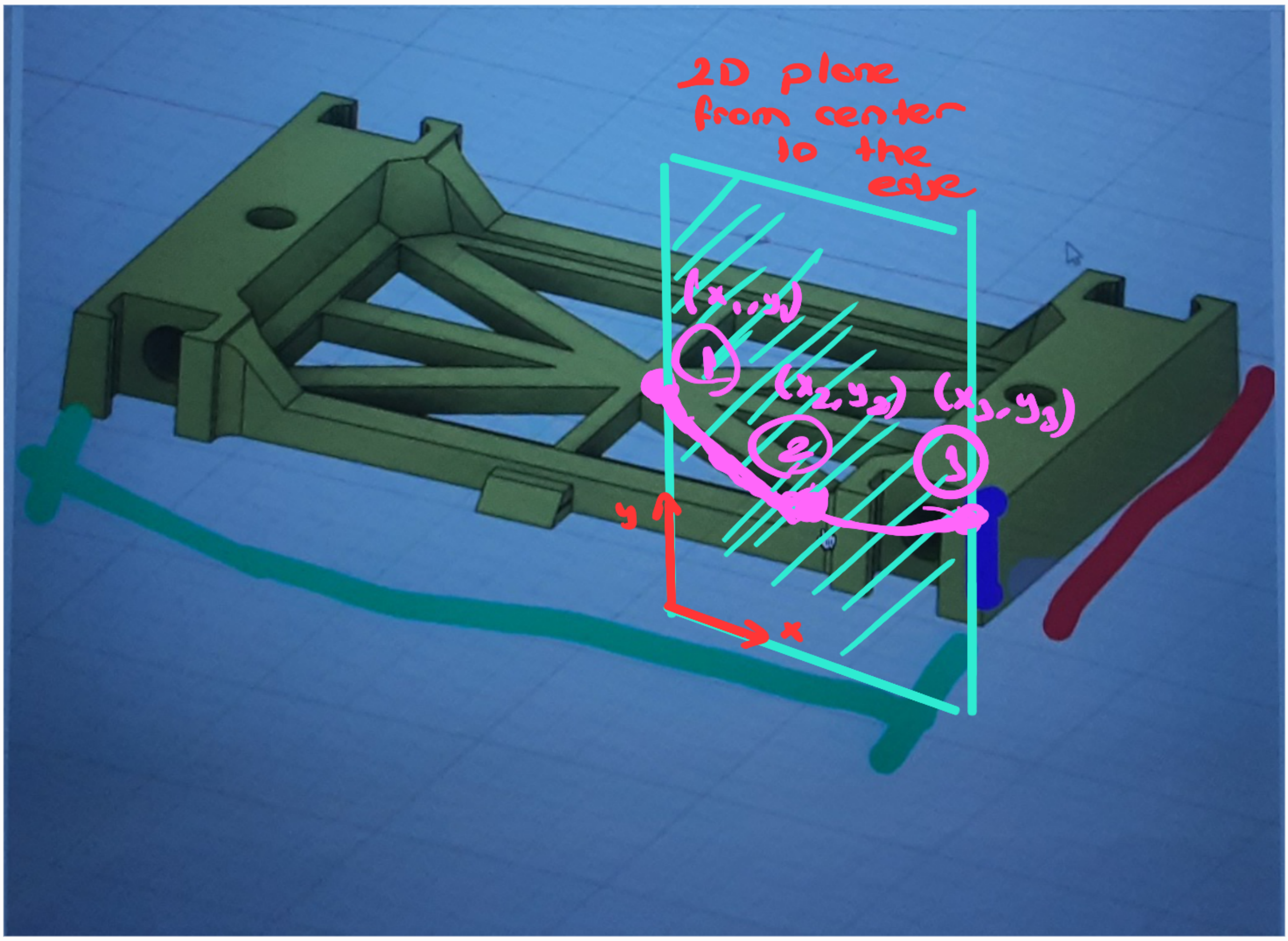
defining a cubic spline ⇒

Hermite Splines ?



given (x_1,y_1) ; (x_2,y_2) and (x_3,y_3)

Joining together at y_2

from 1 to 2 ⇒ $\boxed{ax_2^3+bx_2^2+cx_2+d=y_2}$

from 2 to 3 ⇒ $\boxed{ex_2^3+fx_2^2+gx_2+h=y_2}$

* their first derivatives are also equal to each other.

$$\boxed{3ax_2^2+2bx_2+c=3ex_2^2+2fx_2+g}$$

* their second derivatives are also equal to each other.

$$\boxed{6ax_2+2b=6ex_2+2f}$$

* making natural cubic spline ⇒ their second derivatives are equal to zero at end points ⇒

$$\boxed{\begin{array}{l}6ax_1+2b=0\\6ex_3+2f=0\end{array}}$$

* also,

$$\boxed{\begin{aligned} ax_1^3 + bx_1^2 + cx_1 + d &= y_1 \\ ex^3 + fx_3^2 + gx_3 + d &= y_3 \end{aligned}}$$

then;

matrix-form	variables		values
A	x	$=$	b
8×8	8×1		8×1

$$\Rightarrow A \cdot \underset{\text{coefs}}{x} = b$$

$$\Rightarrow x = A^{-1} \cdot b$$

where a, b, c, d, e, f, g, h stored

$x_1, y_1 = 0, 0$

$x_2, y_2 = 1, 1$

$x_3, y_3 = 2, 2$

solved with matlab $\Rightarrow$

coefs

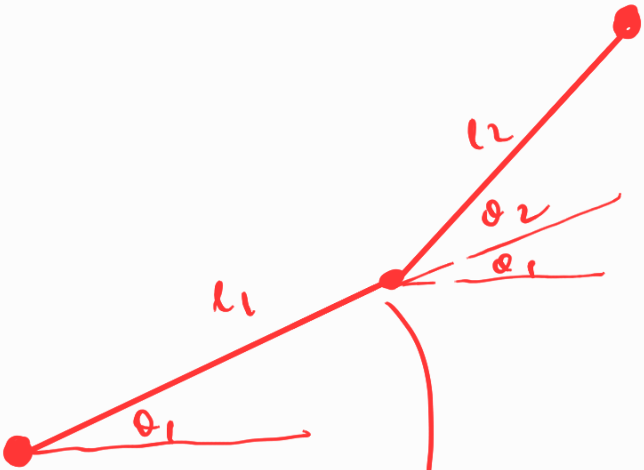
0
0
1
0
0
0
1
0

$ax^3 + bx^2 + cx + d = f_1(1)$

cx

$ex^3 + fx^2 + gx + h = f_2(1)$

gx



$(x_1, y_1) = (0, 0)$

$(x_2, y_2) = (l_1 \cos\theta_1 + l_2 \sin\theta_1)$

$(x_3, y_3) = (l_1 \cos\theta_1 + l_2 \cos(\theta_2 + \theta_1),$

$l_1 \sin\theta_1 + l_2 \sin(\theta_2 + \theta_1))$