Olasılık Teorisi ve İstatistik 2016 örgün Final Çözümleri

1a-10p

$$\int_0^1\int_0^1 f_{XY}(x,y)\,dx\,dy = 1$$

$$\int_0^1\int_0^1 (\alpha xy+\beta)\,dx\,dy = 1$$

$$\int_0^1 \left[\left(\frac{\alpha x^2 y}{2}+\beta y\right)\Big|_0^1\right] dy = 1$$

$$\int_0^1 \left(\frac{\alpha y}{2}+\beta\right) dy = 1$$

$$\frac{\alpha y^2}{4}+\beta y\Big|_0^1 = 1$$

$$\frac{\alpha}{4}+\beta = 1 \rightarrow \underline{\alpha+4\beta=4} \quad (1)$$

$$E(x)=\mu_X = \int_0^1\int_0^1 x\cdot(\alpha xy+\beta)\,dx\,dy$$

$$= \int_0^1 \left(\frac{\alpha x^3 y}{3}+\frac{\beta x^2}{2}\right)\Big|_0^1 dy$$

$$= \int_0^1 \left(\frac{\alpha y}{3}+\frac{\beta}{2}\right) dy$$

$$0.505 = \frac{\alpha}{6}+\frac{\beta}{2} \rightarrow \underline{\alpha+3\beta=3.03} \quad (2)$$

$$\boxed{\text{Using (1) and (2)} \quad \beta=0.97 \text{ and } \alpha=0.12}$$

$\boxed{\text{1b} - 15p}$

$$f_{xy}(x,y) = 0.12xy + 0.97$$

$$f_x(x) = \int_0^1 f_{xy}(x,y)\,dy$$

$$= \boxed{0.06x + 0.97}$$

$\boxed{\text{1c} - 15p}$

$$f_{x|y=0.3} = \frac{f_{xy}(x, 0.3)}{f_y(0.3)}$$

$$= \frac{0.12x \cdot 0.3 + 0.97}{0.06 \times 0.3 + 0.97}$$

$$= \boxed{0.364x + 0.9719}$$

$\boxed{2a - 15p}$

B: Bozuk olma olayı

Y: Yeni makina

E: Eski »

$P(B|Y) = 0.15$ $\qquad P(Y) = 1-p$

$P(B|E) = 0.45$ $\qquad P(E) = p$

$P(B) = 0.15(1-p) + 0.45p = 0.30p + 0.15$

$$P(B|Y) = \frac{0.15(1-p)}{0.15(2p+1)} = 0.4855$$

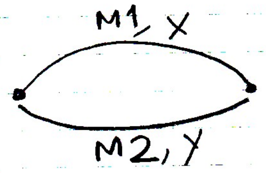
$\Longrightarrow \boxed{p = 0.2610}$

$\boxed{2b - 15p}$ $\qquad P(B'|Y) = 0.85 \quad P(B'|E) = 0.55$

$P(B') = 1 - (0.30 \times 0.2532 + 0.15) = 0.7740$

$$P(E|B') = \frac{P(B'|E)\,P(E)}{P(B')} = 0.1799$$

$\boxed{3a-15p}$



$(\lambda T)_x = 30 \times 0.23 = 6.9$

$(\lambda T)_y = 20 \times 0.52 = 10.4$

X: M_1'deki çukur sayısını gösteren R.D., $(\lambda T)_x = 6.9$

Y: M_2 " " " " " $(\lambda T)_y = 10.4$

M1: M1'i seçme olayı $P(M1) = 0.7$

M2: M2'yi " " $P(M2) = 0.3$

Ç: 2 veya 3 çukur görme olayı

$$P(Ç|M1) = P(X=12) + P(X=13)$$
$$= e^{-6.9}\left(\frac{6.9^{12}}{12!} + \frac{6.9^{13}}{13!}\right) = 0.0375$$

$$P(Ç|M2) = P(Y=12) + P(Y=13)$$
$$= e^{-10.4}\left(\frac{10.4^{12}}{12!} + \frac{10.4^{13}}{13!}\right) = 0.1831$$

$$P(Ç) = P(Ç|M_1)P(M1) + P(Ç|M2)P(M2)$$
$$= 0.0812$$

3-b 15p

$\lambda T = 5000 \times 0.23 = 1150 \gg 10 \qquad \sqrt{\lambda T} = 33.911$

$$P\left(\frac{475-0.5-1150}{33.911} < z < \frac{575+0.5-1150}{33.911}\right)$$

$$= P(-19.919 < z < -16.941) \approx 0$$