

# COUNTERS

Counters WITH INPUTS

Kinds of Counters

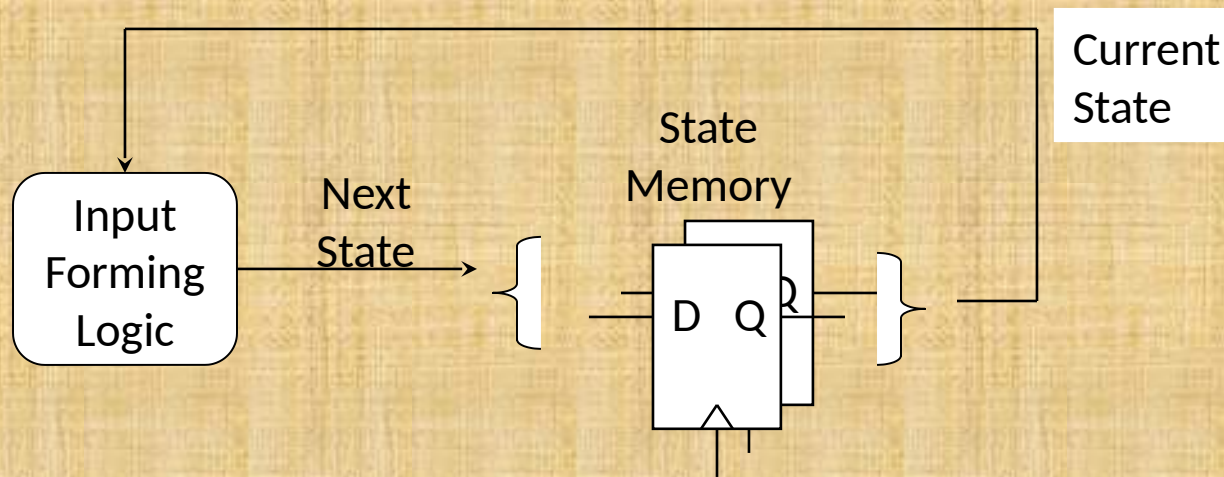
Synchronous Counters

Ripple Counters (Asynchronous counter)

Cascaded Counters (Modulo counter)

Hybrid counters

# General Sequential Systems



# Counters

- Counters are registers that go through a predefined sequence of states when inputs are applied
- Many counters follow the binary number sequence
  - For example, a 3-bit binary ripple counter goes through the following state transitions when the clock is asserted:  
 $000 \rightarrow 001 \rightarrow 010 \rightarrow 011 \rightarrow 100 \rightarrow 101 \rightarrow 110 \rightarrow 111 \rightarrow 000 \dots$
- Counters are said to overflow when their sequence is complete
- Overflow causes counters to repeat a sequence of values over time



# Types of Counters

- Two types of counters exist:

1. Synchronous Counters

2. Ripple Counters

Synchronous counters are triggered by a common clock

Ripple counters use flip-flop output transitions to serve as the trigger source for other flip-flops

- For example, a 1 to 0 transition on flip-flop <sub>$i$</sub>  triggers a toggling transition on flip-flop <sub>$i+1$</sub>

Ripple counters are not triggered by a common clock

# Synchronous Counters

- In a synchronous counter, all flip flops are clocked by the same clock signal
  - They all change at the same time
- Synchronous counters can be cascaded to create larger counters that are also globally synchronous

# Transition Table for 2-Bit Counter

Current State	Next State
00	01
01	10
10	11
11	00

Current State		Next State	
Q1	Q0	N1	N0
0	0	0	1
0	1	1	0
1	0	1	1
1	1	0	0

It is the truth table for the input forming logic...

It describes what the *next state* values  
are as a function of the *current state*  
(clock is assumed)



# Implementation of 2-Bit Counter

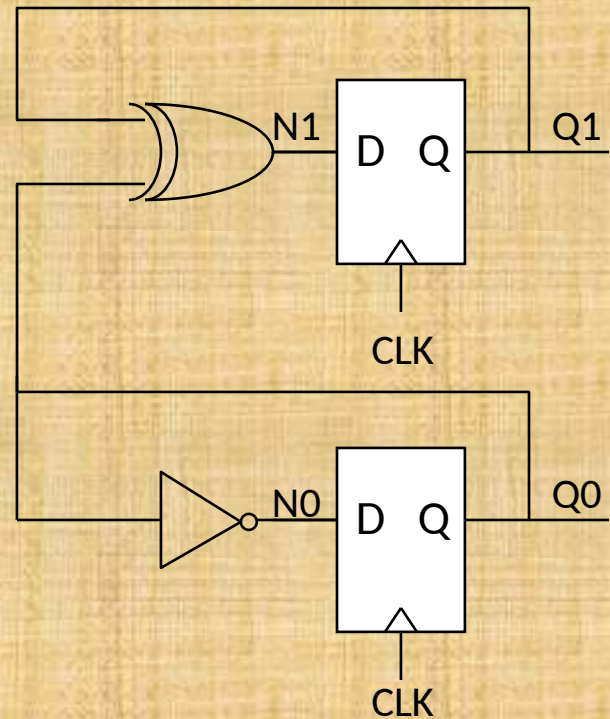
Current State		Next State	
Q1	Q0	N1	N0
0	0	0	1
0	1	1	0
1	0	1	1
1	1	0	0

Q0 \ Q1	0	1
0	1	1
1	0	0

$N0 = Q0'$

Q0 \ Q1	0	1
0	0	1
1	1	0

$N1 = Q1 \oplus Q0$



# Example 2 – A Gray Code Counter

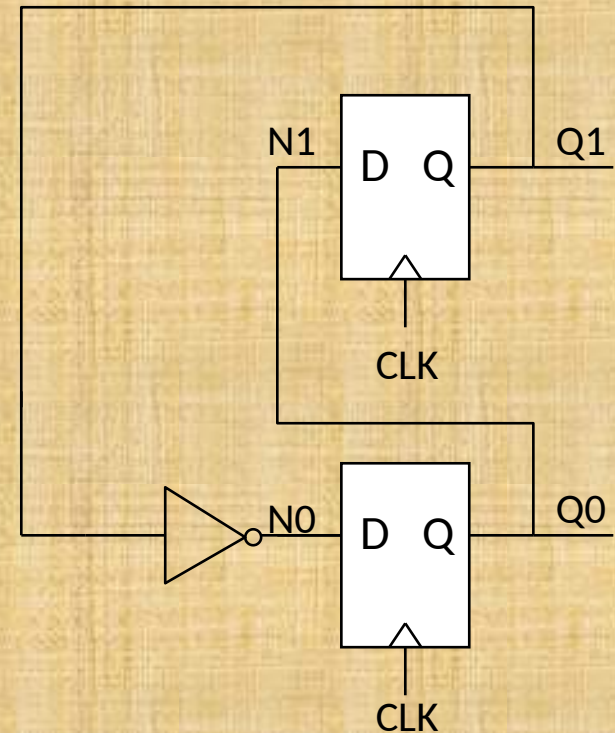
Q1	Q0	N1	N0
0	0	0	1
0	1	1	1
1	0	0	0
1	1	1	0

		Q1	
Q0		0	1
		0	0
	1	1	1

$$N1 = Q0$$

		Q1	
Q0		0	1
		1	0
	1	1	0

$$N0 = Q1'$$





# Example 3 – Not All Count Values Used

Desired count sequence = 00 - 01 - 11 - 00 ...

