Stochastic Interest Rate Modeling: Vasicek vs. CIR Models

Comparative Analysis with Excel Simulations and MLE Calibration

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Abstract

This project develops and implements the Vasicek and Cox–Ingersoll–Ross (CIR) stochastic interest rate models using Excel. Both models capture mean-reverting behavior of short-term interest rates, with Vasicek allowing negative rates and CIR constraining rates to remain positive. Market calibration was performed using Maximum Likelihood Estimation (MLE) to estimate key parameters including mean reversion speed, long-term equilibrium rate, and volatility. Simulations were carried out with 20 calibration for each model, followed by a comparative analysis of model suitability under different market conditions.

Contents

1 Introduction

Interest rate modeling plays a crucial role in financial engineering, risk management, and pricing of interest rate derivatives. Stochastic models such as Vasicek and CIR incorporate randomness and mean reversion to capture realistic economic conditions. This report demonstrates the implementation of these models in Excel and highlights their differences.

2 Model Formulation

2.1 Vasicek Model

The Vasicek model is defined as:

$$dr_t = a(b - r_t)dt + \sigma dW_t$$

where:

- a: Speed of mean reversion
- b: Long-term mean level
- σ: Volatility
- W_t: Wiener process

2.2 CIR Model

The Cox–Ingersoll–Ross (CIR) model is expressed as:

$$dr_t = a(b - r_t)dt + \sigma\sqrt{r_t}dW_t$$

This formulation ensures $r_t > 0$, making it more realistic for modeling interest rates.

3 Methodology

- **Data Source:** Daily U.S. Treasury yield data was collected from the U.S. Department of the Treasury.
- Implemented both Vasicek and CIR models in Excel with daily time steps.
- Conducted 20 simulations for each model to analyze rate dynamics.
- Applied Maximum Likelihood Estimation (MLE) to estimate key parameters (a, b, σ).
- Validated model fit by comparing simulated paths with observed U.S. Treasury yield behavior.

