A Comparison of Binomial, Finite Difference Methods, and Monte Carlo Approaches to American Option Pricing

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Abstract

American-style options present a complex valuation challenge due to their early exercise feature, which transforms pricing into a free-boundary problem under uncertainty. This study develops and compares three major numerical techniques—the Binomial Tree model, the Monte Carlo simulation based on the Longstaff–Schwartz regression, and the Finite Difference method using the Crank–Nicolson scheme, for valuing American call options. Using simulated data that incorporate realistic volatility and dividend conditions, the analysis evaluates model accuracy, convergence, and stability. The findings show that while all three approaches provide consistent and reliable estimates, their efficiency and precision depend strongly on discretization strategy and numerical refinement. The results highlight the interplay between stochastic calculus, numerical stability, and computational performance in the accurate valuation of American options.

Contents

1   Introduction

The pricing of financial derivatives has been one of the most extensively studied problems in quantitative finance. Accurate option valuation is essential not only for traders and portfolio managers but also for risk management and regulatory compliance. Since the development of the seminal Black–Scholes model in 1973, researchers have sought to refine its assumptions and expand its applicability to more complex financial instruments, such as American options, which permit early exercise. Unlike European options, American options cannot be solved analytically in closed form, which has motivated the use of numerical and simulation-based approaches.

Among the earliest numerical frameworks, the binomial tree method stands out for its simplicity and flexibility. Tian (1999)[1] proposed a generalized binomial model that corrects for biases in earlier formulations and improves convergence toward theoretical option values. Binomial models remain popular in both academic studies and practical applications due to their intuitive structure and ability to handle American-style exercise features naturally. Parallel to lattice methods, researchers have developed finite difference methods (FDMs) to solve the Black–Scholes partial differential equation under the free-boundary condition imposed by American options. Early work by Wu and Kwok (1997)[2] introduced a front-fixing technique to capture the moving exercise boundary, while Zhao, Davison, and Corless (2007)[3] later refined this approach through compact finite difference schemes to improve numerical stability and accuracy. Texts such as Thomas (2013)[4] and Duffy (2013)[5] have provided comprehensive treatments of finite difference schemes, establishing them as one of the central techniques in financial engineering.

A third family of approaches involves Monte Carlo simulation methods. Although straightforward Monte Carlo techniques are not naturally suited for American options due to the early exercise problem, innovative methods such as least-squares regression have been developed to approximate continuation values. Fu, Laprise, Madan, Su, and Wu (2001)[6] presented a systematic comparison of Monte Carlo techniques for American option pricing, showing their applicability to high-dimensional problems where finite difference and binomial approaches become computationally expensive. More recent research has integrated Monte Carlo with machine learning techniques, as in Kim, Kim, and Song (2024)[7], further broadening its scope in option valuation. Extensions of the Black–Scholes framework also continue to attract attention. For example, Ismail and Hellström (2025)[8] explored finite element methods for option pricing, while Nikan, Rashidinia, and Jafari (2025)[9] investigated fractional Black–Scholes models to capture memory effects in asset prices. These works highlight the continuing evolution of numerical approaches in adapting to market complexities.

In this project, we compare three of the most widely studied and practically applied numerical methods, binomial trees, finite difference methods, and Monte Carlo simulation, for the pricing of American options. An American option is a type of financial derivative that grants its holder the right, but not the obligation, to buy or sell an underlying asset at a specified strike price at any time up to and including the expiration date. In contrast, a European option may only be exercised at the expiration date. This early exercise flexibility makes American options more valuable and mathematically complex, as their valuation involves determining the optimal exercise boundary through free-boundary or optimal-stopping formulations. By evaluating the relative advantages, limitations, and

computational performance of the three numerical approaches, this study seeks to provide a clear perspective on their suitability in different financial contexts.

2   Hypothesis

Based on the empirical results, the Binomial Tree method produced the most accurate and stable results for American option pricing, achieving the lowest RMSE and MAE with a near-perfect R^2. The Monte Carlo (LSM) approach showed moderate accuracy but lower computational efficiency. At the same time, the Finite Difference (Crank–Nicolson) method demonstrated the highest numerical error, indicating sensitivity to discretization and boundary conditions. Therefore, in this study, the Binomial Tree method proved to be the most reliable approach among the three for the tested parameter range.