Q1	Q0	N1	N0
0	0	0	1
0	1	1	1
1	0	?	?
1	1	0	0

What should next state for 10 be?

# Example 3 – Not All Count Values Used

Q1	Q0	N1	N0
0	0	0	1
0	1	1	0
1	0	X	X
1	1	0	0

		Q1	
		0	1
Q0	0	0	X
	1	1	0

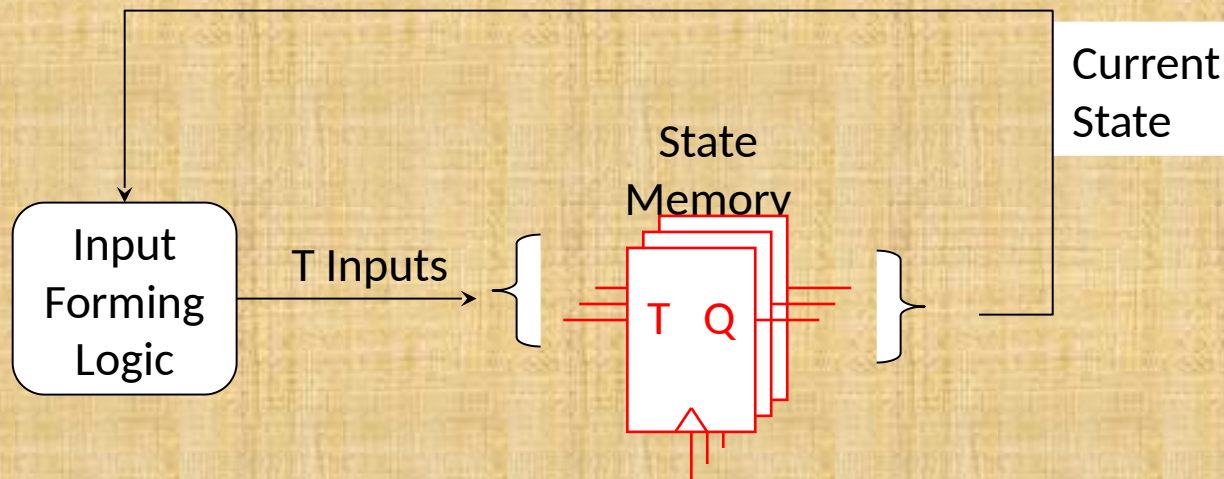
$$N1 = Q0 \bullet Q1'$$

		Q1	
		0	1
Q0	0	1	X
	1	0	0

$$N0 = Q0'$$

Do the normal K-map minimization with don't cares

# Counters With Alternative FF's





# Counter Design Procedure

## Introduction

The process is a special case of the general sequential circuit design procedure.

no decisions on state assignment or transitions  
current state is the output

*Example:* 3-bit Binary Upcounter

*Decide to implement with  
Toggle Flipflops*

Present state			Next state		
0	0	0	0	0	1
0	0	1	0	1	0
0	1	0	0	1	1
0	1	1	1	0	0
1	0	0	1	0	1
1	0	1	1	1	0
1	1	0	1	1	1
1	1	1	0	0	0

What inputs must be presented to the T FFs to get them to change to the desired state bit?

We need to use the T FF excitation table to translate the present/next state values to FF inputs

# TFF Counter Design Using Augmented Transition Table

Current State			Next State			TFF Inputs		
Q2	Q1	Q0	N2	N1	N0	T2	T1	T0
0	0	0	0	0	1			
0	0	1	0	1	0			
0	1	0	0	1	1			
0	1	1	1	0	0			
1	0	0	1	0	1			
1	0	1	1	1	0			
1	1	0	1	1	1			
1	1	1	0	0	0			

Next state values

Inputs to apply to achieve  
desired next state

# TFF Counter Design Using Augmented Transition Table

Current State			Next State			TFF Inputs		
Q2	Q1	Q0	N2	N1	N0	T2	T1	T0
0	0	0	0	0	1	0	0	1
0	0	1	0	1	0	0	1	1
0	1	0	0	1	1	0	0	1
0	1	1	1	0	0	1	1	1
1	0	0	1	0	1	0	0	1
1	0	1	1	1	0	0	1	1
1	1	0	1	1	1	0	0	1
1	1	1	0	0	0	1	1	1