4 Simulation Results

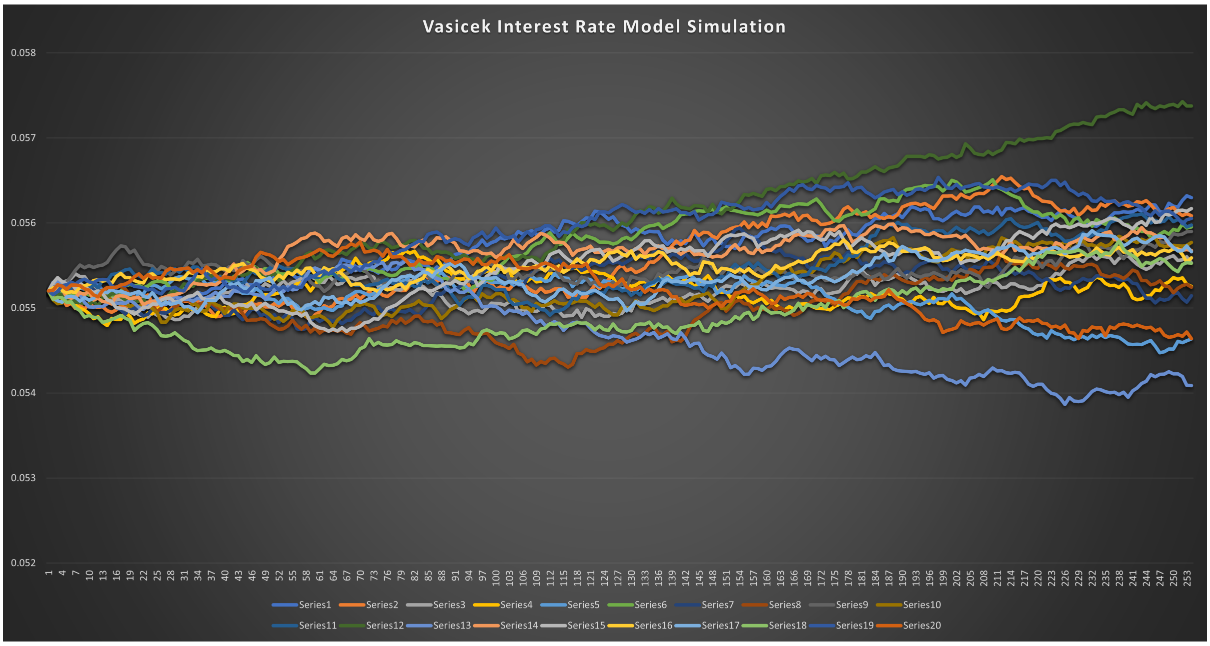
4.1 Vasicek Model Simulation

Days	Simulation 1	Simulation 2	Simulation 3	Simulation 4	Simulation 5	Simulation 6	Simulation 7	Simulation 8	Simulation 9	Simulation 10	Simulation 11	Simulation 12	Simulation 13	Simulation 14	Simulation 15	Simulation 16	Simulation 17	Simulation 18	Simulation 19	Simulation 20
0	0.0552	0.0552	0.0552	0.0552	0.0552	0.0552	0.0552	0.0552	0.0552	0.0552	0.0552	0.0552	0.0552	0.0552	0.0552	0.0552	0.0552	0.0552	0.0552	0.0552
1	0.05519891	0.055199271	0.05528345	0.05518496	0.05527004	0.05523685	0.05516769	0.05509598	0.0552118	0.05520928	0.0552551	0.05517274	0.05521331	0.05515162	0.05516044	0.05520086	0.05516608	0.05525119	0.05520511	0.05519114
2	0.05518643	0.055195399	0.05518009	0.05522073	0.05533766	0.05522201	0.05520124	0.05511189	0.05512549	0.05520576	0.05528194	0.05519921	0.05524465	0.05522233	0.05522758	0.05521295	0.05515955	0.05524949	0.0551799	0.05517173
3	0.05516083	0.05509685	0.05516164	0.0552331	0.05538615	0.05530827	0.05519346	0.05506504	0.05511771	0.05517945	0.05527604	0.05529253	0.05527366	0.05516175	0.05524668	0.05526867	0.05518246	0.05535446	0.05520283	0.05521291
4	0.05515372	0.055073357	0.05510092	0.05515925	0.05539721	0.05525103	0.05515465	0.05512298	0.05515504	0.05518241	0.05527517	0.05526573	0.05534078	0.05512935	0.05523664	0.05522589	0.05521161	0.0553354	0.0552372	0.05523822
5	0.05519341	0.0550748	0.05507865	0.05517233	0.05537789	0.05524538	0.05513122	0.05510028	0.05524458	0.05515158	0.05523321	0.05522309	0.05541035	0.05517138	0.05520731	0.05522615	0.05524485	0.05529572	0.05521312	0.05523825
6	0.05513291	0.055088799	0.05513975	0.05516067	0.05539079	0.05526236	0.0550763	0.05511357	0.05520554	0.05502757	0.05528408	0.05523043	0.05546838	0.05516549	0.05523932	0.05525878	0.05523305	0.05537522	0.05522756	0.05518123
7	0.05515192	0.055106326	0.05510895	0.05521175	0.05533724	0.0553239	0.05500193	0.05510817	0.05517726	0.05500335	0.0552385	0.05516261	0.05544394	0.05510881	0.0552114	0.05522677	0.05525425	0.05535764	0.05522113	0.05518275
8	0.05513114	0.05511487	0.05514661	0.05524474	0.05535051	0.05531304	0.05500417	0.05509711	0.05511325	0.05502026	0.05519103	0.05517459	0.05545785	0.05508114	0.05520245	0.05529634	0.05512898	0.0553382	0.05518628	0.05527719
9	0.05509221	0.05511028	0.05514859	0.05528745	0.05535195	0.05537113	0.05499994	0.05511592	0.05510733	0.05502813	0.05525451	0.05526308	0.0554591	0.0551207	0.05518506	0.0553055	0.05519974	0.05532732	0.05518874	0.05521446
10	0.05504389	0.055074472	0.05519291	0.05536896	0.05537271	0.05537245	0.05505409	0.05511864	0.05517963	0.0549676	0.05529611	0.05521964	0.0553809	0.05515436	0.05516464	0.05534928	0.05516161	0.05528656	0.05524898	0.05523468
11	0.05507318	0.055158952	0.05519345	0.05535708	0.05535963	0.0553075	0.05509917	0.05508728	0.05519862	0.05494919	0.05526676	0.05512158	0.05540327	0.05511034	0.05513026	0.05539013	0.05518027	0.05526433	0.05523143	0.05523104
12	0.05505317	0.055136872	0.05520552	0.05532049	0.05534278	0.05532647	0.05501579	0.05504956	0.05519514	0.05493343	0.05518867	0.05508813	0.05538916	0.05513358	0.05508457	0.05541517	0.05512008	0.05524539	0.05523745	0.05526557
13	0.05510829	0.055198463	0.05522624	0.05532279	0.05538574	0.05530806	0.05501071	0.05503857	0.05524749	0.05489769	0.0552085	0.05504123	0.05534914	0.05507484	0.05503059	0.05543812	0.05510145	0.055251	0.05524539	0.05526721
14	0.05508865	0.055208031	0.05529472	0.05531004	0.05532979	0.0553583	0.05497956	0.05508624	0.05521673	0.05488425	0.05524091	0.0551043	0.05534323	0.05508775	0.05508983	0.05541394	0.0550294	0.05528366	0.0551635	0.05531273
