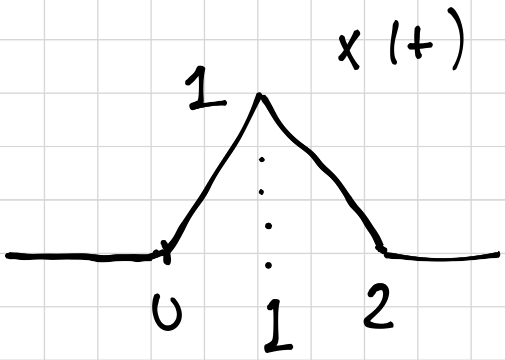
Ex Pr 1.9 @ p25

a) $x(t) = \begin{cases} t, & 0 \leq t \leq 1 \\ 2-t & 1 < t \leq 2 \\ 0, & \text{otherwise} \end{cases}$



• Periodic? No.

$$E = \int_{-\infty}^{+\infty} x^2(t)\, dt = \int_0^1 t^2\, dt + \int_1^2 (2-t)^2 dt$$

$$= \left.\frac{t^3}{3}\right|_0^1 - \left.\frac{(2-t)^3}{3}\right|_1^2 = \frac{1}{3} - \left(0 - \frac{1}{3}\right) = \frac{2}{3}$$

, finite, nonzero $\therefore$ x(t) is an ENERGY Signal.

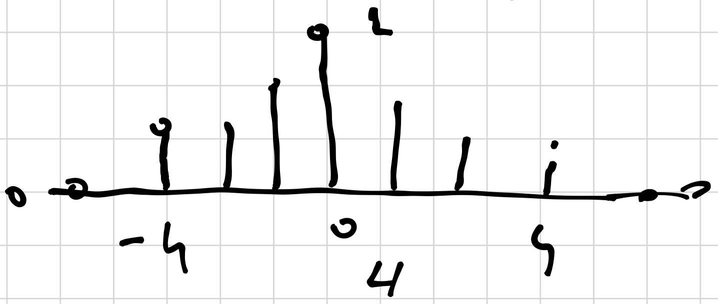
Since it is an energy signal it cannot be a power signal.

• CT signal.

• Neither even nor odd.

• Deterministic.

d) $x[n] = \begin{cases} \cos(\pi n) & , -4 \leq n \leq 4 \\ 0 & , \text{otherwise} \end{cases}$



- Non-periodic.

$$E = \sum_{n=-4}^{4} x^2[n] = \cos^2[4\pi] + \cos^2[-3\pi] + \cdots + \cdots + \cos[4\pi] = \underline{9}$$

Finite non-zero energy

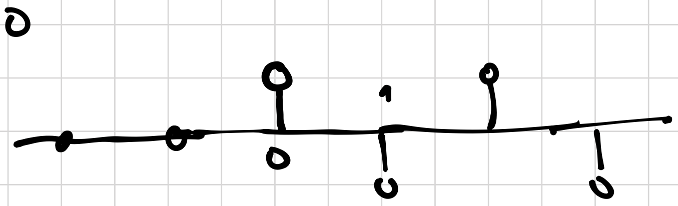
$\therefore$ x[n] is an energy signal.

therefore

Ex

$x[n] = \begin{cases} \cos(\pi n), & n \geq 0 \\ 0, & \text{otherwise} \end{cases}$ non-periodic

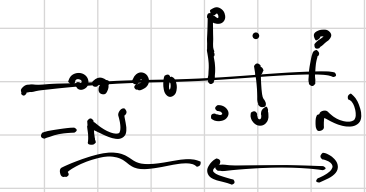
$$E = \sum_{n=0}^{\infty} x^2[n] = \infty$$ Not an energy signal.



...

$\left.\begin{array}{l} \cos^2(\pi n) = 1 \\ \cos(\pi n) = (-1)^n \end{array}\right\}$

$$P = \lim_{N\to\infty} \frac{1}{2N} \sum_{n=-N}^{N} \underbrace{x^2[n]}_{1}$$



$$= \lim_{N\to\infty} \frac{1}{2N} \sum_{n=0}^{N} 1 = \lim_{N\to\infty} \frac{(N+1)}{2N}$$

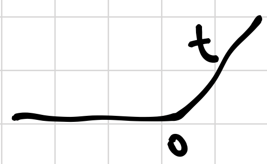
$$= \lim_{N\to\infty} \left(\frac{1}{2} + \frac{1}{2N}\right) = \frac{1}{2} + 0 = 1/2$$

Power is nonzero and finite

$\therefore$ $x[n]$ is a power signal.

