Signal Processing (İkinci Öğretim)
Midterm Exam Solutions

Istanbul University - Computer Engineering Department - FALL 2016

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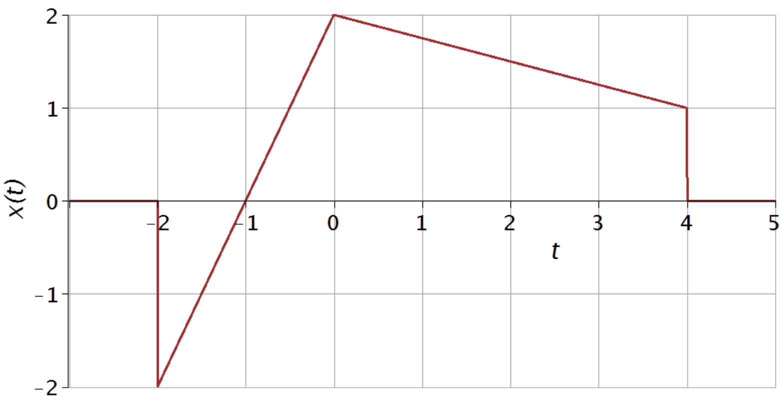
Q1: Consider the following CONTINUOUS TIME signals and answer the following questions.

$$x(t) = \begin{cases} 2t+2 & , \quad -2 \leq t < 0 \\ \dfrac{-t}{4}+2 & , \quad 0 \leq t < 4 \\ 0 & , \quad \text{elsewhere} \end{cases}$$

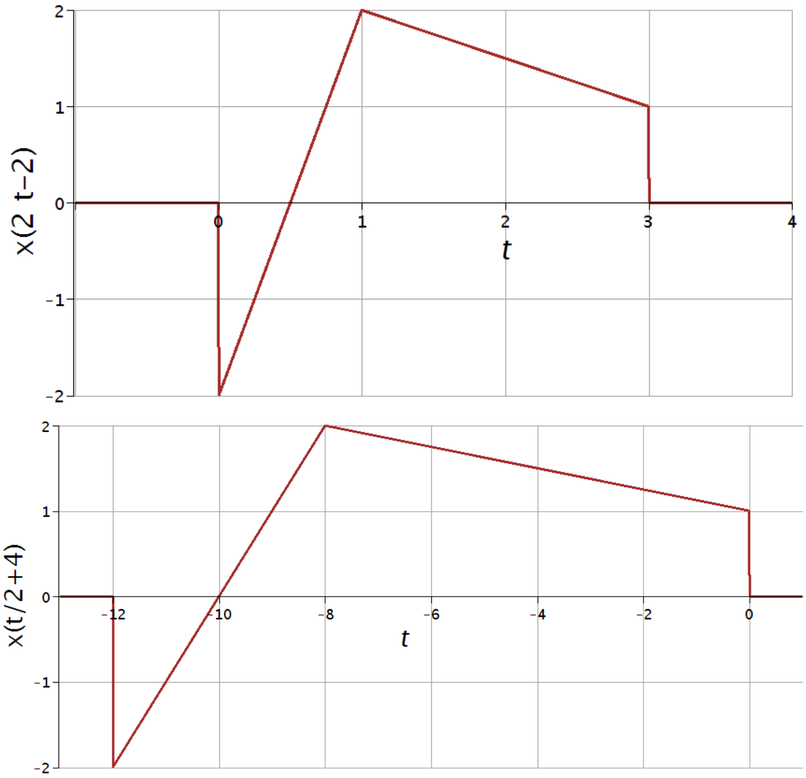
a) (20 pts) Please carefully sketch $x(2t-2) + x(\frac{t}{2}+4)$. Show your steps to receive credit.

Solution 1a:

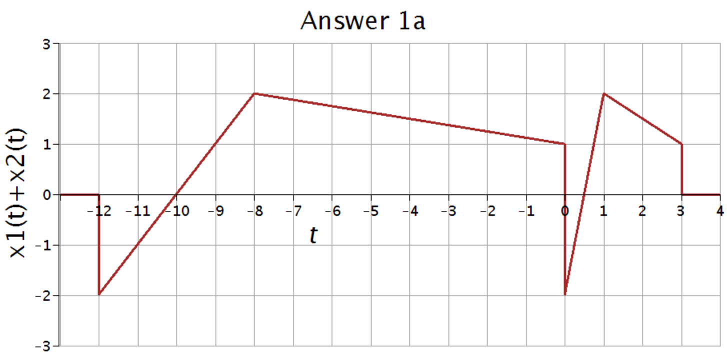
Let's sketch $x(t)$.



Let's say $x_1(t) = x(2t-2)$ and $x_2(t) = x(\frac{t}{2}+4)$. We plot these separately.



Therefore,



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b) (15 pts) Please determine whether $x(t)$ is an energy or power signal. Calculate its power or energy, whichever applies.

