# Convolution Sum (Continued)

Summary

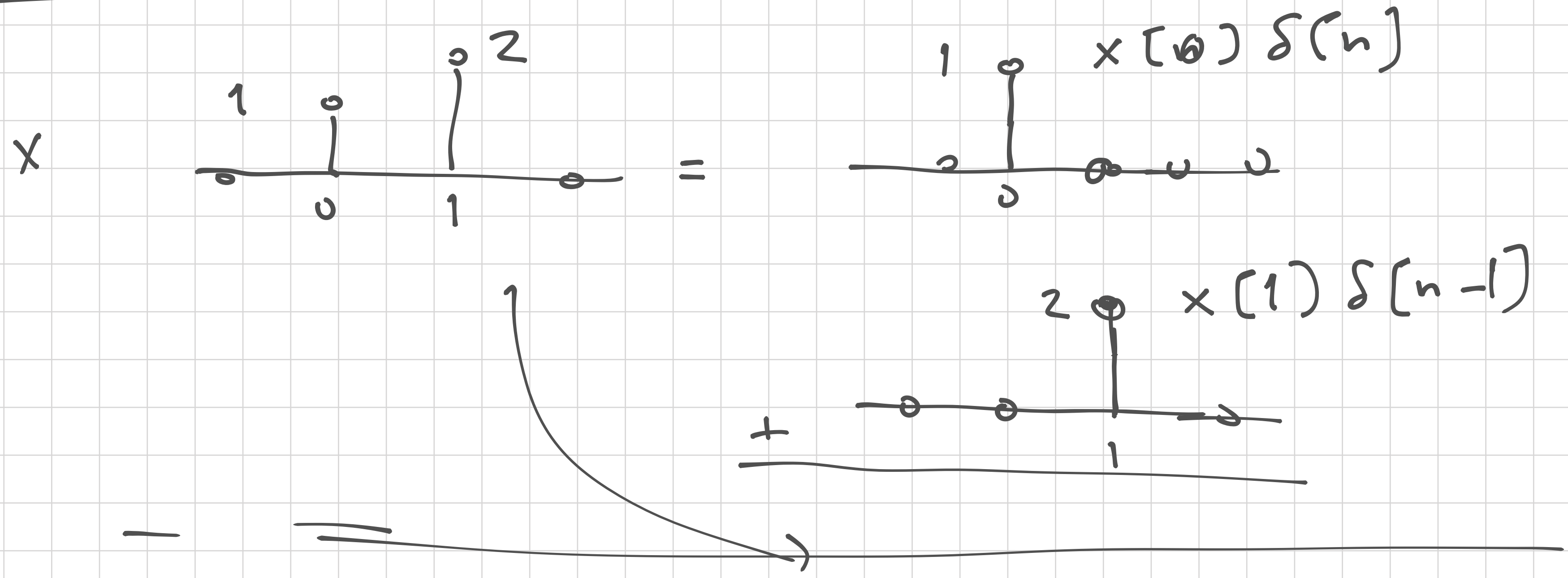
$$x[n] = \sum_{k=-\infty}^{+\infty} x[k] \cdot \delta[n-k]$$

