29.12.2015

NEURAL NETWORKS - FINAL EXAM

1) Define the differences between Artificial Neural Networks(ANNs) and Cellular Neural Networks(CNNs). (10 p.)

2) For a Cellular Neural Network with $r = 1$ neighborhood, give the state equations for all cells and define the nonlinear differential equation of this neural network in vector-matrix form. (20 p.)

$$T : \left\{ \begin{matrix} 0 & b & 0 \\ c & a & c \\ 0 & b & 0 \end{matrix} \right\}$$

3) Consider the Hopfield Neural Network that described by the following set of differential equations :

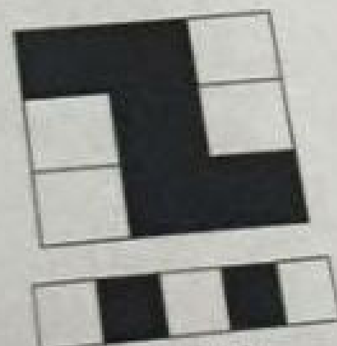
$$\dot{x}(t) = -Ax(t) + Wf(x(t)) + W^{\tau} f(x(t-\tau)) + I$$

$0 = -Au^{*} + wf(z^{*}) + w^{*}f(z^{*}) + I$

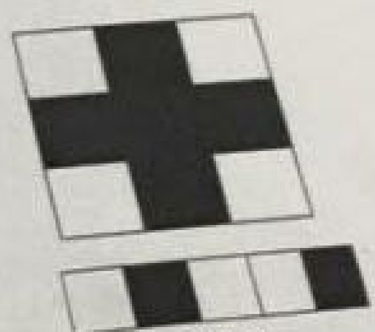
a) Define the equilibrium equation for this neural network model. (10 p.)
b) Shift the equilibrium point of the neural network system to the origin. (20 p.)

4) Consider the BAM Neural Network that trained by the following set of pattern pairs.

Pair(1)

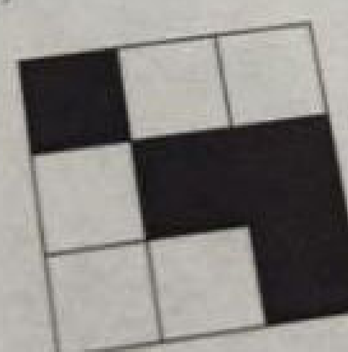


Pair(2)



Pair(3)

a) Give the general architectural graph of this BAM Neural Network. (10 p.)
b) Find the weight matrix of the network after storage phase (15 p.)
c) Process the retrieval phase by showing the steps of retrieval of pairs for the following key pattern: (15 p.)



1 -1 1 1 -1
-3 3 -3 -1 1
3 -3 3 -3 1
-1 1 -1 -1 1
-1 1 -1 -1 1

Duration : 90 min.

(FINAL EXAM NEURA NETWORK)

3

SORU 2 For a Cellular Network with $r=1$ neighborhood give the state equations for all and define the nonlinear differential equations of this neural network in vector matrix form. (20 Puan)

$$T = \begin{Bmatrix} 0 & b & 0 \\ e & a & e \\ 0 & b & 0 \end{Bmatrix} \quad \begin{matrix} \to ① & ② & ③ \\ \to ④ & ⑤ & ⑥ \\ ⑦ & ⑧ & ⑨ \end{matrix}$$

→ Amaç, merkezdeki a her fini sırasıyla numaralı yerlerde konuşlandırarak çözüm bulmak.

a'yı 1. numaraya taşıdık

$$\Rightarrow \begin{Bmatrix} a & e & 0 \\ b & 0 & 0 \\ 0 & 0 & 0 \end{Bmatrix} \to \dot{x}_1 \Rightarrow -x_1 + \overset{\text{1. numarada}}{a.y(x_1)} + e.y(x_2) + b.y(x_4) + u_1$$

$$\Rightarrow \begin{Bmatrix} e & a & e \\ 0 & b & 0 \\ 0 & 0 & 0 \end{Bmatrix} \to \dot{x}_2 \Rightarrow -x_2 + e.y(x_1) + \overset{\text{2 numarada}}{a.y(x_2)} + e.y(x_3) + b.y(x_5) + u_2$$

a'yı 2'ye taşıdık.

