① (20p)

Using the definitons for PMF

$$P(X=x)= f(x) = \sum_{k=-\infty}^{+\infty} f(x) = 1$$

Note
$$\ell n\,(a) + \ell n\,(b) = \ell n\,(ab)$$

$\therefore \quad f(2) + f(3) + f(4) = 1$

$\therefore \quad \alpha\,(\ell n\,2 + \ell n\,3 + \ell n\,4) = 1$

$\therefore \quad \alpha\,\ell n\,(2.3.4) = 1$

$\therefore \quad \alpha\,\ell n\,(24) = 1$

$\alpha = {}^1/\ell n\,(24) \rightarrow \boxed{\alpha \tilde{=} 0.3147}\;\checkmark$

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②
(25p)

$$F(x) = \sum_{k=-\infty}^{x} f(k)$$

Note: Do not forget F(x) is used for CDF and f(x) is " for PMF & PDF !!!

For $x < 2 \rightarrow F(x) = 0$

For $2 \leqslant x < 4 \rightarrow F(x) = \sum_{k=2}^{x} f(k) = \sum_{k=2}^{x} \alpha\,\ell n\,(k)$

$\therefore \qquad F(2) = f(2) = 0.2181$ ①

$\qquad F(3) = f(2) + f(3) = 0.2181 + 0.3457$

$\qquad\qquad\qquad\qquad = 0.5638$ ②

$\qquad F(4) = f(4) + F(3) = 1$ ③

③ (30p)

$$E(X) = \mu_X = \sum_{x=2}^{4} x \cdot \alpha \cdot \ln(x)$$

$$= 2 \cdot f(2) + 3 \cdot f(3) + 4 \cdot f(4)$$

$$= 2 \times 0.2181 + 3 \times 0.3457 + 4 \times 0.4362$$

$$\mu_X = \boxed{3.2181} \quad \text{✓} \quad ①$$

$$V(X) = E(X^2) - \mu_X^2$$

$$E(X^2) = \sum_{x=2}^{4} x^2 \cdot f(x)$$

$$= 4 \cdot f(2) + 9 \cdot f(3) + 16 \cdot f(4)$$

$$= 4 \times 0.2181 + 9 \times 0.3457 + 16 \times 0.4362$$

$$\boxed{E(X^2) = 10.9629} \quad ②$$

using ① ②

$$V(X) = E(X^2) - \mu_X^2 = 10.9629 - (3.2181)^2$$

$$\boxed{V(X) = 0.6067} \quad \text{✓}$$

④ (25p)

$$E[h(X)] = E(X^2 + 2X - 1) = E(X^2) - 2E(X) - 1$$

using ① ②

$$= 10.9629 - 2 \times 3.2181 - 1$$

$$\boxed{E[h(X)] = 5.5267} \quad \text{✓}$$