Solution 1b:

Let's calculate its energy.

$$E = \int_{-\infty}^{\infty} x^2(t)\,\mathrm{d}t$$
$$= \int_{-2}^{0} (2t+2)^2\,\mathrm{d}t + \int_{0}^{4} (-t/4+2)^2\,\mathrm{d}t$$

Using $\int (ax+b)^k \, \mathrm{d}x = \dfrac{(ax+b)^{k+1}}{a(k+1)} + c$

$$
\begin{aligned}
E &= \frac{1}{6}\,(2t+2)^3\Big|_{-2}^{0} - \frac{4}{3}\,(-\frac{1}{4}t+2)^3\Big|_{0}^{4} \\
E &= \frac{8}{3} + \frac{28}{3} \\
E &= 12 < \infty \qquad \therefore \qquad x(t) \text{ is an energy signal.} \qquad \blacksquare
\end{aligned}
$$

Q2: Consider the following CT signal and answer the following questions.

$$x(t) = \cos^2(\frac{2}{3}\pi t)$$

a) `(20 pts)` Is $x(t)$ periodic? If so, calculate its fundamental period, frequency and angular frequency.

Solution 2a:

We know that

$$
\begin{aligned}
x(t) &= \cos^2(\frac{2}{3}\pi t) \\
x(t) &= \frac{1}{2}\left\{ \cos\left(\frac{4}{3}\pi t\right) + 1 \right\}
\end{aligned}
$$

Therefore $x(t)$ is periodic and $\omega = \frac{4}{3}\pi$ rad/sec, $f = \frac{2}{3}$ Hz and $T = 1.5$ seconds $\blacksquare$.

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b) `(15 pts)` Please determine whether $x(t)$ is an even signal, odd signal or neither.

Solution 2b:

$$
\begin{aligned}
x(t) &= \cos^2(\frac{2}{3}\pi t) \\
x(-t) &= \cos^2(-\frac{2}{3}\pi t) \\
x(-t) &= \cos^2(\frac{2}{3}\pi t) \\
\therefore \qquad x(-t) &= x(t)
\end{aligned}
$$

So, $x(t)$ is an EVEN signal.

Q3: (30 pts) The systems below show the input as $x(t)$ or $x[n]$ and the output as $y(t)$ or $y[n]$. For each system, determine whether it is **(i)** (2 pts each) memoryless, **(ii)** (3 pts each) stable, **(iii)** (2 pts each) causal, **(iv)** (4 pts each) linear, and **(v)** (4 pts each) time-invariant.

$$y[n] = 2x[n]u[n-1]$$
$$y(t) = \frac{\mathrm{d}}{\mathrm{d}t}\{e^{-t}x(t)\}$$

Solution 3:

$$\boxed{y[n] = 2x[n]u[n-1]}$$

(i): Memoryless

(ii): Assuming $|x[n]| \leq M_x < \infty$.

$$\begin{aligned}
|y[n]| &= |2x[n]u[n-1]| \\
|y[n]| &= 2|x[n]||u[n-1]| \\
|u[n-1]| &\leq 1 \\
|y[n]| &\leq 2M_x \\
|y[n]| &\leq M_y < \infty
\end{aligned}$$

Therefore, the system is stable.

(iii) Causal

(iv) Homogenity:

$$\mathcal{H}\{ax[n]\} = 2ax[n]u[n-1] = ay[n]$$

Homogenity is satisfied.
Superposition

$$\begin{aligned}
\mathcal{H}\{x_1[n] + x_2[n]\} &= 2(x_1[n] + x_2[n])u[n-1] \\
&= 2x_1[n]u[n-1] + 2x_2[n]u[n-1] \\
&= y_1[n] + y_2[n]
\end{aligned}$$

Superposition is satisfied. Therefore, $\mathcal{H}$ is LINEAR.

(v):

$$\mathcal{H}\{x[n-n_0]\} = 2x[n-n_0]u[n-1] \neq 2x[n-n_0]u[n-n_0-1]$$

Therefore $\mathcal{H}$ is NOT TIME INVARIANT.

$$\boxed{y(t) = \frac{\mathrm{d}}{\mathrm{d}t}\{e^{-t}x(t)\}}$$

(i): Not Memoryless

(ii):

$$\begin{aligned} y(t) &= \frac{\mathrm{d}}{\mathrm{d}t}\{e^{-t}x(t)\} \\ &= e^{-t}x'(t) - e^{-t}x(t) \\ &= e^{-t}[x'(t) - x(t)] \end{aligned}$$

Since $\lim_{t\to-\infty} e^{-t} = \infty$, this system is not stable.

(iii): Not causal

(iv): Checking both homogenity and superposition

$$\begin{aligned} \mathcal{H}\{ax_1(t) + bx_2(t)\} &= \frac{\mathrm{d}}{\mathrm{d}t}\{e^{-t}[ax_1(t) + bx_2(t)]\} \\ &= a\frac{\mathrm{d}}{\mathrm{d}t}\{e^{-t}x_1(t)\} + b\frac{\mathrm{d}}{\mathrm{d}t}\{e^{-t}x_2(t)\} \\ &= ay_1(t) + by_2(t) \end{aligned}$$

Therefore, $\mathcal{H}$ is LINEAR.

(v):

$$\begin{aligned} \mathcal{H}\{x(t-t_0)\} &= \frac{\mathrm{d}}{\mathrm{d}t}\{e^{-t}x(t-t_0)\} \\ y(t-t_0) &= \frac{\mathrm{d}}{\mathrm{d}t}\{e^{-t+t_0}x(t-t_0)\} \\ \mathcal{H}\{x(t-t_0)\} &\neq y(t-t_0) \end{aligned}$$

Therefore, $\mathcal{H}$ is NOT TIME-INVARIANT.