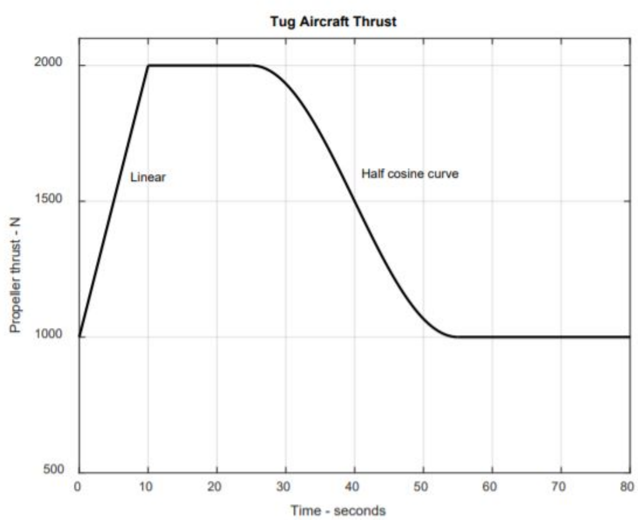
Question: An unpowered sail plane (glider) is being towed aloft by a po...

An unpowered sail plane (glider) is being towed aloft by a powered tug aircraft following a straight line trajectory. We wish to model the forward motion of both the tug plane and the sail plane during an event where the thrust from the tug aircraft is increased and then decreased as shown below. Both aircraft experience an airspeed dependent drag. Before $t = 0$ the thrust has been constant at 1000 N for sufficient time for steady conditions to have been achieved. Under these conditions, both aircraft will have the same airspeed.



Mass of tug plane	840 kg
Mass of sail plane	285 kg
Stiffness of (massless) tow cable (neglect sag)	0.46 kN/m
Tug plane drag	$0.52 v_{tug}^2$ N, v_{tug} is airspeed in m/s
Sail plane drag	$0.13 v_{sailplane}^2$ N, $v_{sailplane}$ is airspeed in m/s

Write a Matlab script (using the differential equation solver ode45) to numerically solve for the resulting forward motion (displacement and velocity) of both aircraft. Plot the motion (velocity and displacement) as a function of time for $0 < t < 80$ sec.

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Expert Answer

Sasanka answered this
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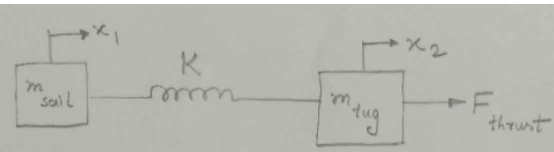
The equations of motion have been derived in the handwritten notes below. The MATLAB code follows after that.



where m_{sail} = mass of sail plane = 285 kg

m_{tug} = mass of tug plane = 840 kg

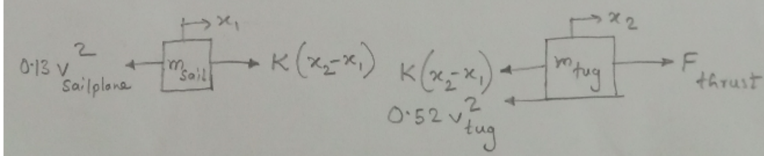
K = stiffness of tow cable = 0.46 kN/m.

Tug plane drag = $0.52\, v_{tug}^2$ N

v_{tug} = airspeed of tug plane = $\dot{x}_2$

Sail plane drag = $0.13\, v_{sailplane}^2$ N

$v_{sailplane}$ = airspeed of sail plane = $\dot{x}_1$



Equations of motion:

$$m_{sail}\, \ddot{x}_1 = K(x_2-x_1) - 0.13\, \dot{x}_1^2 \quad \text{--- } (1)$$

$$m_{tug}\, \ddot{x}_2 = F_{thrust} - K(x_2-x_1) - 0.52\, \dot{x}_2^2 \quad \text{--- } (2)$$

Before $t=0$, $F_{thrust} = 1000$ N, $\dot{x}_1 = \dot{x}_2 = \dot{x}$, $\ddot{x}_1 = \ddot{x}_2 = 0$

So,

$$K(x_2-x_1) - 0.13\, \dot{x}^2 = 0 \quad \text{--- } (3)$$

$$1000 - K(x_2-x_1) - 0.52\, \dot{x}^2 = 0 \quad \text{--- } (4)$$

Adding both equations,

$$1000 - (0.13 + 0.52)\, \dot{x}^2 = 0$$

$$\Rightarrow \dot{x} = \sqrt{\frac{1000}{0.69}} \text{ m/s}$$

$$= \underline{38 \text{ m/s}}$$

Substituting $\dot{x} = 38$ m/s in (3), we get

$$x_2 - x_1 = \frac{0.13 \times 38^2}{0.46 \times 10^3} \text{ m}$$

$$= 0.408 \text{ m}$$

So, initial conditions for simulation are

$x_1 = 0$, $x_2 = 0.408$ m.

