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ARTIFICIAL INTELLIGENCE & EXPERT SYSTEMS

Adversarial Search

GAMES

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Generalizing Search Problems

- So far: our search problems have assumed agent has complete control of environment.
  - state does not change unless the agent changes it.
  - All we need to compute is a single path to a goal state.

- Assumption not always reasonable
  - stochastic environment (e.g., the weather, traffic accidents).
  - other agents whose interests conflict with yours
  - In both cases you might not traverse the path you are expecting.

Generalizing Search Problems

- In these cases, we need to generalize our view of search to handle state changes that are not in the control of the agent.
- One generalization yields game tree search
  - agent and some other agents.
  - The other agents are acting to maximize their profits
    - this might not have a positive effect on your profits.

Generalizing Search Problems

- **Multiagent environments** – any given agent should consider the actions of other agents and how they affect its own welfare.
  - Cooperative multiagent environments
    - e.g. in taxi-driving, to avoid collisions, all agents try to maximize the performance measures.
  - Competitive multiagent environments
    - e.g. in chess, the agent tries to maximize its performance mesaure while minimizing the other agent's performance mesaure.

GAMES

- Competitive environments, lead to **adversarial search** problems.
  - also known as **games.**
- In AI, "games" are called **zero-sum games** of **perfect information**.
  - deterministic, fully observable environments.
  - two agents whose actions must alternate.
  - utility values at the end of the game are always equal and opposite.
    - e.g. if one player wins a game of chess (+1), the other player necessarily loses (-1).
  - This opposition between the agents' utility functions makes the situation **adversarial**.

Two-person Zero-Sum Games

- chess, checkers, tic-tac-toe, backgammon, go, Doom, "find the last parking space"
- Your winning means that your opponent looses, and vice-versa.
- Zero-sum means the sum of your and your opponent's payoff is zero---any thing you gain come at your opponent's cost (and vice-versa). Key insight:
  - how you act depends on how the other agent acts (or how you think they will act)
  - and vice versa (if your opponent is a rational player)

More General Games

- What makes something a game?
  - there are two (or more) agents influencing state change
  - each agent has their own interests
    - e.g., goal states are different; or we assign different values to different paths/states
- Each agent tries to alter the state so as to best benefit itself.

More General Games

- What makes games hard?
  - how you should play depends on how you think the other person will play; but how they play depends on how they think you will play; so how you should play depends on how you think they think you will play; but how they play should depend on how they think you think they think you will play; …

More General Games

- Zero-sum games are "fully competitive"
  - if one player wins, the other player loses
  - e.g., the amount of money I win (lose) at poker is the amount of money you lose (win)
- More general games can be "cooperative"
  - some outcomes are preferred by both of us, or at least our values aren't diametrically opposed
- We'll look in detail at zero-sum games
  - but first, some examples of simple zero-sum and cooperative games

Game 1: Rock, Paper, Scissors

- Scissors cut paper, paper covers rock, rock smashes scissors
- Represented as a matrix: Player I chooses a row, Player II chooses a column
- Payoff to each player in each cell (Pl.I/ Pl.II)
- 1: win, 0: tie, -1: loss
  - so it's zero-sum

Player II

Player I	R	P	S
R	0/0	-1/1	1/-1
P	1/-1	0/0	-1/1
S	-1/1	1/-1	0/0

Game 2: Prisoner's Dilemma

- Two prisoner's in separate cells, there are not enough evidence to convict them
- If one confesses, other doesn't:
  - confessor goes free
  - other sentenced to 4 years
- If both confess (both defect)
  - both sentenced to 3 years
- Neither confess (both cooperate)
  - sentenced to 1 year on minor charge
- Payoff: 4 minus sentence

	Coop	Def
Coop	3/3	0/4
Def	4/0	1/1

Game 3: Battlebots

Two robots: Blue (Craig's), Red (Fahiem's)

- one cup of coffee, one tea left
- both C, F prefer coffee (value 10)
- tea acceptable (value 8)

- Both robot's go for Cof
  - collide and get no payoff
- Both go for tea: same
- One goes for coffee, other for tea:
  - coffee robot gets 10
  - tea robot gets 8

	C	T
C	0/0	10/8
T	8/10	0/0

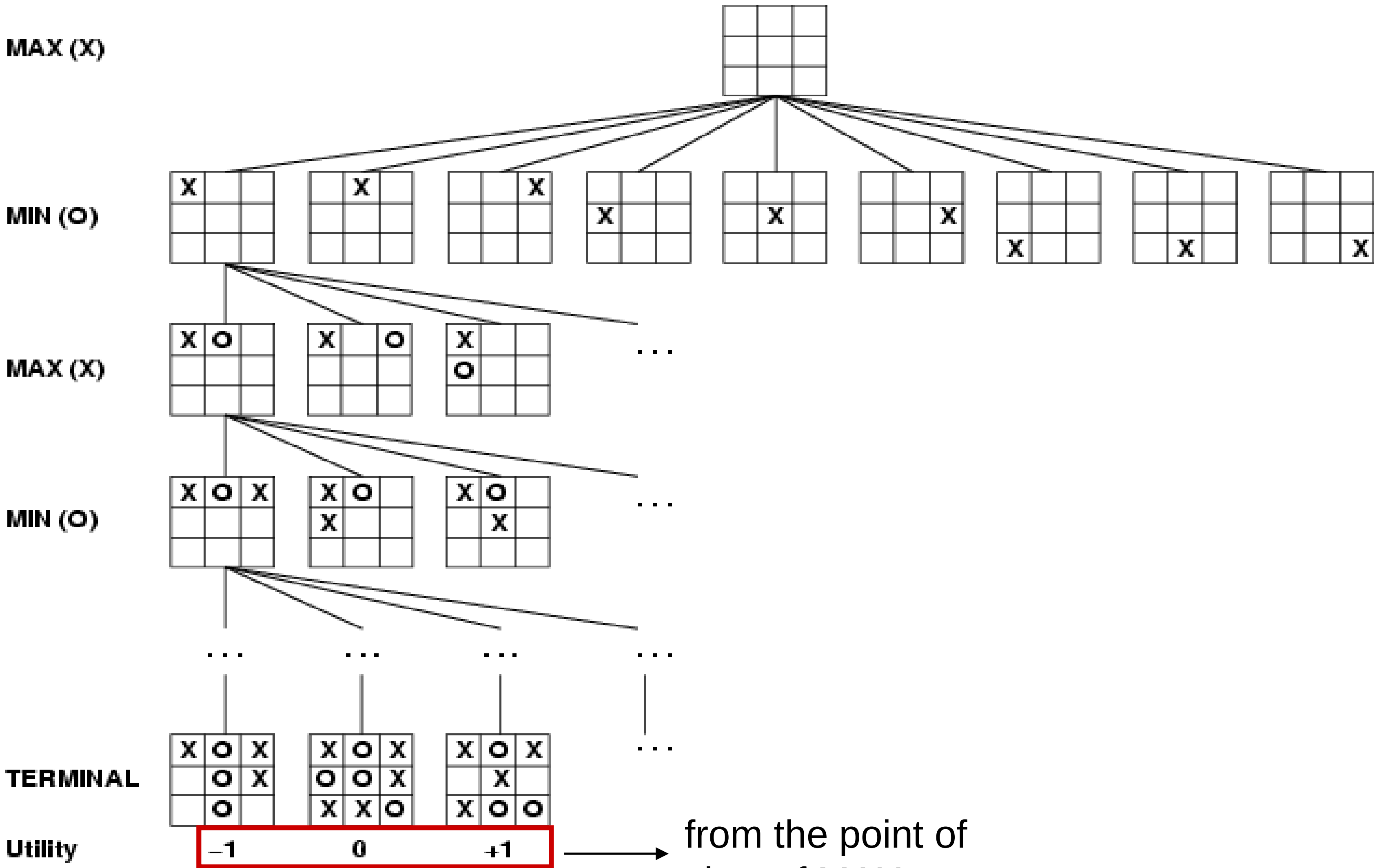
OPTIMAL DECISIONS in GAMES

- Consider games with 2 players: **MAX** and **MIN**
- **MAX** moves first and then they take turns moving until the game is over.
- At the end, points are given to the winning player as a prize and penalties are given to the loser.
- **Game Definition**- as a kind of search problem :
  - **initial state** – board position and which player to move.
  - **successor function** – a list of (*move,state*) pairs.
  - **terminal test** – determines when the game is over.
    - States where the game has ended are called terminal states.
  - **utility function** – gives a numeric value for terminal states.
    - e.g. in chess, the outcome is a win, loss or draw with values +1, -1, or 0

GAME TREE

- The initial state and the legal moves for each side define the **game tree**.
  - e.g. game tree for tic-tac-toe (noughts and crosses) 3x3 board
  - Initial state → MAX has 9 possible moves
  - Play alternates between MAX's placing a X and MIN's placing an O until we reach leaf nodes
  - Terminal states → one player has 3 in a row, column or diagonal OR all the squares are filled.