⇒ diğerlerini de bu şekilde numaralı yerlere taşıyacağız.

$$\to \dot{x}_3 \Rightarrow -x_3 + e.y(x_2) + \overset{\text{3 numarada}}{a.y(x_3)} + b.y(x_6) + u_3$$

$$\to \dot{x}_4 \Rightarrow -x_4 + b.y(x_1) + \overset{\text{4. numarada}}{a.y(x_4)} + e.y(x_5) + b.y(x_7) + u_4$$

$$\to \dot{x}_5 \Rightarrow -x_5 + b.y(x_2) + e.y(x_4) + \overset{\text{5. numarada}}{a.y(x_5)} + e.y(x_6) + b.y(x_8) + u_5$$

$$\to \dot{x}_6 \Rightarrow -x_6 + b.y(x_3) + e.y(x_5) + \overset{\text{6. numarada}}{a.y(x_6)} + b.y(x_9) + u_6$$

$$\to \dot{x}_7 \Rightarrow -x_7 + b.y(x_4) + a.y(x_7) + e.y(x_8) + u_7$$

$$\to \dot{x}_8 \Rightarrow -x_8 + b.y(x_5) + e.y(x_7) + a.y(x_8) + e.y(x_9) + u_8$$

$$\to \dot{x}_9 \Rightarrow -x_9 + b.y(x_6) + e.y(x_8) + a.y(x_9) + u_9$$

Vector-Matris formu

$$\dot{x} = -x + A.y(x) + u$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \\ \dot{x}_5 \\ \dot{x}_6 \\ \dot{x}_7 \\ \dot{x}_8 \\ \dot{x}_9 \end{bmatrix} = - \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \\ x_8 \\ x_9 \end{bmatrix} + \begin{bmatrix} a & e & 0 & b & 0 & 0 & 0 & 0 & 0 \\ e & a & e & 0 & b & 0 & 0 & 0 & 0 \\ 0 & e & a & 0 & 0 & b & 0 & 0 & 0 \\ b & 0 & 0 & a & e & 0 & b & 0 & 0 \\ 0 & b & 0 & e & a & e & 0 & b & 0 \\ 0 & 0 & b & 0 & e & a & 0 & 0 & b \\ 0 & 0 & 0 & b & 0 & 0 & a & e & 0 \\ 0 & 0 & 0 & 0 & b & 0 & e & a & e \\ 0 & 0 & 0 & 0 & 0 & b & 0 & e & a \end{bmatrix} \cdot \begin{bmatrix} y(x_1) \\ y(x_2) \\ y(x_3) \\ y(x_4) \\ y(x_5) \\ y(x_6) \\ y(x_7) \\ y(x_8) \\ y(x_9) \end{bmatrix} + \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \\ u_6 \\ u_7 \\ u_8 \\ u_9 \end{bmatrix}$$

⇒ Bu şekilde gösterip bitiriyoruz ....

4. Sorunun (b) şıkkının devamıdır ......

2

→ Bulduğumuz $a^1(b^1)^T + a^2(b^2)^T + a^3(b^3)^T$ matrislerini topluyoruz ve (W) buluyoruz.

$$\underset{\text{Weight Matrix}}{W} = \begin{bmatrix} 1 & -1 & 1 & 1 & -1 \\ -3 & 3 & -3 & -1 & 1 \\ 3 & -3 & 3 & -3 & 1 \\ -1 & 1 & -1 & -1 & 1 \\ -1 & 1 & -1 & -1 & 1 \\ -1 & 1 & -1 & -1 & 1 \\ 3 & -3 & 3 & -1 & 1 \\ -3 & 3 & -3 & 1 & -1 \\ 1 & -1 & 1 & 1 & -1 \end{bmatrix}_{9\times5} \qquad \underset{\text{(Lazım olacak)}}{W^T} \Rightarrow \begin{bmatrix} 1 & -3 & 3 & -1 & -1 & -1 & 3 & -3 & 1 \\ -1 & 3 & -3 & 1 & 1 & 1 & -3 & 3 & -1 \\ 1 & -3 & 3 & -1 & -1 & -1 & 3 & -3 & 1 \\ 1 & -1 & -3 & -1 & -1 & -1 & -1 & 1 & 1 \\ -1 & 1 & 1 & 1 & 1 & 1 & 1 & -1 & -1 \end{bmatrix}_{5\times9}$$

b şıkkı cevabıdır

(c) Bu şıkta bize key paternı şekil olarak vermiş bunu matrise dönüştüreceğiz

 $\Rightarrow a^1 = [1\ -1\ -1\ -1\ 1\ 1\ -1\ -1\ 1]^T_{(1\times9)} \Rightarrow$ key pattern

$b^2 = sgn(W^T \cdot a^1)$ $\Rightarrow$ W^T ile a^1 matrisini çarpıyoruz. $\Rightarrow \begin{bmatrix} 1 \\ -1 \\ 1 \\ 5 \\ -3 \end{bmatrix} \Rightarrow$ Sgn işlemi olduğundan pozitifse "1", değil ise "-1" yazılır. $\rightarrow b^2 = \begin{bmatrix} 1 \\ -1 \\ 1 \\ 1 \\ -1 \end{bmatrix}_{(5\times1)}$

