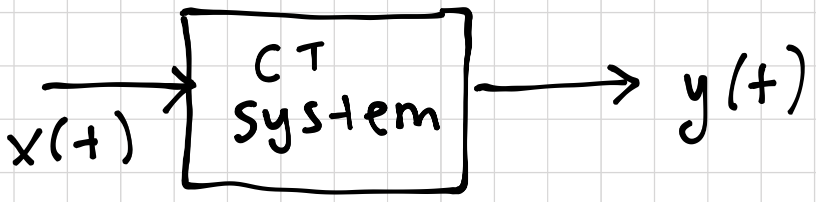
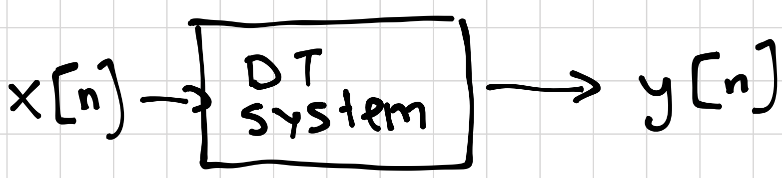


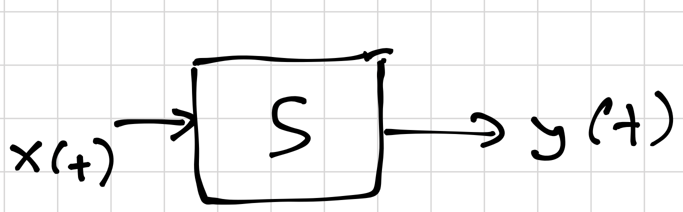


$$y(t) = \mathcal{H}\left\{x(t)\right\}$$

describes the system

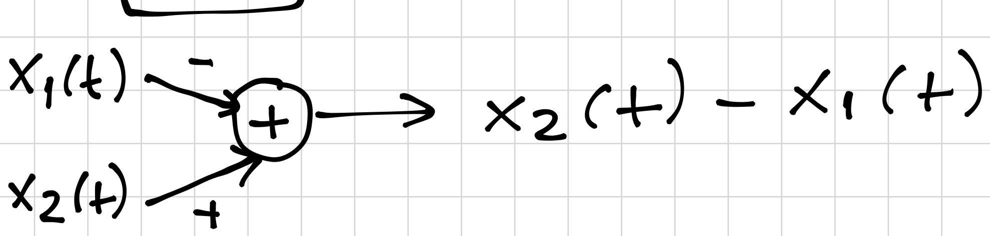
Interconnection of Systems

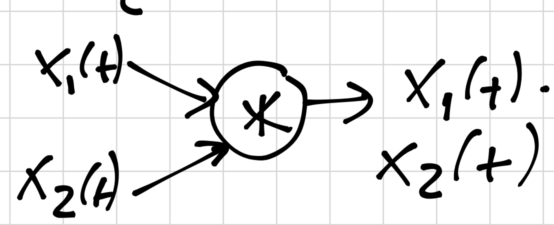
We can view the system as interconnection of operations. We can also represent the systems using block diagrams.



:

$$y(t) = S\left\{x(t)\right\}$$





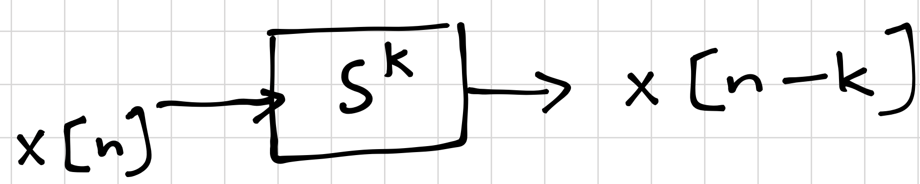
Ex : Moving Average System

Consider a DT system

$$y[n] = \frac{1}{3}\left(x[n] + x[n-1] + x[n-2]\right)$$

Show a block diagram representation of this system.

