Matrix Assignment

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***Problem Statement* - ABC is a triangle right angled at C. A line through the mid-point M of hypotenuse AB and parallel to BC intersects AC at D. Show that**

1. **D is the mid-point of AC**
2. **MD $\perp$ AC**
3. **CM = MA = $\frac{1}{2}$AB**

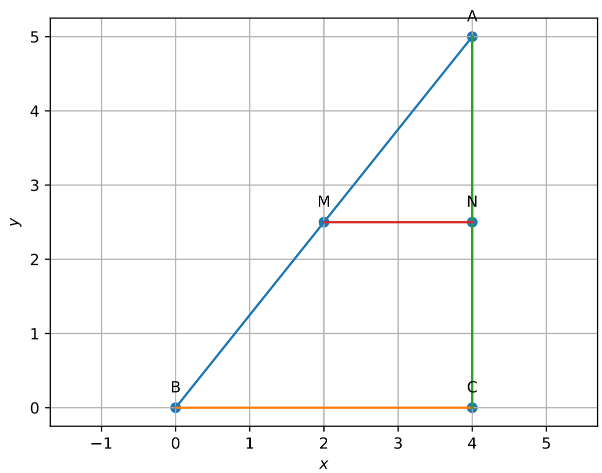

Figure 1: Triangle ABC right angled at C

Solution

Part 1

Given MN is $\|$ to BC, hence

$$\frac{AD}{DC} = \frac{AM}{MB} \tag{1}$$

Since M is the mid-point of AB, $\frac{AM}{MB} = 1$. Substituting in (1), we get

$$AD = DC \tag{2}$$

Therefore, D is the midpoint of AC.

Part 2

Since ABC is right angled at C,

$$(\boldsymbol{C} - \boldsymbol{A})^T(\boldsymbol{C} - \boldsymbol{B}) = 0 \tag{3}$$

Given that MD is parallel to BC, so

$$(\boldsymbol{C} - \boldsymbol{B}) = \lambda(\boldsymbol{M} - \boldsymbol{D}) \tag{4}$$

Substituting (4) in (3) and dividing by λ, we get

$$(\boldsymbol{C} - \boldsymbol{A})^T(\boldsymbol{M} - \boldsymbol{B}) = 0 \tag{5}$$

From (5) it can be concluded that MD $\perp$ AC.

Part 3

Let

$$\boldsymbol{M} - \boldsymbol{D} = \boldsymbol{p} \tag{6}$$
$$\boldsymbol{C} - \boldsymbol{D} = \boldsymbol{q} \tag{7}$$
$$\boldsymbol{A} - \boldsymbol{D} = \boldsymbol{r} \tag{8}$$

Then vectors along CM and AM can be written as

$$\boldsymbol{C} - \boldsymbol{M} = \boldsymbol{q} - \boldsymbol{p} \tag{9}$$
$$\boldsymbol{A} - \boldsymbol{M} = \boldsymbol{r} - \boldsymbol{p} \tag{10}$$

The magnitudes of CM and AM are therefore

$$||\boldsymbol{C} - \boldsymbol{M}|| = ||\boldsymbol{q} - \boldsymbol{p}|| = (\boldsymbol{q} - \boldsymbol{p})^T(\boldsymbol{q} - \boldsymbol{p}) \tag{11}$$
$$||\boldsymbol{A} - \boldsymbol{M}|| = ||\boldsymbol{r} - \boldsymbol{p}|| = (\boldsymbol{r} - \boldsymbol{p})^T(\boldsymbol{r} - \boldsymbol{p}) \tag{12}$$
$$\tag{13}$$

Upon expanding the vector products, the terms $\boldsymbol{q}^T\boldsymbol{p}$ and $\boldsymbol{r}^T\boldsymbol{p}$ evaluate to 0 (from eq.5). From eq.2,

$$||\boldsymbol{q}|| = ||\boldsymbol{r}||$$

. Therefore,

$$||\boldsymbol{C} - \boldsymbol{M}|| = ||\boldsymbol{q}|| + ||\boldsymbol{p}|| \tag{14}$$

$$||\boldsymbol{A} - \boldsymbol{M}|| = ||\boldsymbol{r}|| + ||\boldsymbol{p}|| = ||\boldsymbol{q}|| + ||\boldsymbol{p}|| \tag{15}$$

Equating (14) and (15), we get

$$AM = CM \tag{16}$$

Since given that M is midpoint of AB,

$$AM = \frac{1}{2}AB \tag{17}$$

Equating (16) and (17), the result is proved.

Construction

The input parameters are the lengths a and c.

Symbol	**Value**	**Description**
a	4	BC
c	5	AC
k	1	$\frac{AM}{MB} = \frac{AD}{DC}$
θ	$\arctan(\frac{c}{a})$	$\angle$B
A	$\sqrt{a^2+c^2}\begin{pmatrix} cos\theta \\ sin\theta \end{pmatrix}$	Point A