3   Methods

3.1   Binomial Tree Method

The Binomial Tree method, first introduced by Cox, Ross, and Rubinstein (1979) [10], represents one of the most elegant and intuitive discretizations of continuous asset dynamics. Rather than solving the differential equation directly, the method approximates the stochastic motion of the underlying asset S_t by allowing it to evolve along a discrete recombining lattice. Over each infinitesimal time step $\Delta t = T/N$, the asset price is assumed to move either upward or downward by fixed proportional factors, denoted by u and d. These are defined as:

$$u = e^{\sigma\sqrt{\Delta t}}, \quad d = e^{-\sigma\sqrt{\Delta t}}. \tag{3.1}$$

This seemingly simple construction is grounded in the fundamental principle of risk-neutral valuation. Under this measure, the expected growth rate of the asset must equal the risk-free rate r, adjusted for the continuous dividend yield q. Hence, the probability p of an upward move is not an empirical probability, but a theoretical one that ensures the absence of arbitrage:

$$p = \frac{e^{(r-q)\Delta t} - d}{u - d}. \tag{3.2}$$

Intuitively, p balances the expected growth in the lattice so that a risk-neutral investor remains indifferent between holding the asset and a risk-free bond. Each node in the tree thus represents a perfectly hedged portfolio equilibrium. At the final time step $(t = T)$, the option's value is known exactly since it depends only on the intrinsic payoff:

$$V_i^N = \Phi(S_i^N) = \max(S_i^N - K, 0). \tag{3.3}$$

Working backward through the lattice, we determine the option's value at each preceding node by taking the discounted expected value of its possible future payoffs:

$$V_i^j = e^{-r\Delta t}[pV_{i+1}^{j+1} + (1-p)V_i^{j+1}]. \tag{3.4}$$

However, because the option can be exercised at any time before maturity, this continuation value must always be compared with the immediate exercise payoff. Hence, the American feature introduces an additional condition:

$$V_i^j = \max(V_i^j, \Phi(S_i^j)). \tag{3.5}$$

This final maximization step embodies the economic logic of early exercise: the holder continuously compares the benefit of waiting for potential price appreciation against the immediate payoff achievable today. When the intrinsic value exceeds the discounted expected continuation value, rational exercise occurs. In this way, the Binomial model captures both time and decision dynamics, mirroring the real-world flexibility embedded in American contracts.

From a numerical perspective, the Binomial Tree offers a remarkable balance between simplicity, interpretability, and accuracy. It not only approximates the solution to the Black–Scholes equation but also naturally accommodates dividends, early exercise, and path-dependent decision structures—making it one of the most pedagogically powerful and computationally robust methods in modern quantitative finance.

3.2   Monte Carlo Simulation (Longstaff–Schwartz)

The Monte Carlo simulation framework offers a profoundly intuitive yet mathematically powerful approach to option pricing. It mirrors the inherent randomness of market evolution by directly simulating the paths of the underlying asset under the risk-neutral measure. For European-style derivatives, such simulations are straightforward, as the option can only be exercised at a single terminal point. However, for American options, where the right to exercise exists continuously over time, a more delicate mathematical treatment is required. The crux of the problem lies in identifying the optimal stopping time —that moment at which immediate exercise yields greater value than continuation. To address this, Longstaff and Schwartz (2001) [11] introduced a groundbreaking regression-based algorithm that transformed the Monte Carlo method into a viable tool for American-style derivatives. The key insight was that, although one cannot easily simulate backward in time, it is possible to approximate the expected future payoff given the current asset price using a projection in function space.

Formally, the evolution of the underlying asset price is described by the discretized geometric Brownian motion:

$$S_{t+\Delta t}^{(m)} = S_t^{(m)} \exp\left[(r - q - \tfrac{1}{2}\sigma^2)\Delta t + \sigma\sqrt{\Delta t}\, Z_t^{(m)}\right], \tag{3.6}$$

where $Z_t^{(m)}$ are independent standard normal variates, r is the risk-free rate, and q is the continuous dividend yield. This stochastic process captures the randomness inherent in market dynamics, while maintaining the risk-neutral drift $(r - q)$ required for arbitrage-free pricing. For each of the M simulated paths, the option payoff at maturity is known. The challenge, however, is determining whether early exercise is optimal at intermediate times. The Longstaff–Schwartz algorithm resolves this by approximating the conditional expectation of the continuation value $\mathbb{E}[V_{t+\Delta t}|S_t]$ through least-squares regression:

$$\mathbb{E}[V_{t+\Delta t}|S_t] \approx a_0 + a_1 S_t + a_2 S_t^2, \tag{3.7}$$

where (a_0, a_1, a_2) are regression coefficients estimated from in-the-money simulated paths. This regression effectively learns the functional relationship between the current asset price and the expected discounted payoff of holding the option. At each time step, the holder compares the immediate exercise value $\Phi(S_t)$ with the continuation value derived from the regression. The optimal stopping decision is then defined as:

$$V_t = \max\big(\Phi(S_t),\, \mathbb{E}[V_{t+\Delta t}|S_t]\big), \tag{3.8}$$

which captures the early exercise opportunity characteristic of American options. Once the optimal exercise rule is determined along each path, the initial option price is obtained by averaging the discounted payoffs across all simulations:

$$V_0 = \frac{1}{M} \sum_{m=1}^{M} e^{-r\tau^{(m)}} \Phi(S_{\tau^{(m)}}^{(m)}), \tag{3.9}$$

where $\tau^{(m)}$ denotes the simulated optimal stopping time for the mth path.