15	0.05500385	0.055285217	0.0552469	0.05526086	0.05529522	0.05538744	0.05498067	0.05510409	0.05518838	0.05488015	0.0552504	0.05511554	0.05536628	0.05503438	0.05507214	0.05538688	0.05501362	0.05517113	0.05520583	0.0553828
16	0.05503225	0.055205101	0.05521496	0.05524989	0.05537068	0.05540497	0.05496792	0.0551698	0.05514218	0.05480681	0.05520993	0.05503854	0.055416	0.05502471	0.05507234	0.055355	0.05504858	0.05510513	0.05520693	0.0553851
17	0.05504384	0.055178643	0.05521025	0.05522232	0.0553213	0.05543285	0.05493032	0.05519075	0.05523105	0.05483266	0.05519128	0.0550027	0.05538575	0.05508576	0.05505008	0.05538454	0.0550657	0.05511887	0.05514242	0.0554453
18	0.05501901	0.055207783	0.05522149	0.05524165	0.05536471	0.05542073	0.05494814	0.05519123	0.05516851	0.05484444	0.05517455	0.0549904	0.05540206	0.0551231	0.05501053	0.05539331	0.05502617	0.0551324	0.05509159	0.05539305
19	0.05498444	0.055211066	0.05526386	0.05522763	0.05529992	0.05544441	0.05490394	0.05513636	0.05511316	0.05491016	0.05516568	0.05495289	0.0553509	0.05513206	0.05499351	0.05533921	0.0550543	0.05512523	0.05503045	0.05537112
20	0.05498156	0.055237156	0.05522969	0.05525931	0.05528117	0.05548327	0.05492478	0.05507903	0.05520125	0.05495731	0.05519844	0.05491898	0.05530184	0.05514588	0.05495133	0.05538586	0.05496665	0.05509416	0.05508838	0.05527958
21	0.05498582	0.055239317	0.05522395	0.05529811	0.05532075	0.05551164	0.05492827	0.05506614	0.05514856	0.05493282	0.05510024	0.05491911	0.05532207	0.05513454	0.05494733	0.05535424	0.05495835	0.05518254	0.05505619	0.05525691
22	0.05501313	0.055294873	0.05525248	0.05535772	0.0552822	0.05551857	0.05491358	0.05512827	0.05520757	0.05485456	0.05500924	0.05500252	0.05532156	0.05517979	0.05493524	0.05531235	0.05501425	0.05516783	0.05505126	0.05534645
23	0.05503596	0.055391753	0.05528357	0.05545866	0.05526259	0.05552833	0.05496114	0.05516602	0.05523791	0.05493892	0.05501759	0.05498869	0.05531473	0.0552034	0.05493143	0.05530621	0.05498578	0.05519832	0.05505617	0.05537102
24	0.05504305	0.055393732	0.05517861	0.05547366	0.05529064	0.05556366	0.0549218	0.05523697	0.05517008	0.05495243	0.05499752	0.05502797	0.05527806	0.055153	0.05493474	0.05529743	0.05487898	0.05522201	0.05500995	0.05537052
25	0.05505687	0.055386563	0.05518221	0.05549501	0.05519765	0.0555837	0.05496114	0.05522071	0.05512248	0.05491945	0.05503016	0.05499116	0.05530816	0.05525091	0.05495505	0.05527911	0.05499522	0.05527308	0.05498182	0.05539643
26	0.05504329	0.055409869	0.05515916	0.05549928	0.05518993	0.05565968	0.05502141	0.05526789	0.05513639	0.05489776	0.05505774	0.05507076	0.05534455	0.05528684	0.05492181	0.05531453	0.05496688	0.05533021	0.05500852	0.05542918
27	0.05501529	0.055327223	0.05518309	0.05550715	0.05520368	0.05564521	0.05500506	0.05523662	0.05514904	0.05492783	0.05506648	0.0550325	0.05527902	0.05535474	0.05488971	0.05531583	0.05491223	0.05531938	0.05503955	0.05544702
28	0.05500238	0.055310099	0.05518685	0.0554441	0.05513711	0.05567869	0.05497575	0.05523311	0.05513894	0.05490766	0.05504985	0.05497414	0.0553082	0.05539551	0.05486138	0.05527393	0.05494626	0.05533058	0.055065	0.05544284
29	0.05502167	0.055285781	0.05517583	0.05546292	0.055183	0.05575812	0.05496331	0.05515494	0.05522097	0.05488666	0.05506707	0.05497849	0.05533405	0.05535447	0.05489872	0.05522407	0.0549284	0.05528056	0.05510158	0.05542282
30	0.0550826	0.055266138	0.05509778	0.05542623	0.05522918	0.05575897	0.0549439	0.05521915	0.05517514	0.05487551	0.05508424	0.05494835	0.05527249	0.0553766	0.05489149	0.05520518	0.05483118	0.0553435	0.05507448	0.05537141
31	0.05506465	0.055190211	0.05511872	0.05541632	0.05530817	0.05569682	0.05498404	0.05520819	0.0550716	0.05489259	0.05509016	0.05499028	0.05531427	0.05535886	0.05493218	0.05519378	0.0548451	0.05541454	0.05497839	0.05534945