Game tree (2-player, deterministic, turns)

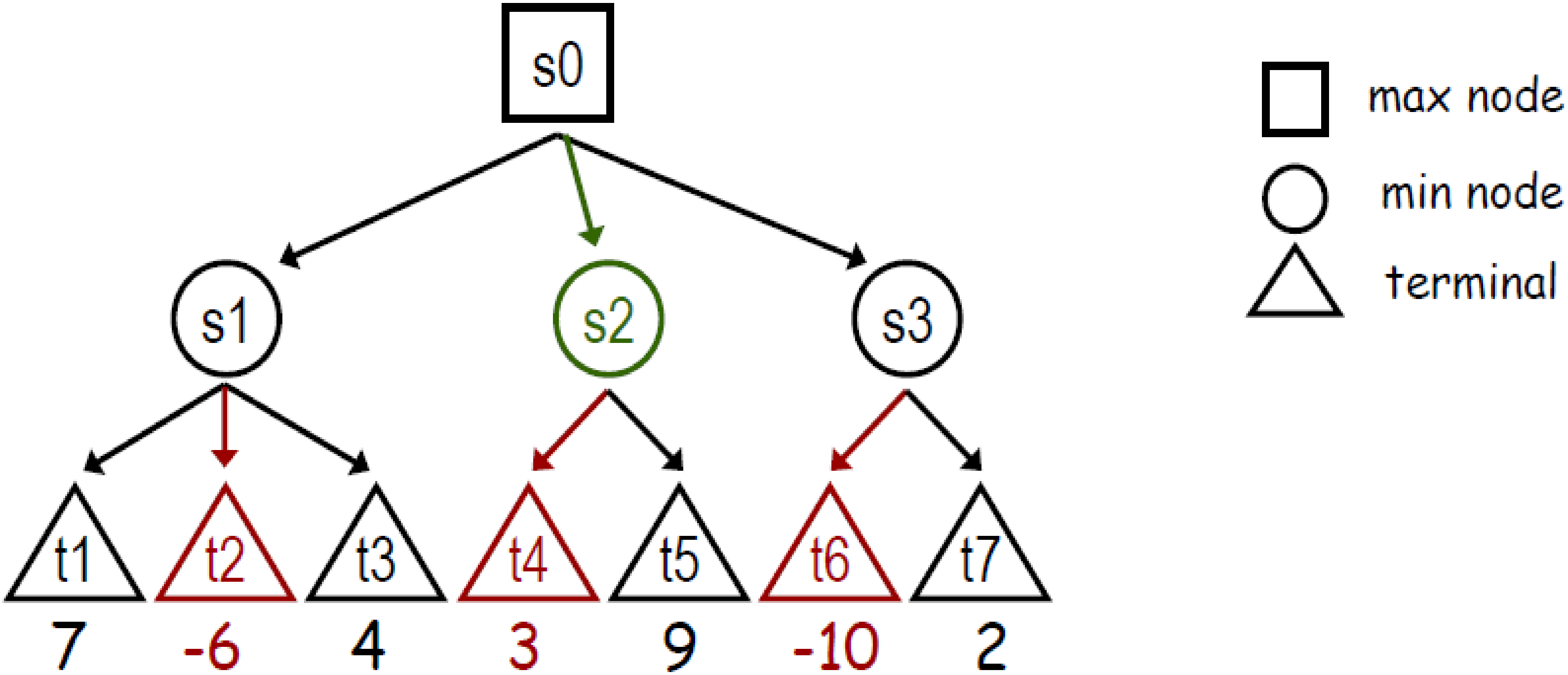


from the point of view of MAX

Optimal Strategies

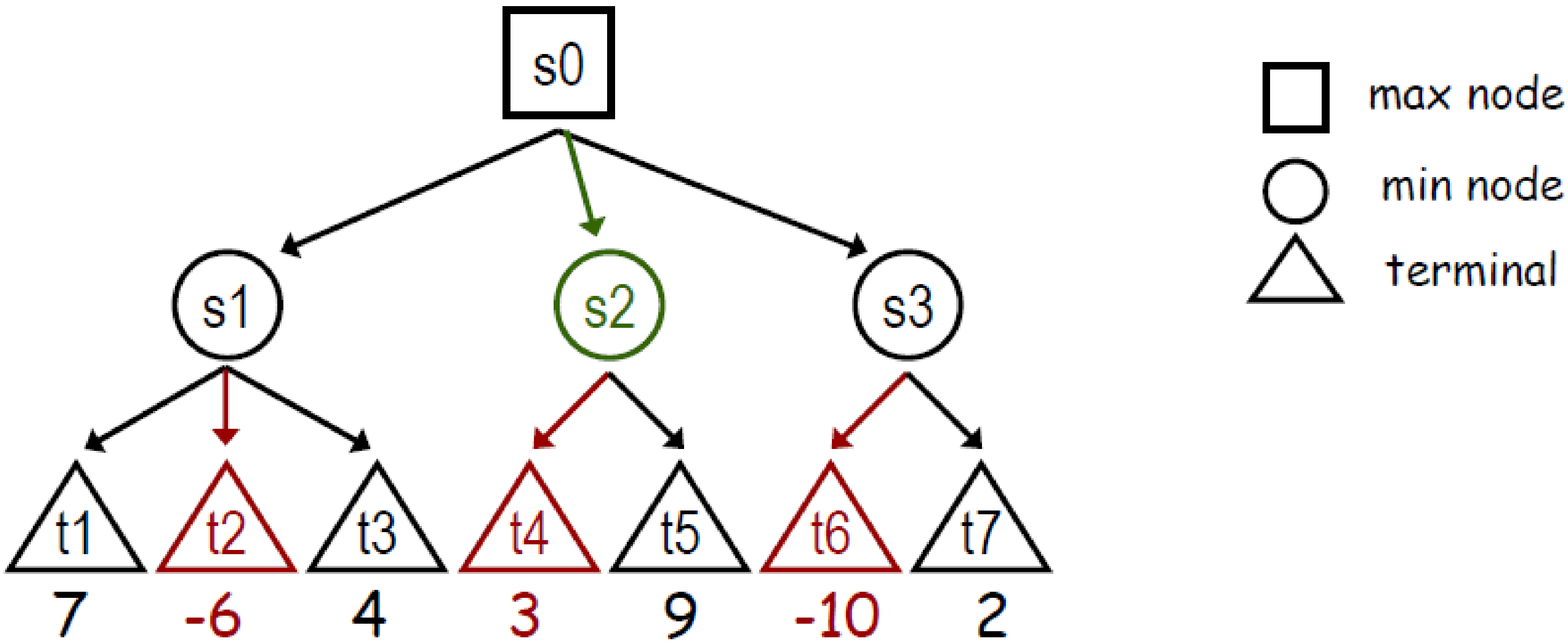
- In a search problem → optimal solution will be a sequence of moves leading to a goal state
- In a game → the opponent has something to say about it.
  - e.g. MAX must find a contingent strategy – specifies MAX's move in the **initial state**
    - then MAX's moves in the states resulting from every possible response by MIN, and so on..
- What is a reasonable strategy?

Minimax Strategy



The terminal nodes have utilities.
But we can compute a "utility" for the non-terminal states, by assuming both players always play their best move.

