Even - Odd Decomposition

Given an arbitrary signal $x(t)$, we can decompose it to its even and odd components.

① $x(t) = \underbrace{x_e(t)}_{\text{Even component}} + \underbrace{x_o(t)}_{\text{Odd Component.}}$

$$x_e(t) = x_e(-t)$$
$$+\ x_o(t) = -x_o(-t)$$

$$x(t) = x_e(t) + x_o(t) = x_e(-t) - x_o(-t)$$
$$x(t) = x_e(-t) - x_o(-t)$$

Replacing $-t$ with t

② $x(-t) = x_e(t) - x_o(t)$

$$x_e(t) = \frac{1}{2}\left[x(t) + x(-t)\right]$$
$$x_o(t) = \frac{1}{2}\left[x(t) - x(-t)\right]$$

Ex Find the even and odd component of the following signal.

$$x(t) = e^{-2t} \cdot \cos(t)$$

Replacing t with $-t$:

$$x(-t) = e^{2t} \cdot \cos(-t)$$
$$x(-t) = e^{2t} \cdot \cos(t)$$

$$x_e(t) = \frac{1}{2}\left[e^{-2t}\cos t + e^{2t}\cos t\right]$$
$$= \cos(t) \cdot \boxed{\frac{1}{2}\left[e^{-2t} + e^{2t}\right]}$$
$$= \cosh(2t) \cdot \cos(t)$$

$$x_o(t) = \frac{1}{2}\left[e^{-2t}\cos t - e^{2t}\cos t\right]$$

$$x_o(t) = -\sinh(2t) \cdot \cos(t)$$

symmetry for complex signals

- A complex-valued signal $x(t)$ is said to be "conjugate symmetric" if

$$x(-t) = x^*(t)$$

$$x(t) = \underline{a(t)} + j\,\underline{b(t)} \qquad (j = \sqrt{-1})$$

$$x^*(t) = a(t) - j\,b(t)$$

— If $x(t)$ is conjugate symmetric

$$\underline{a(-t)} + j \cdot b(-t) = \underline{a(t)} - j\,b(t)$$

$\Rightarrow$
- The real part of $x(t)$ is EVEN
- The imaginary " " " is ODD

$\Rightarrow$ SAME REMARKS APPLY TO DISCRETE-TIME SIGNALS.

③ Periodic and non-periodic signals

A periodic signal $x(t)$ satisfies the condition

$$x(t) = x(t+T) \quad , \forall t$$

where T is a positive constant.

- If the condition is satisfied for let's say $T = T_o$, then it will also be satisfied for $T = 2T_o, 3T_o, 4T_o, \ldots$

$$x(t) = x(t+T_o) = x(t+2T_o) = \ldots$$

- The smallest value of T that satisfies the condition is called "<u>the fundamental period</u>".

- Frequency

$$f = \frac{1}{T} \quad (Hz)$$

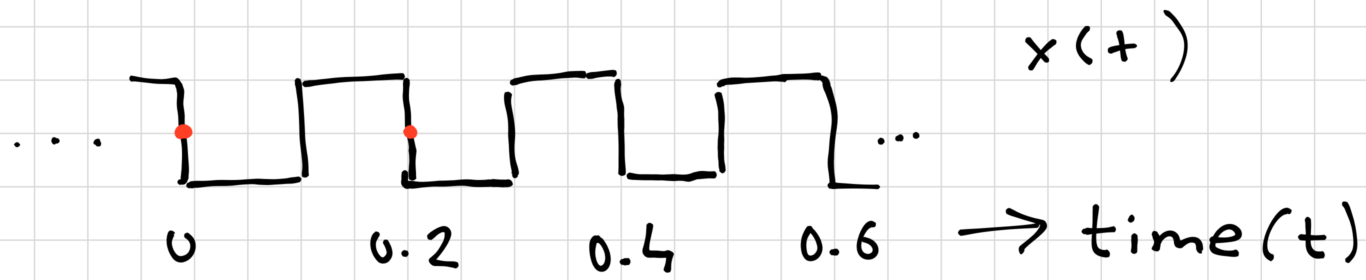
T → seconds.