$$x[n] \cdot \delta[n-k] = x[k] \cdot \delta[n-k]$$

$$x[n] \longrightarrow \boxed{\mathcal{H}} \longrightarrow y[n]$$

$$y[n] = \mathcal{H}\{x[n]\} = \sum_{k=-\infty}^{+\infty} x[k]\, h[n-k]$$

$$h[n] = \mathcal{H}\{\delta[n]\} \qquad = x[n] * h[n]$$



# Convolution Sum Evaluation Procedure

$$y[n] = x[n] * h[n]$$

$$= \sum_{k=-\infty}^{+\infty} x[k] \cdot h[n-k]$$

$$w_n[k] = x[k] \cdot h[n-k]$$

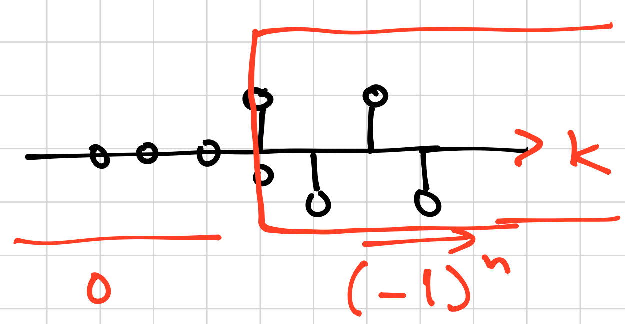


***Procedure 2.1: Reflect and Shift Convolution Sum Evaluation***

1. Graph both $x[k]$ and $h[n-k]$ as a function of the independent variable $k$. To determine $h[n-k]$, first reflect $h[k]$ about $k = 0$ to obtain $h[-k]$. Then shift by $-n$.
2. Begin with $n$ large and negative. That is, shift $h[-k]$ to the far left on the time axis.
3. Write the mathematical representation for the intermediate signal $w_n[k]$.
4. Increase the shift $n$ (i.e., move $h[n-k]$ toward the right) until the mathematical representation for $w_n[k]$ changes. The value of $n$ at which the change occurs defines the end of the current interval and the beginning of a new interval.
5. Let $n$ be in the new interval. Repeat steps 3 and 4 until all intervals of time shifts and the corresponding mathematical representations for $w_n[k]$ are identified. This usually implies increasing $n$ to a very large positive number.
6. For each interval of time shifts, sum all the values of the corresponding $w_n[k]$ to obtain $y[n]$ on that interval.

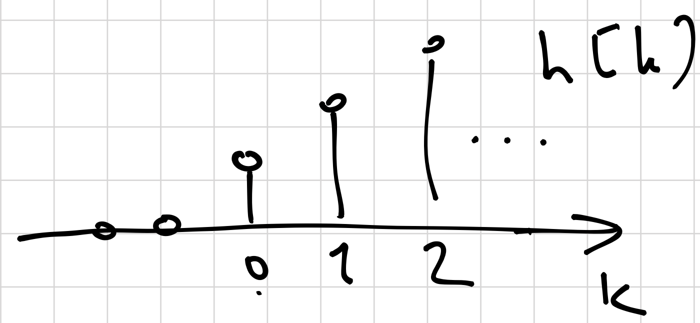
$$x[n] = (-1)^n u[n]$$

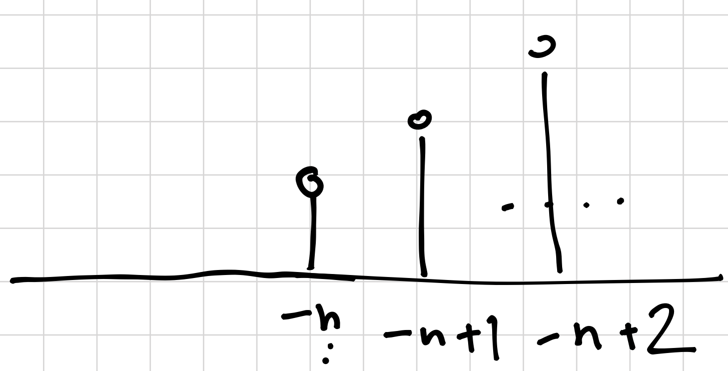
0 | $(-1)^n$



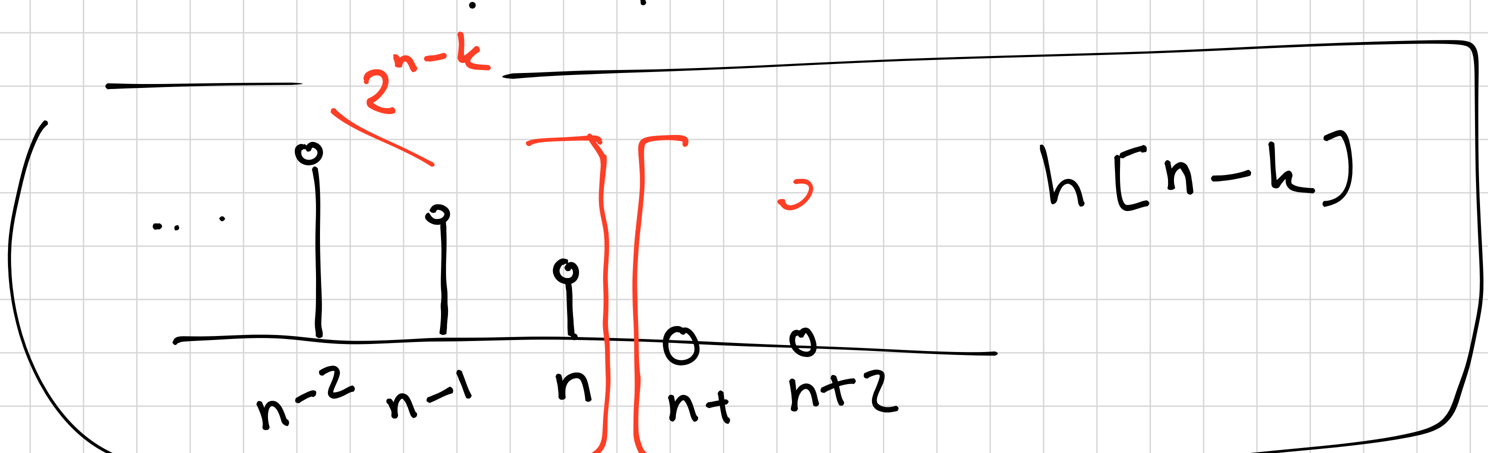

$$h[n-k] = h[-k+n]$$

$$h[n] = 2^n u[n]$$




$h[k+n]$



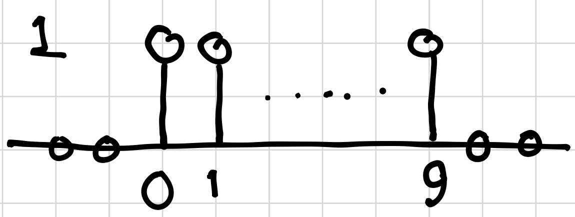
## Ex 2.3 Moving Average System.