Minimax Strategy



If Max goes to s1, Min goes to t2
* U(s1) = min{U(t1), U(t2), U(t3)} = -6

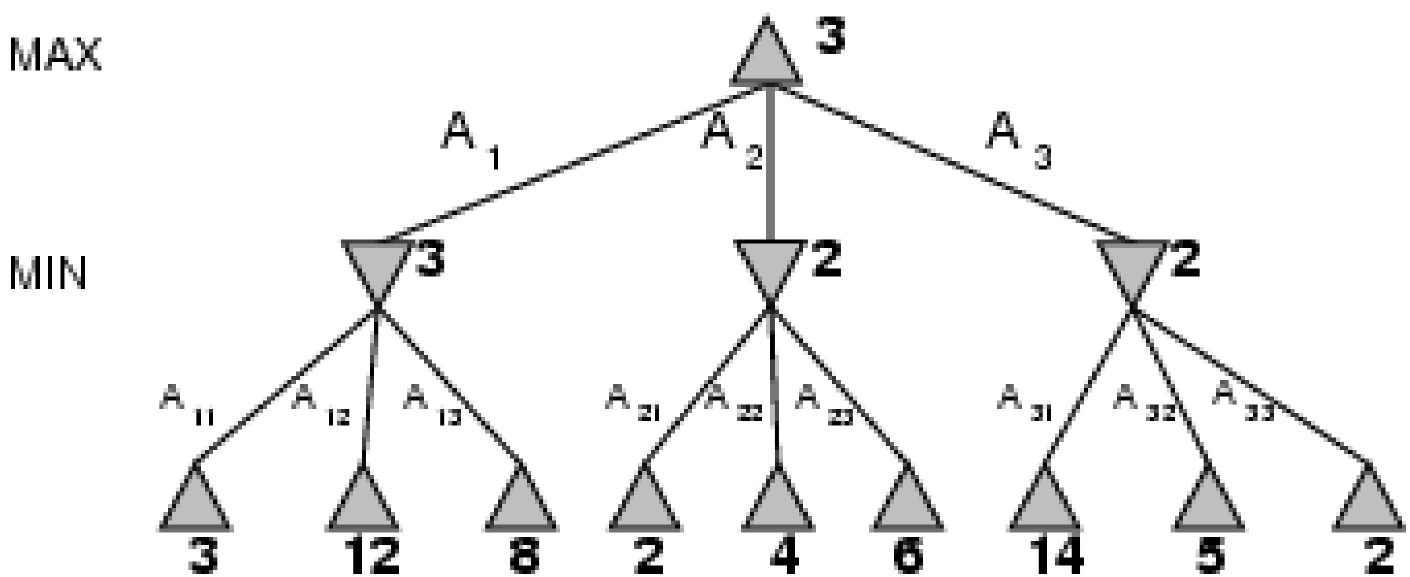
If Max goes to s2, Min goes to t4
* U(s2) = min{U(t4), U(t5)} = 3

If Max goes to s3, Min goes to t6
* U(s3) = min{U(t6), U(t7)} = -10

So Max goes to s2: so
U(s0)
= max{U(s1), U(s2), U(s3)}
= 3

Minimax

- Perfect play for deterministic games
- Idea: choose move to position with highest minimax value
  = best achievable payoff against best play
- E.g., 2-ply game:



Minimax

- Given a choice, MAX will prefer to move to a state of maximum value, whereas MIN prefers a state of minimum value:

$$\text{MINIMAX} - \text{VALUE}(n) = \begin{cases} \text{UTILITY}(n) & \text{if } n \text{ is a terminal state} \\ \max_{s \in \text{Successors}(n)} \text{MINIMAX-VALUE}(s) & \text{if } n \text{ is a MAX node} \\ \min_{s \in \text{Successors}(n)} \text{MINIMAX-VALUE}(s) & \text{if } n \text{ is a MIN node} \end{cases}$$

Minimax Strategy

- Build full game tree (all leaves are terminals)
  - root is start state, edges are possible moves, etc.
  - label terminal nodes with utilities
- Back values *up* the tree
  - $U(t)$ is defined for all terminals (part of input)
  - $U(n) = \min \{U(c) : c \text{ a child of } n\}$ if n is a min node
  - $U(n) = \max \{U(c) : c \text{ a child of } n\}$ if n is a max node

Properties of minimax

- Minimax algorithm performs a complete **depth-first** exploration of the game tree
  - m → the max depth of the tree
  - b → legal moves of each point
- Complete? Yes (if tree is finite)
- Optimal? Yes (against an optimal opponent)
- Time complexity? $O(b^m)$
- Space complexity? $O(bm)$

- For chess, $b \approx 35$, $m \approx 100$ for "reasonable" games
  → exact solution completely infeasible

Pruning

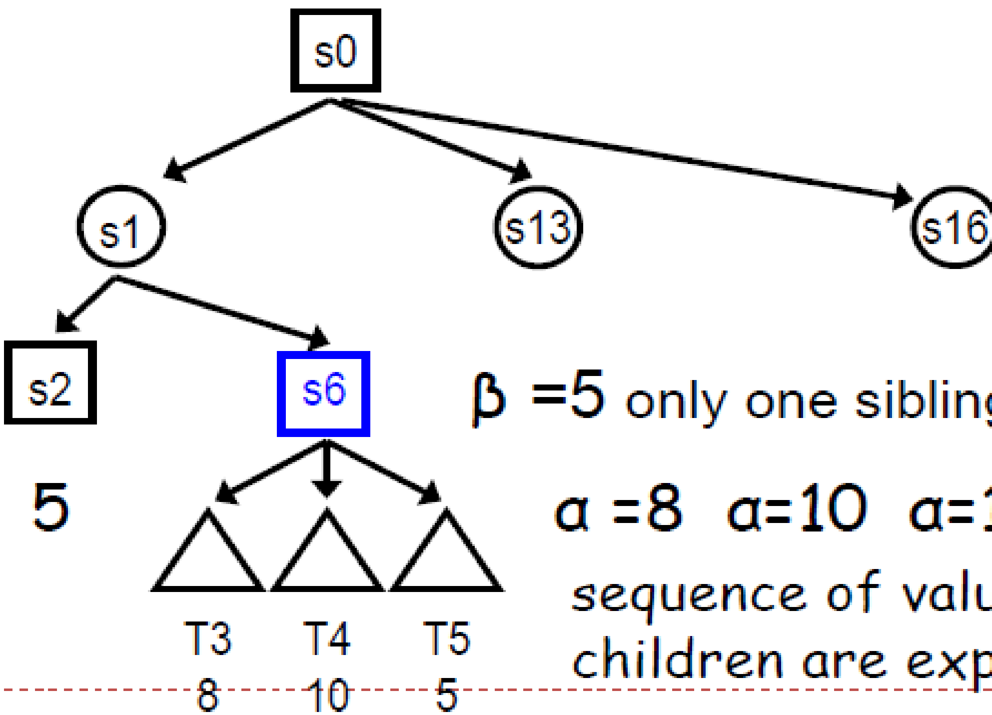
- It is not necessary to examine entire tree to make correct minimax decision
- Assume depth-first generation of tree
  - After generating value for only *some* of *n*'s children we can prove that we'll never reach *n* in a MinMax strategy.
  - So we needn't generate or evaluate any further children of *n* !
- Two types of pruning (cuts):
  - pruning of max nodes (α-cuts)
  - pruning of min nodes (β-cuts)

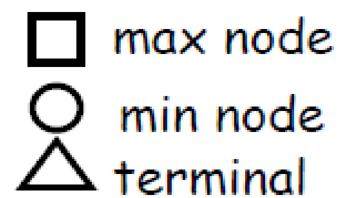
α-β Pruning

- The problem with minimax search → the number of game states is exponential in the number of moves
  - Solution is to cut it in half → compute the correct minimax decision without looking at every node
  - **Pruning** → eliminate large parts of the game tree
  - This technique is called α-β Pruning
    - **it returns the same move as minimax would, but prunes away branches that cannot possibly influence the final decision.**

Cutting Max Nodes (Alpha Cuts)

- At a Max node *n*:
  - Let β be the lowest value of *n*'s siblings examined so far (siblings to the left of *n* that have already been searched)
  - Let α be the highest value of *n*'s children examined so far (changes as children examined)