$(5\times9).(9\times1) \Rightarrow [5\times1]$

$a^3 = sgn(W \cdot b^2)$ $\Rightarrow$ W ile b^2 matrisini çarpıyoruz. $\Rightarrow \begin{bmatrix} 5 \\ -11 \\ 5 \\ -5 \\ -5 \\ -5 \\ 7 \\ -7 \\ 5 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 \\ -1 \\ 1 \\ -1 \\ -1 \\ -1 \\ 1 \\ -1 \\ 1 \end{bmatrix}_{(9\times1)} \rightarrow a^3$

$(9\times5)(5\times1) \Rightarrow [9\times1]$

$b^4 = sgn(W^T \cdot a^3)$ $\Rightarrow$ W^T ile a^3 matrisini çarpıyoruz. $\Rightarrow \begin{bmatrix} 17 \\ -17 \\ 17 \\ 1 \\ -3 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 \\ -1 \\ 1 \\ 1 \\ -1 \end{bmatrix}_{(5\times1)} \rightarrow b^4$

$(5\times9)(9\times1) \Rightarrow [5\times1]$

eşit çıkmıştır.

eşit olur.

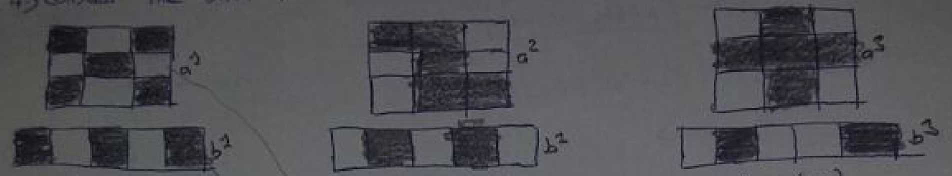
Sonucu: $b^2 \rightarrow a^3 \rightarrow b^4 \rightarrow a^5$ = { b^2 ile b^4 eşit çıkarsa a^3 ile a^5'de eşit olur. }

eşit

$\Rightarrow \boxed{b^2 = b^4,\ a^3 = a^5}$

$\Rightarrow$ 2. örüntü çiftini key patern için çağırdı diyeceğiz.

4.) Consider the BAM Neural Network that trained by the following set of pattern pairs.

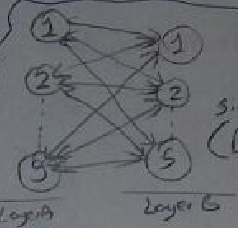


Pair (1) Pair (2) Pair (3)

a) Give the general architectural graph of this BAM Neural Network (10 puan)

$\Rightarrow a_i ; 1, 2, \ldots, n \rightarrow n = 9 \rightarrow$ number of neurons of Layer A

$b_j ; 1, 2, \ldots, m \rightarrow m = 5 \rightarrow$ number of neurons of Layer B $\Rightarrow$



simplified graph (Bu şekilde olacak)

b.) Find the weight matrix of the network storage phase? (15 Puan)
[Doluysa = 1 / Boşsa = -1 yazacağız]

Pair (1) $\Rightarrow a^1 = [1\ -1\ 1\ -1\ 1\ -1\ 1\ -1\ 1]^T$

$b^1 = [1\ -1\ 1\ -1\ 1]^T$

Pair (2) $\Rightarrow a^2 = [1\ 1\ -1\ -1\ 1\ -1\ -1\ 1\ 1]^T$

$b^2 = [-1\ 1\ -1\ 1\ -1]^T$

Pair (3) $\Rightarrow a^3 = [-1\ 1\ -1\ 1\ 1\ 1\ -1\ 1\ 1]^T$

$b^3 = [-1\ 1\ -1\ -1\ 1]^T$

- $W = \sum_{i=1}^{P} a^{(i)} \cdot (b^{(i)})^T$

$\boxed{a^1 \cdot (b^1)^T + a^2 (b^2)^T + a^3 (b^3)^T \Rightarrow W}$

↳ BUNU BULMALIYIZ.

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→ Weight matrisimiz 9x5 boyutunda olacaktır.