32	0.05502745	0.055126606	0.0551725	0.05536081	0.05531713	0.0556793	0.0550096	0.05520608	0.05511628	0.05483834	0.05509736	0.05498504	0.05529361	0.05537978	0.05494065	0.05513186	0.05483787	0.05541386	0.05501003	0.05541664
33	0.05508972	0.055165627	0.05518321	0.05534517	0.05529072	0.05578082	0.05505401	0.05523496	0.05505949	0.05491238	0.05512115	0.05499476	0.05526874	0.05535952	0.05500147	0.05508596	0.05488827	0.05547529	0.05492921	0.05539979
34	0.05510153	0.055183003	0.05519106	0.05534223	0.05530025	0.0558743	0.05506734	0.05521465	0.05501198	0.0550017	0.05511413	0.05499464	0.05526218	0.05536636	0.05498696	0.05508555	0.05495874	0.05539771	0.05492991	0.05538205
35	0.05509799	0.055203246	0.05515143	0.055398	0.05529726	0.05584588	0.0550664	0.05513409	0.0550257	0.05501565	0.05518532	0.05498538	0.05529446	0.05528829	0.05494392	0.05505399	0.05498408	0.05541595	0.05490379	0.05532747
36	0.05510372	0.055179393	0.05513727	0.05544801	0.05536432	0.05588875	0.05501938	0.05511348	0.05504691	0.05501682	0.05518452	0.05493665	0.05529111	0.05530021	0.0550278	0.05509308	0.05495595	0.05539342	0.05495232	0.05537713
37	0.05514356	0.05504307	0.055099	0.05550942	0.0553714	0.05589497	0.05506217	0.05510137	0.05507709	0.05505725	0.05522033	0.05494457	0.05527859	0.0552909	0.05503364	0.05504533	0.05494641	0.05538808	0.05495779	0.05532436
38	0.05521391	0.055036339	0.05515534	0.05544281	0.05541834	0.05590978	0.05499471	0.05507597	0.05501887	0.05510309	0.05526343	0.05493802	0.05525671	0.05530175	0.05507629	0.05508427	0.05495309	0.05541635	0.05492876	0.05527818
39	0.0552319	0.054992329	0.05519817	0.0554165	0.05546347	0.05596759	0.05495444	0.05503398	0.05505398	0.05511316	0.05525347	0.05496236	0.05525779	0.05534331	0.05509234	0.05499605	0.05499512	0.0554125	0.05499303	0.05525833
40	0.05520024	0.054972866	0.05519475	0.05539095	0.0554506	0.05589445	0.05493401	0.05503834	0.05501564	0.0551364	0.05531758	0.05504898	0.05521963	0.05535826	0.05515878	0.05501621	0.05500294	0.05542891	0.05499014	0.05521054
41	0.05523081	0.054973728	0.05522429	0.05549484	0.05546408	0.05589886	0.05493627	0.05506497	0.0550424	0.05504856	0.05534498	0.05513854	0.0552952	0.05536421	0.05520021	0.05502082	0.0550648	0.055389	0.05502319	0.05523636
42	0.05530212	0.055016351	0.05524128	0.05544454	0.05547925	0.05587049	0.05493318	0.05509881	0.05500403	0.05510193	0.05532424	0.05516374	0.05528364	0.05538927	0.05520646	0.05506881	0.05501142	0.05535202	0.05503288	0.0552057
43	0.05536028	0.05502444	0.05520256	0.0555236	0.05548843	0.05589988	0.05495871	0.05505296	0.05501937	0.05511333	0.05521039	0.05512694	0.05527818	0.05531802	0.05525573	0.05499801	0.05501811	0.05538638	0.05496795	0.05512893
44	0.05534345	0.055068532	0.05521219	0.05548738	0.05548319	0.0558819	0.05492134	0.05503557	0.05500383	0.05511285	0.05524224	0.05509788	0.05527224	0.05527535	0.05532195	0.05507459	0.0549928	0.0553568	0.05499049	0.05510982
45	0.05539406	0.055074947	0.05527282	0.05553385	0.0555091	0.05587077	0.05494007	0.05503679	0.05505715	0.05504767	0.05524956	0.05501149	0.05528627	0.05527238	0.055313	0.05508794	0.05497214	0.05536399	0.05508892	0.05516303
46	0.05539704	0.055052277	0.05535171	0.05558479	0.05555554	0.05591488	0.05492154	0.05505322	0.05504844	0.05503605	0.05528335	0.05510266	0.05530332	0.05528356	0.05529477	0.05498972	0.05500109	0.05542361	0.05514765	0.05513131
47	0.05542017	0.055065213	0.05537124	0.05561115	0.05555996	0.05594221	0.05490618	0.05506946	0.05505822	0.05509827	0.05528128	0.05513022	0.05527109	0.0553102	0.05528362	0.05493612	0.0549448	0.05539403	0.05519654	0.05507836