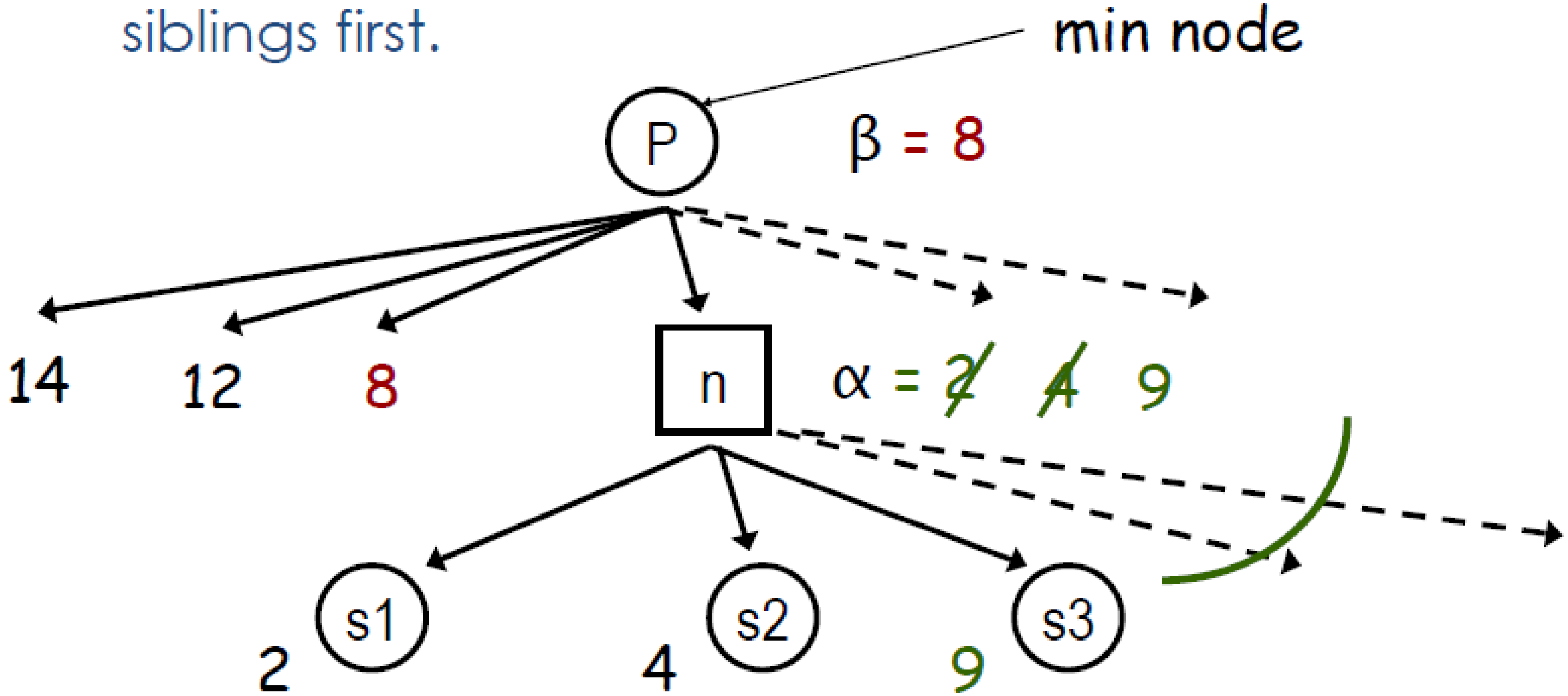
β =5 only one sibling value known

α =8 α=10 α=10

sequence of values for α as s6's children are explored

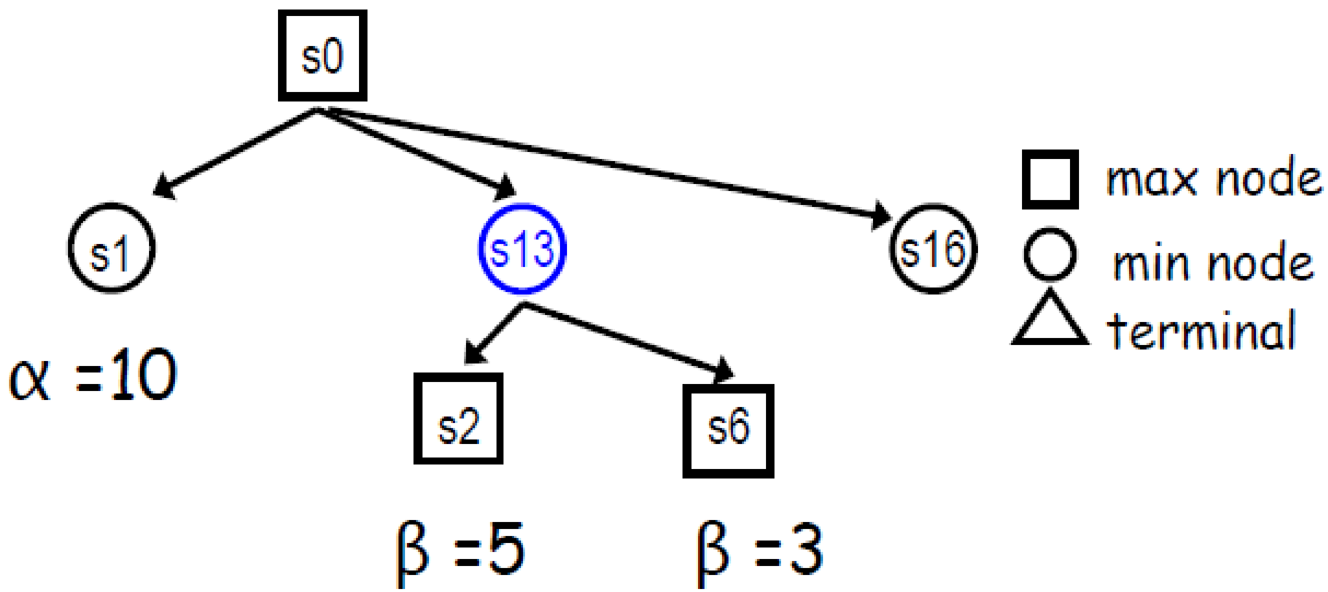
Cutting Max Nodes (Alpha Cuts)

- If α becomes $\geq \beta$ we can stop expanding the children of *n*
  - Min will never choose to move from *n*'s parent to *n* since it would choose one of *n*'s lower valued siblings first.



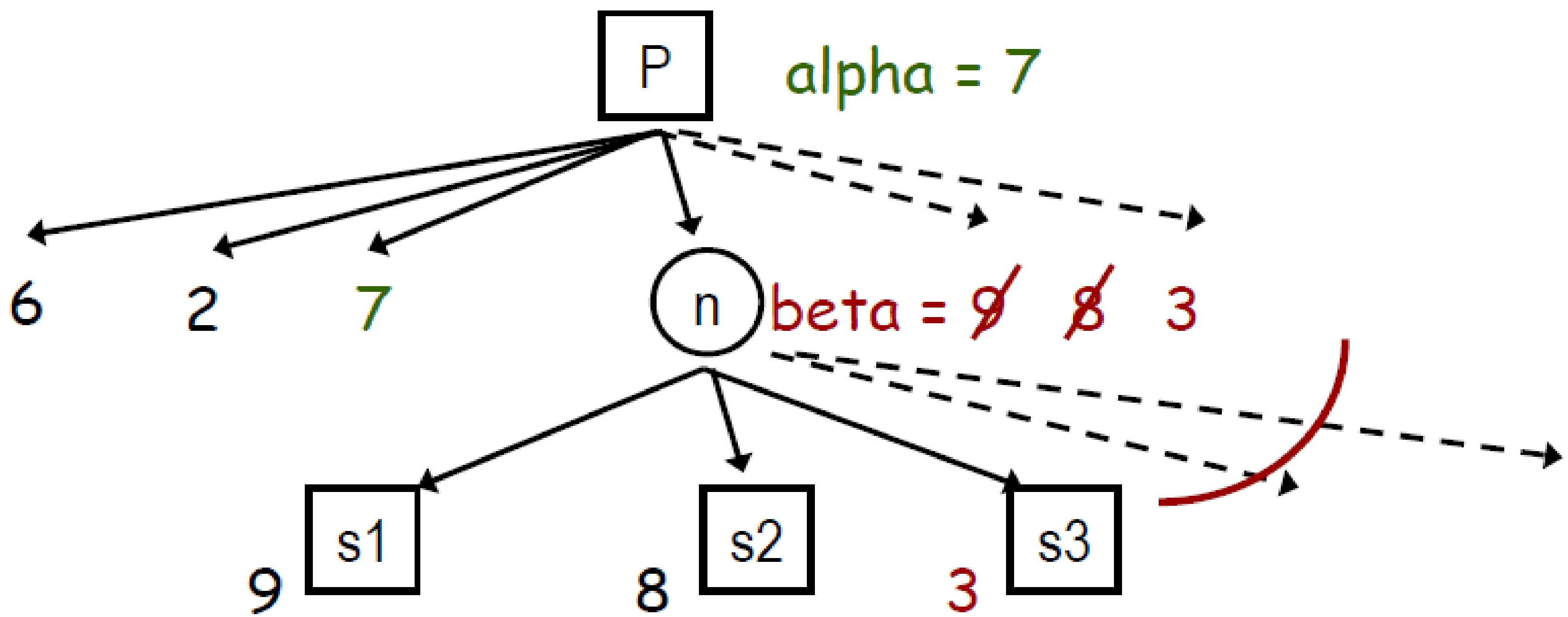
Cutting Min Nodes (Beta Cuts)