$\dot{x}_1 = \dot{x}_2 = 38$ m/s

From graph,

$$F_t = \begin{cases} (100t + 1000)\ \mathrm{N}, & 0 \leq t < 10\ \mathrm{s} \\ 2000\ \mathrm{N}, & 10 \leq t < 25\ \mathrm{s} \\ 1000\left[1 + \cos\left(\frac{\pi}{60}(t-25)\right)\right], & 25 \leq t < 55\ \mathrm{s} \\ 1000\ \mathrm{N}, & 55 \leq t < 80\ \mathrm{s} \end{cases}$$

The MATLAB code to generate the plots is:

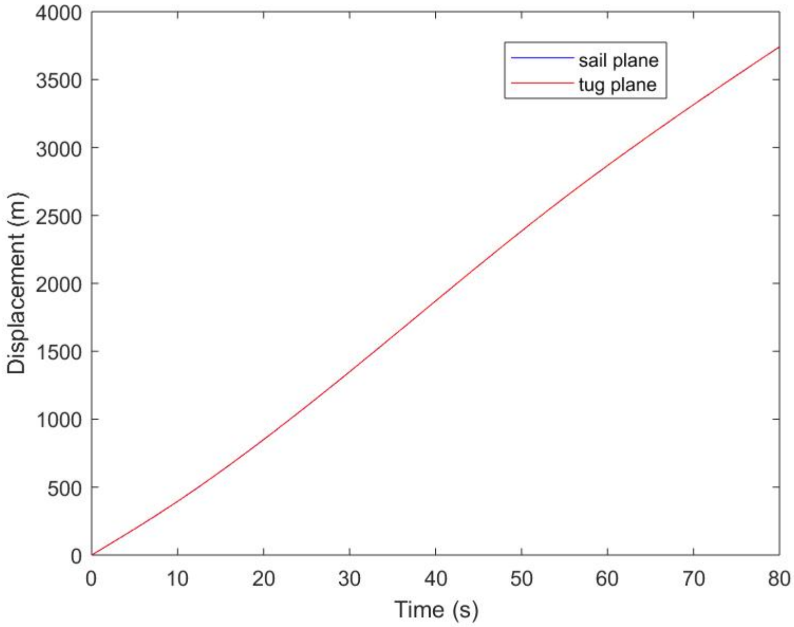
```
clear;clc;

tspan = linspace(0,80,1000);
y0 = [0;0.408;38;38];
[t,y] = ode45(@odefun1,tspan,y0);

% plotting displacements
figure()
plot(t,y(:,1),'b',t,y(:,2),'r')
xlabel('Time (s)')
ylabel('Displacement (m)')
legend('sail plane','tug plane')

% plotting velocities
figure()
plot(t,y(:,3),'b',t,y(:,4),'r')
xlabel('Time (s)')
ylabel('Velocity (m/s)')
legend('sail plane','tug plane')

%----------------------------------------------
```

The function file is

```
function dydt = odefun1(t,y)
K = 460;
m_tug = 840;
m_sail = 285;

% calculate thrust force from graph
% piecewise relations used

if t<10
F_thrust = 100*t+1000;
elseif (t>=10 && t<25)
F_thrust = 2000;
elseif (t>=25 && t<55)
F_thrust = 1000*(1+cos(pi*(t-25)/60));
else
F_thrust = 1000;
end

% state-space equations
dydt = [y(3);y(4); (K*(y(2)-y(1))-0.13*y(3)^2)/m_sail;(F_thrust-K*(y(2)-y(1))-0.52*y(4)^2)/m_tug];

%---------------------------------------------------------------------------------------------------
```

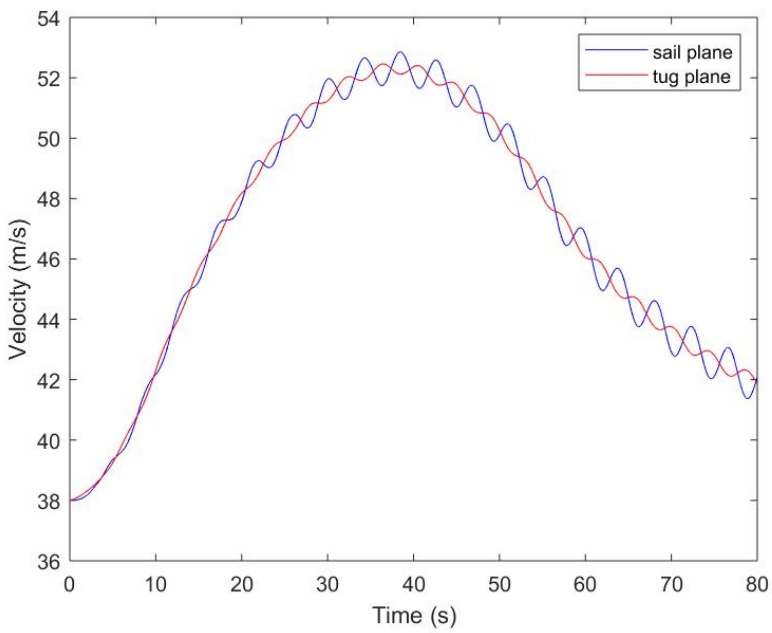
The plots generated are

Displacement



Both the displacement plots almost coincide.

Velocity



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Questions viewed by other students

Q: Can you derive the differential equation to the attached question

A: See answer

Q: A motor delivers 10 Hp to a the shaft driving the fan blades. The shaft is rotating at 500 RPM. Design a solid forro Shaft from Aluminum 6061 Using a factor Safety (F.s) against yeild of 2. Determine the shaft angle of twist occuring for your design in degrees. Fan. I 10 Hp & shaff: 36" & Shafts 10 HP motor aft = 364

A: See answer 100% (1 rating)

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