Random and Stochastic Processes - 2. Öğretim - Midterm Exam

İstanbul University Computer Engineering - Spring 2013

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LÜTFEN OKUYUN: Bu sınavın süresi 90 dakikadır. Lütfen soruları TÜRKÇE kısa ve anlaşılır olarak cevaplayınız. Anlaşılmayan, muğlak ifadeler kullanmak, kötü yazı yazmak notunuza negatif olarak etki edecektir. Sınavda 1 adet hesap makinasi, HER İKİ YÜZÜNE notlarinizi el yazinizla yazdiginiz fotokopi/printout olmayan 1 adet A4 sayfası ve Kitaptan Appendix A Table III'de CSNF'i gösteren 2 sayfalık tabloyu kullanabilirsiniz. **SINAVDA KENDİ CEVAP KAĞITLARINIZI KULLANABİLİRSİNİZ. BU KAĞITLAR KARELİ A4 OLMALIDIR**. Bunların dışında her türlü defter, kitap, notlar, sözlük ve elektronik sözlük yasaktır. Materyalin paylaşılması yasaktır. **Hesap makinasi ve silgi paylaşmak kopya sayılacaktır!** Bilgisayar, PDA, cep telefonu türünden elektronik cihazlar kullanmak yasaktır. Soruları çözmeye başlamadan lütfen okuyun. Soru kağıtlarını çıkarken LÜTFEN KAĞIDINIZLA BİRLİKTE İADE EDİNİZ. Bu sınavda toplam 100 puanlık soru vardır. **SINAVDA KOPYA ÇEKENLER, KOPYA VERENLER VE BUNLARA TEŞEBBÜS EDENLER SINAVDAN "0" ALACAKTIR VE DEKANLIĞA ŞİKAYET EDİLECEKLERDİR!**

Q1: Consider the following probability density function for a CT random variable, X. Answer the following questions. *Hint:* $\int \ln(x)dx = (x-1)\ln(x)$

$$f(x) = \begin{cases} \beta \ln(x) & 1.2 < x < 1.8 \quad \beta \in \Re^+ \\ 0 & \text{otherwise} \end{cases}$$

(a) (`10 pts`) Calculate the value of β.

(b) (`10 pts`) Calculate the expected value and standard deviation of X.

(c) (`15 pts`) Consider a function of X, given as $h(X) = X^2 + X + 1$. Calculate the expected value and variance of $h(X)$.

Q2: According to FIFA regulations, a football must have a circumference (*çevre*) of 69 cm with an error margin of ± 1 cm and the ball pressure must be 0.8 bar with an error margin of ± 0.1 bar. A company which manufactures footballs and the circumference and pressure of the balls that they produce follow a normal distribution. The circumference has a mean of 69 cm and standard deviation of 0.96 cm; the pressure has a mean of 0.8 bar with a standard deviation of 0.12 bars. Assume that the ball pressure and circumference are independent.

(a) (`15 pts`) What is the probability that any randomly chosen ball will fulfill FIFA's circumference requirement?

(b) (`15 pts`) What is the probability that any randomly chosen ball will fulfill FIFA's pressure requirement?

(c) (`5 pts`) What is the probability that any randomly chosen ball will fulfill FIFA's both requirements?

Q3: Asbestos fibers in a dust sample are identified by an electron microscope after sample preparation. Suppose that the number of fibers is a Poisson random variable and the mean number of fibers per square centimeter of surface dust is 100. A sample of 8000 square centimeters of dust is analyzed. Assume a particular grid cell under the microscope represents 1/16,000 of the sample (*i.e. A grid cell for this particular sample is 8000/16,000 square centimeter*).

(a) (`15 pts`) What is the probability that no more than 45 fiber is visible in the grid cell? (Use Normal Approximation and apply continuity correction)

(b) (`15 pts`) If 100 grid cells are analyzed and the fibers on grid cells are independent from each other what is the probability that at most 20 grid cells would have no more than 45 fibers visible on them? (Use Normal Approximation and apply continuity correction)