- At a Min node *n*:
  - Let β be the lowest value of *n*'s children examined so far (changes as children examined)
  - Let α be the highest value of *n*'s sibling's examined so far (fixed when evaluating *n*)

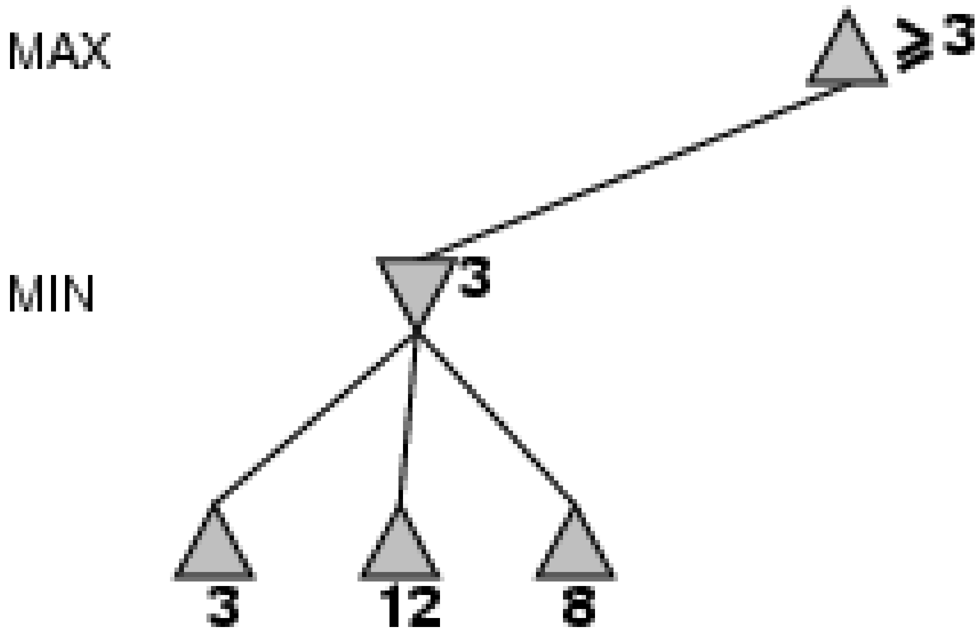


Cutting Min Nodes (Beta Cuts)

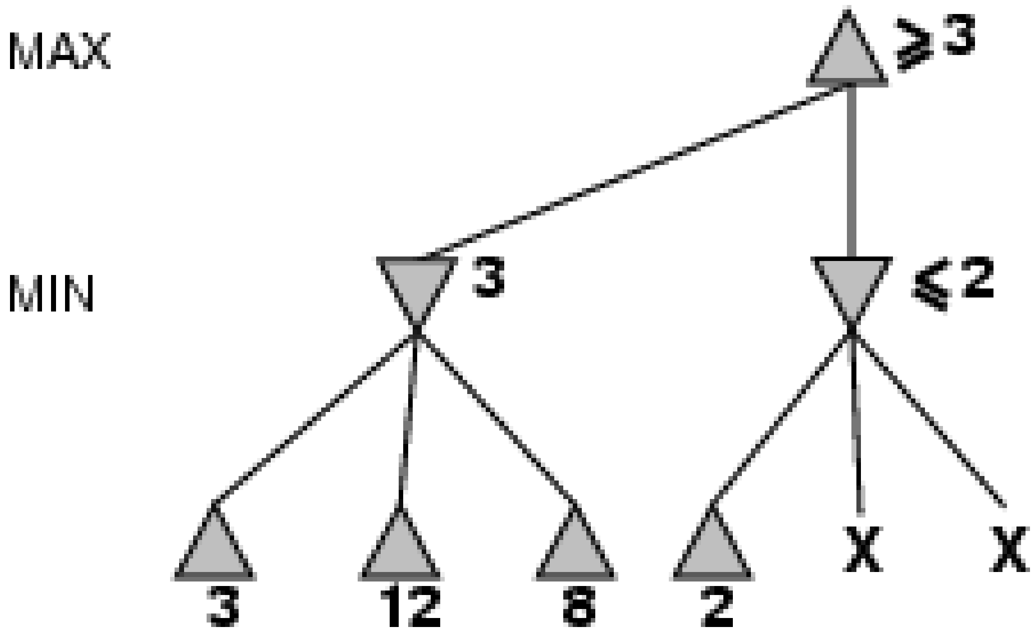
- If β becomes ≤ α we can stop expanding the children of *n*.
  - Max will never choose to move from *n*'s parent to *n* since it would choose one of *n*'s higher value siblings first.



