Signal Processing (Örgün Öğretim)
Final Exam Solutions

Istanbul University - Computer Engineering Department - FALL 2016

January 5$^{\text{th}}$, 2017

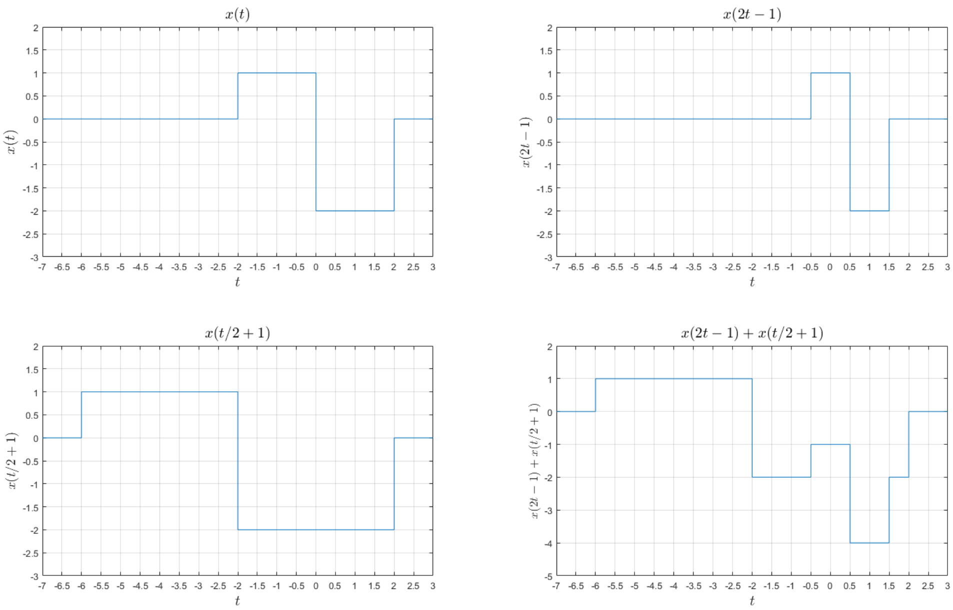
Q1: Consider the following CONTINUOUS TIME signals and answer the following questions.

$$x(t) = \begin{cases} 1 & , \quad -2 \le t < 0 \\ -2 & , \quad 0 \le t < 2 \\ 0 & , \quad \text{elsewhere} \end{cases}$$

$$w(t) = \sum_{k=-\infty}^{\infty} x(t-4k)$$

(a) `(10 pts)` Please carefully sketch $x(2t-1) + x(\frac{t}{2}+1)$. Show your steps to receive credit.

Solution 1a:



(b) `(10 pts)` Please determine whether $w(t)$ is an energy or power signal. Calculate its power or energy, whichever applies.

Solution 1b:

$w(t)$ is periodic with T=4 seconds.

$$P = \frac{1}{4}\Big(\int_{-2}^{0} 1^2 \mathrm{d}t \ + \ \int_{0}^{2} (-2)^2 \mathrm{d}t\Big)$$
$$P = 2.5 \qquad \blacksquare$$

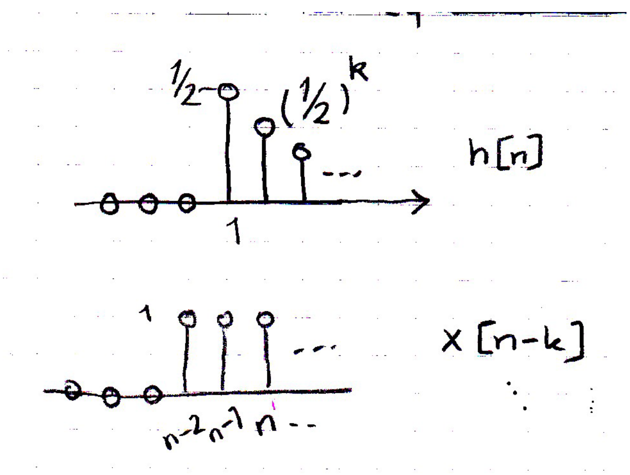
Q2: `(25 pts)` Find the DISCRETE TIME convolution sum of the following two signals.

$$x[n] = u[2-n]$$
$$h[n] = \left(\frac{1}{2}\right)^n \times u[n-1]$$

Solution 2:

Since $x[n] * h[n] = h[n] * x[n]$, we can flip and shif $x[n]$, isntead of $h[n]$, since it would be asier to do it this way.

$$x[n-k] = u[2-(n-k)]$$
$$= u[k-(n-2)]$$



For $n-2 < 1$, which is $n < 3$

$$y[n] = \sum_{k=1}^{\infty} \left(\frac{1}{2}\right)^k = 1$$

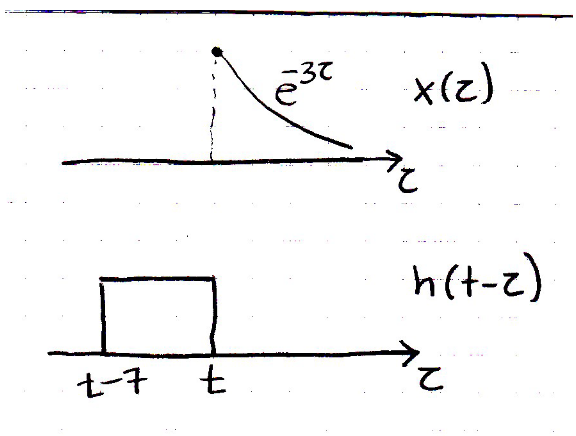
For $n \geq 3$

$$y[n] = \sum_{k=n-2}^{\infty} \left(\frac{1}{2}\right)^k = 2^{3-n}$$

Q3: `(25 pts)` Find the CONTINUOUS TIME convolution integral of the following two signals.

$$x(t) = e^{-3t} \times u(t)$$
$$h(t) = u(t) - u(t-7)$$

Solution 3:



For $t < 0$,

$$y(t) = 0$$

For $0 \leq t < 7$,

$$y(t) = \int_0^t \mathrm{e}^{-3\tau} \mathrm{d}\tau = \frac{1}{3}(1 - \mathrm{e}^{-3t})$$

For $t \geq 7$,

$$y(t) = \int_{t-7}^{t} \mathrm{e}^{-3\tau} \mathrm{d}\tau = \frac{1}{3}(\mathrm{e}^{21-3t} - \mathrm{e}^{-3t})$$

Q4: `(15 pts)` Based on the impulse response, $h[n]$, given in Q2, determine the step response of the corresponding system.

Solution 4:

For $n < 1$,

$$s[n] = 0$$

For $n \geq 1$

$$s[n] = \sum_{k=1}^{n} (\frac{1}{2})^k = 1 - 2^{-n}$$

Q5: `(15 pts)` Based on the impulse response, $h(t)$, given in Q3, determine the step response of the corresponding system.

Solution 5:

For $t < 0$

$$s(t) = 0$$

For $0 \leq t < 7$,

$$s(t) = \int_0^t 1 \; \mathrm{d}\tau = t$$

For $t \geq 7$,

$$s(t) = \int_0^7 1 \; \mathrm{d}\tau = 7$$