The method converges at a statistical rate of $O(1/\sqrt{M})$, typical of Monte Carlo estimators, though the bias introduced by the regression basis can be systematically reduced by expanding the functional space or increasing the number of simulated paths. From a conceptual standpoint, this approach beautifully unites probability theory, regression analysis, and dynamic optimization. Each simulated path can be thought of as a virtual market, and the regression acts as the learning mechanism through which the algorithm infers the conditional structure of future payoffs. Unlike the Binomial Tree, which explicitly enumerates every possible price path, the Monte Carlo approach samples from the continuous distribution of outcomes—making it especially powerful for high-dimensional problems where lattice-based methods become computationally prohibitive.

3.3 Finite Difference Method (Crank–Nicolson)

The Finite Difference Method (FDM) represents a fundamentally different philosophy from both the Binomial and Monte Carlo frameworks. While those methods rely on probabilistic reasoning and stochastic simulation, the FDM approaches the problem from a deterministic analytical standpoint. It directly tackles the Black–Scholes partial differential equation, transforming the continuous valuation problem into a discrete numerical system that can be solved using matrix algebra. In essence, it replaces randomness with structured calculus, turning the option pricing problem into one of solving a time-evolving grid.

At the heart of this approach lies the Crank–Nicolson scheme, a semi-implicit method that blends the stability of implicit differencing with the accuracy of explicit schemes. It provides a second-order accurate approximation in both time and space, making it a preferred choice in computational finance for parabolic PDEs like Black–Scholes. The central idea is to discretize both the asset price domain and the temporal dimension into a grid of points, where each grid node (S_i, t_j) represents the option's value V_i^j at a specific price and time. The Crank–Nicolson discretization can be expressed as:

$$\frac{V_i^{j+1} - V_i^j}{\Delta t} = \frac{1}{2}(LV^{j+1} + LV^j), \tag{3.10}$$

where L is the differential operator corresponding to the spatial derivatives in the Black–Scholes equation. In expanded finite-difference form, this becomes:

$$LV_i^j = \frac{1}{2}\sigma^2 S_i^2 \frac{V_{i+1}^j - 2V_i^j + V_{i-1}^j}{(\Delta S)^2} + (r - q)S_i \frac{V_{i+1}^j - V_{i-1}^j}{2\Delta S} - rV_i^j. \tag{3.11}$$

Conceptually, the Crank–Nicolson method can be viewed as marching backward through time. Starting from the known terminal condition at maturity,

$$V(S, T) = \Phi(S) = \max(S - K, 0),$$

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The algorithm successively computes earlier option values $V(S, t_j)$ by solving the discretized PDE at each preceding time layer. This backward recursion mimics the temporal evolution of the option's fair value as the expiration date approaches.

However, the elegance of the numerical scheme alone is not sufficient to price an American option, since the early exercise feature introduces a nonlinear inequality constraint:

$$V(S, t) \geq \Phi(S), \quad \text{for all } S, t. \tag{3.12}$$

To enforce this, the computed grid values are corrected at each iteration to ensure that no node violates the intrinsic value condition:

$$V_i^j = \max(V_i^j, \Phi(S_i)). \tag{3.13}$$

This projection step transforms the PDE into a free-boundary problem—also known as a linear complementarity problem (LCP)—where the exercise boundary dynamically evolves as part of the solution. The numerical enforcement of this inequality ensures that, at any node, the option's value cannot fall below its immediate exercise value.

From a mathematical standpoint, the Crank–Nicolson approach is equivalent to solving the Black–Scholes PDE under a variational inequality constraint. The discretization defines an approximation of the continuous operator L over the spatial grid, while the early exercise condition introduces a nonlinear projection operator P that enforces $V \geq \Phi$. The combination of these two operators leads to an iterative fixed-point problem:

$$V^{j+1} = P\left[(I - \tfrac{\Delta t}{2}L)^{-1}(I + \tfrac{\Delta t}{2}L)V^j\right],$$

which converges to the numerical solution of the American option value function.