**T2**

	<b>Q<sub>2</sub></b>	
	0	1
<b>Q<sub>1</sub>Q<sub>0</sub></b>	00	
	01	
	11	1
	10	

$$T2 = Q1 \bullet Q0$$

$$T1 = Q0$$

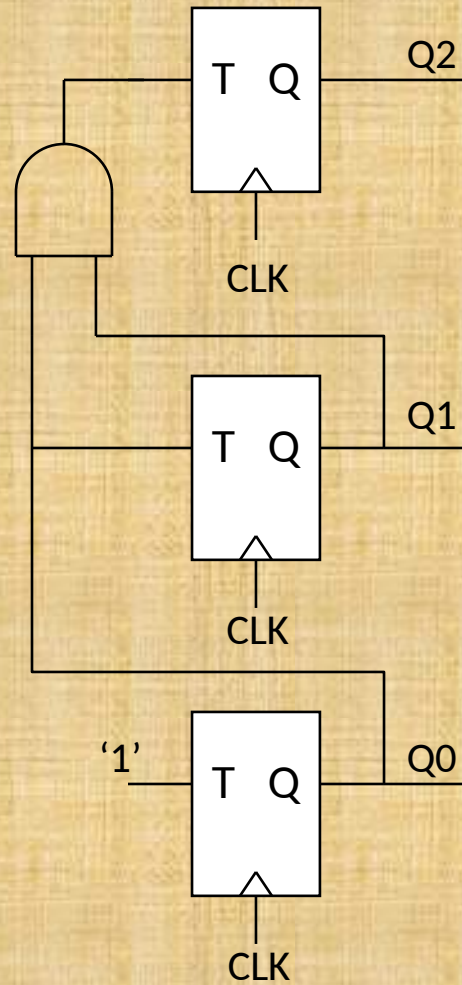
$$T0 = '1'$$

Next state values

Inputs to apply to achieve desired next state



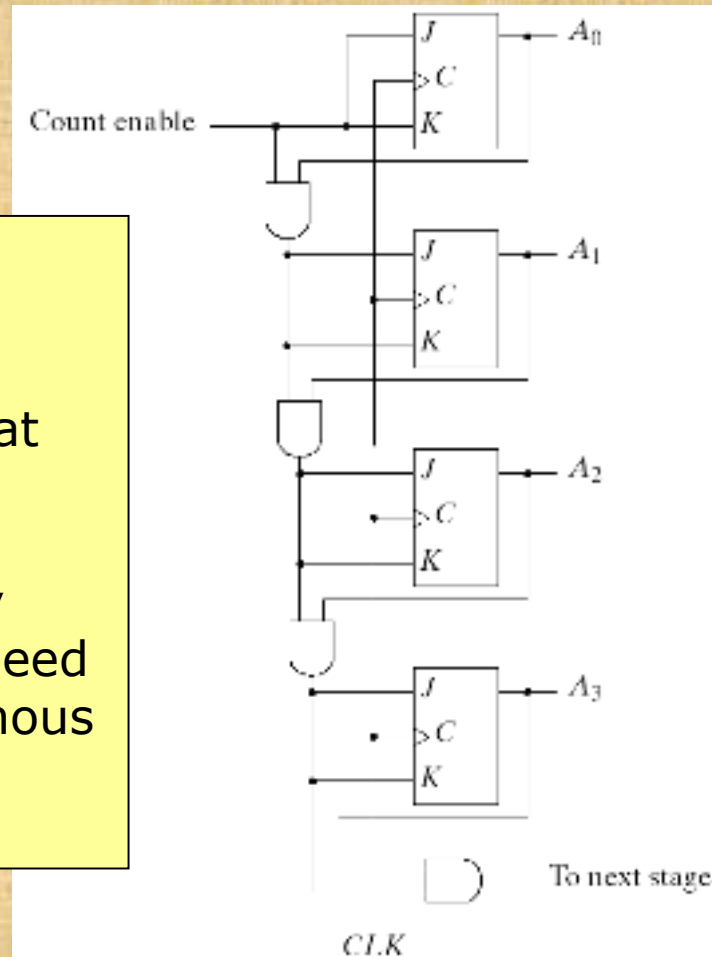
# TFF Counter Design



# 4-Bit Synchronous Binary Counter

## NOTE:

This is a well-designed circuit. It can operate at high clock frequencies. The maximum clock frequency can be easily calculated. For high-speed digital design, synchronous binary counters are preferred.



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Fig. 6-12 4-Bit Synchronous Binary Counter

# JKFF Gray Code Counter Design

Q2	Q1	Q0	N2	N1	N0	J2	K2	J1	K1	J0	K0
0	0	0	0	0	1						
0	0	1	0	1	1						
0	1	0	1	1	0						
0	1	1	0	1	0						
1	0	0	0	0	0						
1	0	1	1	0	0						
1	1	0	1	1	1						
1	1	1	1	0	1						

Next state values

Inputs to apply to achieve  
desired next state



# JKFF Gray Code Counter Design

Q2	Q1	Q0	N2	N1	N0	J2	K2	J1	K1	J0	K0
0	0	0	0	0	1	0	X	0	X	1	X
0	0	1	0	1	1	0	X	1	X	X	0
0	1	0	1	1	0	1	X	X	0	0	X
0	1	1	0	1	0	0	X	X	0	X	1
1	0	0	0	0	0	X	1	0	X	0	X
1	0	1	1	0	0	X	0	0	X	X	1
1	1	0	1	1	1	X	0	X	0	1	X
1	1	1	1	0	1	X	0	X	1	X	0

$$J2 = Q1 \bullet Q0'$$

$$K2 = Q1' \bullet Q0'$$

$$J1 = Q2' \bullet Q0$$

$$K1 = Q2 \bullet Q0$$

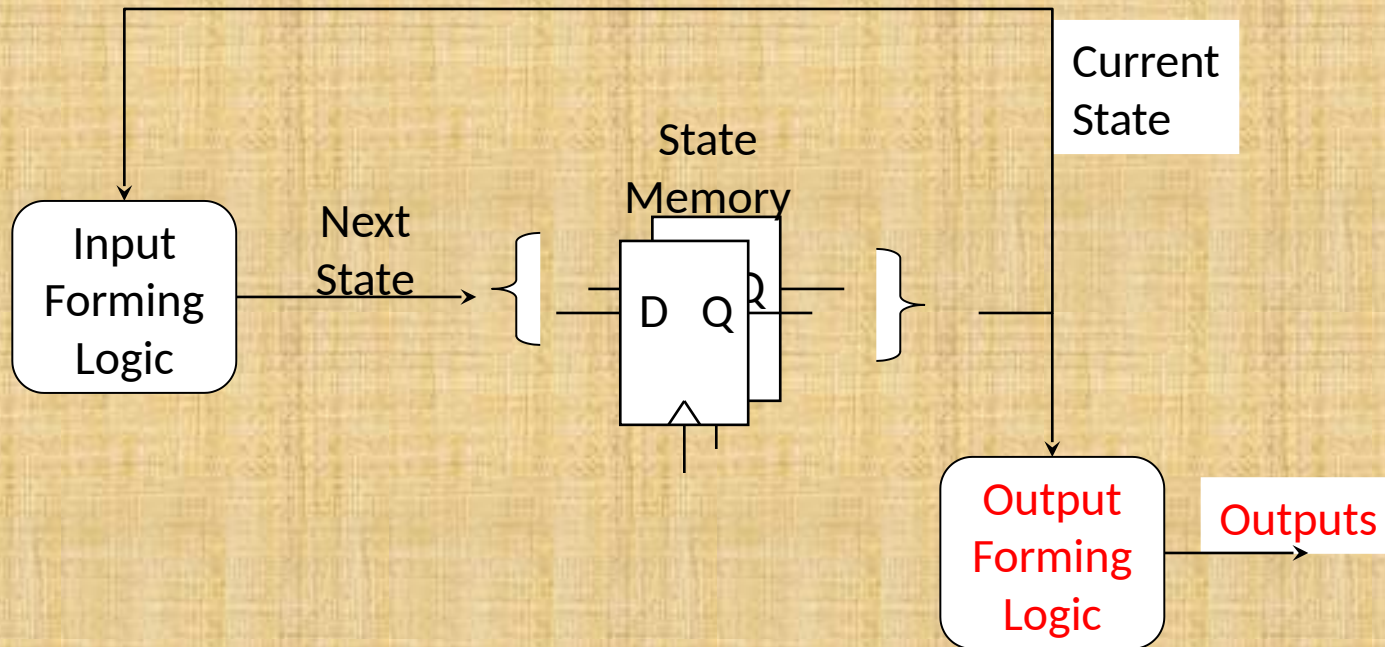
$$J0 = Q2 \bullet Q1 + Q2' \bullet Q1' = K0'$$

$$K0 = Q2' \bullet Q1 + Q2 \bullet Q1' = Q2 \oplus Q1$$

Next state values

Inputs to apply to achieve  
desired next state

# Counters With Outputs



$$\text{Outputs} = f(\text{CurrentState})$$

# Counters With Outputs

$Z=1$  when  $\text{count}=\{0,3,6\}$

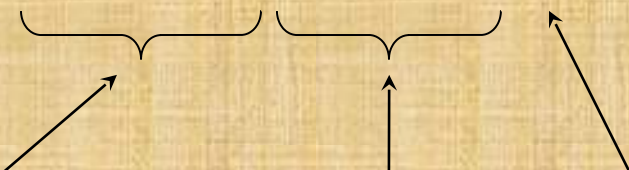
Q2	Q1	Q0	Z
0	0	0	1
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	0

$Z$  is called a *Moore* or *static* output.  
It is a function only of the current state.