- $a^1(b_1)^T \Rightarrow \overset{a_1}{\begin{bmatrix} 1 \\ -1 \\ 1 \\ -1 \\ 1 \\ -1 \\ 1 \\ -1 \\ 1 \end{bmatrix}} \cdot \overset{(b_1)^T}{\begin{bmatrix} 1 & -1 & 1 & -1 & 1 \end{bmatrix}} \Rightarrow \begin{bmatrix} 1 & -1 & 1 & -1 & 1 \\ -1 & 1 & -1 & 1 & -1 \\ 1 & -1 & 1 & -1 & 1 \\ -1 & 1 & -1 & 1 & -1 \\ 1 & -1 & 1 & -1 & 1 \\ -1 & 1 & -1 & 1 & -1 \\ 1 & -1 & 1 & -1 & 1 \\ -1 & 1 & -1 & 1 & -1 \\ 1 & -1 & 1 & -1 & 1 \end{bmatrix}_{(9\times5)} \rightarrow a^1(b^1)^T$

- $a_2(b_2)^T \Rightarrow \overset{(a_2)}{\begin{bmatrix} 1 \\ 1 \\ -1 \\ -1 \\ 1 \\ -1 \\ -1 \\ 1 \\ 1 \end{bmatrix}} \cdot \overset{(b_2)^T}{\begin{bmatrix} -1 & 1 & -1 & 1 & -1 \end{bmatrix}} \Rightarrow \begin{bmatrix} -1 & 1 & -1 & 1 & -1 \\ -1 & 1 & -1 & 1 & -1 \\ 1 & -1 & 1 & -1 & 1 \\ 1 & -1 & 1 & -1 & 1 \\ -1 & 1 & -1 & 1 & -1 \\ 1 & -1 & 1 & -1 & 1 \\ 1 & -1 & 1 & -1 & 1 \\ -1 & 1 & -1 & 1 & -1 \\ -1 & 1 & -1 & 1 & -1 \end{bmatrix}_{9\times5}$

$a^2(b^2)^T$

- $a_3(b_3)^T \Rightarrow \overset{a_3}{\begin{bmatrix} -1 \\ 1 \\ -1 \\ 1 \\ 1 \\ 1 \\ -1 \\ 1 \\ 1 \end{bmatrix}} \cdot \overset{(b_3)^T}{\begin{bmatrix} -1 & 1 & -1 & -1 & 1 \end{bmatrix}}$

$\Downarrow$

$\begin{bmatrix} 1 & -1 & 1 & 1 & -1 \\ -1 & 1 & -1 & -1 & 1 \\ 1 & -1 & 1 & 1 & -1 \\ -1 & 1 & -1 & -1 & 1 \\ -1 & 1 & -1 & -1 & 1 \\ -1 & 1 & -1 & -1 & 1 \\ 1 & -1 & 1 & 1 & -1 \\ -1 & 1 & -1 & -1 & 1 \\ 1 & -1 & 1 & 1 & -1 \end{bmatrix}_{(9\times5)} \leftarrow a^3(b^3)^T$

2015 FINAL EXAM
NEURAL NETWORK

1.

(3) $\dot{x}(t) = -Ax(t) + Wf(x(t)) + W'f(x(t-\tau)) + I$ Vector Matrix Form

(a) Equilibrium equation? (10 Puan)

$$\Rightarrow \quad 0 = -Ax^{*} + Wf(x^{*}) + W'f(x^{*}) + I$$

equilibrium point

(x(t) yerine, x(t-τ) yerine } → x^{*} yazıyoruz)

(b) Shift the equilibrium point of the neural network system to the origin? (20 Puan)

$z(t) = x(t) - x^{*}$

$\dot{z}(t) = \dot{x}(t)$ türevi

$x(t) = z(t) + x^{*}$

$I = Ax^{*} - Wf(x^{*}) - W'f(x^{*})$

$$\Rightarrow \quad \dot{x}(t) = -Ax(t) + Wf(x(t)) + W'f(x(t-\tau)) + I \quad \rightarrow \text{Vector Matrix Form}$$

$$\dot{z}(t) = -A\left(z(t) + x^{*}\right) + Wf(z(t) + x^{*}) + W'f\left(z(t-\tau) + x^{*}\right) + \underbrace{Ax^{*} - Wf(x^{*}) - W'f(x^{*})}_{I}$$

Burdan devam edeyim

$$\dot{z}(t) = -Az(t) - \cancel{Ax^{*}} + W\Big[\underbrace{f(z(t) + x^{*}) - f(x^{*})}_{g(z(t))}\Big] + W'\Big[\underbrace{f(z(t-\tau) + x^{*}) - f(x^{*})}_{g(z(t-\tau))}\Big] + \cancel{Ax^{*}}$$

$$\boxed{\dot{z}(t) = -Az(t) + W\left(g(z(t)) + W'\left(g\left(z(t-\tau)\right)\right)\right)}$$

(b) → CEVABI BUDUR