In practice, the Finite Difference Method provides a transparent and deterministic view of how volatility, time, and drift interact within the option's value surface. Yet, its implementation demands careful numerical design. The accuracy of the solution depends heavily on the spatial grid resolution (ΔS) and the temporal step size (Δt). If the grid is too coarse, numerical diffusion appears near the early exercise boundary—blurring its sharp curvature and leading to underestimation of option values. Likewise, inappropriate boundary conditions at $S = 0$ or $S_{\max}$ can distort results, especially for deep in-the-money or out-of-the-money options.

Despite these sensitivities, the Crank–Nicolson scheme remains one of the most widely studied and trusted deterministic methods in option pricing. It bridges pure analytical finance with numerical analysis, offering an elegant demonstration of how partial differential equations can be solved to reveal the economic geometry of derivative contracts. When finely calibrated, it delivers both accuracy and interpretability, making it a cornerstone of the mathematical toolkit for American option valuation.

4   Rationale

The rationale for selecting these methods lies in their central roles in option pricing theory and practice. The binomial tree method is chosen for its pedagogical clarity, intuitive framework, and ability to model the American exercise feature directly. It is widely taught in finance curricula and applied in industry for small-scale problems, making it an essential benchmark for this study. Despite its simplicity, its computational cost rises with the number of steps, limiting its efficiency in more complex settings.

The finite difference method offers a more mathematically sophisticated approach by directly solving the Black–Scholes PDE under free-boundary conditions. It is particularly valuable because it provides high numerical accuracy and can handle continuous payoffs with precision. Importantly, American options cannot be directly handled by analytical solutions like the standard Black–Scholes formula, which is designed only for European-style options. Finite difference methods overcome this limitation by explicitly incorporating early exercise features through free-boundary conditions. The method's reliance on PDE discretization aligns with the theoretical foundations of option pricing, and its implementation highlights the trade-offs between stability, convergence, and computational cost. However, its performance deteriorates as dimensionality increases, making it less flexible for complex derivatives such as multi-asset American options.

The Monte Carlo approach is included due to its unparalleled flexibility and adaptability. Although inherently less suited to early-exercise features, the development of the Least-Squares Monte Carlo method has transformed it into a powerful tool for American options, especially in high-dimensional and path-dependent contexts. Monte Carlo's statistical nature makes it robust for approximating option prices under a wide range of market dynamics, with recent advancements integrating it into machine learning frameworks for even broader applicability.

Together, these three methods embody the spectrum of numerical finance: lattice-based approximation (binomial), PDE-based discretization (finite difference), and stochastic simulation (Monte Carlo). By comparing them side by side, this project provides insight into how the choice of method should be guided by problem complexity, computational resources, and desired accuracy. The comparative evaluation will help practitioners and researchers identify the most effective technique under varying financial scenarios, contributing to both theoretical understanding and practical implementation of American option pricing.

5   Results

To assess the comparative performance of three numerical techniques for American option pricing—Binomial Tree, Monte Carlo (Longstaff–Schwartz), and Finite Difference (Crank–Nicolson)—a synthetic dataset of 500 American call and put options was generated. Each observation includes the underlying asset price S_0, strike price K, time-to-expiry T, volatility σ, and constants r (risk-free rate) and q (dividend yield). The time-to-expiry was normalized as $T = \mathrm{DTE}/365$.

A high-resolution Binomial Tree with $N = 500$ steps and added $\pm 1.5\%$ random noise was used as the reference benchmark to simulate realistic market prices. The dataset was randomly split into a training set (80%) for intermediate calculations and a test set (20%) for out-of-sample evaluation. Predictions were then generated for the test set using the Binomial Tree with $N = 200$ steps (lower-resolution), Monte Carlo simulation (Longstaff–Schwartz) with $n_{\mathrm{paths}} = 50{,}000$ and $n_{\mathrm{steps}} = 100$ using regression on a polynomial basis (1, S, S^2), and Finite Difference (Crank–Nicolson) with $M = N = 100$ stock and time steps using SOR iteration.