# Combined Transition Table

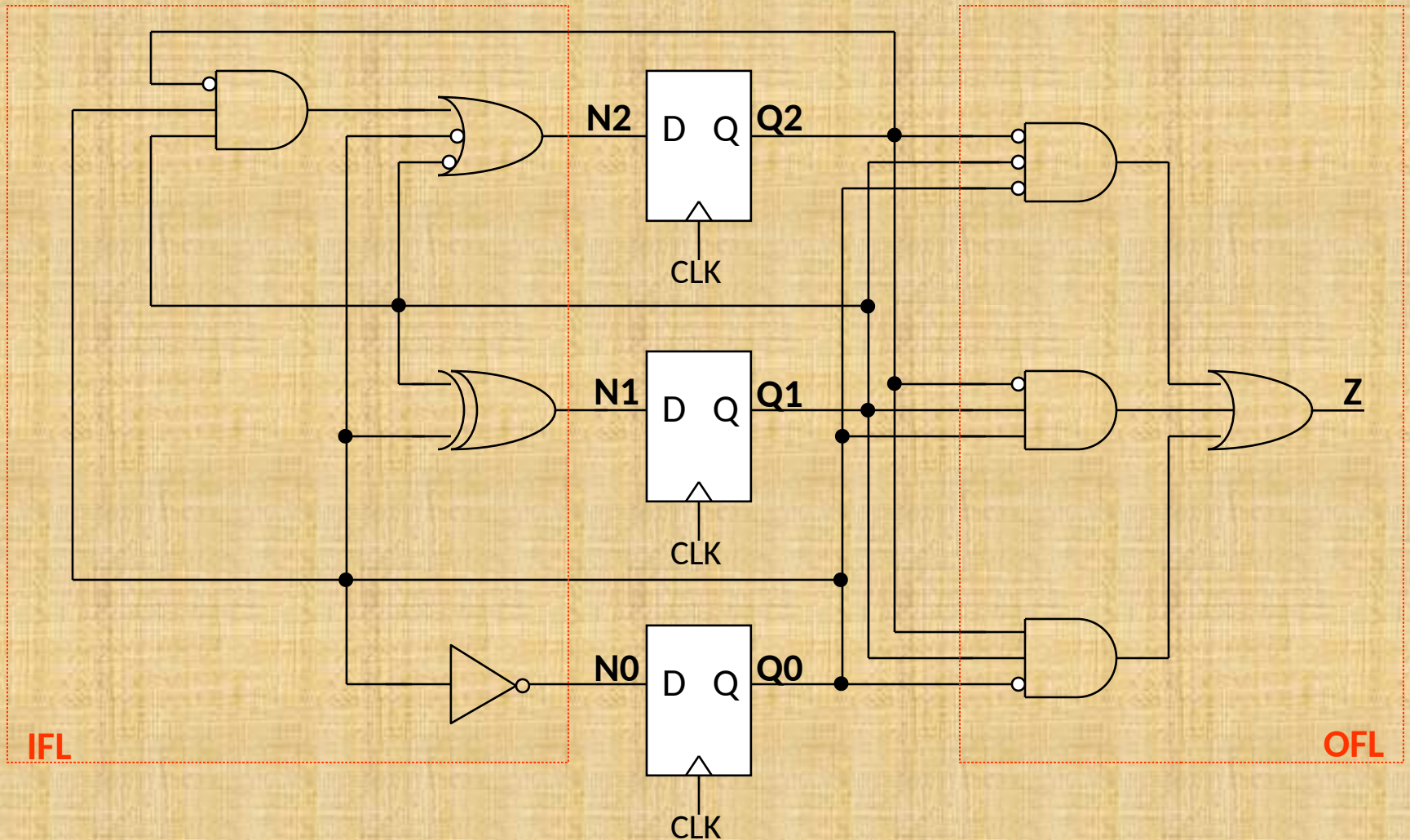
Q2	Q1	Q0	N2	N1	N0	Z
0	0	0	0	0	1	1
0	0	1	0	1	0	0
0	1	0	0	1	1	0
0	1	1	1	0	0	1
1	0	0	1	0	1	0
1	0	1	1	1	0	0
1	1	0	1	1	1	1
1	1	1	0	0	0	0

  
Current state                  Next state                  Output

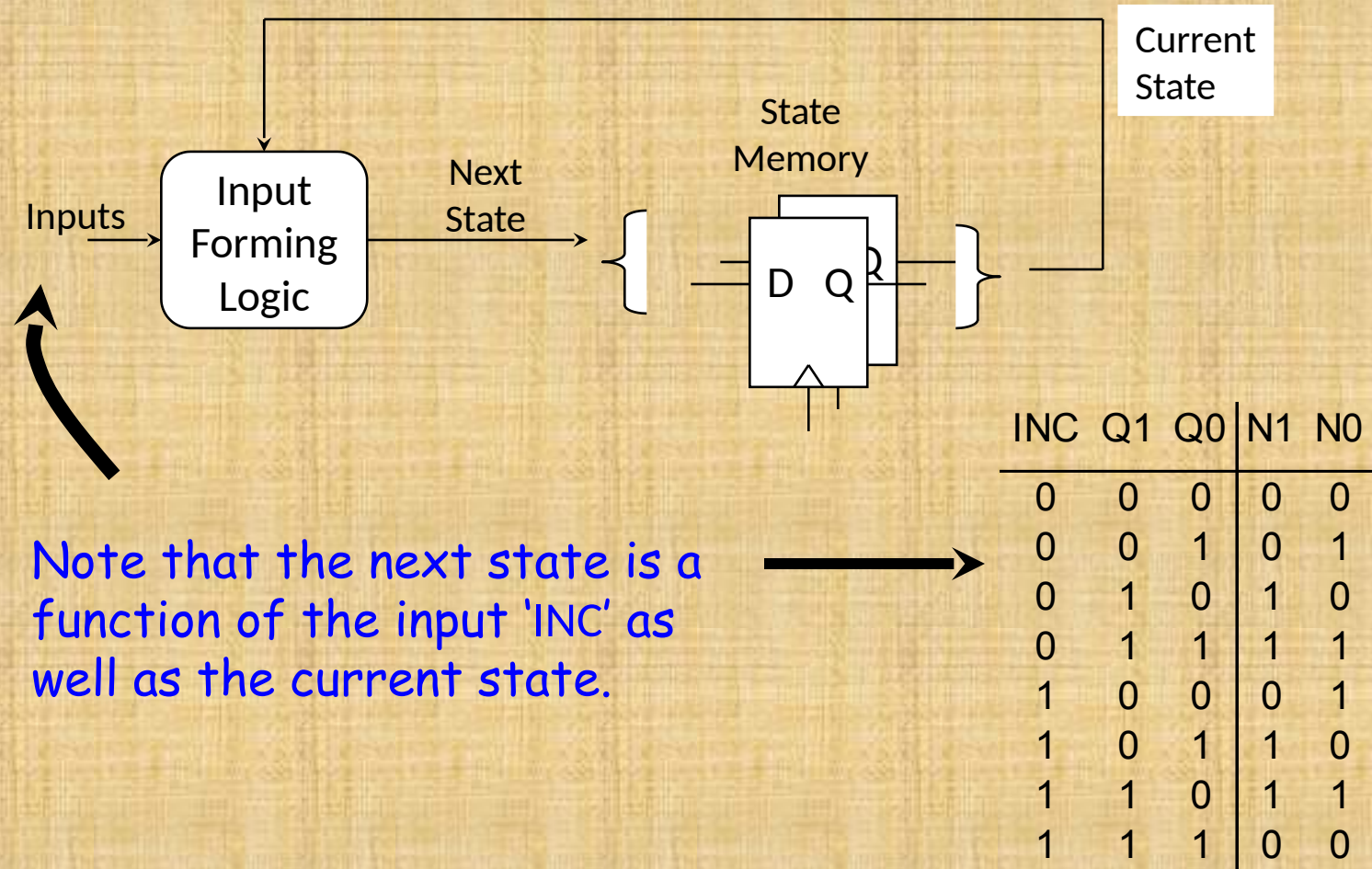
$$Z = Q2' \cdot Q1' \cdot Q0' + Q2' \cdot Q1 \cdot Q0 + Q2 \cdot Q1 \cdot Q0'$$