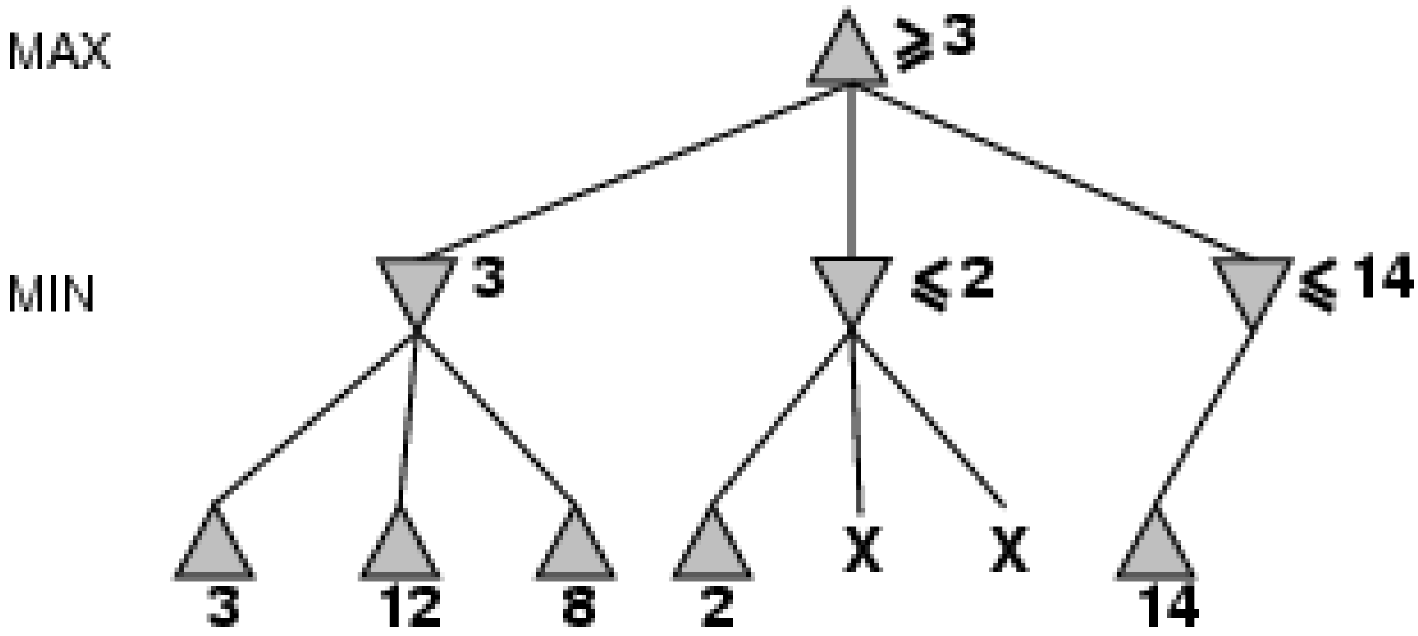
α-β Pruning Example



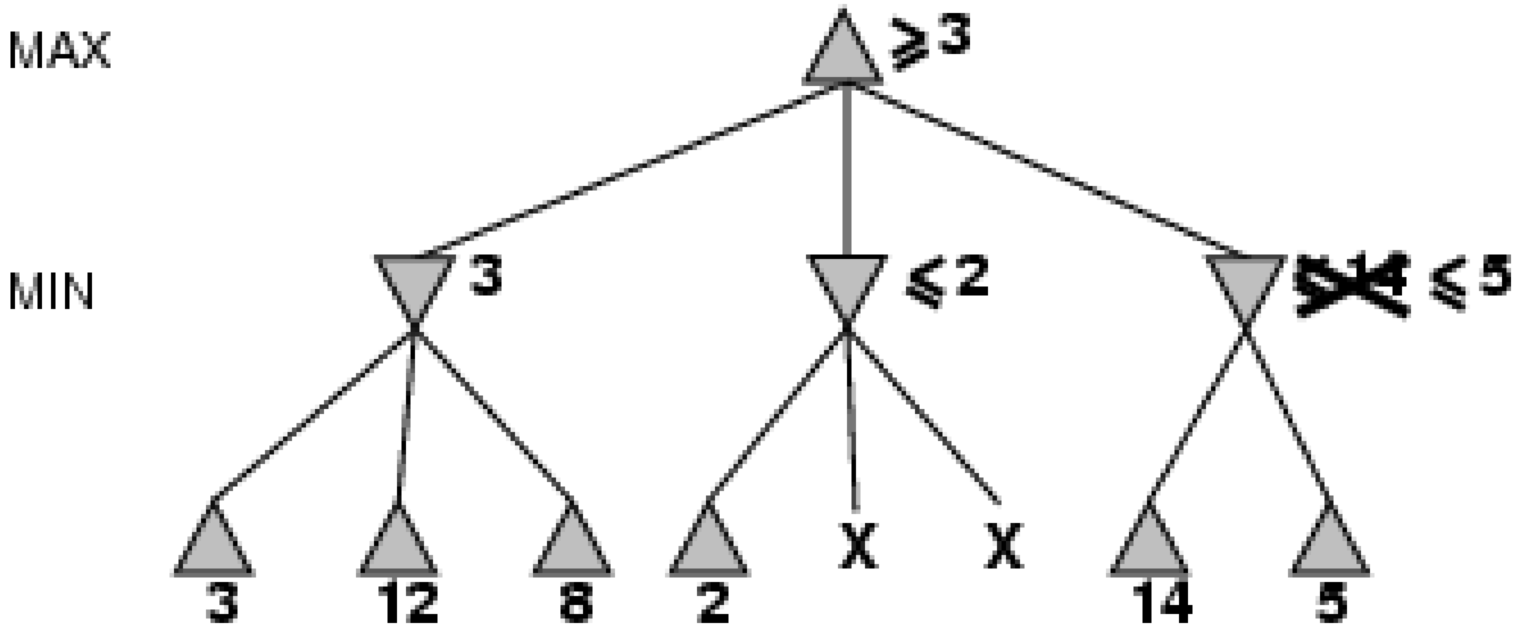
α-β Pruning Example



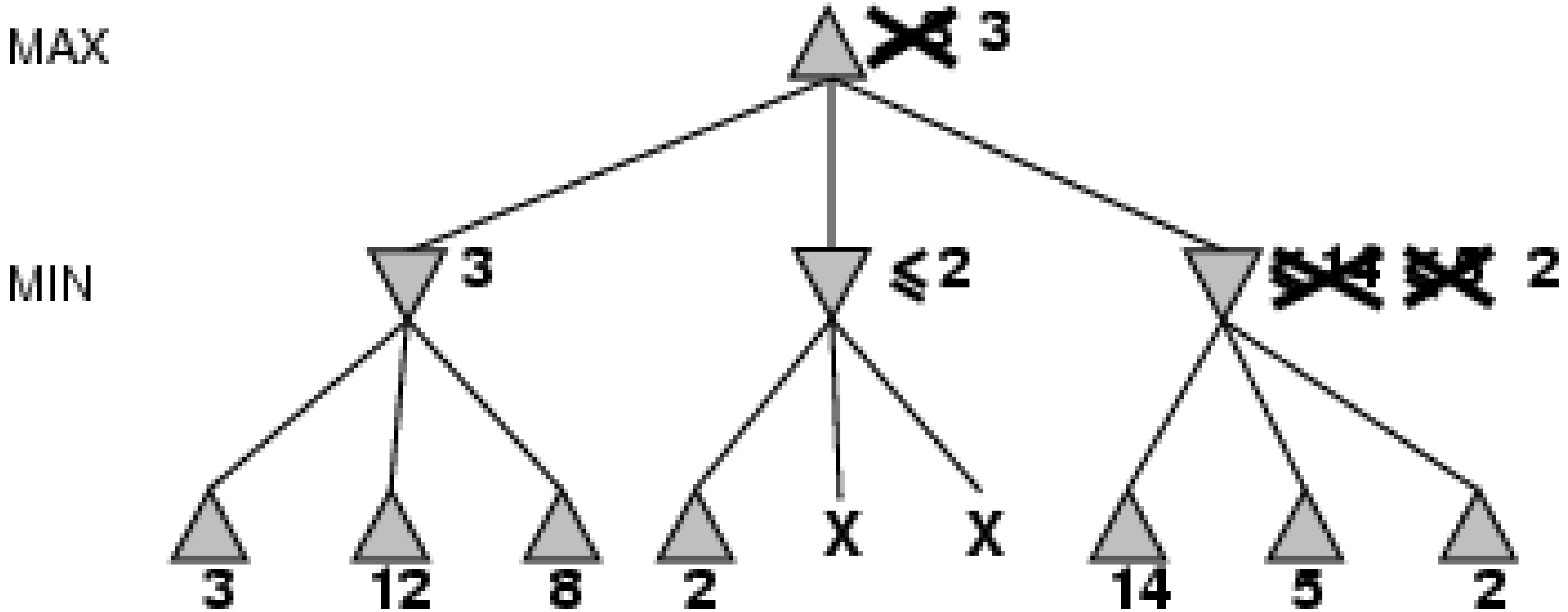
α-β Pruning Example



α-β Pruning Example



α-β Pruning Example



α-β Pruning Example

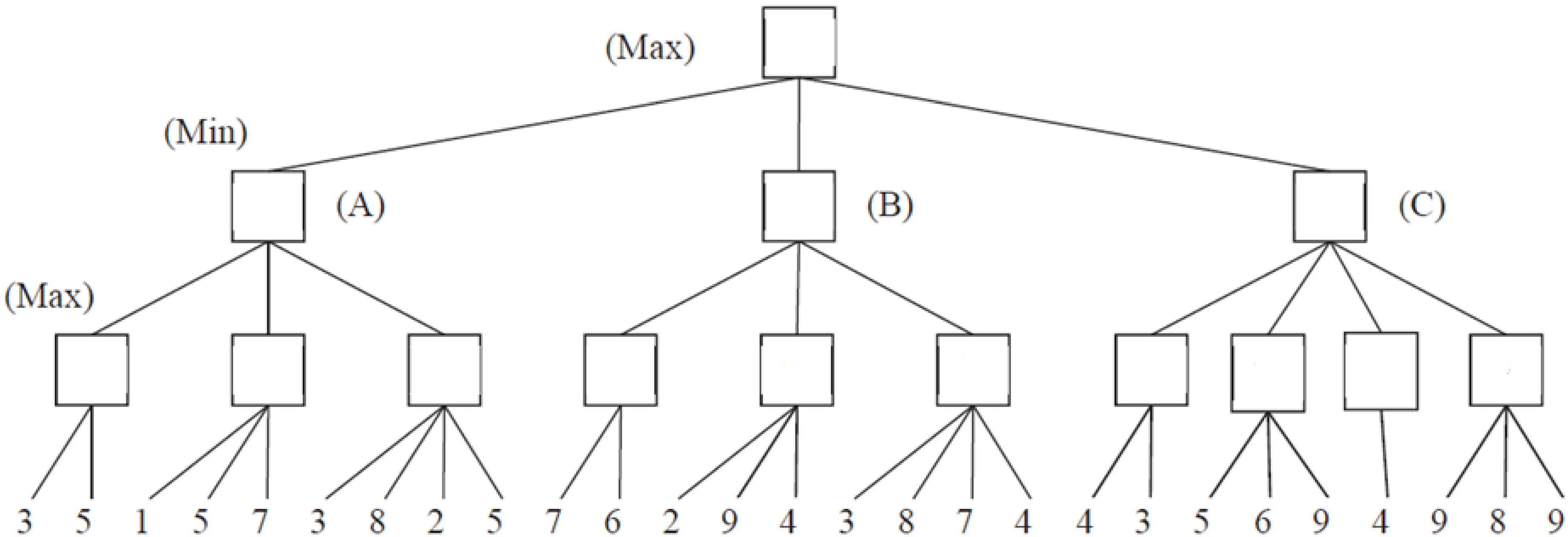
MINIMAX-VALUE (root) = max(min(3,12,8),
min(2,x,y), min(14,5,2))
= max(3,min(2,x,y),2)
= max(3, z, 2) z ≤ 2
= 3.

Properties of α-β

- Pruning does not affect final result
- Good move ordering improves effectiveness of pruning
- With "perfect ordering," time complexity = $O(b^{m/2})$
  → doubles depth of search
- What if your opponent doesn't play rationally? Will your stored strategy still work?