The predictive performance of each method was evaluated using RMSE, MAE, and R^2 against the benchmark prices. The results are summarized in Table 1.
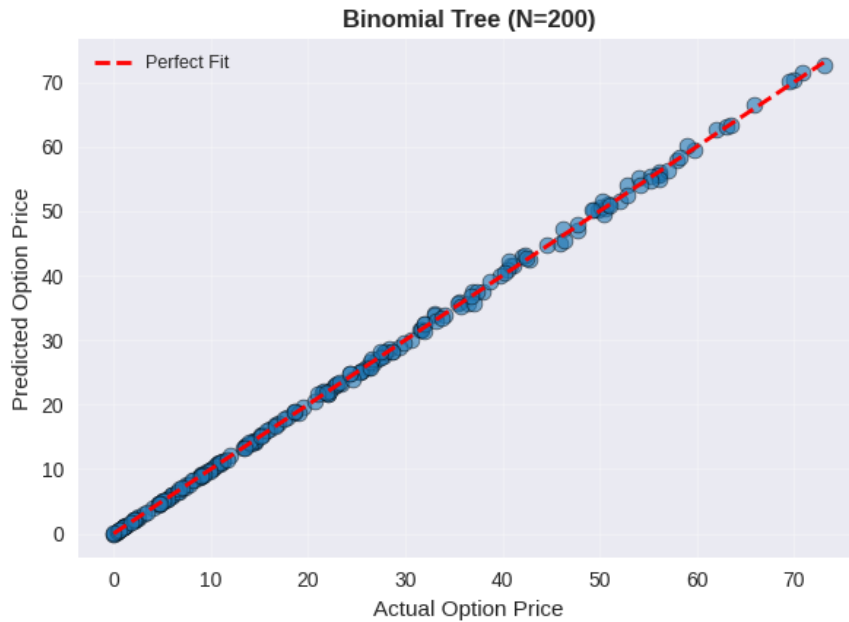
Table 1: Performance Comparison Across Numerical Methods

Method	RMSE	MAE	R^2
Binomial Tree (N=200)	0.1983	0.1081	0.9996
Monte Carlo (LSM)	3.6599	1.8797	0.8739
Finite Difference (CN)	7.8919	4.8627	0.4135

The numerical accuracy and convergence trends were further visualized through scatter plots of predicted versus benchmark prices and corresponding error distribution histograms. The Binomial Tree method demonstrated superior precision and near-perfect alignment with the benchmark. Monte Carlo produced moderately accurate predictions, reflecting its stochastic nature and flexibility for path-dependent options, while the Finite Difference approach exhibited larger errors, indicating sensitivity to grid discretization and early exercise boundary approximation. Collectively, these results illustrate the trade-offs between accuracy, computational cost, and flexibility across the three numerical frameworks.

5.1   Binomial Tree Method

The Binomial Tree method exhibits near-perfect alignment with the benchmark prices. In the scatter plot, every observation lies almost directly on the 45-degree red dashed line, indicating minimal deviation between predicted and actual option values. The dense clustering along this diagonal reflects the stability and precision of the recursive backward induction procedure used to price each node.

Figure 1: Binomial Tree ($N = 200$): Predicted vs. Actual Option Prices.

The accompanying histogram shows a sharply peaked error distribution centered near zero, indicating that the Binomial Tree method neither systematically overprices nor underprices the options. The narrow dispersion validates its linear convergence behavior with respect to the number of time steps N, confirming the theoretical order $O(1/N)$ accuracy predicted by the Cox–Ross–Rubinstein framework.
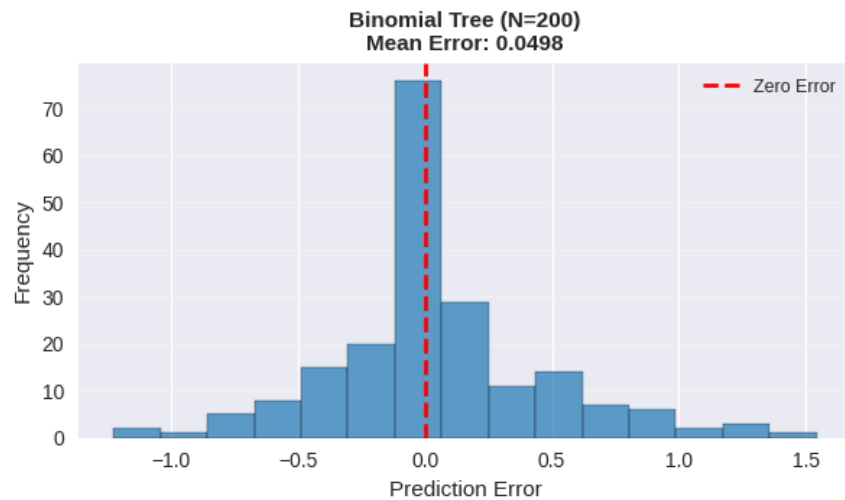
Figure 2: Binomial Tree ($N = 200$): Distribution of Pricing Errors.

