## DISCRETE MATHEMATICS FINAL EXAM

1) The Harmonic numbers  $H_i$ , i = 1,2,3... is defined as follows:

$$H_i = 1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{i}$$

Use mathematical induction to show that

$$H_{2^n} \ge 1 + \frac{n}{2}$$

whenever n is a nonnegative integer.

2) Find an explicit formula for Fibonacci numbers whose recursive relation is:

$$f_n = f_{n-1} + f_{n-2}$$
  $f_0 = 1$  and  $f_1 = 1$ 

- 3) A simple graph is called regular if every vertex of this graph has the same degree. A regular graph is called n-regular if every vertex in this graph has degree n.
- a) How many vertices does a regular graph of degree four with 10 edges have?
- b) Investigate if  $K_n$ ,  $C_n$  and  $W_n$  is a regular graph or not.