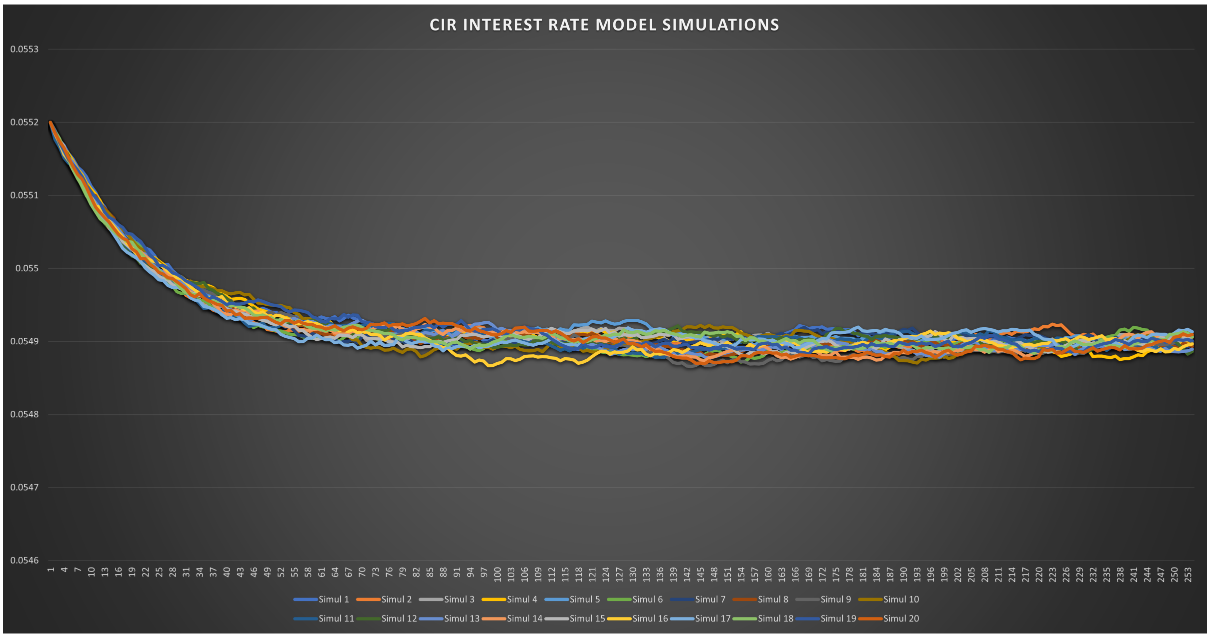
4.2 CIR Model Simulation

Days	Simulation 1	Simulation 2	Simulation 3	Simulation 4	Simulation 5	Simulation 6	Simulation 7	Simulation 8	Simulation 9	Simulation 10	Simulation 11	Simulation 12	Simulation 13	Simulation 14	Simulation 15	Simulation 16	Simulation 17	Simulation 18	Simulation 19	Simulation 20
0	0.0552	0.0552	0.0552	0.0552	0.0552	0.0552	0.0552	0.0552	0.0552	0.0552	0.0552	0.0552	0.0552	0.0552	0.0552	0.0552	0.0552	0.0552	0.0552	0.0552
1	0.05519258	0.05519033	0.05518905	0.05518553	0.05518971	0.05519388	0.0551924	0.05518449	0.05518564	0.05518982	0.05518332	0.05518931	0.05517946	0.05518469	0.05518148	0.05518886	0.05518741	0.05518789	0.05518494	0.05519132
2	0.05518243	0.05517915	0.05517589	0.0551735	0.05517841	0.05518301	0.05518385	0.05517258	0.05517536	0.05517939	0.05516884	0.05517378	0.0551653	0.05517009	0.05516801	0.05517456	0.05517587	0.05517191	0.05516999	0.05517576
3	0.0551699	0.05516461	0.05516269	0.05515892	0.05516435	0.05517223	0.05517199	0.05516017	0.05517006	0.05516644	0.05515159	0.05515494	0.05515345	0.05516053	0.05515262	0.05516171	0.05516483	0.05516274	0.05515404	0.05516299
4	0.05515817	0.05515377	0.05515496	0.05514692	0.05515174	0.05515735	0.05516648	0.05515135	0.05515882	0.05515743	0.05513679	0.05514158	0.05514192	0.05514765	0.05514318	0.05514656	0.05515159	0.05515594	0.05513984	0.05515667
5	0.05514546	0.05514088	0.05514786	0.05513213	0.05514085	0.05514751	0.0551564	0.05514099	0.0551493	0.05514711	0.05512374	0.05512969	0.05513768	0.05513559	0.05513359	0.05513279	0.05514206	0.05514929	0.05512899	0.05514121
6	0.05513679	0.05512818	0.05514101	0.05512367	0.05512945	0.05513512	0.05514137	0.05512898	0.0551402	0.05513797	0.05511274	0.05511818	0.05512504	0.05512511	0.0551219	0.05512156	0.05513152	0.05514303	0.05511735	0.05513594
7	0.05512756	0.05511881	0.05513232	0.05511293	0.05512135	0.05512279	0.05512805	0.05511929	0.05512805	0.05513222	0.05510202	0.05510881	0.0551155	0.05511222	0.05511123	0.05511186	0.05512093	0.05513128	0.05510434	0.05512598
8	0.05511893	0.05511047	0.05512114	0.05509862	0.05511293	0.05510976	0.05511776	0.05511133	0.05511936	0.05512156	0.0550891	0.05509835	0.0551079	0.05509825	0.05509705	0.05510202	0.05511013	0.05511779	0.05509772	0.05511737
9	0.05510673	0.05509946	0.05511446	0.05508672	0.05510088	0.05509857	0.05510972	0.05510037	0.05510537	0.05511213	0.05507743	0.0550896	0.05509611	0.05509051	0.05508783	0.05509299	0.05510243	0.055108	0.05509192	0.0551116
10	0.05509301	0.05509359	0.05510632	0.05507635	0.0550881	0.05508743	0.05510293	0.0550926	0.05510161	0.05510019	0.05506431	0.0550798	0.0550827	0.05507926	0.05508194	0.05508619	0.05509351	0.05510366	0.05508501	0.05510561
11	0.05508535	0.05508274	0.0550953	0.05507061	0.05508014	0.05507993	0.05508991	0.05508656	0.05509309	0.05508679	0.05505502	0.05507089	0.05507462	0.05507052	0.05507309	0.05508077	0.0550841	0.05509074	0.05507566	0.05509799
12	0.05506982	0.05507537	0.05508805	0.05506909	0.05507005	0.05507709	0.05508277	0.05507911	0.05508232	0.055077	0.05504631	0.05506206	0.05506636	0.0550583	0.05506371	0.05507507	0.05507311	0.05507664	0.05507169	0.05508845
13	0.0550646	0.05506608	0.05507596	0.05506138	0.0550622	0.05506834	0.05507696	0.0550701	0.05507209	0.05507228	0.05504196	0.05505541	0.05505915	0.05505026	0.05505315	0.05506645	0.05506427	0.05506451	0.05506575	0.05507577
14	0.0550592	0.05505817	0.05506399	0.0550505	0.05505871	0.05505654	0.05506675	0.05506118	0.05506705	0.05506282	0.05503676	0.05504357	0.05505224	0.0550417	0.0550457	0.0550579	0.05505471	0.05505442	0.05505928	0.0550706