(implement OFL with gates)

# Counter With A Moore Output



# An Incrementable Counter





# Incrementable Counter Derivation

Doing the KMaps for this results in:

INC	Q1	Q0	N1	N0
0	0	0	0	0
0	0	1	0	1
0	1	0	1	0
0	1	1	1	1
1	0	0	0	1
1	0	1	1	0
1	1	0	1	1
1	1	1	0	0

K-Map for N1:

		0	1
INC	Q <sub>1</sub> Q <sub>0</sub>		
	00		
	01		1
	11	1	
	10	1	1

N1

K-Map for N0:

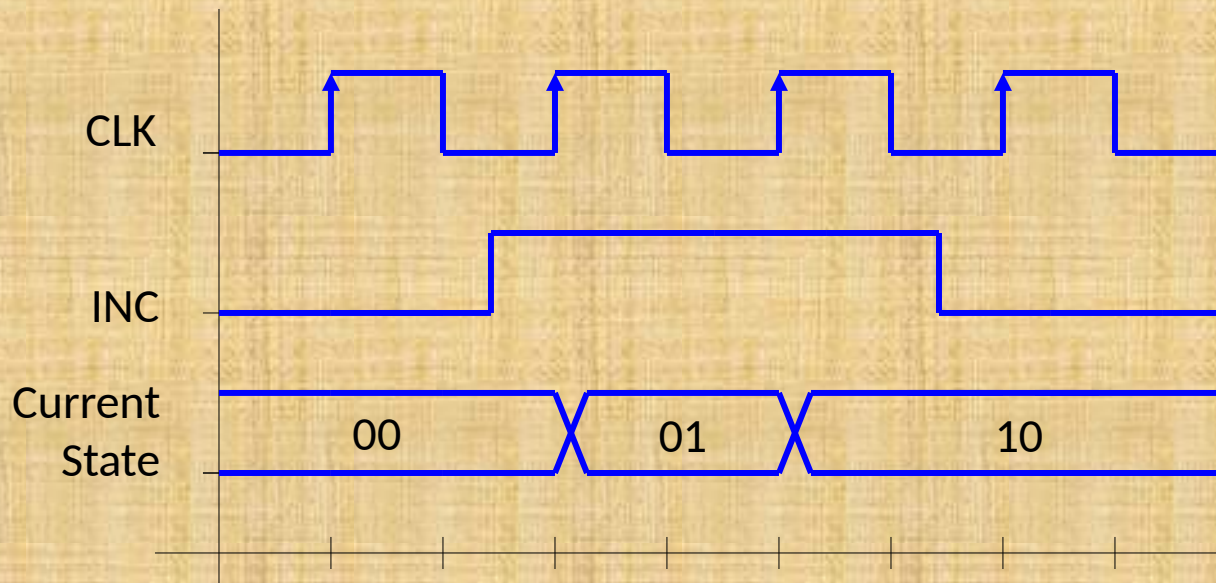
		0	1
INC	Q <sub>1</sub> Q <sub>0</sub>		
	00		1
	01	1	
	11	1	
	10		1

N0

$$N1 = INC \bullet Q1' \bullet Q0 + INC' \bullet Q1 + Q1 \bullet Q0'$$

$$N0 = INC' \bullet Q0 + INC \bullet Q0' = INC \oplus Q0$$

# Incrementable Counter Behavior



The counter increments on the clock edge only when INC is asserted

# Counters With More Inputs

	CLR	INC	Q1	Q0	N1	N0
CLR = INC = 0 No state transition	0	0	0	0		
	0	0	0	1		
	0	0	1	0		
	0	0	1	1		
CLR = 0   INC = 1 Counter Increments	0	1	0	0		
	0	1	0	1		
	0	1	1	0		
	0	1	1	1		
CLR = 1   INC = 0 Counter resets to '00'	1	0	0	0		
	1	0	0	1		
	1	0	1	0		
	1	0	1	1		
CLR = 1   INC = 1 What should it do?	1	1	0	0		
	1	1	0	1		
	1	1	1	0		
	1	1	1	1		



# Counters With More Inputs

	CLR	INC	Q1	Q0	N1	N0
CLR = INC = 0 No state transition	0	0	0	0	0	0
	0	0	0	1	0	1
	0	0	1	0	1	0
	0	0	1	1	1	1
CLR = 0   INC = 1 Counter Increments	0	1	0	0	0	1
	0	1	0	1	1	0
	0	1	1	0	1	1
	0	1	1	1	0	0
CLR = 1   INC = 0 Counter resets to '00'	1	0	0	0	0	0
	1	0	0	1	0	0
	1	0	1	0	0	0
	1	0	1	1	0	0
CLR = 1   INC = 1 What should it do?	1	1	0	0	?	?
	1	1	0	1	?	?
	1	1	1	0	?	?
	1	1	1	1	?	?

# Precedence of INC vs. CLR?

1. Could do nothing
2. Could give INC precedence
3. Could give CLR precedence
4. Assume  $\text{INC}=\text{CLR}=1$  will never occur

You decide when you draw the transition table!

# Case 1 – Do Nothing

CLR	INC	Q1	Q0	N1	N0
0	0	0	0	0	0
0	0	0	1	0	1
0	0	1	0	1	0
0	0	1	1	1	1
0	1	0	0	0	1
0	1	0	1	1	0
0	1	1	0	1	1
0	1	1	1	0	0
1	0	0	0	0	0
1	0	0	1	0	0
1	0	1	0	0	0
1	0	1	1	0	0
1	1	0	0	0	0
1	1	0	1	0	1
1	1	1	0	1	0
1	1	1	1	1	1

		Clr Inc			
Q <sub>1</sub> Q <sub>0</sub>		00	01	11	10
00					
01			1		
11	1			1	
10	1	1	1		

N1

		Clr Inc			
Q <sub>1</sub> Q <sub>0</sub>		00	01	11	10
00			1		
01	1			1	
11	1			1	
10			1		

N0

$$\begin{aligned}
 N1 = & \text{CLR}' \bullet \text{INC} \bullet Q1' \bullet Q0 + \\
 & \text{CLR}' \bullet \text{INC}' \bullet Q1 + \\
 & \text{CLR} \bullet \text{INC} \bullet Q1 + \\
 & \text{INC} \bullet Q1 \bullet Q0'
 \end{aligned}$$

$$\begin{aligned}
 N0 = & \text{CLR}' \bullet \text{INC}' \bullet Q0 + \\
 & \text{CLR} \bullet \text{INC} \bullet Q0 + \\
 & \text{CLR}' \bullet \text{INC} \bullet Q0'
 \end{aligned}$$



# Case 2 – Give INC Precedence

CLR	INC	Q1	Q0	N1	N0
0	0	0	0	0	0
0	0	0	1	0	1
0	0	1	0	1	0
0	0	1	1	1	1
0	1	0	0	0	1
0	1	0	1	1	0
0	1	1	0	1	1
0	1	1	1	0	0
1	0	0	0	0	0
1	0	0	1	0	0
1	0	1	0	0	0
1	0	1	1	0	0
1	1	0	0	0	1
1	1	0	1	1	0
1	1	1	0	1	1
1	1	1	1	0	0

