Optimization Assignment - 1

Mohamed Hamdan

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Problem Statement - The longest side of a triangle is 3 times the shortest side and the third side is 2 cm shorter than the longest side. If the perimeter of the triangle is at least 61 cm, find the minimum length of the shortest side.

Solution

Let x be the largest side, z the shortest side and y be other side of the triangle. The problem can be formulated as

$$P = \min_{x,y,z} z \tag{1}$$

$$x - 3z = 0 \tag{2}$$

$$x - y = 2 \tag{3}$$

$$x + y + z \ge 61\tag{4}$$

which can be expressed in vector form as

$$P = \min_{\mathbf{x}} \begin{pmatrix} 0 & 0 & 1 \end{pmatrix} \mathbf{x} \tag{5}$$

$$\begin{pmatrix} 1 & 0 & -3 \\ 1 & -1 & 0 \end{pmatrix} \mathbf{x} = \begin{pmatrix} 0 \\ 2 \end{pmatrix} \tag{6}$$

$$\begin{pmatrix} 1 & 1 & 1 \end{pmatrix} \mathbf{x} \ge \begin{pmatrix} 61 \end{pmatrix} \tag{7}$$

$$\mathbf{x} \ge \mathbf{0} \tag{8}$$

Solving using cvxpy, we get

$$P_{min} = 9 (9)$$

$$\mathbf{x} = \begin{pmatrix} 27\\25\\9 \end{pmatrix} \tag{10}$$