Applying this to all components:

Component	Failure Rate (per million hours)	Failure Rate (per hour) (λ)
Power Supply	193.94	1.9394×10^{-4}
HUD Interface Unit	87.07	8.707×10^{-5}
Servo Controller	495.5	4.955×10^{-4}
Elevation Servo Actuator	53.67	5.367×10^{-5}
Azimuth Servo Actuator	116.9	1.169×10^{-4}
Ammunition Feed System	77.39	7.739×10^{-5}
ECU	10.39	1.039×10^{-5}
Recoil System	189.63	1.8963×10^{-4}
Temperature Sensor	72.32	7.232×10^{-5}
Airspeed Sensor	84.49	8.449×10^{-5}
Ballistic Load Sensor	125.49	1.2549×10^{-4}

The reliability of a component over time t (assuming an exponential failure distribution) is given by:

$$R(t) = e^{-\lambda t}$$

For each component, we calculate $R(t)$ at both **10 hours and 400 hours**.

Component	$R(10)$	$R(400)$
Power Supply	0.9808	0.9236
HUD Interface Unit	0.9913	0.9155
Servo Controller	0.9951	0.8200
Elevation Servo Actuator	0.9946	0.9790
Azimuth Servo Actuator	0.9884	0.8975
Ammunition Feed System	0.9923	0.9257
ECU	0.9999	0.9959
Recoil System	0.9812	0.9231
Temperature Sensor	0.9928	0.9291
Airspeed Sensor	0.9916	0.9189
Ballistic Load Sensor	0.9875	0.8833

Using the given failure rates for each of the 11 subsystems (see table above), we compute the **system reliability** R_S at 10 hours (a short mission) and 400 hours (extended operation) for each configuration:

Mod 1:
For a series system, the total system reliability is:

$$R_S = \prod_{i=1}^{11} R_i\,(t)$$

Multiplying all the $R_i(10)$ values:

$$R_S(10) = 0.9808 \times 0.9913 \times 0.9951 \times \ldots \times 0.9875 = 0.985$$

Multiplying all the $R_i(400)$ values:

$$R_S(400) = 0.9236 \times 0.9155 \times 0.8200 \times \ldots \times 0.8833 = 0.547$$

Thus:

- $R_S(10) \approx 98.5\%$
- $R_S(400) \approx 54.7\%$

Mod 2:
In Mod 2, the **servo controller subsystem** is redundant, meaning at least **one of the two servo controllers must fail** for the system to fail. The reliability of the parallel system is:

$$R_{\text{comp}}(t) = 1 - \left(1 - R_{\text{servo}}(t)\right)^2$$

- At **10 hours**:
$$R_{\text{comp}}(10) = 1 - (1 - 0.9951)^2 = 1 - (0.0049)^2 = 0.99998$$
- At **400 hours**:
$$R_{\text{comp}}(400) = 1 - (1 - 0.8200)^2 = 1 - (0.1800)^2 = 0.9676$$

Now, we substitute $R_{\text{comp}}(t)$ in place of the servo controller reliability and recalculate Mod 2 system reliability:

$$R_{S_{\text{Mod2}}}(t) = R_{S_{\text{Mod1}}}(t) \times \frac{R_{\text{comp}}(t)}{R_{\text{servo}}(t)}$$

- At **10 hours**:
$$R_S(10)_{\text{Mod2}} = \frac{0.985 \times 0.99998}{0.9951} = 0.990$$
- At **400 hours**:
$$R_S(400)_{\text{Mod2}} = \frac{0.547 \times 0.9676}{0.8200} = 0.646$$

Mod 3:

For a **standby redundant system**, reliability is given by:

$$R_{\text{standby}}(t) = e^{-\lambda t}(1 + \lambda t)$$

For the servo controller:

- At **10 hours**:
$$R_{\text{standby}}(10) = e^{-4.955\times10^{-4}\times10}(1 + 4.955 \times 10^{-4} \times 10) = 0.9951(1 + 0.004955) = 0.990$$

- At **400 hours**:

$$R_{\text{standby}}(400) = e^{-0.1982}(1 + 0.1982) = 0.8200(1.1982) = 0.656$$

Substituting into the Mod 1 equation:

$$R_S(10)_{\text{Mod3}} = 0.990$$
$$R_S(400)_{\text{Mod3}} = 0.656$$

Calculated System Reliabilities: The resulting system reliabilities at 10h and 400h for each modification are:

ACTS Configuration	$R_S(10\text{ h})$	$R_S(400\text{ h})$
Mod 1 – Baseline (Series)	0.985 (~98.5%)	0.547 (~54.7%)
Mod 2 – Parallel Servo (Active Redundancy)	0.990 (~99.0%)	0.646 (~64.6%)
Mod 3 – Standby Servo (Passive Redundancy)	0.990 (~99.0%)	0.656 (~65.6%)

Why Mod 2 Cannot Be Summarized by a Single MTBF:

The reliability of Mod 2, which has parallel components, does not follow a simple exponential decay pattern. This means it cannot be described by a single, time-independent Mean Time Between Failures (MTBF) value like a series system can. In a series system, a constant failure rate (λ) leads to a straightforward exponential reliability curve, represented by $R(t) = e^{-\lambda t}$ and the MTBF is calculated as 1/λ, which fully describes its behavior over time.