Clr Inc					
Q <sub>1</sub> Q <sub>0</sub>		00	01	11	10
00					
01			1	1	
11					
10		1	1	1	

N1

Clr Inc					
Q <sub>1</sub> Q <sub>0</sub>		00	01	11	10
00			1	1	
01		1			
11		1			
10			1	1	

N0

$$N1 = \text{INC} \bullet Q1' \bullet Q0 + \\ \text{CLR}' \bullet \text{INC}' \bullet Q1 + \\ \text{INC} \bullet Q1 \bullet Q0'$$

$$N0 = \text{CLR}' \bullet \text{INC}' \bullet Q0 + \\ \text{INC} \bullet Q0'$$

# Case 3 – Give CLR Precedence

CLR	INC	Q1	Q0	N1	N0
0	0	0	0	0	0
0	0	0	1	0	1
0	0	1	0	1	0
0	0	1	1	1	1
0	1	0	0	0	1
0	1	0	1	1	0
0	1	1	0	1	1
0	1	1	1	0	0
1	0	0	0	0	0
1	0	0	1	0	0
1	0	1	0	0	0
1	0	1	1	0	0
1	1	0	0	0	0
1	1	0	1	0	0
1	1	1	0	0	0
1	1	1	1	0	0

Clr Inc					
Q <sub>1</sub> Q <sub>0</sub>		00	01	11	10
	00				
	01		1		
	11	1			
	10	1	1		

N1

Clr Inc					
Q <sub>1</sub> Q <sub>0</sub>		00	01	11	10
	00		1		
	01	1			
	11	1			
	10		1		

N0

$$\begin{aligned}
 N1 = & \text{CLR}' \bullet \text{INC} \bullet Q1' \bullet Q0 + \\
 & \text{CLR}' \bullet \text{INC}' \bullet Q1 + \\
 & \text{CLR}' \bullet Q1 \bullet Q0'
 \end{aligned}$$

$$\begin{aligned}
 N0 = & \text{CLR}' \bullet \text{INC}' \bullet Q0 + \\
 & \text{CLR}' \bullet \text{INC} \bullet Q0'
 \end{aligned}$$

# Case 4 – Assume INC=CLR=1 Will Never Occur

CLR	INC	Q1	Q0	N1	N0
0	0	0	0	0	0
0	0	0	1	0	1
0	0	1	0	1	0
0	0	1	1	1	1
0	1	0	0	0	1
0	1	0	1	1	0
0	1	1	0	1	1
0	1	1	1	0	0
1	0	0	0	0	0
1	0	0	1	0	0
1	0	1	0	0	0
1	0	1	1	0	0
1	1	0	0	X	X
1	1	0	1	X	X
1	1	1	0	X	X
1	1	1	1	X	X

Clr Inc					
		00	01	11	10
Q <sub>1</sub> Q <sub>0</sub>	00			X	
	01		1	X	
	11	1		X	
	10	1	1	X	

N1

Clr Inc					
		00	01	11	10
Q <sub>1</sub> Q <sub>0</sub>	00		1	X	
	01	1		X	
	11	1		X	
	10		1	X	

N0

$$N1 = \text{CLR}' \bullet \text{INC}' \bullet Q1 + \text{INC} \bullet Q1' \bullet Q0 + \text{CLR}' \bullet Q1 \bullet Q0'$$

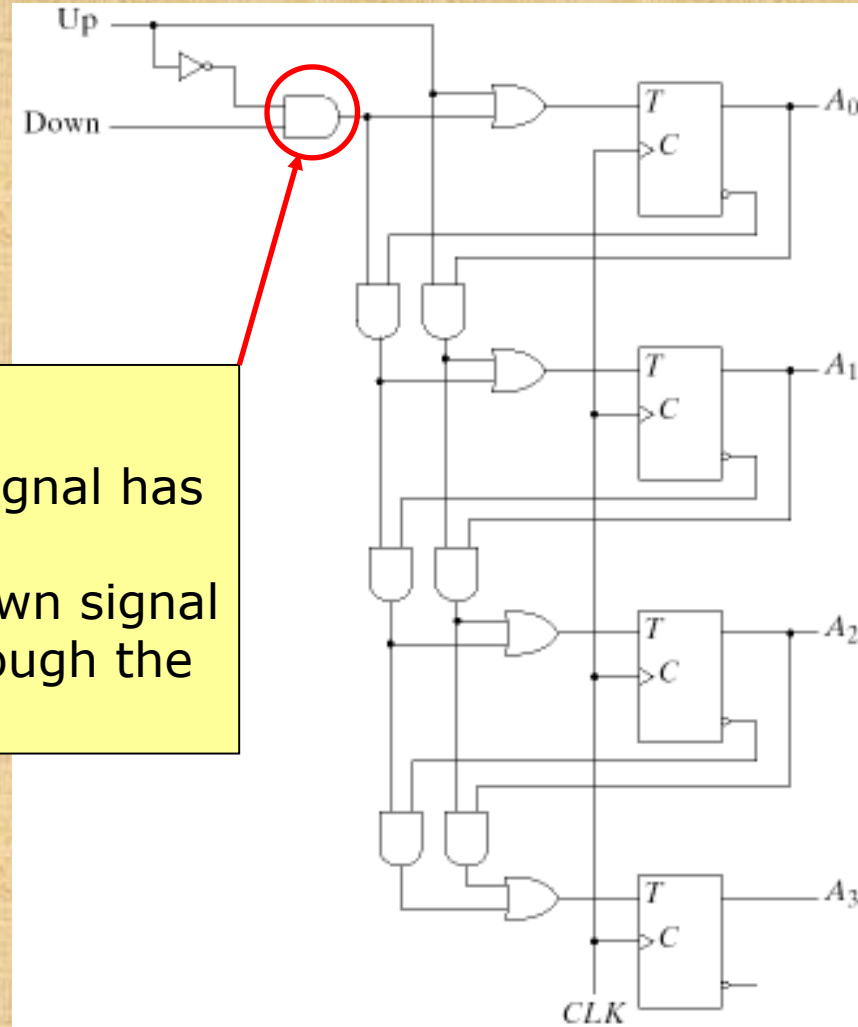
$$N0 = \text{CLR}' \bullet \text{INC}' \bullet Q0 + \text{INC} \bullet Q0'$$

What happens in the real circuit  
when INC=CLR=1?

It depends on the final equations...



