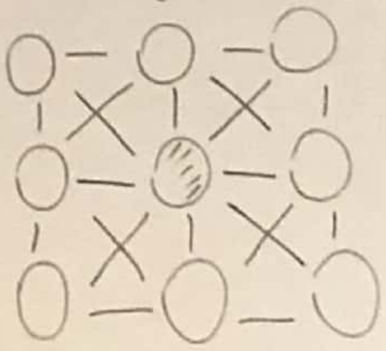
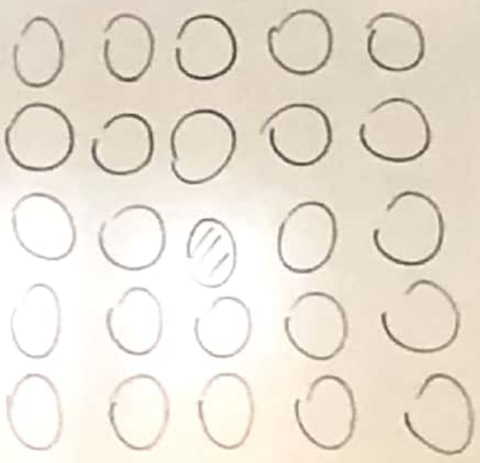
CELLULAR NEURAL NETWORKS

A Cellular Neural Network (CNN) is an array of cells with local connections only. The communication is allowed just between neighbouring units.



r=1



r=2

r: radius of neighbourhood

If r=1 then we call "3x3" neighborhood
r=2 " " " "5x5" "
⋮
r=n " " " "(2n+1)x(2n+1)" "

In order to calculate the state of the cells, it is necessary to define the template of the network.

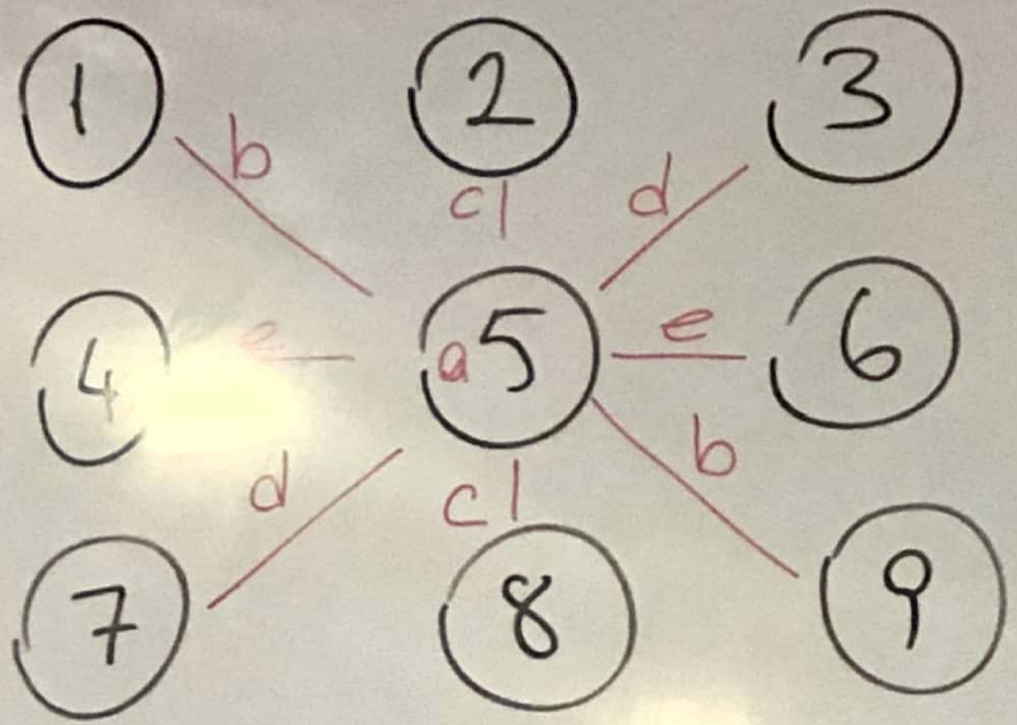
$$T:\begin{Bmatrix} b & c & d \\ e & a & e \\ d & c & b \end{Bmatrix}$$

3x3 neighborhood

T: template (symmetric)

$\underbrace{(2r+1)\times(2r+1)}$

Dimension of the template.



a: feedback of the centrall cell

c: " of the above and below neighbor cells.

d: " of the right-above and left-below " ".

b: " " " left-above and right-below " ".

e: " " " right and left " ".