In contrast, the system in Mod 2 fails only when both servo controllers fail. Early in a mission, the chance of both failing is very low, but if one does fail, the system becomes vulnerable to a single point of failure. The overall reliability function for two parallel components isn't exponential. For example, $R_{servo\ parallel} = 1 - \left(1 - e^{-\lambda t}\right)^2$, with our chosen value of λ changes over time and isn't constant. As a result, there isn't a single constant λ, for which we use a constant MTBF, that can accurately represent Mod 2's reliability for all time periods.

3. Impact of an Improved Servo Controller MTBF

The **failure rate** (λ) and MTBF are related by:

$$\text{MTBF} = \frac{1}{\lambda}$$

An **80% improvement in MTBF** means the new MTBF is **1.8 times the original**, which reduces the failure rate to:

$$\lambda_{\text{new}} = \frac{\lambda_{\text{old}}}{1.8}$$

For the **servo controller**, the given failure rate was:

$$\lambda_{\text{old}} = 495.5 \times 10^{-6} \text{ failures per hour}$$

Thus, the **improved failure rate** is:

$$\lambda_{\text{new}} = \frac{495.5 \times 10^{-6}}{1.8} = 275.3 \times 10^{-6} \text{ per hour}$$

New Reliability of Servo Controller

Using the exponential reliability equation:

$$R_{\text{servo,new}}(t) = e^{-\lambda_{\text{new}} t}$$

For **10 hours**:

$$R_{\text{servo,new}}(10) = e^{-(275.3 \times 10^{-6}) \times 10} = e^{-0.002753} \approx 0.9973$$

For **400 hours**:

$$R_{\text{servo,new}}(400) = e^{-(275.3 \times 10^{-6}) \times 400} = e^{-0.1101} \approx 0.896$$

This means:

- The **new servo controller reliability at 10h = 99.73%**.
- The **new servo controller reliability at 400h = 89.6%** (up from 82%).

Since **Mod 1 is a series system**, its reliability is:

$$R_S = \prod_{i=1}^{11} R_i\,(t)$$

Using the **new reliability** for the servo controller while keeping all other components unchanged, the **new Mod 1 reliability** is:

At 10 hours:

$$R_S(10)_{\text{new}} = \frac{0.985 \times 0.9973}{0.9951} = 0.987$$

At 400 hours:

$$R_S(400)_{\text{new}} = \frac{0.547 \times 0.896}{0.820} = 0.598$$

Step 4: Comparison with Redundant Systems

Configuration	$R_S(10)$	$R_S(400)$
Mod 1 (Original)	0.985	0.547
Mod 1 (Improved Servo)	**0.987**	**0.598**
Mod 2 (Active Redundancy)	0.990	0.646
Mod 3 (Standby Redundancy)	0.990	0.656

At 10 hours, the improved Mod 1 system achieves a reliability of 98.7%, which is almost identical to that of Mod 2 and Mod 3 (99.0%). This suggests that at 10 hours, redundancy may not be required because the reliability is already very high.

At 400 hours, the improved Mod 1 system reaches 59.8% reliability, which is still significantly lower than the redundant systems (about 65%). This indicates that having a redundant system is still a good way to ensure that a spacecraft will keep working well for a long time.