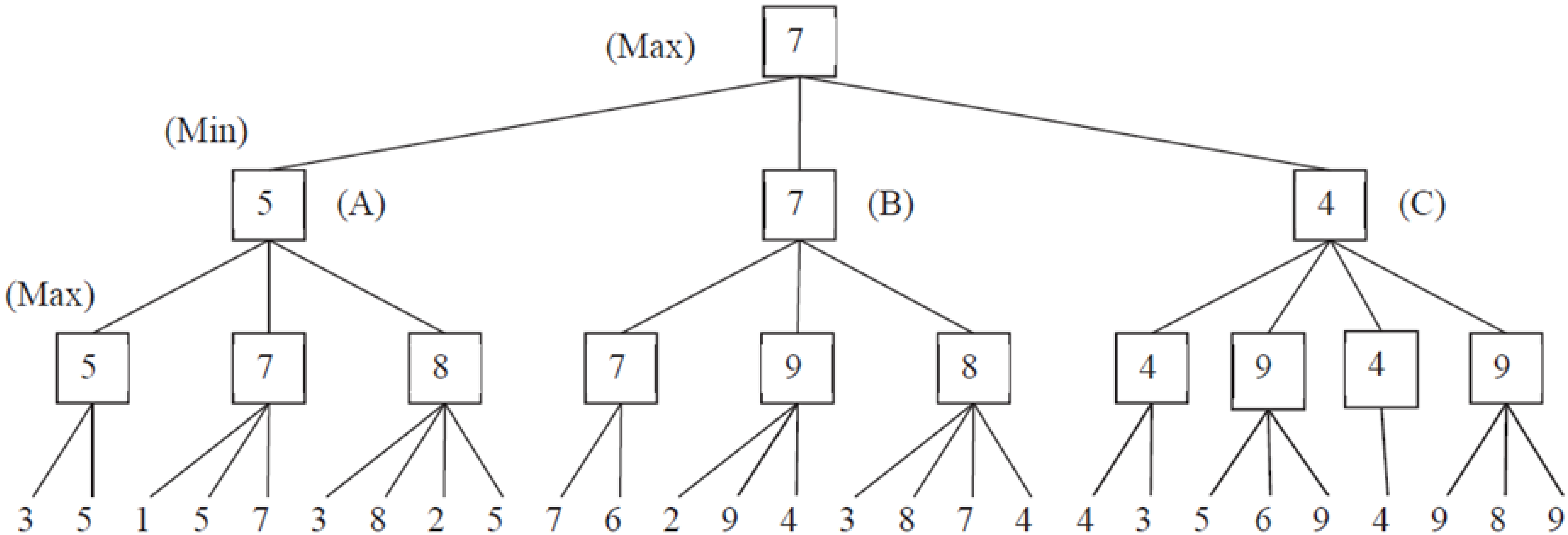
Example

- The game tree illustrates a position reached in the game.
- Process the tree left-to-right. It is **Max**'s turn to move.
- At each leaf node is the estimated score returned by the heuristic static evaluator.


(Max)
(Min)
(A)
(B)
(C)
(Max)
3 5 1 5 7 3 8 2 5 7 6 2 9 4 3 8 7 4 4 3 5 6 9 4 9 8 9

Example cont.

- **Fill in each blank square with the proper mini-max search value.**

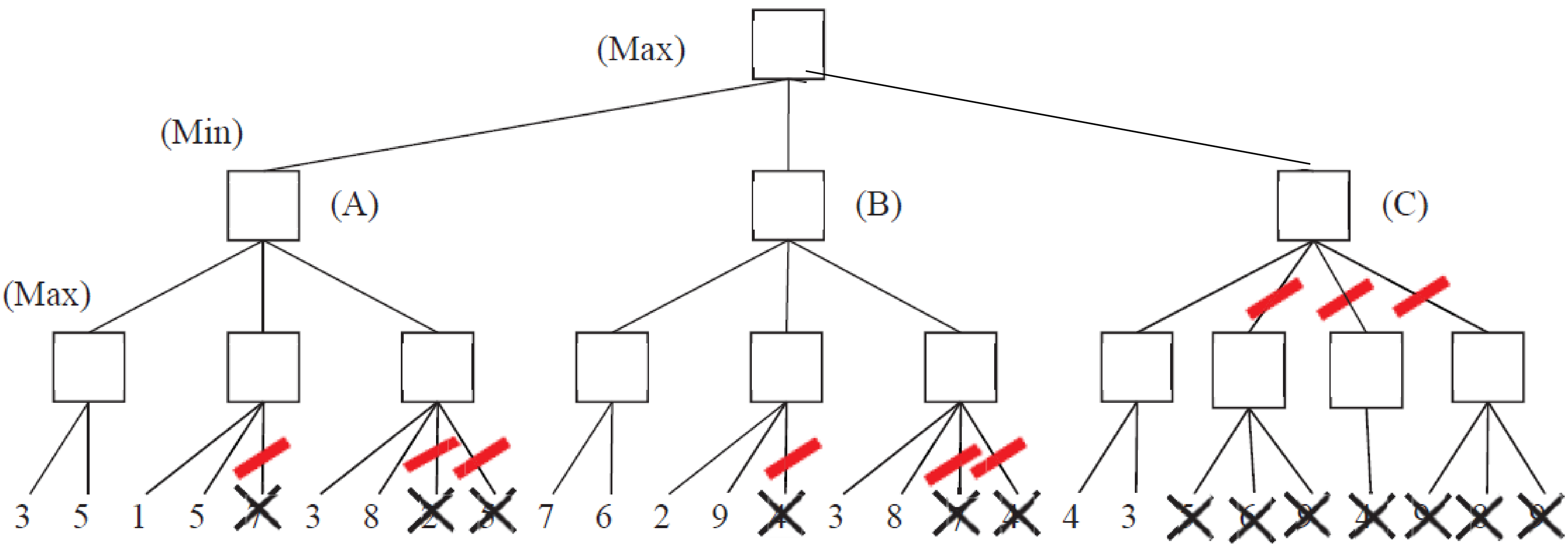


Example cont.

- **What is the best move for Max?** (write A, B, or C)

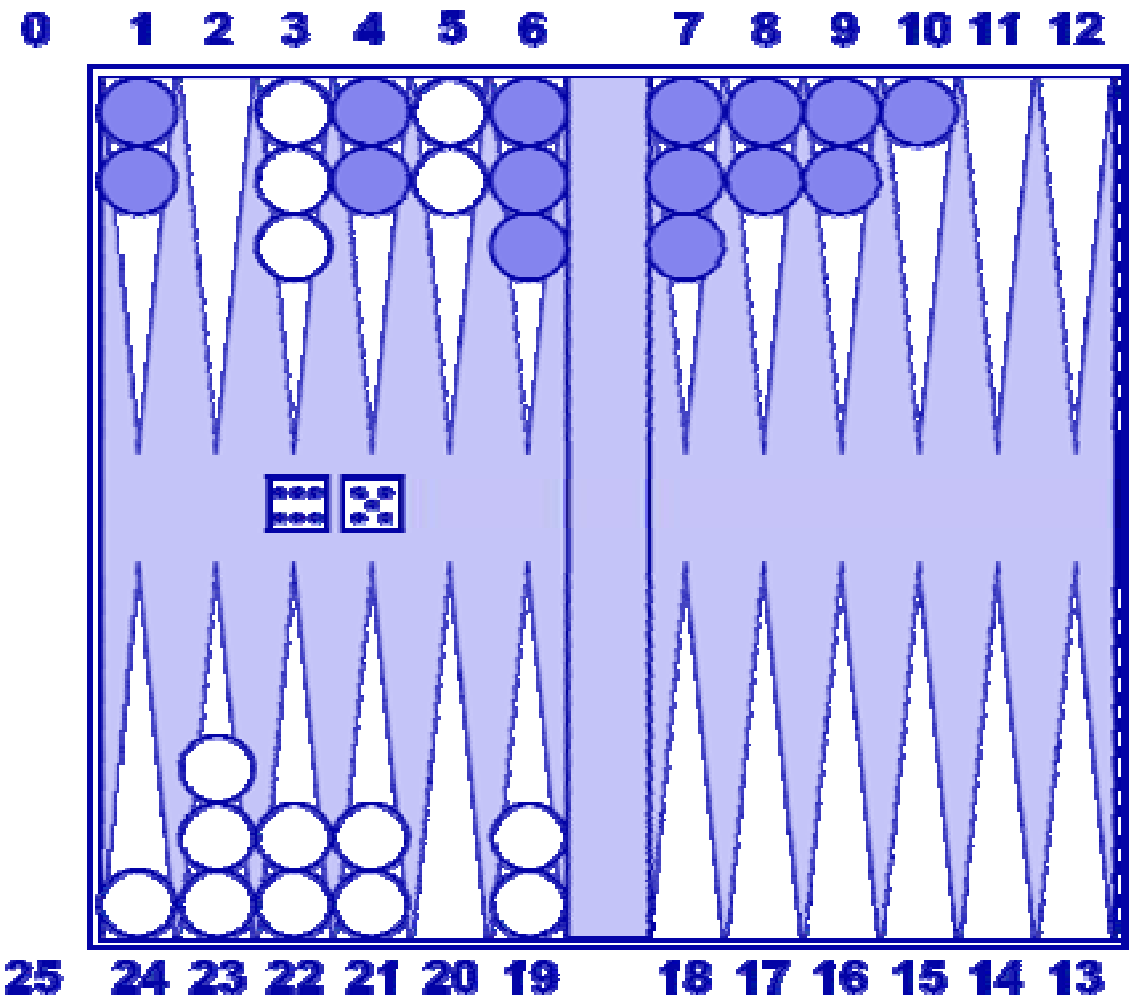
- **What score does Max expect to achieve?**

