Optimization Assignment - 2

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***Problem Statement* - An open topped box is to be constructed by removing equal squares from each corner of a 3 meter by 8 meter rectangular sheet of aluminium and folding up the sides. Find the volume of the largest such box**

Solution

Monomial approximation

Let h be the side of the square removed. Let l and b be the length and breadth of the box. Due to the construction, h is the height of the box. The volume of the box can be expressed as

$$V = lbh \tag{1}$$

where

$$l = 8 - 2h \tag{2}$$
$$b = 3 - 2h \tag{3}$$

Since l and h are positive, from (2) and (3) we get

$$h <= \min(4, 1.5) \tag{4}$$
$$\implies h <= 1.5 \tag{5}$$

The problem can be formulated as

$$P = \max_{l,b,h} lbh \tag{6}$$
$$l + 2h = 8 \tag{7}$$
$$b + 2h = 3 \tag{8}$$
$$h <= 1.5 \tag{9}$$

The above formulation is not a geometric programming problem since the equality constraints (7) and (8) are posynomials. To convert the problem into a geometric programming problem, we approximate the posynomials in the equality constraints as monomials using

$$f(w) \approx f(x) \prod_{i=1}^{n} \left(\frac{w_i}{x_i}\right)^{a_i} \tag{10}$$

where

$$a_i = \frac{x_i}{f(x)} \frac{\partial f}{\partial x_i} \tag{11}$$

Approximating (7) and (8) as $f(l, b, h)$ and $g(l, b, h)$ respectively using (10), the problem is expressed as

$$P = \max_{l,b,h} lbh \tag{12}$$
$$f(l, b, h) = 8 \tag{13}$$
$$g(l, b, h) = 3 \tag{14}$$
$$h >= \frac{h_k}{1 + \Delta} \tag{15}$$
$$h <= \min((1 + \Delta)h_k, 1.5) \tag{16}$$

Where h_k is the current guess of h and Δ is the parameter to determine the trust region around h_k where the approximation in (10) holds valid. The above formulation can be iterated till the problem converges at a local maximum. Taking $n = 100$ and $\Delta = 0.005$, the optimal solution obtained using cvxpy is

$$\boxed{V_{max} = 7.407485} \tag{17}$$
$$\boxed{h = 0.664331} \tag{18}$$

Gradient descent

Let x be the side of each square removed. The volume of the box can be expressed as

$$f(x) = (8 - 2x)(3 - 2x)x \tag{19}$$
$$\implies f(x) = 4x^3 - 22x^2 + 24x \tag{20}$$

The polynomial in (20) has roots at $x = 0$, $x = 1.5$ and $x = 4$. Since the coefficient of x^3 is positive and the roots are distinct, it can be concluded that there exists a local maximum between $x = 0$ and $x = 1.5$. This can be seen in Figure 1. Using gradient ascent method we can find its maxima in the interval $[0, 1.5]$,

$$x_{n+1} = x_n + \alpha \nabla f(x_n) \tag{21}$$
$$\implies x_{n+1} = x_n + \alpha \left(12x_n^2 - 44x_n + 24\right) \tag{22}$$

Taking $x_0 = 0.5, \alpha = 0.001$ and precision $= 0.00000001$, values obtained using python are:

$$\boxed{\text{Maxima} = 7.407407} \tag{23}$$
$$\boxed{\text{Maxima Point} = 0.666666} \tag{24}$$

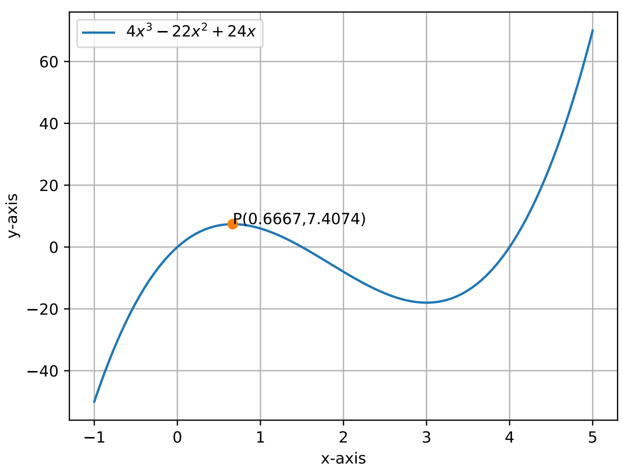

Figure 1: Graph of $f(x)$ in (20)