Let S^k denote the following:



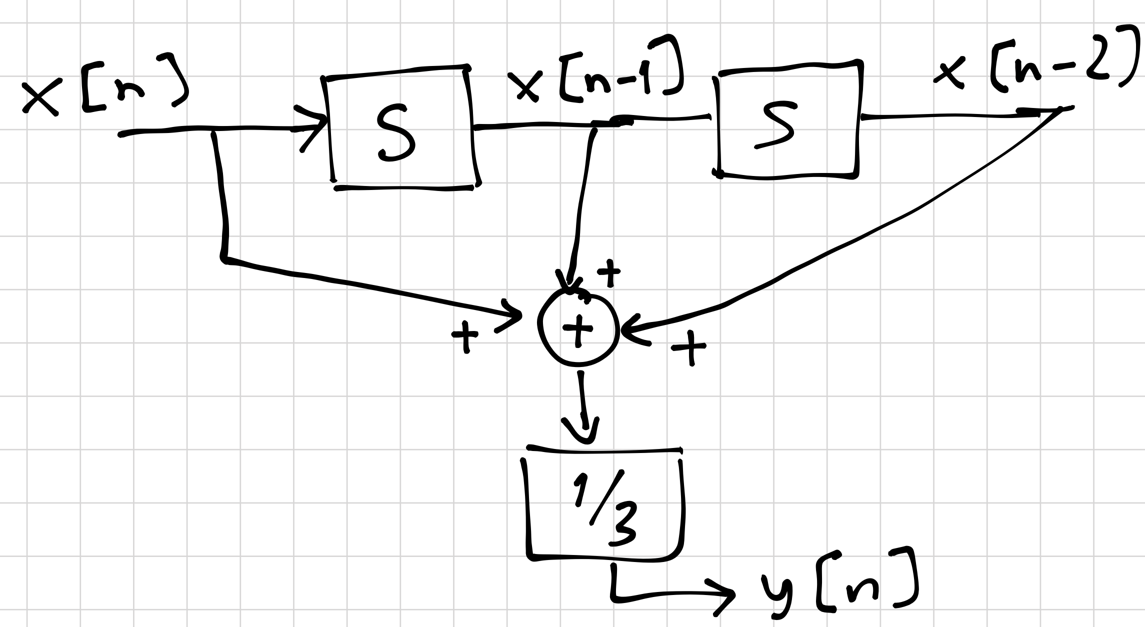
$$y[n] = \mathcal{H}\{x[n]\}$$

$$\mathcal{H} = \frac{1}{3}\left[S^0 + S^1 + S^2\right]$$

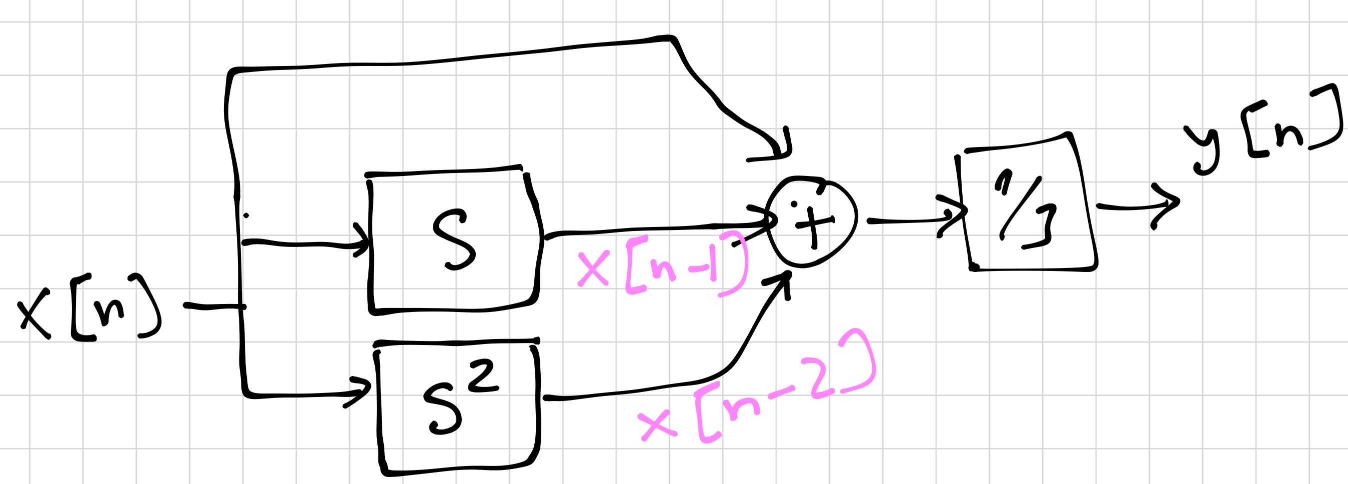
$$= \tfrac{1}{3}\left[1 + S + S^2\right]$$

We can create two different but equivalent implementations.

Cascade Form



Parallel form



Properties of Systems.

① Stability.

A system is <u>bounded-input bounded-output</u> (BIBO) stable <u>iff</u> every bounded input results in a bounded output.