Consider a 4 point moving average system

$$y[n] = \frac{1}{4} \sum_{k=0}^{3} x[n-k]$$

Find the output when

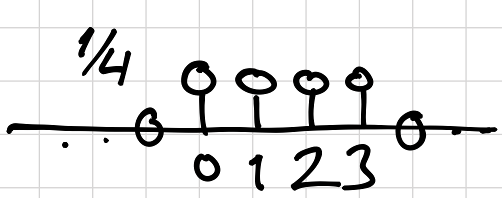
$$x[n] = u[n] - u[n-10]$$




$x[n]$

Impulse response: set $x[n] = \delta[n]$

$$h[n] = \frac{1}{4} \sum_{k=0}^{3} \delta[n-k] \quad \left( = \frac{1}{4} \left( u[n] - u[n-4] \right) \right)$$



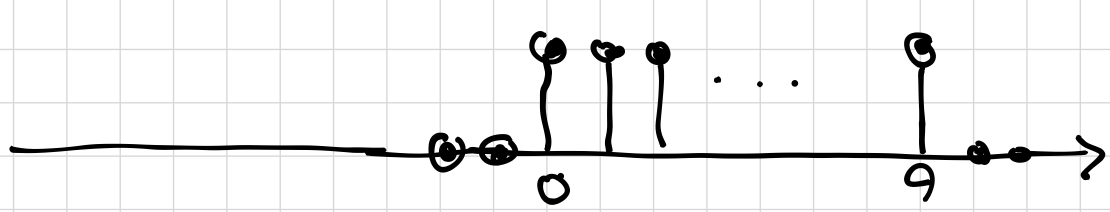

$h[n]$

$y[n] = x[n] * h[n]$

$$y[n] = \sum_{k=-\infty}^{+\infty} x[k] \cdot h[n-k]$$

$$w_n[k] = x[k] \cdot h[n-k]$$

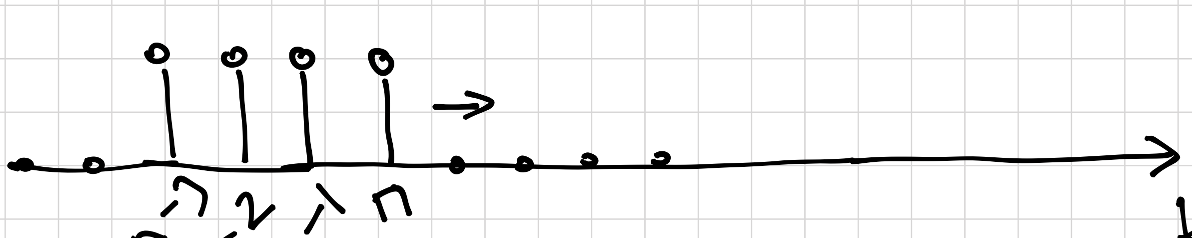
(1) Graph $x[k]$ and $h[n-k]$ w.r.t. k, (with respect to) keep n large and negative