- Angular frequency

$$\omega = 2\pi f = \frac{2\pi}{T} \quad (rad/sec)$$

<u>Ex</u>

periodic

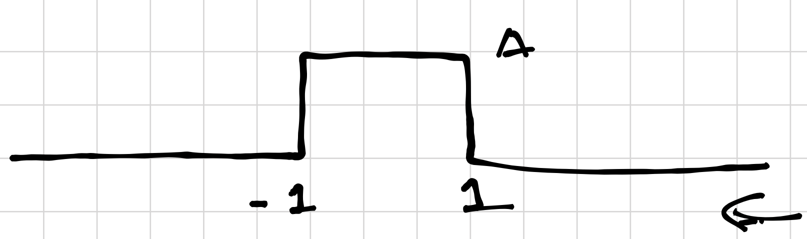


T= 0.2 seconds

$$f = \frac{1}{0.2} = 5\ Hz$$

$$\omega = 2\pi . 5 = 10\pi \ rad/sec.$$

<u>Ex</u>



Non-periodic signal.

Discrete-time signals:

- $x[n]$ is periodic if

$$x[n] = x[n+N] \quad \text{for all } n$$

where N is positive integer.

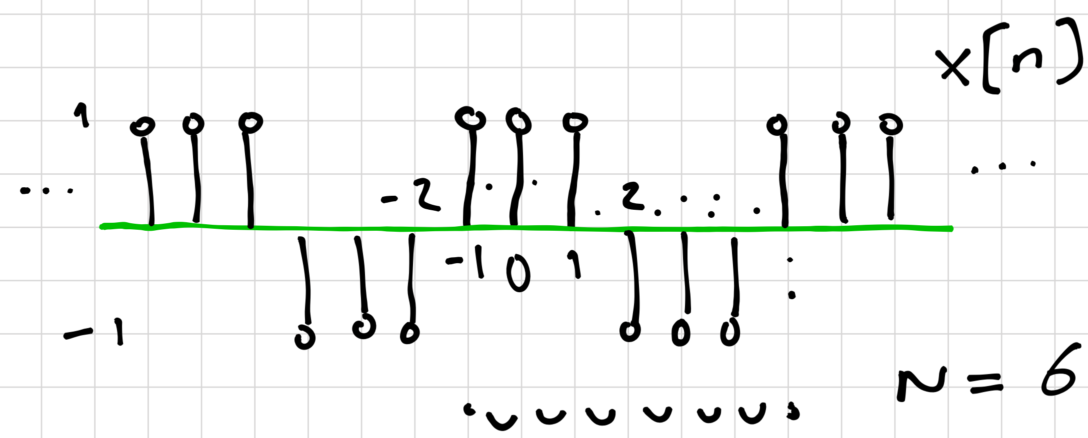
The smallest N that satisfies this condition is called the fundamental period.

The fundamental frequency

$$\Omega = \frac{2\pi}{N} \quad \text{(radians)}$$

must be integer

Ex

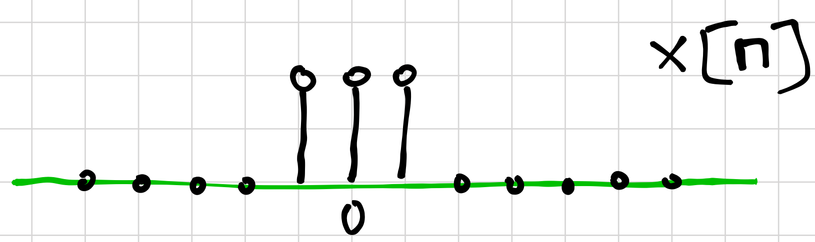


Periodic

$N = 6$

$\Omega = \frac{2\pi}{6} = \frac{\pi}{3}$ rad.

Ex



non-periodic.

④ Deterministic vs Random Signals

⊙ A deterministic signal is a signal about which there is no uncertainty with respect to its value at any time

⊙ A random signal is that about which there is uncertainty before it occurs.

- noise

etc.