- **ALPHA-BETA PRUNING.**
- Process the tree left-to-right.
- **Cross out each leaf node that will be pruned by Alpha-Beta Pruning.**



(Max)
(Min)
(A)
(B)
(C)
(Max)
3 5 1 5 3 8 7 6 2 9 3 8 4 3

Nondeterministic Games

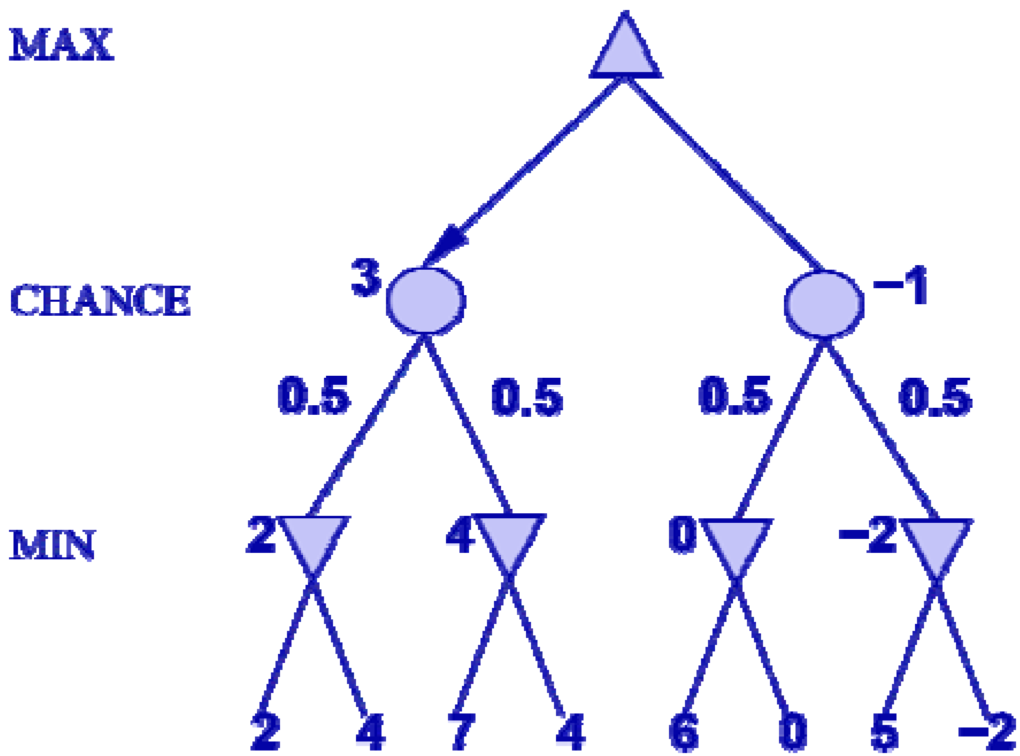


All "real"games are too large to enumerate tree

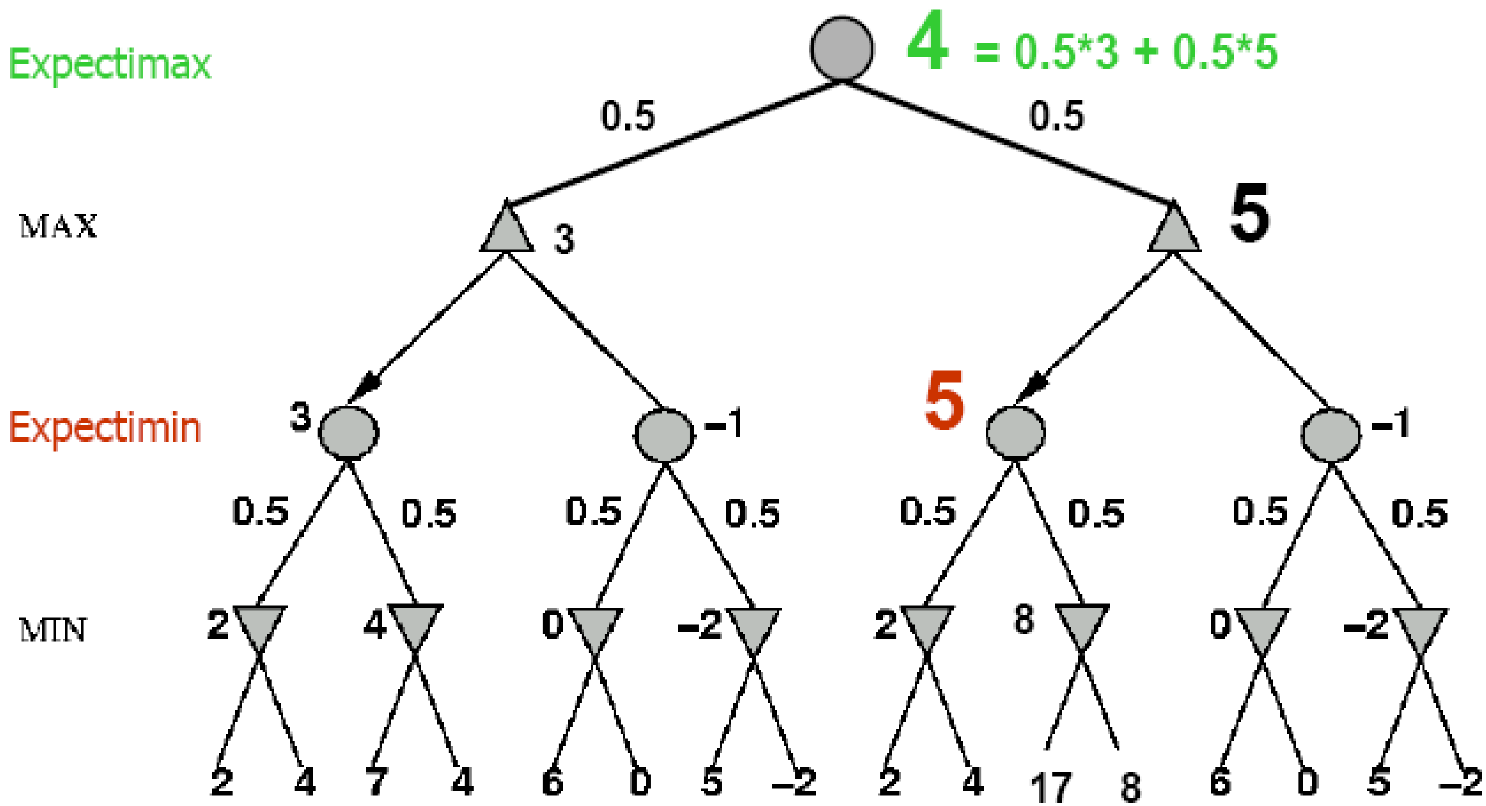
- e.g., chess branching factor is roughly 35
- Depth 10 tree: 2,700,000,000,000,000 nodes
- Even alpha-beta pruning won't help here!

Nondeterministic games: the element of chance

- In nondeterministic games, chance introduced by dice, card-shuffling
- Simplified example with coin-flipping



Nondeterministic games: the element of chance



Summary

- Games are fun to work on!
- They illustrate several important points about AI
- perfection is unattainable → must approximate
- good idea to think about what to think about