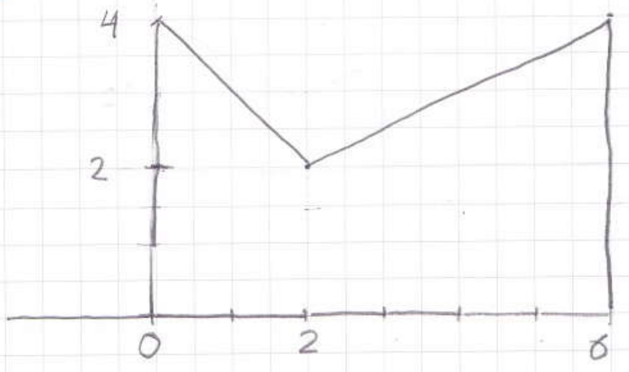
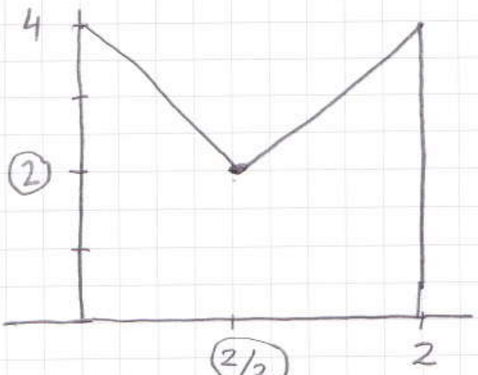
Signal Processing - İ.Ö - 2015 Midterm Exam Solutions

$\boxed{1a - 15}$

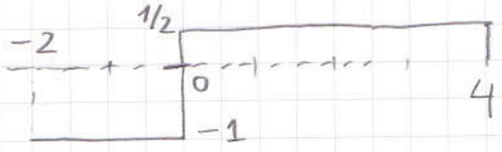


$x(t-2)$



$x(3t-2)$

1b-10



1c-15

$$E = \int_{-2}^{0} (-1)^2 \, dt + \int_{0}^{4} \left(\tfrac{1}{2}\right)^2 dt$$

$$E = t\Big|_{-2}^{0} + \frac{1}{4} t\Big|_{0}^{4}$$

$$= (0-(-2)) + \frac{1}{4}(4-0) = 3$$

Energy Signal

(2) $y[n] = \log_{10}(|x[n]| + 1)$

i) (3) $\boxed{\text{memoryless}}$

ii) 3 $0 \leqslant |x[n]| \leqslant M_x$

Since $|x[n]| \geqslant 0$ $\quad \log_{10}(|x[n]| + 1) \geqslant 0 \quad \forall n$

$$|y[n]| = |\log_{10}(|x[n]| + 1) \leqslant \log_{10}(M_x + 1)$$
$$= M_y \leqslant \infty$$

$\boxed{\text{STABLE}}$

iii) 3 $\boxed{\text{CAUSAL}}$

iv) 3 Homogenity:

$$\mathcal{H}\{a.x[n]\} = \log_{10}(|a x[n]| + 1)$$
$$\neq a \cdot \log_{10}(|x[n]| + 1) \quad \therefore \boxed{\text{NOT-LINEAR}}$$

v) 3

$$\mathcal{H}\{x[n-n_0]\} = \log_{10}(|x[n-n_0]| + 1]$$
$$= y[n-n_0] \quad \therefore \boxed{\text{TIME-INVARIANT}}$$

$$y(t) = \frac{d}{dt}\{e^{-t}x(t)\}$$

a) Definition of a derivative

$$f'(x) = \lim_{a \to 0} \frac{f(x+a) - f(x)}{a}$$

Since $y(t)$ needs to look into the future values of $x[n]$, $\boxed{\mathcal{H} \text{ is. NOT-MEMORYLESS}}$

b) $$\lim_{t \to -\infty} |e^{-t}| = \infty \quad \therefore \quad \boxed{\text{NOT-STABLE}}$$

c) $\boxed{\text{NOT-CAUSAL}}$

d) Homogenity

$$H\{a x(t)\} = \frac{d}{dt}\left(e^{-t} a x(t)\right) = a \cdot \frac{d}{dt}\left(e^{-t} x(t)\right)$$
$$= a \cdot y(t)$$

Superposition

$$H\{x_1(t) + x_2(t)\} = \frac{d}{dt}\{e^{-t}(x_1(t) + x_2(t))\}$$

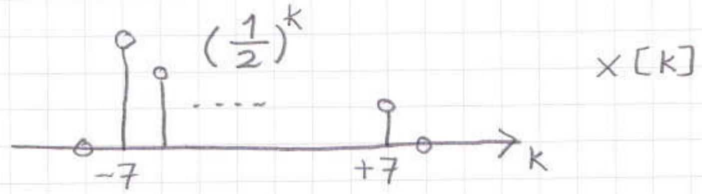
$$= \frac{d}{dt}\{e^{-t} x_1(t)\} + \frac{d}{dt}\{e^{-t} x_2(t)\} = y_1(t) + y_2(t)$$

$\therefore$ $\boxed{\text{LINEAR}}$

e) $$\mathcal{H}\{x(t-t_0)\} = \frac{d}{dt}\left(e^{-t} x(t-t_0)\right) \neq \frac{d}{dt}\left(e^{-(t-t_0)} x(t-t_0)\right)$$

$\therefore$ $\boxed{\text{NOT TI}}$

3-30p



• $n-2 < -7 \rightarrow n < -5 \rightarrow y[n] = 0$

$-7 \leq n-2 < 7 \rightarrow -5 \leq n < 9$

$$y[n] = \sum_{k=-7}^{n-2} \left(\frac{1}{2}\right)^k 2^{n-k} = 2^n \sum_{k=-7}^{n-2} \left(\frac{1}{2}\right)^{2k} = 2^n \sum_{k=-7}^{n-2} \left(\frac{1}{4}\right)^k$$

$$= 2^n \frac{\left(\frac{1}{4}\right)^{-7} - \left(\frac{1}{4}\right)^{n+1}}{1 - 1/4}$$

$$= \boxed{\frac{4 \cdot 2^n}{3} \left(4^7 - 4^{-(n+1)}\right)}$$

$$n \geq 9 \rightarrow \quad y[n] = 2^n \sum_{k=-7}^{7} \left(\frac{1}{2}\right)^{2k} = 2^n \frac{(1/4)^{-7} - (1/4)^8}{1 - 1/4}$$

$$= 2^n \frac{16384 - 15.25 \times 10^{-6}}{3/4}$$

$$= \boxed{21845.33 \times 2^n}$$