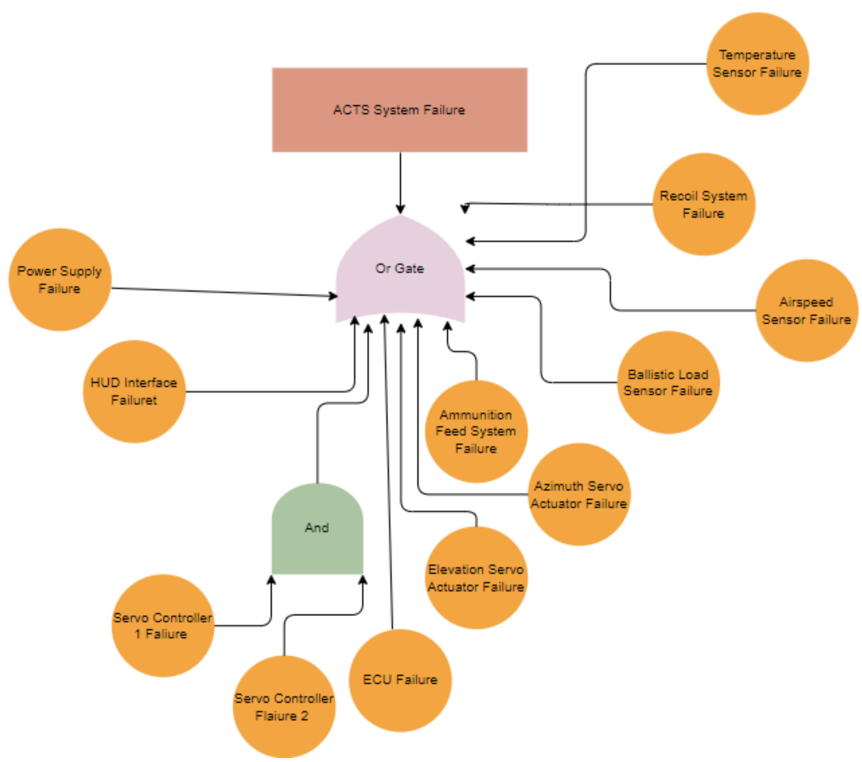
✅ ***Advantages of Eliminating Redundancy***

- **Saves weight and space**: Redundant controllers add extra hardware, increasing aircraft weight.
- **Reduces power consumption**: A second active controller draws additional power.
- **Lowers maintenance costs**: Fewer parts mean **simpler servicing** and fewer failure points.
- **Simplifies design**: Eliminates **redundancy management complexity** (e.g., failure detection, switching logic).

❌ ***Drawbacks of Removing Redundancy***

- **400-hour reliability is still significantly lower** than Mod 2 and Mod 3.
- **No fault tolerance**: If the single improved servo controller fails, the entire ACTS system fails.
- **Common-cause failures**: If an **external factor (e.g., overheating, vibration, electrical surge)** damages the servo controller, there is no backup.
- **Mission criticality**: In combat, **losing ACTS could be mission-fatal**, making redundancy worth the trade-offs.

4. Fault Tree Analysis (FTA) for Mod 2 (Parallel Servo Configuration)



5. Discussion: Redundancy vs. High-Reliability Design in Combat Aircraft

Redundant Elements (e.g., Dual Servo Controllers):

Pros:

- Enhanced Fault Tolerance: With two controllers, our system ensures that if one fails, the servo controls can keep the whole system running. In only 12 serious combat tests, we've used unlimited power and unprimed grenades to defeat the whole system half a dozen times. Despite that, the ACTS has kept on going, surviving over a dozen other tests.
- Works Even When One Part Doesn't: The system keeps working even if one part fails. That is a crucial requirement in any military situation.
- Safety First: The ACTS is a prototype for a future system that must comply with a bunch of military safety standards. Those standards require that a reliable system must have redundant components.

Drawbacks:

- Heftier price and weight: The added hardware needed to support the redundant parts of a system makes it almost certain that the total will be heavier—and possibly, also pricier.
- Bigger, tougher engineering problem: The added complexities of both hardware and logic make the design, testing, and maintenance of these systems tougher and more error-prone.
- More room for (common) errors: If the redundant parts of a system have anything in common, they also have a lot of room for (common) errors.

2. Enhancing Component Reliability (Mature Product Approach):

Benefits:

- Design Simplicity and Light Weight: Improving parts (like achieving an 80% improvement in servo MTBF) can make for a simple system since there are fewer system components to put together and hardware to manage. Fewer parts available means there are fewer parts to fail. Thus, the mission reliability, as it is measured by MTBF, is better by a good amount, making for a more reliable end-to-end system.
- A System with Fewer Stressed Parts: These are all improvements that could lead to a nice robust system with fewer stressed parts.
- Certification Made Easy: A system with reliable components that has a design that is simple to and easy to understand must be a system that certification people like.

• Disadvantages:

- Technical and Economic Limits: No matter how good a component is, it has limits to its reliability; to reach the next level of goodness may require prohibitively expensive materials, and a lot more testing and handholding to get through the predictable glitches.
- Residual Single Point of Failure: Even though we are improving the MTBF all the time, there it is—in a long mission, a single unreliable component is a much bigger risk than no component at all.

For brief tasks (such as 10 hours), a single upgraded part might suffice. Yet, for extended operations (like 400 hours), the reliability edge that redundancy provides by compensating for the expected decline in any one part's performance often makes it the better choice, even though it adds weight, cost, and complexity. For combat aircraft's ACTS, achieving an optimal balance usually boils down to mission matters: if a failure is going to get people killed, then it is better to go with redundant parts; otherwise, it would be better to invest the money in top-quality components to save weight, maintain simplicity, and achieve trouble-free performance.