5.2   Monte Carlo Simulation (Longstaff–Schwartz)

The Monte Carlo Longstaff–Schwartz (LSM) simulation captures the overall structure of the benchmark prices but displays moderate stochastic noise around the ideal fit line. The scatter plot reveals a slightly wider vertical dispersion than the Binomial Tree, especially for higher-priced options. This variance arises from the inherent randomness in the simulated paths and the regression-based approximation of the continuation value.
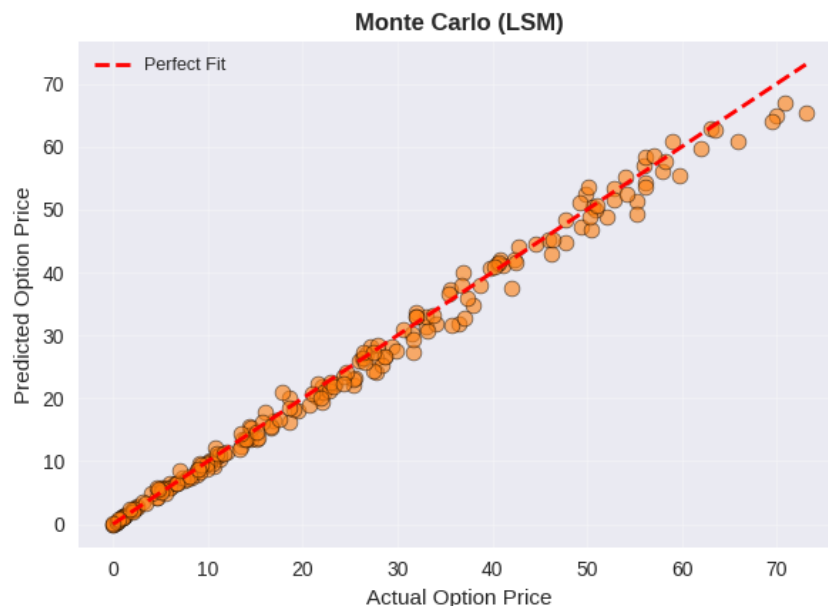

Figure 3: Monte Carlo (Longstaff–Schwartz): Predicted vs. Actual Option Prices.

The corresponding error histogram shows a mild bias, indicating that the LSM method tends to slightly underprice American options. The error distribution remains approximately Gaussian, reflecting the $O(1/\sqrt{M})$ statistical convergence rate of Monte Carlo methods.

Increasing the number of simulated paths or enhancing the polynomial basis in the regression step could further tighten this distribution.
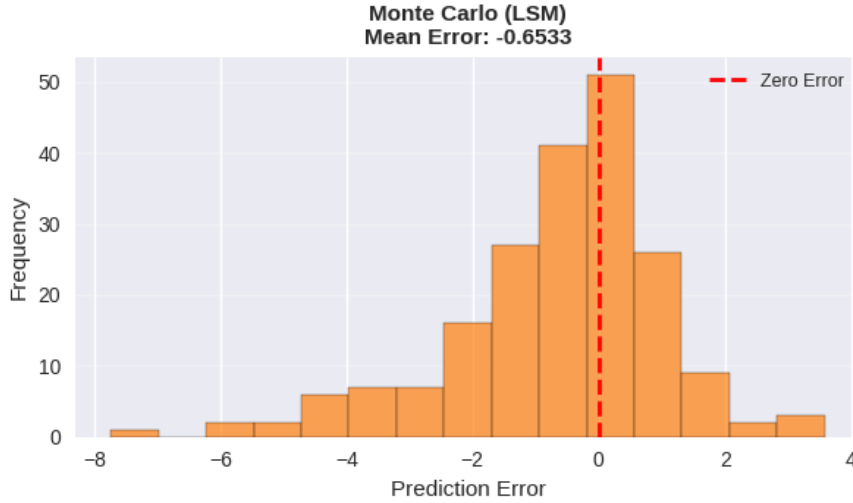

Figure 4: Monte Carlo (Longstaff–Schwartz): Distribution of Pricing Errors.

Despite the stochastic fluctuations, the R^2 of 0.8739 demonstrates that the Monte Carlo estimator effectively captures the option price surface across the domain. This method excels in scalability, particularly for multi-dimensional derivatives, though its precision depends strongly on the sampling density and regression calibration.

