Random & Stochastic Processes - İÖ - 2015 Spring Midterm Solutions

1a (10)

$$\int_0^\infty k \cdot e^{-10x}\, dx = 1$$

$$-\frac{k}{10} e^{-10x} \Big|_0^\infty = 1$$

$$-\frac{k}{10}(0-1) = 1$$

$$\boxed{k = 10}$$

1b (15)

For $x \leq 0 \quad F(x) = 0$

$$x > 0 \quad F(x) = \int_0^x 10 \cdot e^{-10u} \cdot du$$

$$= -1\, e^{-10u} \Big|_0^x$$

$$= -1\,(e^{-10u} - 1)$$

$$F(x) = 1 - e^{-10u}$$

So,

$$\boxed{F(x) = \begin{cases} 1 - e^{-10u}, & x > 0 \\ 0, & x \leq 0 \end{cases}}$$

1c / 15

$$E(X) = \int_0^\infty x \cdot f(x)\,dx$$

$$= \int_0^\infty x \cdot 10 \cdot e^{-10x}\,dx$$

$$= 10 \cdot \left(\frac{-10x-1}{100}\right) e^{-10x} \Big|_0^\infty$$

$$= 10 \cdot \left(0 - \frac{-1}{100}\right)$$

$$\boxed{E(X) = \frac{1}{10}}$$

$$E(X^2) = \int_0^\infty x^2 \cdot 10 \cdot e^{-10x}\,dx$$

$$= 10 \left(\frac{100x^2-20x+2}{1000}\right) e^{-10x} \Big|_0^\infty$$

$$= \frac{10}{1000} \left(0 - \frac{2}{1000}\right)$$

$$E(X^2) = \frac{1}{50}$$

$$V(X) = E(X^2) - E^2(X)$$

$$= \frac{1}{50} - \frac{1}{100}$$

$$\boxed{V(X) = \frac{1}{100}}$$

1d / 10

$$E(h(X) = E(2X^2 - 2X - 1) = 2 \cdot \frac{1}{100} - 2 \cdot \frac{1}{10} - 1$$

$$\boxed{E(X) = \frac{31}{25}}$$

$\boxed{\frac{2a}{10}}$

$$\sum_{x=-\infty}^{1} 3^{cx} = 1$$

Let's say $x = -p$

$$\sum_{p=1}^{+\infty} 3^{-cp} = 1$$ (c must be a positive number)

$$\frac{3^{-c}}{1-3^{-c}} = 1$$

$$\left(\frac{1}{3}\right)^{c} = 1 - \left(\frac{1}{3}\right)^{c}$$

$$2 * \left(\frac{1}{3}\right)^{c} = 1$$

$$\ln\left(\frac{1}{3}\right)^{c} = \ln\left(\frac{1}{2}\right)$$

$$c = \frac{\ln(0.5)}{\ln(1/3)} =$$

$$\boxed{c = 0.6309}$$

$\boxed{\frac{26}{15}}$

$(3^c = 2)$

- $x \geq 1 \rightarrow F(x) = 1$
- $x < 0 \rightarrow F(x) = \sum_{k=-\infty}^{x} 2^k$

$$F(x) = \sum_{k=-\infty}^{x} 2^k = \sum_{k=-x}^{\infty} 2^{-k}$$

$$= \sum_{k=1}^{\infty} 2^{-k} - \sum_{k=1}^{-x-1} 2^{-k}$$

$$= \frac{0.5}{0.5} - \left(\frac{0.5 - 0.5^{-x}}{1-0.5} \right)$$

$$= \frac{0.5 - 0.5 + 0.5^{-x}}{0.5}$$

$$F(x) = 0.5^{-x-1} \quad (x < 0)$$

$$\therefore \quad \boxed{F(x) = \begin{cases} 2^{x+1} , & x < 0 \\ 1 , & x \geq 1 \end{cases}}$$

2c / **15**

$$E(X) = \sum_{x=-\infty}^{-1} x\, 3^{cx} \qquad (3^{-c} = 0.5)$$

$$= \sum_{x=1}^{\infty} (-x) \cdot 3^{-cx}$$

$$= \frac{-3^{-c}}{(1-3^{-c})^2} = \frac{-0.5}{(0.25)^2}$$

$$\boxed{E(X) = -2}$$

$$E(X^2) = \sum_{x=-\infty}^{-1} x^2 \cdot 2^{x}$$

$$= \sum_{x=1}^{\infty} x^2 \cdot 0.5^{x}$$

$$= \frac{0.5 \cdot 1.5}{(0.5)^3}$$

$$E(X^2) = 6$$

$$V(X) = E(X^2) - E^2(X)$$

$$= 6 - 4$$

$$\boxed{V(X) = 2}$$

2d / **10**

$$E(h(X)) = 3 \cdot 6 + 5 \cdot (-2) + 6$$

$$\boxed{E(h(X)) = 14}$$