Using Monte Carlo Method in Nuclear Reactor Applications – One-Dimensional System Modeling

Let's consider a probability distribution function (pdf) defined in the interval [a,b]. Let's call this function p(x) and normalize the function p(x) in this range.

We want to generate random numbers x_i from the distribution function p(x) using the random numbers ηi uniformly distributed in the range [0,1]. So that,

$$p(x)dx = d\eta$$

The cumulative distribution function c(x) is also defined as follows,

$$c(x) = \int_a^x p(x')dx' \rightarrow \frac{dc(x)}{dx} = p(x)$$

If $c(x) = \eta$, then $p(x)\ dx = d\eta$.

Exponential Distribution [0,∞);

$$p(x) = \Sigma \cdot \exp(-\Sigma \cdot x) \rightarrow c(x) = 1 - \exp(-\Sigma \cdot x) = \eta \rightarrow x = -\frac{ln(1-\eta)}{\Sigma}$$

Uniform Distribution [a,b];

$$p(x) = \frac{1}{b-a} \rightarrow c(x) = \frac{x-a}{b-a} = \eta \rightarrow x = a + (b-a)\eta$$

$\Sigma t \rightarrow$ probability of neutron reacting at unit length in a sample

$\exp(-\Sigma tx) \rightarrow$probability of the neutron traveling the length dx without any interaction

$\Sigma t\exp(-\Sigma tx)dx \rightarrow$probability of a neutron making its first interaction in dx = p(x)dx

$$c(x) = 1 - \exp(-\Sigma_t \cdot x) = \eta \rightarrow x = -\frac{ln(1-\eta)}{\Sigma_t}$$

$0 \leq \eta \leq 1$ since it is a uniformly distributed random number in [0,1],

$$x = -\frac{ln(1-\eta)}{\Sigma_t} = x = -\frac{ln(\eta)}{\Sigma_t}$$

The point x is sampled as the point where the neutron will make its first interaction. However, it is not yet known whether this point is inside or outside the defined system or what interaction it will make.

$\Sigma t = 1$

$c = \Sigma s / \Sigma t = 0{,}5$ (Σscattering/ Σtotal)

$pdf(x)\,dx = \Sigma_t e^{-\Sigma t\, x}\,dx$ ⟶ Probability of neutron interacting between x and x+dx

$$\int_0^x pdf(x)\,dx = cdf(x) = \xi$$

ξ: a random number that takes values in the range of [0.1]

$$x = -\frac{1}{\Sigma_t} \ln \xi$$

$$x_i + \mu x_d - L = 0 \quad (\text{for } \mu > 0)$$
$$x_i + \mu x_d = 0 \quad (\text{for } \mu < 0)$$

$$x_1 = -\frac{1}{\Sigma_t} \ln \xi_1$$

xi= the position of the neutron at any moment

xd= maximum distance the neutron can travel

Condition 1:

Neutrons can escape from the system at x=L. If $xi > xd$ and $\mu = 1$, since the neutron moves in the +x direction, it has traveled the minimum distance it can travel in the rod. Therefore, the neutron escaped from the system at x=L. When this condition is met, let nt=nt+1 (nt starts from zero). We have lost the neutron and it is necessary to return to the initial conditions and sample (give birth) the new neutron.

Initial conditions:

xi=0,

xd=L,

μ=1

Condition 2:

Neutrons can escape from the system at x=0. If; If $xi > xd$ and $\mu = -1$, the neutron has traveled the minimum distance it can travel in the rod since it moves in the -x direction. Therefore, the neutron escaped from the system at x=L. When this condition is met, let nr=nr+1 (nr starts from zero). We have lost the neutron and it is necessary to return to the initial conditions and sample (give birth) the new neutron.

Condition 3:

Neutrons can be absorbed by the system (rod). If ; $xi < xd$ means that the neutron will make its first interaction inside the bar. In this case, it is necessary to determine which interaction the neutron will make. If we remember that $c = \Sigma s/\Sigma t = 0.5$, a new random number $\xi 2$ is generated for the neutron that will interact, and if this number is less than 0.5, the neutron has been absorbed. We have lost the neutron and it is necessary to return to the initial conditions and sample (give birth) the new neutron.

Condition 4:

Neutrons can perform scattering interactions. If ; $xi < xd$ means that the neutron will make its first interaction inside the bar. In this case, it is necessary to determine which interaction the neutron will make. If we remember that $c = \Sigma s/\Sigma t = 0.5$, a new random number $\xi 2$ is generated for the neutron that will interact, and if this number is greater than 0.5, the neutron has scattered interaction. This is the only interaction in this system where we don't lose the neutron. The new direction of the neutron as a result of the scattering reaction is found by generating a different random number $\xi 3$. If this number is less than 0.5, then $\mu = \mu$ (forward scattering) otherwise $\mu = -\mu$ (backward scattering). So the neutron is still in the bar but in a new location.

$$x_i^{new} = x_i^{old} + \mu\, x_l$$

Since some distance travels in the neutron bar, xi will change, so xd will change. This procedure is carried out until the neutron is lost.

$$x_d = L - x_i \quad (\mu = 1)$$
$$x_d = x_i \quad (\mu = -1)$$

n is the total number of neutrons lost in the system; n=nr+nt+na,

PR =nr/n

PT =nr/n

PA =nr/n

n	P_R	P_A	P_T
10^2	0.13	0.85	0.02
10^3	0.166	0.811	0.023
10^4	0.1679	0.8023	0.0298
10^5	0.1698	0.8009	0.0293
10^6	0.1714	0.8009	0.0284

We will see in the code that as the number of sampling increases, more realistic results will be obtained.

The example we will do in our program is as follows;

Consider the one-dimensional neutron transport system given in the figure above. Single-energy neutrons are born at a random point ($x0 = L * \xi$) in a rod of length L. The rod is made of neutron absorber material only ($\Sigma s = 0$). There is a reflective material at x=L and x=0 points, and neutrons arriving at these points are reflected back to the rod with certain probabilities ($\alpha 1$, $\alpha 2$). the rest passes through the reflectors and disappears. The physical parameters are given below.

$L = 1\ cm$

$\Sigma a = 0.2\ cm^{-1}$

$\alpha 1 = 0.5$

$\alpha 2 = 0.6$

In this case, since there are no scattered neutrons in our system, the total cross section is equal to the absorption cross section.

$\Sigma t = \Sigma a + \Sigma s$

$\Sigma s = 0$

$\Sigma t = \Sigma a$