The CNN dynamics are described by a system of nonlinear differential equations:

$$\dot{x} = -x + Ay(x) + u$$

or

$$\dot{x}_i = -x_i + \sum_{J=1}^{n} a_{ij} \cdot y(x_j) + u_i \,, \quad i = 1, 2, \ldots, n$$

where

$x = [x_1 \; x_2 \ldots x_n]^T$: State of cell

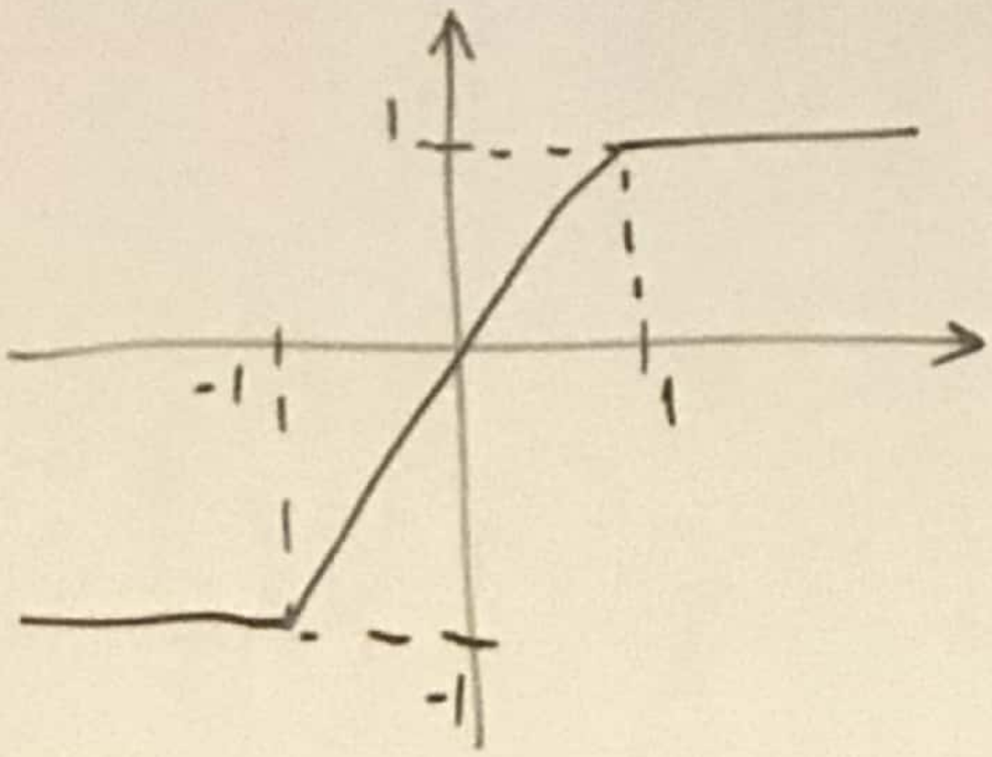
$A = \{a_{ij}\}_{n \times n}$: Feedback matrix that contains the weights of the network

$u = [u_1 \; u_2 \ldots u_n]^T$: Input

$y(x) = [y(x_1) \; y(x_2) \ldots y(x_n)]^T$: Output

The expression for the output y is:

$$y(x_i) = \frac{1}{2}\left[|x_i+1| - |x_i-1|\right]$$



piece-wise linear function

Ex.

$$T:\begin{Bmatrix} b & c & d \\ e & a & e \\ d & c & b \end{Bmatrix}$$

① ② ③
④ ⑤ ⑥
⑦ ⑧ ⑨

The state equations of the CNN:

$$\dot{x}_1 = -x_1 + a\cdot y(x_1) + e y(x_2) + c y(x_4) + b y(x_5) + u_1$$
$$\dot{x}_2 = -x_2 + e y(x_1) + a y(x_2) + e y(x_3) + d y(x_5) + c y(x_6) + u_3$$
$$\dot{x}_3 = -x_3 + e y(x_2) + a y(x_3) + d y(x_5) + c y(x_7) + b y(x_8) + u_4$$
$$\dot{x}_4 = -x_4 + c y(x_1) + d y(x_2) + a y(x_4) + e y(x_5) + c y(x_7) + b y(x_8) + u_5$$
$$\dot{x}_5 = -x_5 + b y(x_1) + c y(x_2) + d y(x_3) + e y(x_4) + a y(x_5) + e y(x_6) + d y(x_7) + c y(x_8) + b y(x_9) + u_6$$
$$\dot{x}_6 = -x_6 + b y(x_2) + c y(x_3) + e y(x_5) + a y(x_6) + d y(x_8) + c y(x_9) + u_7$$
$$\dot{x}_7 = -x_7 + c y(x_4) + d y(x_5) + a y(x_7) + e y(x_8) + u_8$$
$$\dot{x}_8 = -x_8 + b y(x_4) + c y(x_5) + d y(x_6) + e y(x_7) + a y(x_8) + e y(x_9) + u_9$$
$$\dot{x}_9 = -x_9 + b y(x_5) + c y(x_6) + e y(x_8) + a y(x_9) + u_9$$

$\dot{X} = -$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \\ \dot{x}_5 \\ \dot{x}_6 \\ \dot{x}_7 \\ \vdots \end{bmatrix}$$

$$\dot{x} = -x + A \cdot y(x) + u$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \\ \dot{x}_5 \\ \dot{x}_6 \\ \dot{x}_7 \\ \dot{x}_8 \\ \dot{x}_9 \end{bmatrix} = - \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \\ x_8 \\ x_9 \end{bmatrix} + \begin{bmatrix} a & e & 0 & c & b & 0 & 0 & 0 & 0 \\ e & a & e & d & c & b & 0 & 0 & 0 \\ 0 & e & a & 0 & d & c & 0 & 0 & 0 \\ c & d & 0 & a & e & 0 & c & b & 0 \\ b & c & d & e & a & e & d & c & b \\ 0 & b & c & 0 & e & a & 0 & d & c \\ 0 & 0 & 0 & c & d & 0 & a & e & 0 \\ 0 & 0 & 0 & b & c & d & e & a & e \\ 0 & 0 & 0 & 0 & b & c & 0 & e & a \end{bmatrix} \begin{bmatrix} y(x_1) \\ y(x_2) \\ y(x_3) \\ y(x_4) \\ y(x_5) \\ y(x_6) \\ y(x_7) \\ y(x_8) \\ y(x_9) \end{bmatrix} + \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \\ u_6 \\ u_7 \\ u_8 \\ u_9 \end{bmatrix}$$

$\dot{x}_9) + u_5$

scribed by a
tial equations:

, $i=1,2,\dots,n$

.11
t contains
x

Ex.

$$T:\begin{Bmatrix} b & c & d \\ e & a & e \\ d & c & b \end{Bmatrix}$$

①	②	③
④	⑤	⑥
⑦	⑧	⑨

$$\begin{bmatrix} a & e & d & c & b & 0 & 0 & 0 & 0 \\ e & a & e & d & c & b & 0 & 0 & 0 \\ 0 & e & a & 0 & d & c & 0 & 0 & 0 \end{bmatrix}$$

The state equations of the CNN:

$$\dot{x}_1 = -x_1 + a\cdot y(x_1) + e y(x_2) + c y(x_4) + b y(x$$
$$\dot{x}_2 = -x_2 + e y(x_1) + a y(x_2) + e y(x_3) + d y($$
$$\dot{x}_3 = -x_3 + e y(x_2) + a y(x_3) + d y(x_5) + c y($$
$$\dot{x}_4 = -x_4 + c y(x_1) + d y(x_2) + a y(x_4) + e y($$
$$\dot{x}_5 = -x_5 + b y(x_1) + c y(x_2) + d y(x_3) + e y(x_4) +$$
$$\dot{x}_6 = -x_6 + b y(x_2) + c y(x_3) + e y(x_5) + a y(x_6)$$
$$\dot{x}_7 = -x_7 + c y(x_4) + d y(x_5) + a y(x_7) + e y(x_8)$$
$$\dot{x}_8 = -x_8 + b y(x_4) + c y(x_5) + d y(x_6) + e y(x_7) +$$
$$\dot{x}_9 = -x_9 + b y(x_5) + c y(x_6) + e y(x_8) + a y(x_9$$