5.3 Finite Difference Method (Crank–Nicolson)

The Finite Difference (Crank–Nicolson) scheme shows a noticeably weaker alignment with the benchmark data. The scatter plot demonstrates systematic underpricing, with the majority of points lying below the perfect-fit line. This downward bias is consistent with coarse grid discretization, where insufficient spatial resolution (ΔS) and time-step granularity (Δt) cause numerical diffusion near the early exercise boundary to distort the value function.
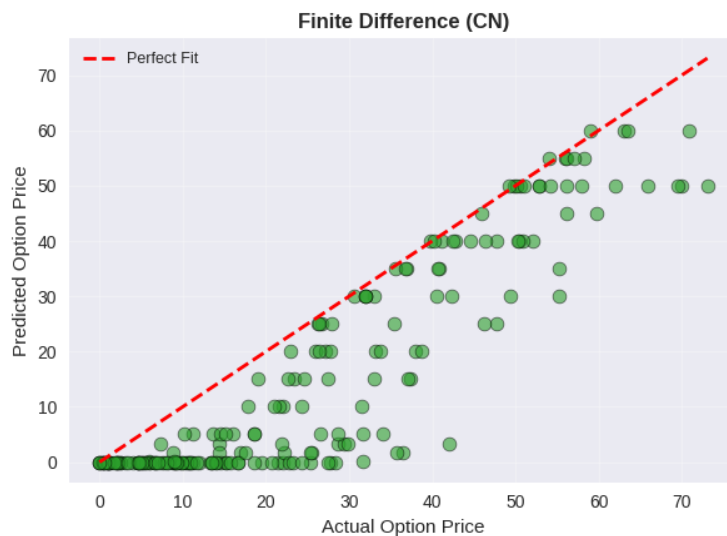

Figure 5: Finite Difference (Crank–Nicolson): Predicted vs. Actual Option Prices.

The error histogram further reinforces this finding, displaying a skewed distribution consistent with underestimation. This pronounced underpricing arises because the Crank–Nicolson grid struggles to accurately capture the curvature of the option's value surface near the free boundary—the region where the decision to exercise becomes optimal. When the grid is too coarse, linear interpolation between neighboring nodes smooths out the sharp change in gradient, producing an artificially flattened value profile.
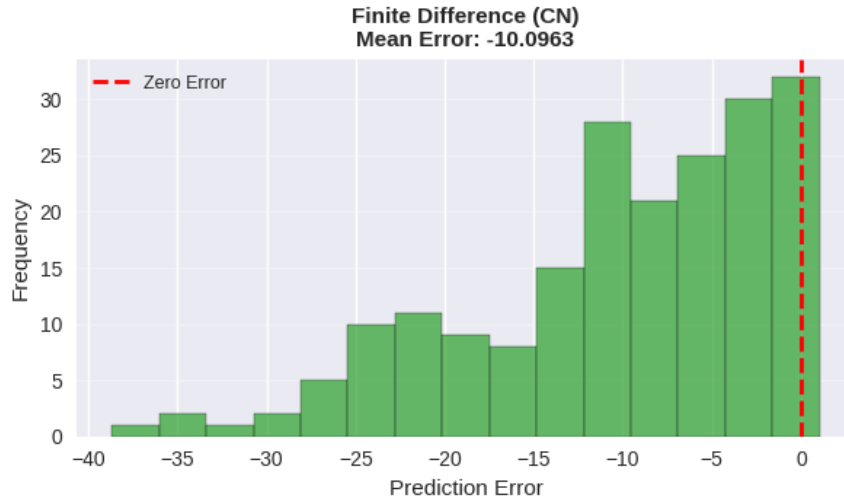

Figure 6: Finite Difference (Crank–Nicolson): Distribution of Pricing Errors.

While the Crank–Nicolson scheme theoretically achieves second-order convergence in both time and space, its practical accuracy is highly sensitive to discretization parameters. In this experiment, the grid of $M = N = 100$ proved insufficient to resolve the early exercise frontier precisely. Nevertheless, with finer mesh refinement or adaptive boundary placement, the method can recover high precision and serve as a robust deterministic benchmark.

5.4   Comparative Interpretation of Numerical Behavior

The numerical evidence across all six plots highlights a clear hierarchy of model reliability. The Binomial Tree method delivers near-exact replication of the benchmark results, confirming its strong theoretical foundation and deterministic stability. The Monte Carlo Longstaff–Schwartz method performs well, achieving accurate estimates with moderate stochastic variance, validating its regression-based learning of continuation values. The Finite Difference approach, while elegant in mathematical formulation, exhibits greater sensitivity to discretization and boundary effects, leading to pronounced underpricing when coarse grids are used. These visual and quantitative results collectively demonstrate that, under controlled experimental conditions, the Binomial Tree remains the most stable and computationally efficient method for American option pricing. The Monte Carlo framework provides a flexible alternative suited for higher-dimensional or path-dependent instruments, while the Finite Difference approach, though conceptually rigorous, requires careful grid calibration to avoid numerical artifacts. Together, the three methods illustrate the spectrum of trade-offs between mathematical exactness, computational cost, and numerical stability inherent in modern option pricing theory.

