1

a-15p

$$P(X>-1) = 1 - P(X \leq -1)$$
$$= 1 - F(-1) = 0.3934693$$
$$= 1 - e^{-c} = \text{''}$$
$$\therefore \quad e^{-c} = 1 - 0.3934693$$
$$c = -\ln(0.6065307)$$
$$\boxed{c \cong 0.5}$$

b-15p

$$f(x) = \frac{d}{dx} F(x)$$
$$\therefore \quad f(x) = \begin{cases} c\, e^{cx}, & x < 0 \\ 0, & x > 0 \end{cases}$$

2

a) $\mu = E(X) = \int_{-\infty}^{0} y \cdot 3 . e^{3y}\, dy = 3 \left[\frac{(3y-1)}{9}\, e^{3y}\right]_{-\infty}^{0}$

$$= 3\left[-\frac{1}{9} - 0\right] = \boxed{-\frac{1}{3}\ /\!L}$$

$$V(X) = E(Y^2) - \mu^2 = \int_{-\infty}^{0} y \cdot 3 . e^{3y}\, dy \ - \mu^2$$

$-\infty$

$$E(X^2) = 3\left[\frac{9y^2 - 6y + 2}{27}\, e^{3y}\right]_{-\infty}^{0} = \frac{3 \times 2}{27} = \frac{2}{9}$$

$$V(Y) = E(X^2) - \mu^2 = \frac{2}{9} - \frac{1}{9} = \boxed{\frac{1}{9}\ /\!L}$$

b) $E(h(Y)) = \underbrace{E(Y^2)}_{2/9} + 4\, \underbrace{E(Y)}_{-1/3} + 1$

$$= \frac{2}{9} - \frac{4}{3} + 1 = \boxed{-\frac{1}{9}\ /\!L}$$

Q3 /20pts

For $x=0$ $\quad f(x-1)=f(-1)=0 \quad f(0)>0$

$f(1).0=0<f^2(0)$

For $x>0$

$$\binom{n}{x-1} p^{x-1}(1-p)^{n-x+1} \times \binom{n}{x+1} p^{x+1}(1-p)^{n-x-1}$$

$$= \binom{n}{x-1}\binom{n}{x+1} p^{x}(1-p)^{n-x}$$

$$\binom{n}{x-1}\binom{n}{x+1} = \frac{n!}{(x-1)!\,(n-x+1)!} \cdot \frac{n!}{(x+1)!\,(n-x-1)!}$$

$$= (n!)^2 \frac{x}{x!} \cdot \frac{1}{(x+1)\,x!} \cdot \frac{n-x-1}{(n-x)!} \cdot \frac{1}{(n-x+1)(n-x)!}$$

$$= \frac{x}{x+1} \cdot \frac{(n-x)-1}{(n-x)+1} \cdot \left(\frac{n!}{(n-x)!\,x!}\right)^2$$

$$\therefore \quad f(x-1)f(x+1) = f^2(x) \cdot \underbrace{\frac{x}{x+1}}_{<1} \; \underbrace{\frac{n-x-1}{n-x+1}}_{<1}$$

$$\therefore \quad f(x-1)f(x+1) \leq f^2(x)$$ ▲ Q.E.D.

Q4 - 20 pts

X: poisson, λ = $\lambda = 5 \cdot t$ errors
$t = ?$ (minutes)

$P(X < 450) = 0.4$ → Find λ first

$$P(X \leq 450) = P(X \leq 450.5) = P\left(z < \frac{450.5 - \lambda}{\sqrt{\lambda}}\right) = 0.4$$

Using the table $\dfrac{450.5 - \lambda}{\sqrt{\lambda}} = -0.25$

Let's say $m = \sqrt{\lambda}$

$$450.5 - m^2 = -0.25\,m$$

$$\therefore \quad m^2 - 0.25\,m - 450.5 = 0$$

$$m = \frac{0.25 \mp \sqrt{0.25^2 - 4 \times (-450.5)}}{2}$$

$$= \frac{0.25 \mp \sqrt{0.0625 + 1802}}{2}$$

$$m = \frac{0.25 + 42.4507}{2} = 21.3504$$

$\lambda = m^2 = 455.8374$ errors / t

$\lambda = 5t$ = 5 errors/minute × t

$$t = \frac{455.8374}{5} = \boxed{91.1675} \text{ minutes}$$ (sufficient answer)

But for the obsessive :)

t = 1 hrs 31 m 10.049 sec.