Formally for a system, defined as $y(t) = \mathcal{H}\{x(t)\}$, is BIBO-stable

if $$|y(t)| \leq M_y < \infty \quad \text{for } \forall t,$$ M_y is some finite positive number

<u>when</u> $$|x(t)| \leq M_x < \infty \quad \text{for } \forall t$$

M_x is some finite positive number.

- Same applies to a DT system.

Ex. $$y[n] = \frac{1}{3}\left(x[n] + x[n-1] + x[n-2]\right) = \mathcal{H}\{x[n]\}$$

Is this system stable?

Assume $|x[n]| \leq M_x < \infty \quad \forall n \in \mathbb{Z}$

$$|y[n]| = \left|\frac{1}{3}\left(x[n] + x[n-1] + x[n-2]\right)\right|$$

/* $a + b = c$

$|a| + |b| \geq |c|$ */

$$|y[n]| \leq \frac{1}{3}\left(\underbrace{|x[n]|}_{\leq M_x} + \underbrace{|x[n-1]|}_{\leq M_x} + \underbrace{|x[n-2]|}_{\leq M_x}\right)$$

$$\leq \frac{1}{3}\left(M_x + M_x + M_x\right) = M_x$$

$$|y[n]| \leq M_x < \infty$$

$\boxed{\mathcal{H} \text{ is STABLE.}}$

M_x

$$y[n] = \mathcal{H}\{x[n]\} = r^n\, x[n] \quad , r > 1$$

Show that this system is unstable.

- Assume $|x[n]| \leqslant M_x < \infty$

$$|y[n]| = |r^n \cdot x[n]|$$

$$= |r^n| \cdot \underbrace{|x[n]|}_{M_x}$$

$$|y[n]| \leqslant \boxed{|r^n|} \cdot M_x$$

r^n goes to infinity as $n \to \infty$, so we cannot guarantee $|y[n]|$ will be finite. Therefore $\mathcal{H}$ is unstable

If $0 < r \leqslant 1$ $\quad n \to \infty \quad r^n$ converges to zero.

Ex

$$y(t) = \frac{1}{t-1} \cdot x(t) \qquad \text{unstable}$$

② Memory

A system is said to be memoryless if its output signal depends on the current value of its input signal.

Ex

$$y[n] = \left(2\, x[n] - x^2[n]\right)^2$$

Memoryless system.

Ex

$$y[n] = x[n-1] \quad \rightarrow \text{has memory.}$$

Ex

$$y[n] = (n-1)\, x[n] \quad \rightarrow \text{Memoryless system.}$$

③ Causality

A system is "causal" if the current output of the system depend only on past and/or present values of the input.

$$y[n] = x^2[n-1] + x[n] \quad : \text{causal}$$

$$y(t) = x(t+\underline{1}) \quad : \text{non-causal.}$$

↳ 1 second in the future

4. Invertibility

A system is invertible if distinct inputs lead to distinct outputs, that is the input of the system can be recovered from the output.

$$x(t) \to \boxed{\mathcal{H}} \xrightarrow{y(t)} \boxed{\mathcal{H}^{inv}} \to x(t)$$

If H^{inv} exists then $\mathcal{H}$ is invertible.

$$\mathcal{H}^{inv}\{y(t)\} = \mathcal{H}^{inv}\{\mathcal{H}\{x(t)\}\}$$

$$= \mathcal{H}^{inv}\mathcal{H}\{x(t)\}$$

$$I = \mathcal{H}^{inv} \cdot \mathcal{H} \quad : \text{identity system.}$$

$$I\{x(t)\} = x(t)$$

Ex

$$\left.\begin{aligned} y(t) &= 2x(t) = \mathcal{H}\{x(t)\} \\ x(t) &= \frac{1}{2}y(t) = \mathcal{H}^{inv}\{y(t)\} \end{aligned}\right\} \mathcal{H} \text{ is invertible}$$

Ex

$$y(t) = x^2(t) = \mathcal{H}\{x(t)\}$$

Since $x(t)$ and $-x(t)$ produce the same output $\mathcal{H}$ is not invertible.

5 Time Invariance

A system is said to be time invariant if a time delay or time advance of the input signal leads to an identical time shift in the output signal.

Consiter a system

$$y(t) = \mathcal{H}\{x(t)\}$$

If for any $t_0 \in \mathbb{R}$

$$y(t-t_0) = \mathcal{H}\{x(t-t_0)\}, \quad \forall t$$

then $\mathcal{H}$ is time invariant.
Otherwise $\mathcal{H}$ is time-variant.

Ex

$$y(t) = \mathcal{H}\{x(t)\} = \int_{-\infty}^{t} x(\tau)\, d\tau$$

is $\mathcal{H}$ Time-Invariant?