5 Energy Signals vs Power Signals.

Instantaneous power of a CT-signal, $x(t)$,

$$p(t) = x^2(t)$$

Total Energy of $x(t)$:

$$E = \lim_{T \to \infty} \int_{-T/2}^{T/2} x^2(t)\, dt$$

$$E = \int_{-\infty}^{+\infty} x^2(t)\, dt$$

Average Power

$$P = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} x^2(t)\, dt$$

If the signal is periodic with a fundamental period T

$$P = \frac{1}{T} \int_{-T/2}^{T/2} x^2(t)\, dt$$

For DT signals:

Total energy $E = \sum_{n=-\infty}^{+\infty} x^2[n]$

Average Power $\Big\}$ $P = \lim_{N \to \infty} \frac{1}{2N} \sum_{n=-N}^{N} x^2[n]$

If $x[n]$ is periodic then its average power: $P = \frac{1}{N} \sum_{n=0}^{N-1} x^2[n]$

A signal is called an ENERGY signal if $0 < E < \infty$

A signal is called a POWER " if $0 < P < \infty$

$\Rightarrow$ A signal cannot be both power and energy signal.

Basic Operations on Signals

$x(t)$ $\quad$ $x[n]$

dependent variable

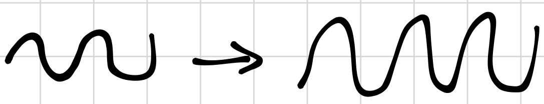
independent variable

- Operations Performed on the Dependent Variable

Amplitude Scaling

$x(t)$

$$y(t) = c \cdot x(t) \quad , \quad c \text{ is a real number}$$



the scaling factor.

$$y[n] = c \cdot x[n]$$

〃

Addition

$$y(t) = x_1(t) + x_2(t)$$
$$y[n] = x_1[n] + x_2[n]$$

Multiplication

$$y(t) = x_1(t) \cdot x_2(t)$$
$$y[n] = x_1[n] \cdot x_2[n]$$

Differentiation (Applies to CT signals)

$$y(t) = \frac{d}{dt} x(t)$$

Integration (Applies to CT signals)

$x(t)$ is a CT signal $\Rightarrow$ $$y(t) = \int_{-\infty}^{t} x(\tau)\, d\tau$$

Operations performed on the independent variable

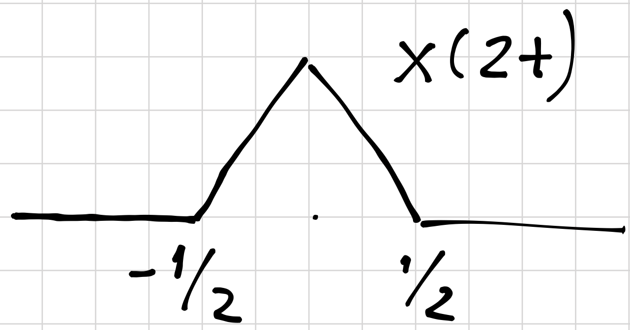
Time Scaling: $x(t)$ is a CT signal.

