Computational Methods Project Part-3

Student ID: XYZ = 293

Problem: 3

Given drill tip distance as a function of time;

Time (sec)	0	1	2	3	4	5	6	7	8	9	10	11	12
Depth (meters)	0	2	5	8	15	28	32	49	57	68	110	109	130

Velocity and Acceleration Using Forward Difference Method:

$$u'(x_i) \approx \frac{u(x_i + \Delta x) - u(x_i)}{\Delta x} = \frac{u_{i+1} - u_i}{\Delta x}$$

Here depth, u is a function of time, x (variable). $\Delta x = 1$

Velocity:

$$V_{f(i)} = \frac{Depth(i+1) - Depth(i)}{\Delta x}$$

For i = 0-11

$$V_{f(0)} = \frac{Depth(0+1) - Depth(0)}{1} = \frac{2-0}{1} = 2\ m/s$$

Acceleration:

$$A_{f(i)} = \frac{Velocity(i+1) - Velocity(i)}{\Delta x}$$

For i = 0-10

$$A_{f(0)} = \frac{Velocity(0+1) - Velocity(0)}{Time\ (0+1) - Time(0)} = \frac{3-2}{1} = 1\frac{m}{s^2}$$

Using Backward Difference Method:

$$u'(x_i) \approx \frac{u(x_i) - u(x_i - \Delta x)}{\Delta x} = \frac{u_i - u_{i-1}}{\Delta x}$$

Velocity:

$$V_{b(i)} = \frac{Depth(i) - Depth(i-1)}{\Delta x}$$

For i = 1-12

$$V_{b(1)} = \frac{Depth(1) - Depth(1-1)}{1} = \frac{2-0}{1} = 2\ m/s$$

Acceleration:

$$A_{b(i)} = \frac{Velocity(i) - Velocity(i-1)}{\Delta x}$$

For i = 2-12

$$A_{b(2)} = \frac{Velocity(2) - Velocity(2-1)}{1} = \frac{3-2}{1} = 1\frac{m}{s^2}$$

Using Central Difference Method:

$$u'(x_i) \approx \frac{u(x_i + \Delta x) - u(x_i - \Delta x)}{2\Delta x} = \frac{u_{i+1} - u_{i-1}}{2\Delta x}$$

Velocity:

$$V_{c(i)} = \frac{Depth(i+1) - Depth(i-1)}{2\Delta x}$$

For i = 1-11

$$V_{c(1)} = \frac{Depth(1+1) - Depth(1-1)}{2*1} = \frac{5-0}{2} = 2.5\, m/s$$

Acceleration:

$$A_{c(i)} = \frac{Velocity(i+1) - Velocity(i-1)}{2\Delta x}$$

For i = 2-10

$$A_{c(2)} = \frac{Velocity(2+1) - Velocity(2-1)}{2*1} = \frac{5-2.5}{2} = 1.25\frac{m}{s^2}$$

velocity at other values of i is calculated which is shown in table below;

Time (s)	Depth (m)	Forward Difference		Backward Difference		Central Difference	
		Velocity (m/s)	Acceleration (m/s^2)	Velocity (m/s)	Acceleration (m/s^2)	Velocity (m/s)	Acceleration (m/s^2)
0	0	2	1	NA	NA	NA	NA

1	2	3	0	2	NA	2.5	NA
2	5	3	4	3	1	3	1.25
3	8	7	6	3	0	5	3.5
4	15	13	-9	7	4	10	1.75
5	28	4	13	13	6	8.5	0.25
6	32	17	-9	4	-9	10.5	2
7	49	8	3	17	13	12.5	-0.5
8	57	11	31	8	-9	9.5	7
9	68	42	-43	11	3	26.5	5.5
10	110	-1	22	42	31	20.5	-8.25
11	109	21	NA	-1	-43	10	NA
12	130	NA	NA	21	22	NA	NA

Problem: 4

My Student ID: XYZ = 293

Top of oil reservoir follows the function:

$$f(x) = -3.8(x - XYZ)^2 - 8.6(x - XYZ) - 500 - XYZ$$

Bottom of oil reservoir follows the function:

$$f(x) = 3.8(x - XYZ)^2 + 8.6(x - XYZ) - 800 - XYZ$$

Where x = Distance from current location

y = Depth in the Ground

To find the interaction point:

$$-3.8(x - 293)^2 - 8.6(x - 293) - 500 - 293 = 3.8(x - 293)^2 + 8.6(x - 293) - 800 - 293$$

$$7.6(x - 293)^2 + 17.2(x - 293) - 300 = 0$$

It is a quadratic equation by solving it:

$$7.6x^2 + (17.2 - 15.2 * 293)x + [(7.6 * 293^2) - (17.2 * 293) - 300] = 0$$

$$7.6x^2 - 4436.4x + 647112.8 = 0$$

By solving the above quadratic equation

$$x_1 = 285.484, \;\; x_2 = 298.252$$

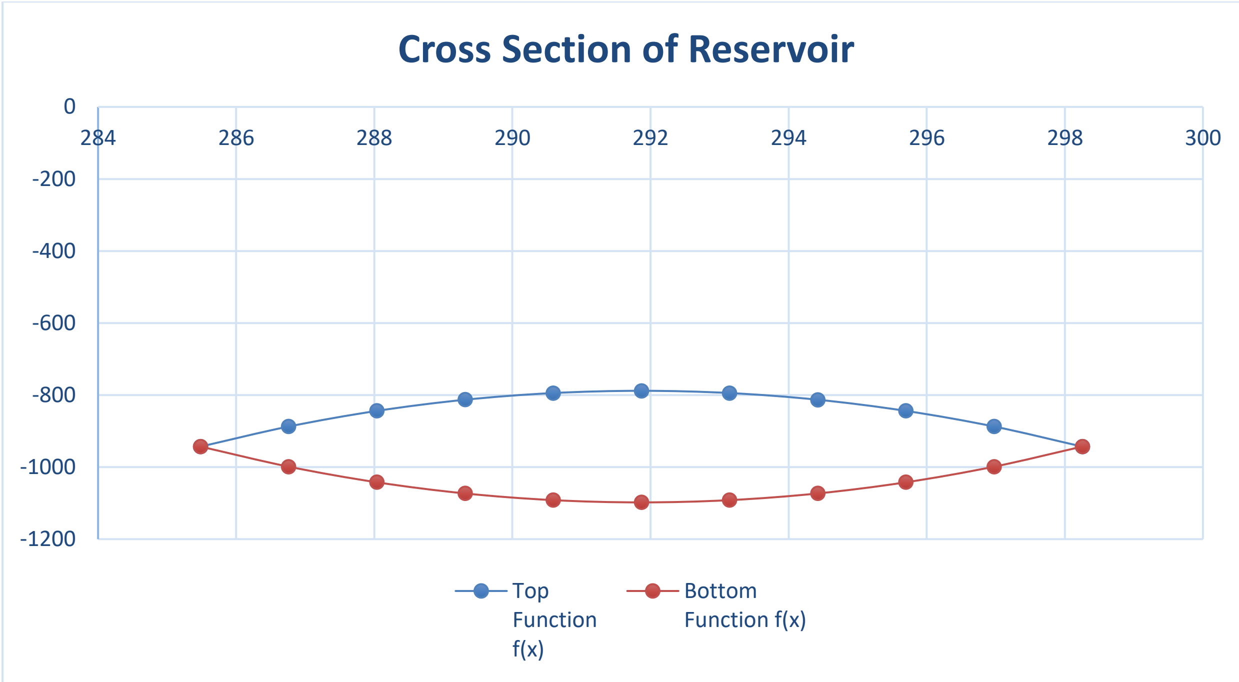
To calculate area between x_1 and x_2 lets take 10 divided small area between both so;

$$h = \frac{x_1 - x_2}{10} = \frac{298.252 - 285.484}{10} = 1.277$$

$$x_i = x_o + (i * h)$$

Values calculated between x1 and x2:

t	x	Top Function f(x)	Bottom Function f(x)
0	285.484	-943.025	-942.975
1	286.761	-887.26	-998.74
2	288.038	-843.888	-1042.112
3	289.315	-812.91	-1073.09
4	290.592	-794.325	-1091.675
5	291.869	-788.134	-1097.866
6	293.146	-794.337	-1091.663
7	294.423	-812.933	-1073.067
8	295.7	-843.922	-1042.078
9	296.977	-887.305	-998.695
10	298.254	-943.082	-942.918



1. By Composite Trapezoidal:

$$\int_a^b f(x)\,dx \approx T_n = \frac{\Delta x}{2}\left[f(x_0) + 2f(x_1) + 2f(x_2) + \cdots + 2f(x_{n-1}) + f(x_n)\right],$$

$$A_{Top} = \frac{h}{2}[f(x_o) + 2 * \{f(x_1) + f(x_2) + \cdots \dots \dots \dots f(x_9)\} + f(x_{10})]$$

$$A_{Bottom} = \frac{h}{2}[g(x_o) + 2 * \{g(x_1) + g(x_2) + \cdots \dots \dots \dots g(x_9)\} + g(x_{10})]$$

After calculation;

		TOP Area	Bottom Area	Net Area
Composite Trapezoidal		-10737.102	-13347.118	**2610**

2. By Simpson's 1/3 rule:

$$\int_{x_0}^{x_{2n}} f(x)\,dx = \frac{1}{3} h[f_0 + 4(f_1 + f_3 + \ldots + f_{2n-1}) + 2(f_2 + f_4 + \ldots + f_{2n-2}) + f_{2n}] - R_n,$$

$$A_{Top} = \frac{h}{3}[f(x_o) + 4 * \{f(x_1) + f(x_3) + f(x_5) + f(x_7) + f(x_9)\} + 2 * \{f(x_2) + f(x_4) + f(x_6) + f(x_8)\} + f(x_{10})]$$

$$A_{Bottom} = \frac{h}{3}[g(x_o) + 4 * \{g(x_1) + g(x_3) + g(x_5) + g(x_7) + g(x_9)\} + 2 * \{g(x_2) + g(x_4) + g(x_6) + g(x_8)\} + g(x_{10})]$$

After Calculation using above formula Area;

		TOP Area	Bottom Area	Net Area
Simpson's 1/3		-10723.914	-13360.306	2636.4

3. Rectangular Method

$$A_{Top} = h[f(x_o) + f(x_1) + \cdots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots f(x_9) + f(x_{10})]$$

$$A_{Bottom} = h[g(x_o) + g(x_1) + \cdots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots + g(x_9) + g(x_{10})]$$

		TOP Area	Bottom Area	Net Area
Rectangular		-11941.382	-14551.26	2609.9