$x[k]$

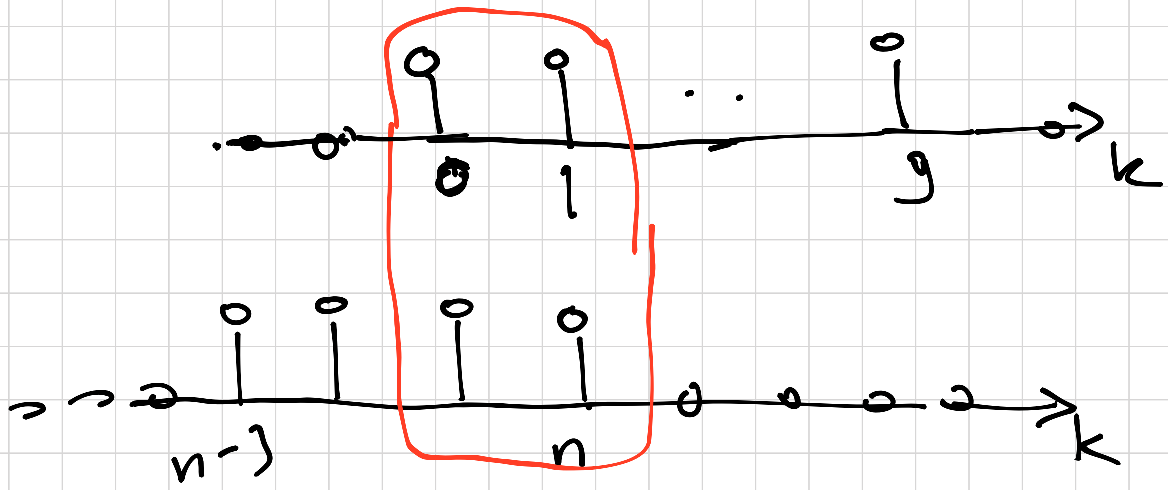
k

$h[n-k]$




k

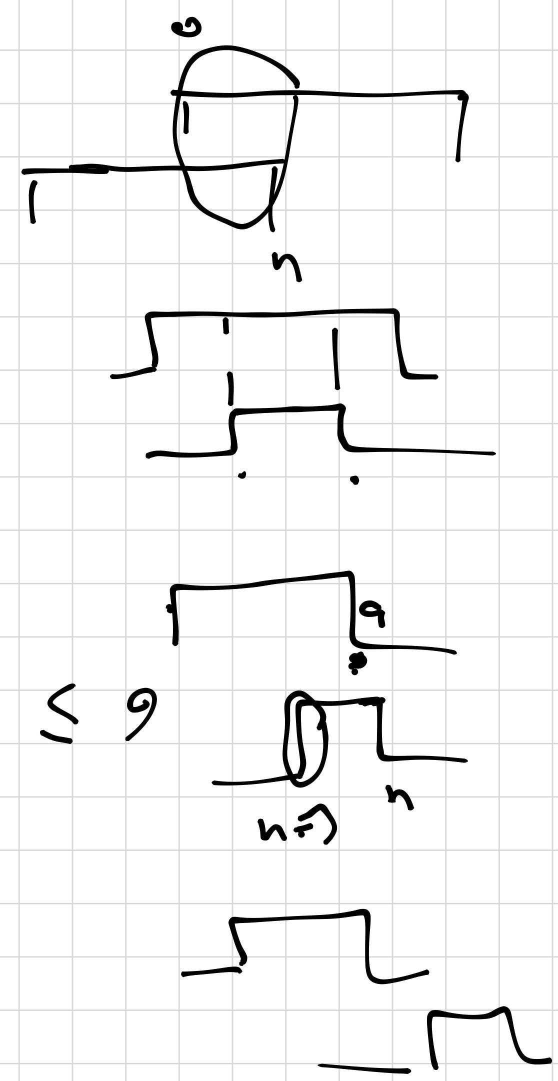
$n < 0 \qquad x[k] \cdot h[n-k] = 0$




$n \geq 0$

$n - 3 < 0$

$$w_n[k] = \begin{cases} 1/4 & 0 \leq k < n \\ 0, & \text{otherwise} \end{cases}$$




$n - 3 \geq 0$

$n \leq 9$

$$w_n[k] = \begin{cases} 1/4, & n-3 \leq k \leq n \\ 0, & 0 \end{cases}$$

$$\left.\begin{matrix} n > 9 \\ n-3 \leq 9 \end{matrix}\right\} \quad w_n[k] = \begin{cases} 1/4 & n-3 \leq k \leq 9 \end{cases}$$

$n - 3 > 9 \qquad w_n[k] = 0$

1) $n < 0$ $\qquad w_n[k] = 0 \qquad y[n] = \sum_{k=-\infty}^{+\infty} w_n[k] = 0$

2) $0 \leq n < 3$

$$w_n[k] = \begin{cases} 1/4, & 0 \leq k \leq n \\ 0, & \text{otherw} \end{cases}$$

$$y[n] = \sum_{k=0}^{n} 1/4 = \frac{(n+1)}{4}$$

3) $3 \leq n \leq 9$

$$w_n[k] = \begin{cases} 1/4 & n-3 \leq k \leq n \\ 0, & \text{other} \end{cases}$$

$$y[n] = \sum_{k=n-3}^{n} 1/4 = 1$$

$9 < n \leq 12$

$$w_n[k] = \begin{cases} 1/4 & n-3 \leq k \leq 9 \\ 0 & \text{otherwise} \end{cases}$$

$$y[n] = \sum_{k=n-3}^{9} 1/4 = \frac{13-n}{4}$$

$n > 12 \Rightarrow w_n[k] = 0$

$y[n] = 0$

$$y[n] = \begin{cases} \frac{n+1}{4}, & 0 \leq n \leq 3 \\ 1, & 3 < n \leq 9 \\ \frac{13-n}{4}, & 9 < n \leq 12 \\ 0, & \text{otherwise.} \end{cases}$$