$$y(t) = x(at)$$

If $a > 1 \Rightarrow y(t)$ is a compressed version of $x(t)$

$0 < a < 1 \Rightarrow y(t)$ is an expanded (stretched) version of $x(t)$

Ex

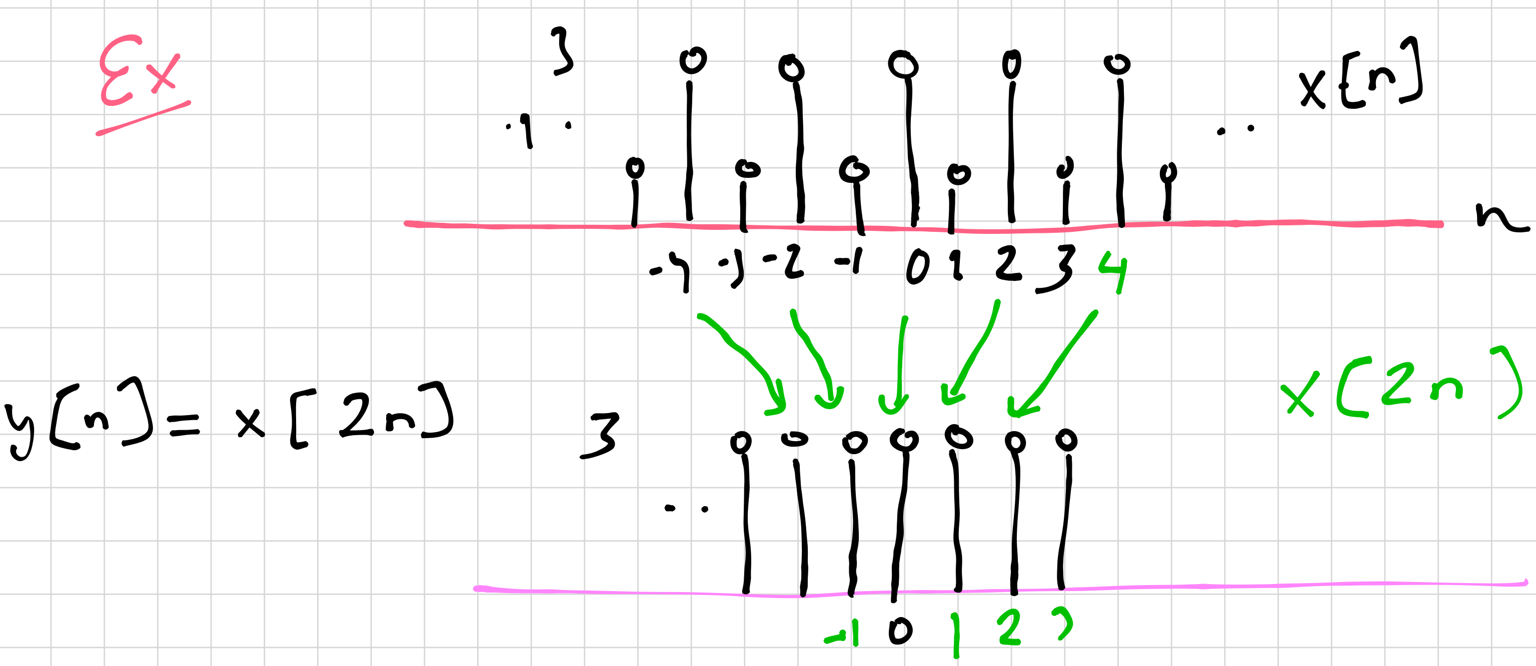


For the discrete-time signals

$$y[n] = x[\underbrace{kn}], \quad k>0, \; k \in \mathbb{Z}^+$$

$k > 1 \Rightarrow$ some values of the DT signal is lost.

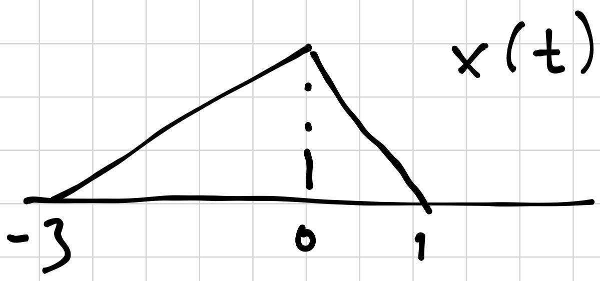
Ex



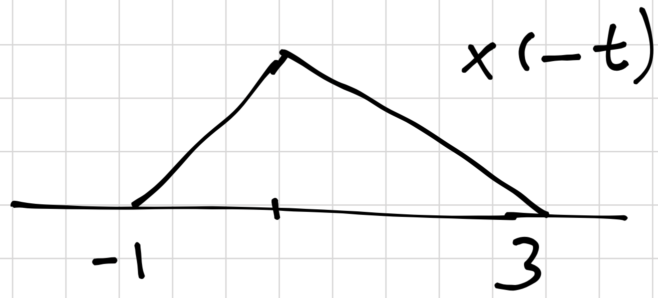
Reflection

$$y(t) = x(-t)$$

is a reflected version of $x(t)$



$\Rightarrow$



Same applies to DT signals.