Let's first shift the input

$$y_2(t) = \mathcal{H}\{x(t-t_0)\}$$

$$y_2(t) = \int_{-\infty}^{t} x(\tau - t_0)\, d\tau$$

$$y_1(t) = y(t-t_0) = \int_{-\infty}^{t-t_0} x(\tau)\, d\tau$$

$$\tau' \triangleq \tau - t_0 \qquad \tau = \tau' + t_0$$

$$d\tau' = d\tau$$

$$y_2(t) = \int_{-\infty}^{t-t_0} x(\underbrace{\tau' + t_0}_{\tau} - t_0)\, d\tau'$$

$$= \int_{-\infty}^{t-t_0} x(\tau')\, d\tau' = y_1(t)$$

Therefore $\mathcal{H}$ is $\boxed{\text{time-invariant}}$

Ex $y[n] = \mathcal{H}\{x[n]\} = r^n \cdot x[n]$ T.I ?

$y_1[n] = \mathcal{H}\{x[n-n_0]\} = r^n \, x[n-n_0]$

$y_2[n] = y[n-n_0] = r^{n-n_0} \cdot x[n-n_0]$

$y_1[n] \neq y_2[n]$ $\therefore$ not T.I

Time-Variant System.

$\boxed{\text{Adobe Scan}}$ } PDF $\rightarrow$ Share $\rightarrow$ piazza

numara.pdf

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⑥ Linearity

A system $\mathcal{H}$, is said to be linear if it satisfies the following two conditions.

6.1 Superposition

Let for any signal $x_1(t)$ and $x_2(t)$

$$y_1(t) = \mathcal{H}\{x_1(t)\}$$
$$y_2(t) = \mathcal{H}\{x_2(t)\}$$

Let $y(t) = y_1(t) + y_2(t)$

and $x(t) = x_1(t) + x_2(t)$

If $y(t) = \mathcal{H}\{x(t)\}$ then the system $\mathcal{H}$ satisfies the principle of superposition.

6.2 Homogeneity

Let $y(t) = \mathcal{H}\{x(t)\}$

If $\alpha\, y(t) = \mathcal{H}\{\alpha\, x(t)\}$ for $\forall \alpha \in \mathbb{R}$

Then $\mathcal{H}$ satisfies the property of homogeneity.

Let $x(t) = \sum_{i=1}^{N} \alpha_i \cdot x_i(t)$

where $\alpha_1, \alpha_2, \ldots, \alpha_N$ are constants, $N \in \mathbb{Z}^+$

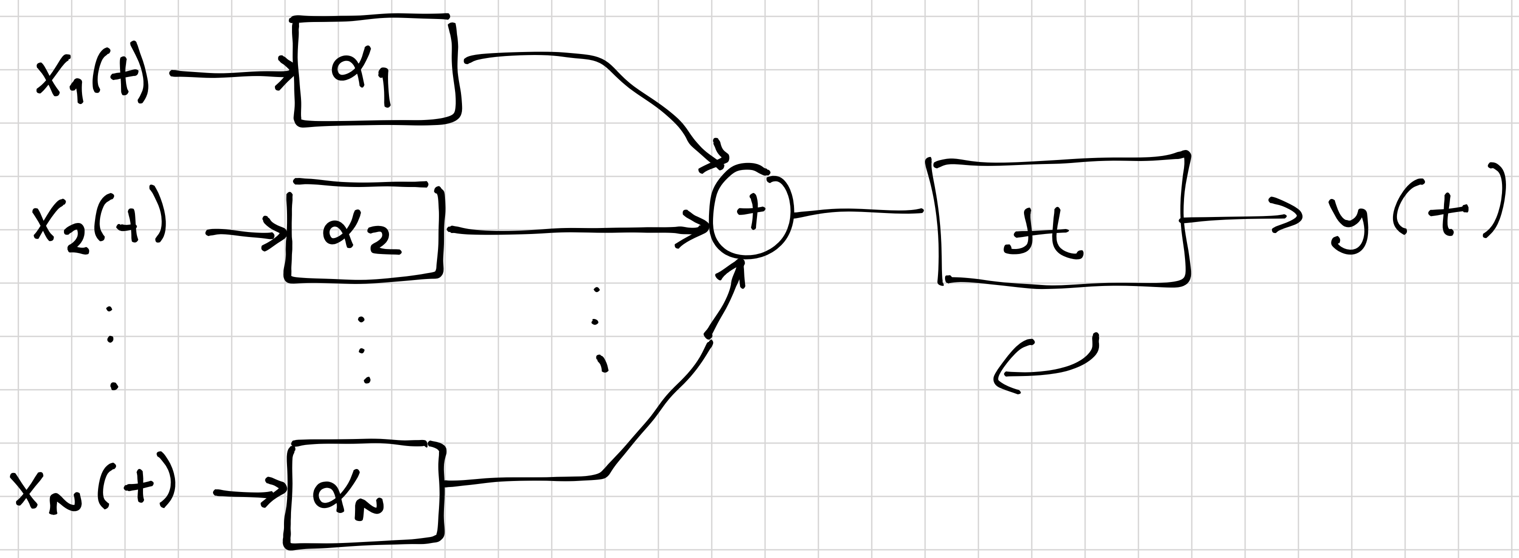
$$y(t) = \mathcal{H}\{x(t)\} = \mathcal{H}\left\{ \sum_{i=1}^{N} \alpha_i \, x_i(t) \right\}$$