# 4-Bit Up-Down Binary Counter

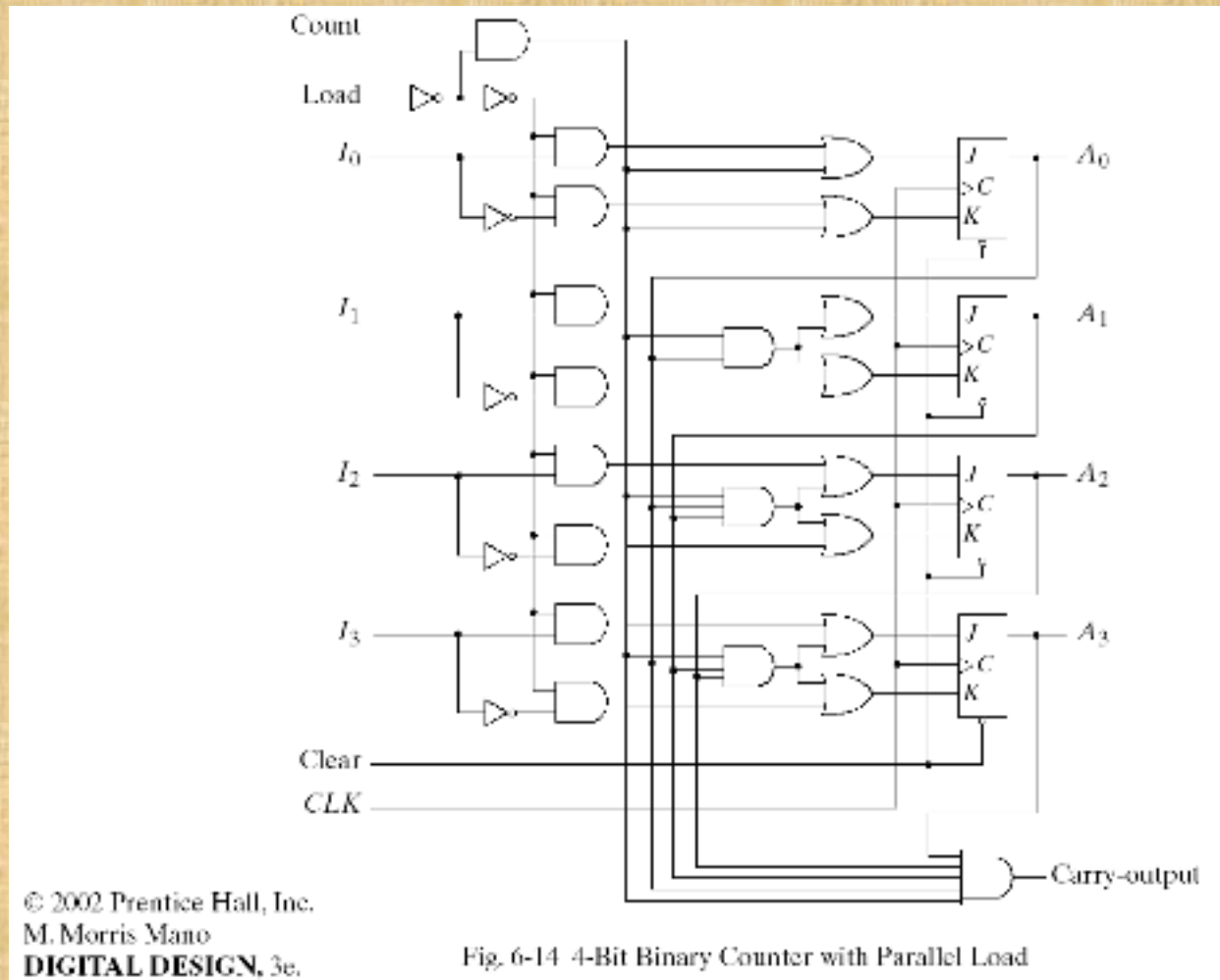


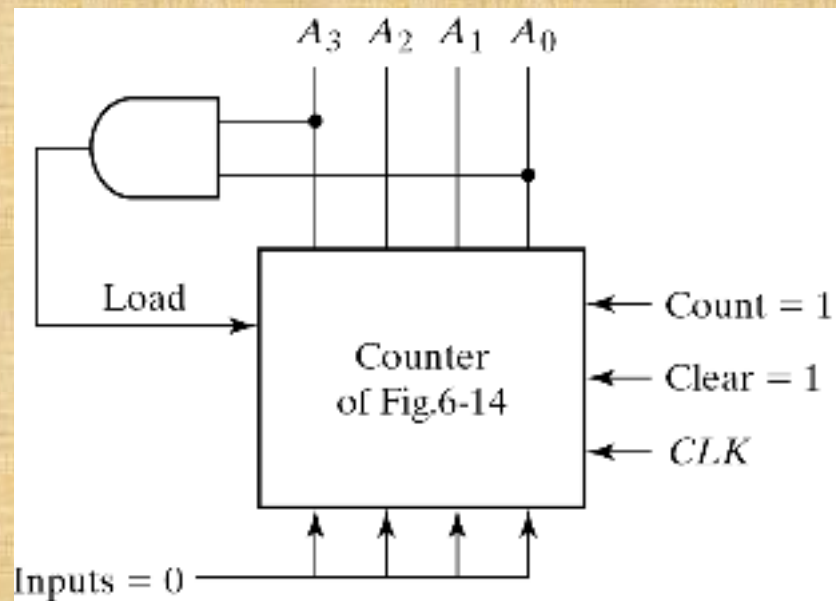
## NOTE:

The Up control signal has priority. If Up is asserted, the Down signal will not pass through the AND gate.

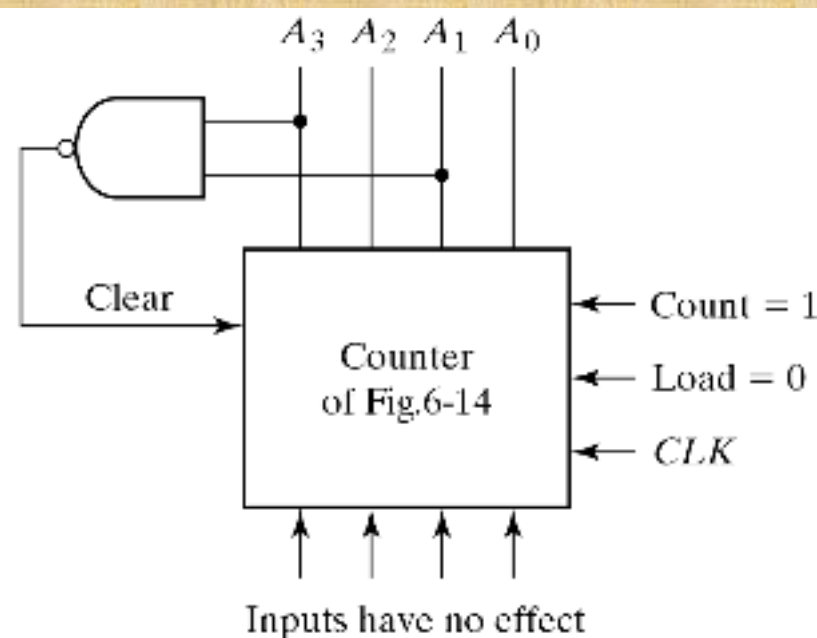
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## 4-Bit Binary Counter with Parallel Load