15	0.05505305	0.05505363	0.05505389	0.0550468	0.05505047	0.05504816	0.05505833	0.05505131	0.05506516	0.05505215	0.05503105	0.05503863	0.05504786	0.05503363	0.05504023	0.05505226	0.05504685	0.05504482	0.05505288	0.05506727
16	0.0550429	0.05504291	0.05504607	0.05504254	0.05504267	0.05504135	0.0550498	0.05504867	0.05505779	0.0550426	0.05502398	0.05503399	0.05503824	0.05502875	0.05503565	0.05504801	0.05504227	0.05504035	0.05504076	0.05506287
17	0.05503973	0.05503486	0.05504459	0.05503537	0.05503571	0.05503635	0.05504412	0.05504342	0.05504747	0.05504137	0.05501844	0.055024	0.05503535	0.05502797	0.05502299	0.05503943	0.05503561	0.05503916	0.05503476	0.0550555
18	0.05503522	0.05503057	0.0550418	0.05503199	0.05503132	0.05502545	0.05503634	0.05503612	0.05504085	0.05503092	0.05501168	0.05501499	0.05502777	0.05502435	0.05501629	0.05503321	0.05502699	0.05503289	0.05502528	0.05504704
19	0.05503453	0.05502441	0.05503273	0.05502247	0.05502351	0.05502191	0.05503082	0.0550343	0.05503161	0.05502496	0.05500699	0.05501188	0.05502241	0.0550186	0.05500865	0.05503179	0.05502156	0.05502323	0.05501981	0.05504134
20	0.05502583	0.05501617	0.05503211	0.05501882	0.05501736	0.05501672	0.05502787	0.05503133	0.05502379	0.05501832	0.05500552	0.05500593	0.05501903	0.05501477	0.05500038	0.05502853	0.055015	0.05501558	0.05500788	0.05503546
21	0.05502057	0.05501479	0.05502427	0.05501326	0.05500934	0.05501398	0.0550199	0.05502513	0.05501463	0.05501278	0.05500189	0.05499325	0.05501426	0.05500689	0.0549993	0.05502368	0.05500556	0.05500635	0.05499846	0.05502973
22	0.05501761	0.05500608	0.05502078	0.05501198	0.05500447	0.05500772	0.05500834	0.05501967	0.05501094	0.05500833	0.05499573	0.05499056	0.05500839	0.05500398	0.05499793	0.05502295	0.05499801	0.05499812	0.05499351	0.05502363
23	0.0550135	0.05500067	0.05501316	0.05500468	0.05500148	0.05500455	0.05499978	0.0550104	0.05500273	0.05500315	0.05498878	0.0549885	0.05500338	0.05499662	0.05499305	0.05502223	0.05499398	0.0549911	0.05499052	0.05501613
24	0.05501183	0.05499204	0.05500466	0.05499914	0.05499436	0.05500159	0.05499442	0.05500939	0.05499862	0.05499924	0.05498249	0.05498449	0.05499663	0.05498904	0.05498706	0.05501554	0.0549928	0.054985	0.05498717	0.0550144
25	0.05500647	0.05498483	0.05500042	0.0549927	0.05498854	0.05499909	0.05498818	0.05500578	0.05499329	0.05498889	0.05498345	0.0549775	0.05499608	0.05498442	0.0549802	0.0550123	0.0549868	0.05497659	0.0549832	0.05500738
26	0.05499986	0.05497861	0.05499294	0.05498908	0.0549835	0.05499193	0.05498444	0.0550023	0.05499154	0.05498386	0.05497854	0.05497739	0.05499695	0.05497934	0.05497623	0.05500961	0.0549849	0.05497558	0.05498087	0.05500377
27	0.05499974	0.05497488	0.05499507	0.05499158	0.05497812	0.05498526	0.05498406	0.05499961	0.05498295	0.05497877	0.05497618	0.05497782	0.0549937	0.05497398	0.05497016	0.05500416	0.05498113	0.05497316	0.05497631	0.05500098
28	0.05499498	0.0549692	0.05499128	0.05498669	0.05497555	0.05497718	0.05498342	0.05499502	0.05497503	0.05497618	0.05496848	0.0549757	0.0549853	0.05497018	0.05496943	0.05500156	0.05497544	0.05496994	0.05497283	0.05499653
29	0.05499599	0.05496991	0.05498538	0.05498047	0.05496851	0.05497241	0.05498067	0.0549901	0.05497066	0.0549719	0.05496609	0.0549734	0.05498288	0.05496921	0.05496778	0.05499884	0.05496982	0.05496219	0.05496982	0.05498851
30	0.05499129	0.05496981	0.05498209	0.05497757	0.05496499	0.05496901	0.05497742	0.05498662	0.05496812	0.05497011	0.05496277	0.05496859	0.05498126	0.05496774	0.05496385	0.05499722	0.05496565	0.05496147	0.05496863	0.05498379
31	0.05498667	0.05496763	0.0549781	0.0549741	0.05496576	0.05496281	0.05497675	0.0549885	0.05496332	0.05496756	0.05496055	0.05496708	0.05497852	0.05496652	0.05495681	0.05498973	0.0549646	0.05495718	0.05496906	0.05497801
