# Homework #1

**Due: Sunday, January 24, 11:59pm**

**Note**: you are expected to write MATLAB programs (and include them with your submission) in problems 2(b) and 3. Problems 1 and 2(a) need to be done by hand with the calculator. Plotting can be done in MATLAB for all the problems.

1. (10 points) You are given a toy calculator that has only one function of multiplying two numbers, both are restricted to be between 0.000 and 0.999 with only a 3-digit accuracy. Moreover, after performing the calculation, the machine will retain only 3 digits (to the right of the floating point) as its outcome. For instance, given A = 0.318, the precise value of $A \times A$ should be 0.101124 while the calculator will produce 0.101. For the calculation of $A \times A \times A$, the process will unfold as the following

   $A = 0.318$

   $A \times A = 0.101124 \Rightarrow$ calculator retains $\underline{0.101}$

   $A \times A \times A = (A \times A) \times A = 0.101 \times 0.318 = 0.032118 \Rightarrow$ calculator retains $\underline{0.032}$ as final answer

   The underlined numbers are those that have been trimmed by the calculator. Note that the exact value of $A \times A \times A$ is 0.032157432.

   Using this toy calculator, **and given $\mathbf{A = 0.629}$**, evaluate $A^2$, $A^3$, $A^4$, ..., to $A^{10}$. Compare the results with those evaluated by using a real calculator (or MATLAB). Treat the latter as the "true" values to evaluate the "true percent relative error" produced by the toy calculator. Plot the error as a function of $N$, the exponent of A (e.g., $N = 4$ for $A^4$).

   ***Note*:** consider two cases of round-off – chopping and rounding. Compare the two cases (i.e. plot the error as a function of $N$ for two cases on the same plot). Discuss your results.

2. (15 points) The Maclaurin series expansion of $\sin(x)$ is given by

   $$\sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \frac{x^9}{9!} \cdots$$

   (a) Use the first three terms in this equation to calculate the value of $\sin(\pi/3)$. Use your calculator to estimate the true value of $\sin(\pi/3)$. Calculate the truncation error (true percent relative error).

   (b) Now write a MATLAB program that adds the terms until the approximate percent relative error (error between current and previous estimate) falls below 0.01%. Document the final value, the percent relative error (approximate and true), and the number of iterations it took.

3. (10 points) The infinite series

   $$f(n) = \sum_{k=1}^{n} \frac{1}{k^4}$$

   converges on a value $f(n) = \pi^4/90$ as $n$ approaches infinity. Write a program to calculate $f(n)$ for $n = 10,000$ by computing the sum from $k$=1 to 10,000. Then repeat the calculations but in reverse order - that is, from $k$=10,000 to 1 using increments of -1. In each case, compute the true percent relative error after each term is added. Compare the final error in the end of the calculations for the two cases. Explain the results.