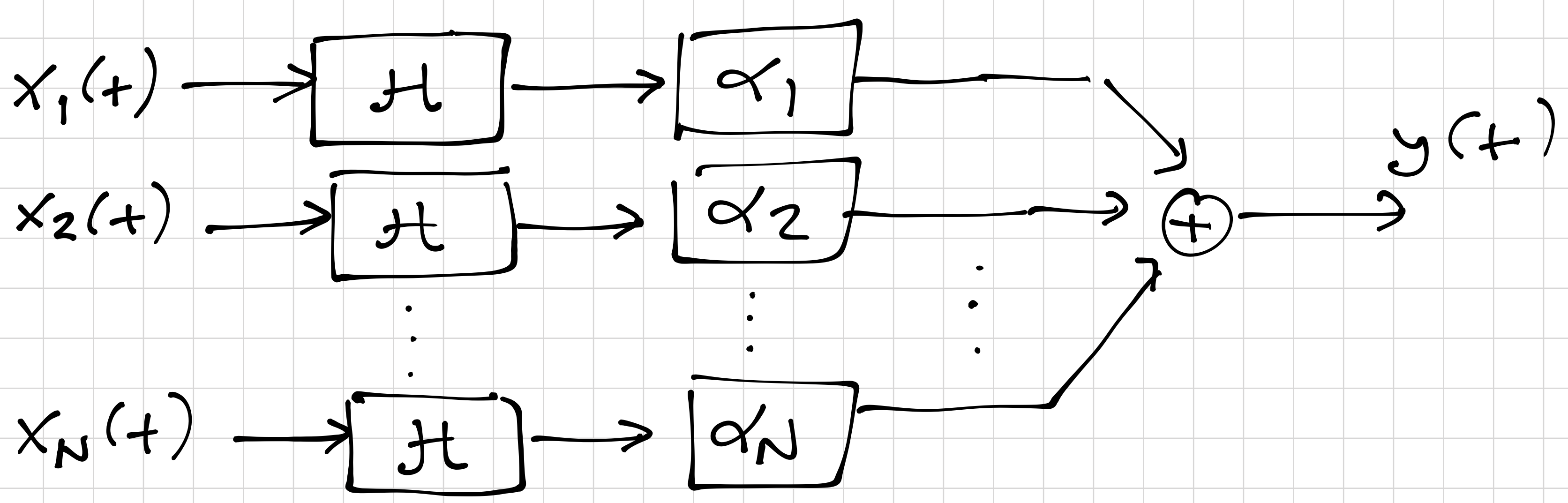
If the system is linear then we can express the output of the system as

$$y(t) = \sum_{i=1}^{N} \alpha_i \, y_i(t)$$

where

$$y_i(t) = \mathcal{H}\{x_i(t)\} \quad , i = 1, \ldots, N$$





Ex

$$y(t) = (t+1)^2 \, x(t) \qquad \text{Linear?}$$

$\boxed{\text{Homogeneity}}$

$$\mathcal{H}\{\alpha \cdot x(t)\} = (t+1)^2 \, \alpha \cdot x(t)$$
$$= \alpha \left[(t+1)^2 x(t) \right] = \alpha \cdot y(t)$$

Homogeneity is satisfied.

Superposition

$$\mathcal{H}\{x_1(t)\} = y_1(t)$$
$$\mathcal{H}\{x_2(t)\} = y_2(t)$$

$$\mathcal{H}\{x_1(t) + x_2(t)\} = (t+1)^2\left[x_1(t) + x_2(t)\right]$$

$$= \underbrace{(t+1)^2 x_1(t)} + \underbrace{(t+1)^2 x_2(t)}$$

$$= y_1(t) + y_2(t)$$

$\therefore$ Superposition satisfied.

$\therefore$ This system is LINEAR.