32	0.05498375	0.05496501	0.05497579	0.05497019	0.05496209	0.05496305	0.05497172	0.0549868	0.05495825	0.0549665	0.05496116	0.05496138	0.05497298	0.05496519	0.05495438	0.05498591	0.05496558	0.0549522	0.05496562	0.0549737
33	0.05497938	0.05495836	0.05497256	0.05496713	0.05496242	0.05495732	0.05496605	0.05498419	0.05495496	0.05496797	0.05496078	0.05495832	0.05496838	0.05496132	0.05495131	0.05497851	0.05496802	0.0549503	0.05496497	0.05496652
34	0.05497728	0.054953	0.05496492	0.05496635	0.05496024	0.05495275	0.05496	0.05498	0.05495338	0.05496585	0.05495967	0.05495909	0.05496069	0.05495674	0.05495066	0.05497396	0.05496521	0.05494695	0.05495769	0.0549633
35	0.05497209	0.0549496	0.05496659	0.05496484	0.05495613	0.05495193	0.05495623	0.05497743	0.05494581	0.05496415	0.05495334	0.05495793	0.05495477	0.0549562	0.05494506	0.0549729	0.05495942	0.05494405	0.05495185	0.05495707
36	0.0549702	0.05495374	0.0549604	0.05496336	0.05495878	0.05494728	0.05495364	0.05497261	0.05494366	0.05496106	0.05495323	0.05495065	0.05494722	0.05495168	0.05494417	0.05497047	0.05495806	0.05494657	0.05495246	0.05495703
37	0.05496319	0.05494984	0.05496098	0.05496101	0.05495557	0.05494571	0.05495558	0.0549707	0.0549396	0.05495571	0.05495224	0.05494941	0.05494128	0.05494789	0.0549448	0.05496376	0.05495961	0.05494641	0.0549514	0.05495281
38	0.0549612	0.05494338	0.05495512	0.05496172	0.05495256	0.05494763	0.05495477	0.05497456	0.05493585	0.05494925	0.05494788	0.05494828	0.05493877	0.05494564	0.05493877	0.054963	0.05495634	0.05494221	0.05495	0.05494761
39	0.05495883	0.05494124	0.05495353	0.05495821	0.05495005	0.05494628	0.0549549	0.05496912	0.05493404	0.05494539	0.05494556	0.0549489	0.05494005	0.05494246	0.05494302	0.05495832	0.05495224	0.05494228	0.05494489	0.05494531
40	0.05495699	0.0549401	0.05495296	0.05495897	0.05494731	0.05494333	0.05494986	0.05496257	0.05493404	0.05494271	0.05494185	0.05494303	0.05493734	0.05494195	0.05494103	0.05495811	0.05494936	0.05494155	0.05494228	0.05494698
41	0.05495655	0.05494172	0.05495072	0.05495404	0.05494886	0.05493894	0.05494992	0.05496236	0.05493232	0.05493933	0.05494377	0.05494313	0.05493677	0.05494097	0.05494215	0.05495834	0.05495188	0.0549397	0.05494188	0.05494781
42	0.05495002	0.05494316	0.05494964	0.0549546	0.05494745	0.05493388	0.05494658	0.05495839	0.05493312	0.05493744	0.05493977	0.05494623	0.05493491	0.05494091	0.05494235	0.05495605	0.05495096	0.05493601	0.05494195	0.05494861
43	0.05494944	0.05494415	0.05494905	0.05495808	0.05494387	0.05493459	0.05494027	0.0549507	0.05493091	0.05493524	0.05493729	0.05494129	0.05493048	0.05493288	0.05493883	0.05495572	0.05494608	0.05493362	0.05493887	0.05494712
44	0.05494123	0.05494385	0.05494535	0.05495665	0.05493454	0.05493142	0.05493955	0.05494584	0.05493272	0.05493487	0.05493569	0.05493585	0.05492952	0.05493633	0.05493262	0.05495736	0.05494802	0.05493087	0.0549346	0.05494371
45	0.05493995	0.05494391	0.05494714	0.05495536	0.05494092	0.05492991	0.05493839	0.05494242	0.05492959	0.05493503	0.0549338	0.05493823	0.05492289	0.05493563	0.0549323	0.05495847	0.05494399	0.05492547	0.05493203	0.05494319
46	0.0549351	0.05493598	0.05494768	0.05495705	0.05494248	0.05492588	0.05493696	0.05494097	0.05492753	0.05493277	0.05492727	0.05493658	0.05492105	0.05493425	0.05493032	0.05495688	0.05494303	0.05491548	0.0549352	0.05493617
47	0.05493495	0.05493521	0.05494185	0.05495617	0.05493859	0.05492446	0.05493498	0.05494337	0.05492489	0.05493051	0.05492681	0.05493355	0.05492196	0.05493361	0.05492178	0.05495561	0.05493941	0.05491694	0.05492958	0.05493625