Time shifting

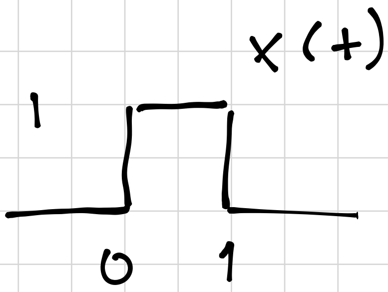
$y(t) = x(t - t_0)$ is a time-shifted version of $x(t)$, t_0: real number

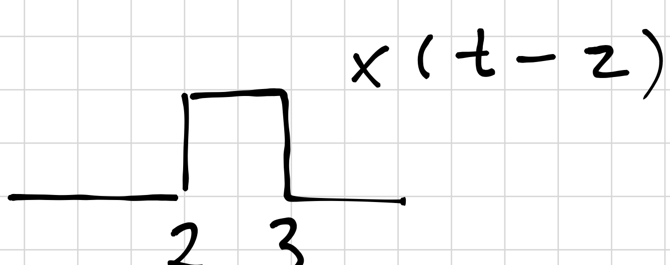
$y[n] = x[n - n_0]$ 〃, n_0: integer

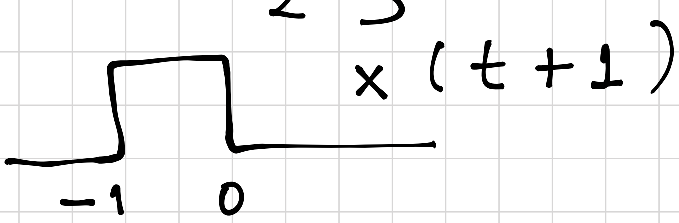
$t_0 > 0 \Rightarrow$ shift right

$t_0 < 0 \Rightarrow$ 〃 left

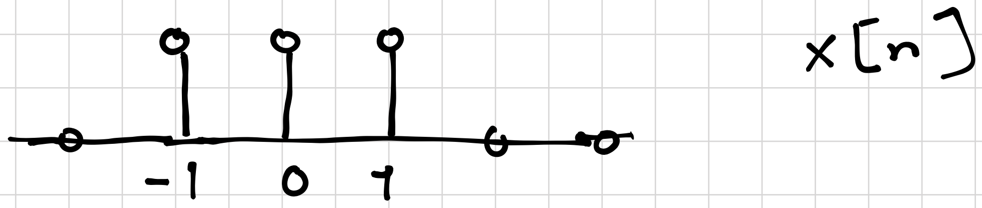
Ex

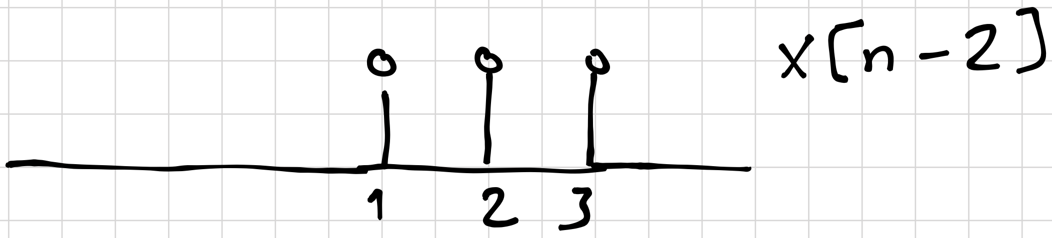






Ex





Precedence Rule for Time-Shifting and Time Scaling.

$x(t)$ is a CT signal

$$y(t) = x(at - b)$$

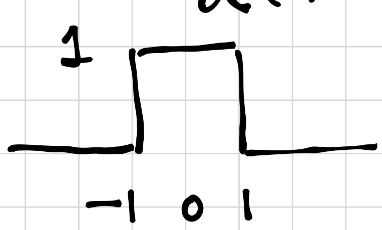
$$\begin{cases} y(0) = x(-b) \\ y\left(\frac{b}{a}\right) = x(0) \end{cases}$$

Define a intermediate signal

$$v(t) = x(t-b) \Rightarrow \text{shift}$$

$$y(t) = v(at) = x(at-b) \Rightarrow \text{Scale.}$$

Ex $x(t)$ $\quad y(t) = x(2t+3)$



① Shift

$v(t)$