Ex

$$x(t) = \begin{cases} t, & t \geq 0 \\ 0, & \text{elsewhere} \end{cases}$$



Non-periodic.

$$E = \int_0^\infty t^2 \, dt = \left.\frac{t^3}{3}\right|_0^\infty = \infty$$

Not an energy signal.

$$P = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} t^2 \, dt = \lim_{T\to\infty} \frac{1}{T} \int_0^{T/2} t^2 \, dt$$

$$= \lim_{T\to\infty} \frac{1}{T} \cdot \frac{T^3}{6} = \lim_{T\to\infty} T^2/6 = \infty$$

$\therefore$ Not a power signal.

Ex Two periodic CT signals are given:

$x_1(t)$ periodic with a (fundamental) period T_1

$x_2(t)$ 〃 〃 〃 〃 T_2

Determine under what condition

$$x(t) = x_1(t) + x_2(t)$$

is periodic and if so what is the period?

$$x_1(t) = x_1(t+T_1) = x_1(t + m \cdot T_1) \quad , \quad m \in \mathbb{Z}^+$$
$$x_2(t) = x_2(t+T_2) = x_2(t + k \cdot T_2) \quad , \quad k \in \mathbb{Z}^+$$

$$x(t) = x_1(t + m T_1) + x_2(t + k \cdot T_2)$$

Let's write

$$x(t+T) = x_1(t+T) + x_2(t+T)$$
$$= x_1(t + m T_1) + x_2(t + k T_2)$$

For peridicity

$$T = m T_1 = k T_2 \Rightarrow \frac{k}{m} = \frac{T_1}{T_2}$$

$\frac{T_1}{T_2}$ must be a rational number.

If k/m is not rational then $x(t)$ is not periodic.

The fundamental period of $x(t)$ is

$\{T = LCM(T_1, T_2)\}$ and $T = m T_1 = k T_2$

when m and k are relatively prime

(that is $GCD(m,k) = 1$

(*) The period of $x(t) + c$, $c \in \mathbb{R}$ is the same as $x(t)$

Ex DT

$x_1[n]$: periodic, N_1

$x_2[n]$: " , N_2

Under what condition $x[n] = x_1[n] + x_2[n]$ is periodic ?

$$x_1[n] = x_1[n + m \cdot N_1] \quad , \quad m \in \mathbb{Z}^+$$
$$x_2[n] = x_2[n + k \cdot N_1] \quad , \quad k \in \mathbb{Z}^+$$
$$x[n] = x_1[n + m N_1] + x_2[n + k N_2]$$

for some N :

$$x[n+N] = x_1[n+N] + x_2[n+N]$$

So if $x[n]$ is periodic

$$N = m N_1 = k N_2 \qquad \frac{m}{k} = \frac{N_2}{N_1}$$

must be rational

This equation is always satisfied

$\therefore$ $x[n]$ is periodic.

$$N = \mathrm{LCM}(N_1, N_2)$$

$\boxed{\text{Systems}}$

/* SISO: single input single-output system */

$$x(t) \longrightarrow \boxed{\text{CT-system}} \longrightarrow y(t)$$

input $\qquad\qquad$ $\mathcal{H}$ $\qquad\qquad$ output

$$x[n] \longrightarrow \boxed{\text{DT-System}} \longrightarrow y[n]$$

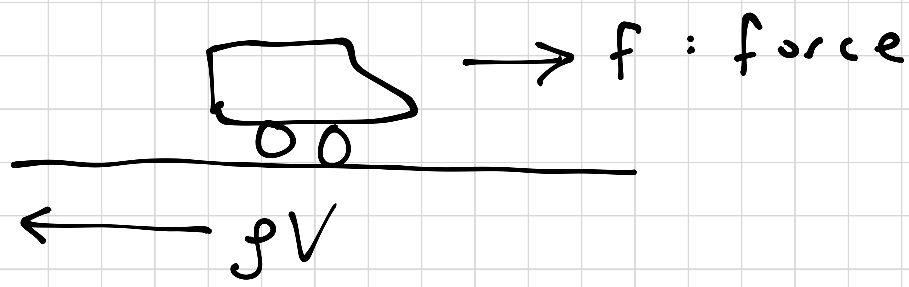
$\mathcal{H}$

$$x(t) \xrightarrow{\quad \mathcal{H} \quad} y(t) \quad *$$

$$y(t) = \mathcal{H}\{x(t)\}$$

We will use the operator $\mathcal{H}\{\ \}$ to denote the action of a system.

Ex



V: velocity

ρV : frictional force

m: mass

$v(t) \longrightarrow f(t)$

$$\frac{dv(t)}{dt} = \frac{1}{m}\left[f(t) - \rho v(t)\right]$$

$$\left[\frac{dv(t)}{dt} + \frac{\rho}{m} v(t)\right] \cdot m = f(t)$$

$$\mathcal{H}\{f(t)\} = v(t)$$

$$f(t) \xrightarrow{\mathcal{H}} v(t)$$