4.3 Historical Fit (Vasicek)



4.4 Historical Fit (CIR)



5 Calibration via MLE

MLE was applied to estimate parameters a, b, σ for each model. Results are summarized in Table 1.

Table 1: MLE Calibration Results

Model	a (Speed)	b (Mean)	σ (Volatility)
Vasicek	0.102324189463251	0.057406085645852	0.0182799831393017%
CIR	11.0784180763815	0.0548973529828751	0.0180933604968501%

6 Comparative Analysis

- **Vasicek:** Easier to implement, but allows negative rates.
- **CIR:** Ensures positivity of rates, making it more realistic in stressed conditions.
- **Market Suitability:** Vasicek preferred for short horizons or low-vol regimes; CIR more robust for risk management.

7 Limitations of the Vasicek and CIR Models

7.1 Vasicek (Ornstein–Uhlenbeck) Model

- **Negative rates possible:** Normal innovations imply r_t can become negative, which may be unrealistic in many markets.
- **Constant volatility:** Assumes homoskedastic shocks; empirical rate volatility is typically state– and regime–dependent.
- **Single-factor short-rate:** One factor often cannot match the full term-structure dynamics (level, slope, curvature) without time-varying parameters.
- **Weak fit in low-rate regimes:** Normality and constant σ can misprice options and caps/floors when rates cluster near the zero lower bound.
- **Parameter stability:** Calibrations can drift over time; constant a, b, σ ignore macro regime shifts and policy breaks.

7.2 Cox–Ingersoll–Ross (CIR) Model

- **Feller condition sensitivity:** Positivity is ensured only if $2ab \geq \sigma^2$; violations during calibration are common in low-rate data.
- **Volatility too low near zero:** The $\sigma\sqrt{r_t}$ term suppresses volatility as $r_t \to 0$, often underestimating observed variability near the ZLB.
- **Single-factor limitation:** Like Vasicek, a single state variable struggles to fit the entire yield curve and its dynamics.
- **Numerical issues:** Discretizations can become biased near the boundary; ad-hoc fixes (e.g., truncation) can distort likelihood estimates.
- **Rigid functional form:** Mean reversion and diffusion are fixed; cannot capture jumps, fat tails, or regime changes without extensions.

7.3 Limitations Common to Both

- **Stationary, time-invariant parameters:** Both assume constant a, b, σ; real markets exhibit structural breaks and regime shifts.
- **No jumps or macro events:** Gaussian/diffusion-only dynamics miss policy surprises and crisis jumps.
- **Short-rate perspective:** Pricing the full term structure requires additional assumptions; multi-factor (e.g., Hull–White, multi-factor CIR) often fit better.
- **Calibration risk:** MLE fits can be sample- and frequency-dependent; out-of-sample performance may degrade.
- **Spreadsheet implementation caveats:** Euler discretization and finite precision may introduce bias; use sufficiently small Δt and check discretization robustness.

8 Conclusion

The Vasicek and CIR models provide valuable insights into interest rate dynamics. While Vasicek offers analytical simplicity, CIR adds realism by preventing negative rates. This project highlighted the practical use of Excel in stochastic modeling and demonstrated how MLE calibration aligns models with observed data.

References

1. Vasicek, O. (1977). An equilibrium characterization of the term structure. *Journal of Financial Economics.*
2. Cox, J., Ingersoll, J., & Ross, S. (1985). A theory of the term structure of interest rates. *Econometrica.*
3. Brigo, D., & Mercurio, F. (2006). *Interest Rate Models: Theory and Practice.*