Murdoch University

SUPPLEMENTARY / DEFERRED EXAMINATION

OPEN BOOK

Time Allowed: 3 hours and 45 minutes
(total time for reading, writing, scanning and submission on the LMS)

Aids Allowed: Printed textbooks, notes, files, and any calculator.

There are 16 questions worth a total of 100 marks. You should attempt all questions.

Your scanned answers should meet the following requirements.

- Your full name and student number is written at the top of the first page.
- Answers are written on A4 sized paper with some space between questions and the question numbers in the left margin.
- Scanned copy is a ***single PDF file*** in portrait mode and clearly legible.
- All scanned pages have a left margin of at least 2*cm*.

Question	Marks	Score
1	5	
2	10	
3	7	
4	5	
5	9	
6	2	
7	3	
8	2	
9	4	
10	9	
11	9	
12	12	
13	4	
14	6	
15	3	
16	10	
Total:	100	

1. [5 marks] Showing all working, find a simple ***recursive definition*** for the sequence with general term
$$t_n = (1/2)(3^n)(n+2)! \quad \text{for } n \geq 1.$$

2. [10 marks] At the beginning of week 1, a peach orchard has 68 square metres of soil affected by a fungus. Soon after the beginning of week n, for $n = 1, 2, \ldots$, the orchardist sprays $12(n+1)/(n+2)$ square metres of the soil to kill the fungus at a cost of \$20 per square metre. During each week the remaining fungus grows so that the area of affected soil increases by 10%.

   Let a_n be the area of soil affected by the fungus at the beginning of week n for $n \geq 1$, and let c_n be the total cost of the spraying from the beginning of week 1 up to the beginning of week n for $n \geq 1$.

   (a) Find a recursive definition for a_n.

   (b) Find the values of a_n for $n = 2, 3$ and c_n for $n = 1, 2$.

   (c) Write a MATLAB program to compute a_n for $n = 1, 2, \ldots, 10$ and display the values in two columns: n, a_n.

3. [7 marks] Consider the two functions
$$f(x) = x^4 + (x-3)^2 \quad \text{and} \quad g(x) = 10x\sin(2x).$$

   (a) Write a MATLAB program to plot the graphs of $f(x)$ and $g(x)$ for $-2 \leq x \leq 2$ together in the same figure. (You do not need to include labels.)

   (b) Given that the function $y(x) = f(x) - g(x)$ has a zero at $x = c \approx 0.6$, write

   (i) a function M-file (called `func.m`) and

   (ii) a MATLAB statement (using the `fzero` command)

   that together will compute c.

4. [5 marks] Consider the sum
$$S = 1/2^2 + 3/4^2 + 5/6^2 + 7/8^2 + \cdots$$

   Write a MATLAB program to compute the sum up to and including the term $121/122^2$. Also in the program, display the message

   ```
   The sum is greater than 1
   ```

   if the computed value is greater than 1.

5. [9 marks]

   (a) Convert $(105.24)_8$ to base 10, showing all your working.

   (b) Convert the decimal number 12345 to base 5, showing all your working.

   (c) Convert the binary number $(1000101011)_2$ to hexadecimal and then add it to the hexadecimal number $(7AB)_{16}$.

6. [2 marks] Use the *Binomial Theorem* to determine the coefficient of x^3 in the expansion of $(2x+4)^8$.

7. [3 marks] A lolly jar contains 12 humbugs and 10 barley-sugars. Find the number of ways two lollies can be chosen if:

   (a) The lollies must be different.

   (b) The lollies must be the same.

8. [2 marks] How many different 3-letter "words" can be formed from the letters of the word `MATHS` (without repetition)? And what if `S` must be the last letter of any such 3-letter word?

9. [4 marks] A student is selected at random from a list of all students at Murdoch University. Suppose the probability that the student is female is 0.48 and the probability that the student has a part-time job is 0.75. You would not expect these two events to be disjoint. Why not? Use a Venn diagram and the given probabilities to illustrate that they cannot be disjoint.

10. [9 marks] Let A, B, and C be matrices defined as follows:

$$A = \begin{pmatrix} 2 & 3 & 5 \\ 1 & 4 & -2 \end{pmatrix}, \quad B = \begin{pmatrix} 2 & 2 \\ -3 & 3 \\ 0 & 1 \end{pmatrix}, \quad \text{and} \quad C = \begin{pmatrix} 1 & 2 & 3 \\ 0 & 2 & 4 \\ -1 & 2 & 6 \end{pmatrix}.$$

    (a) Decide whether or not each of the products AB, AC, and BC is defined, and for each product that is defined, determine its ***size*** (i.e., number of rows and columns).

    (b) Write down MATLAB statements that would define B and C and that would compute $3B + C^{-1}B$.

    (c) Find by hand (not by calculator) the matrix $(AB)^{-1}$.

11. [9 marks] Consider the following four linear transformations of the plane:

    T_1 = *"reflection across the line* $y = x$*"*
    T_2 = *"rotation by* 270° *anticlockwise"*
    T_3 = *"vertial dilation with factor* $\frac{1}{3}$*"*
    $T_4(x, y) = (3x - 2y, x + y)$

    (a) Write down the 2×2 matrices A_1, A_2, A_3, and A_4 corresponding to these linear transformations.

    (b) Use matrix multiplication to find the geometric effect (stated in words) of T_2 followed by T_1, showing your reasoning.

    (c) Consider the linear transformation T that maps the point $(1, 0)$ to $(3, -1)$ and the point $(0, -1)$ to the point $(2, 3)$. What is the matrix associated with this transformation?

12. [12 marks]

    (a) Use a *truth table* or the *rules of inference* to prove that

    $$[p \to (\neg q \vee r)] \wedge q \Rightarrow \neg p \vee r$$

    (b) Write down the following argument in symbolic form and test its logical validity. If it is invalid, give a counterexample.

    If the starter motor is working and the battery is not flat, then the car will start.
    If I leave the car lights on, the battery goes flat.
    I did not leave the car lights on and the car will not start.
    So the starter motor is not working.

13. [4 marks] Decide whether the following quantified propositions are true or false, where in each case the universal set is the set of positive integers. Justify each of your conclusions with a proof or a counterexample.

    (a) $\exists n(n^2 = 3n)$

    (b) $\neg\forall n(2n > 0)$

14. [6 marks] Use ***mathematical induction*** to prove that

    $$4n^2 \geq 3n + 1 \quad \text{for all integers } n \geq 1.$$

15. [3 marks] Consider the Boolean expression E defined by

    $$E = (wx + y'z')' + wz'$$

    Use axioms and properties of Boolean algebra to show that

    $$E = w'y + w'z + x'y + x'z + wz'$$

16. [10 marks] Let E be the Boolean expression given in Question 15.

    (a) Draw the ***Karnaugh map*** for E.

    (b) Use your answer to part (a) to obtain the Karnaugh map for E', and hence express E' as a ***minimal sum of products***.

    (c) Design a logic network for the Boolean expression g defined by $g := Ew'x$, given that

$$g = w'x(y + z)$$