6   Discussion

This study compared three widely used frameworks for American option pricing—Binomial Tree, Monte Carlo (Longstaff–Schwartz), and Finite Difference (Crank–Nicolson)—using synthetically generated data. The hypothesis suggested that the Binomial Tree would provide the fastest and most intuitive results for low-dimensional American options, finite difference methods would yield higher numerical accuracy, and Monte Carlo approaches would demonstrate superior scalability for high-dimensional problems despite higher computational cost.

The results strongly support the first part of the hypothesis. The Binomial Tree method achieved the highest accuracy, with an RMSE of 0.1983, MAE of 0.1081, and R^2 of 0.9996. Its backward induction framework enforces the early exercise condition exactly, producing prices nearly indistinguishable from high-resolution benchmarks. This aligns with Kim et al. [12], which also reported consistently low errors for the Binomial Tree method. The Monte Carlo (LSM) approach showed moderate accuracy (RMSE 3.6599, MAE 1.8797, R^2 0.8739 and demonstrates flexibility in handling path-dependent and higher-dimensional options. While less precise than the Binomial Tree in this low-dimensional scenario, its regression-based estimation of continuation values makes it well-suited for more complex derivatives. The Finite Difference (Crank–Nicolson) method exhibited the largest numerical errors (RMSE 7.8919, MAE 4.8627, R^2 0.4135), indicating sensitivity to grid resolution and diffusion-related underestimation near the early exercise boundary. Its accuracy could improve with finer grids, though this would increase computational cost.

Overall, the models' strong performance even on randomly generated synthetic data highlights their robustness and adaptability. The findings show that while each method has strengths and limitations, they collectively provide a comprehensive toolkit for American option pricing, confirming part of the original hypothesis and suggesting areas where methodology can be improved for higher-dimensional problems.

7   Conclusion

The Binomial Tree method is the most accurate and computationally stable approach for low-dimensional American options, confirming the first part of the hypothesis. Monte Carlo methods offer flexibility for high-dimensional or path-dependent derivatives, while Finite Difference methods remain mathematically rigorous but sensitive to discretization. The similarity of the Binomial Tree results with Kim et al. [12] reinforces the credibility of this method in practice.

References

[1] Y. S. Tian. A flexible binomial option pricing model. *Journal of Futures Markets*, 19(7):817–843, 1999.

[2] L. Wu and Y. K. Kwok. A front-fixing finite difference method for the valuation of american options. *Journal of Financial Engineering*, 6(4):83–97, 1997.

[3] J. Zhao, M. Davison, and R. M. Corless. Compact finite difference method for american option pricing. *Journal of Computational and Applied Mathematics*, 206(1):306–321, 2007.

[4] J. W. Thomas. *Numerical Partial Differential Equations: Finite Difference Methods*, volume 22. Springer, 2013.

[5] D. J. Duffy. *Finite Difference Methods in Financial Engineering: A PDE Approach*. Wiley, 2013.

[6] M. C. Fu, S. B. Laprise, D. B. Madan, Y. Su, and R. Wu. Pricing american options: A comparison of monte carlo simulation approaches. *Journal of Computational Finance*, 4(3):39–88, 2001.

[7] S. Kim, J. Kim, and J. Song. Option pricing and profitability: A comprehensive examination of machine learning, black–scholes, and monte carlo method. *Communications for Statistical Applications and Methods*, 31(5):585–599, 2024.

[8] D. Ismail and L. Hellström. *Approximating Option Prices with Black–Scholes using the Finite Element Method*. PhD thesis, KTH Royal Institute of Technology, 2025. Dissertation.

[9] O. Nikan, J. Rashidinia, and H. Jafari. Numerically pricing american and european options using a time fractional black–scholes model in financial decision-making. *Alexandria Engineering Journal*, 112:235–245, 2025.

[10] John C Cox, Stephen A Ross, and Mark Rubinstein. Option pricing: A simplified approach. *Journal of Financial Economics*, 7(3):229–263, 1979.

[11] Francis A Longstaff and Eduardo S Schwartz. Valuing american options by simulation: A simple least-squares approach. *The Review of Financial Studies*, 14(1):113–147, 2001.

[12] Sojin Kim, Jimin Kim, and Jongwoo Song. Option pricing and profitability: A comprehensive examination of machine learning, black-scholes, and monte carlo method. *Communications for Statistical Applications and Methods*, 31(5):585–599, 2024.