(a) Using the load input

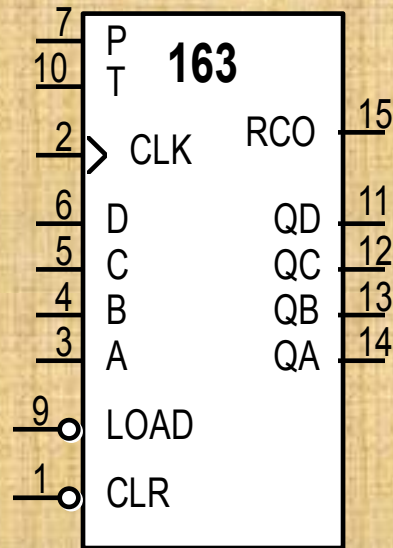


(b) Using the clear input

Fig. 6-15 Two ways to Achieve a BCD Counter Using a Counter with Parallel Load



# A common 4-bit counter



74163 Synchronous  
4-Bit Upcounter

Synchronous Load and Clear Inputs

Positive Edge Triggered FFs

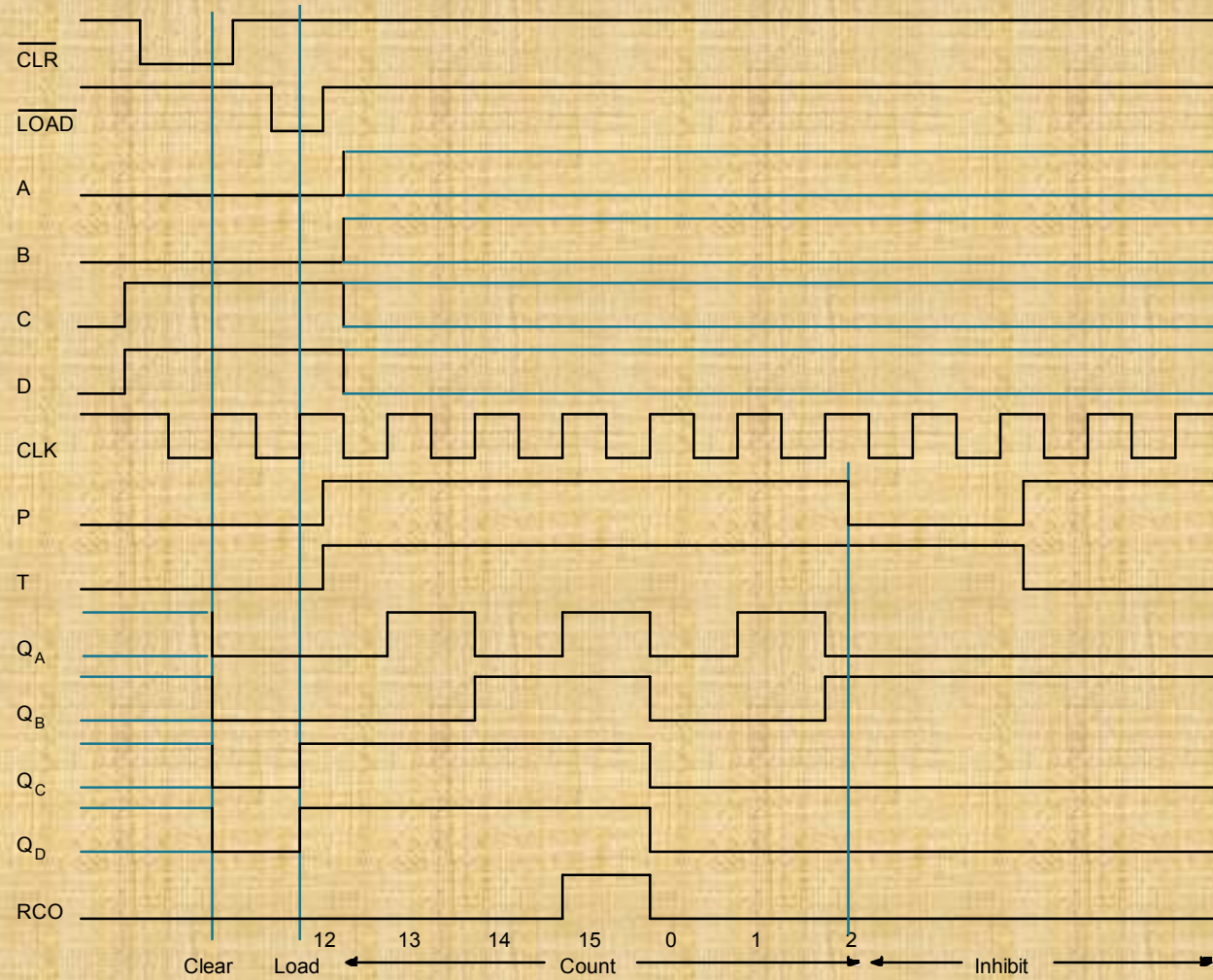
Parallel Load Data from D, C, B, A

P, T Enable Inputs: both must be asserted to enable counting

RCO: asserted when counter enters its highest state 1111, used for cascading counters  
*"Ripple Carry Output"*

74161: similar in function, asynchronous load and reset

# 74163 Detailed Timing Diagram



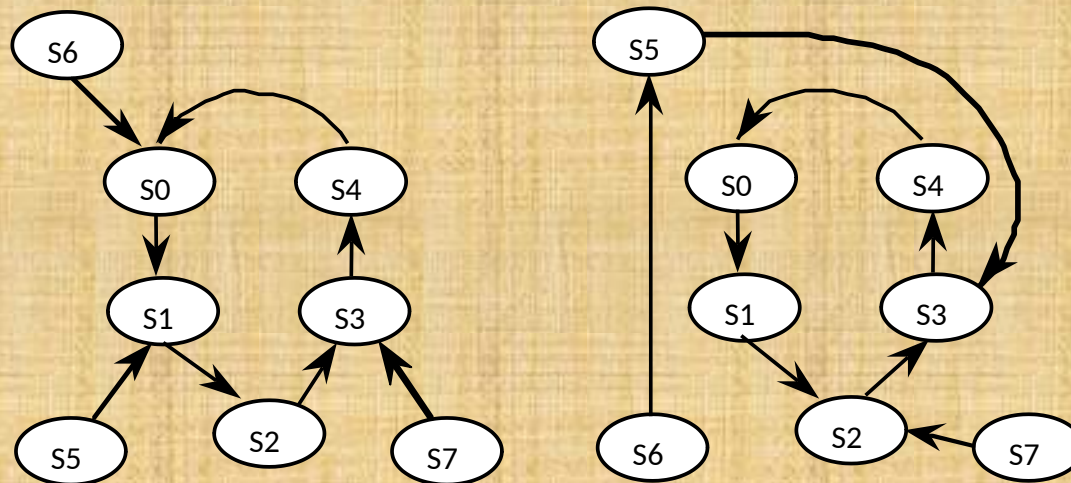
# Self-Starting Counters

## Start-Up States

At power-up, counter may be in any possible state  
Designer must guarantee that it (eventually) enters a valid state  
Especially a problem for counters that validly use a subset of states

## Self-Starting Solution:

Design counter so that even the invalid states  
eventually transition to valid state



Two Self-Starting State Transition Diagrams



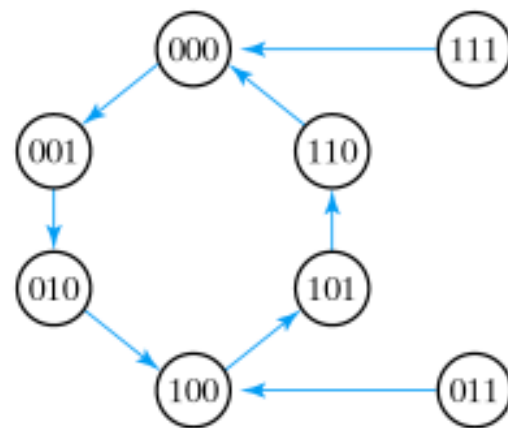
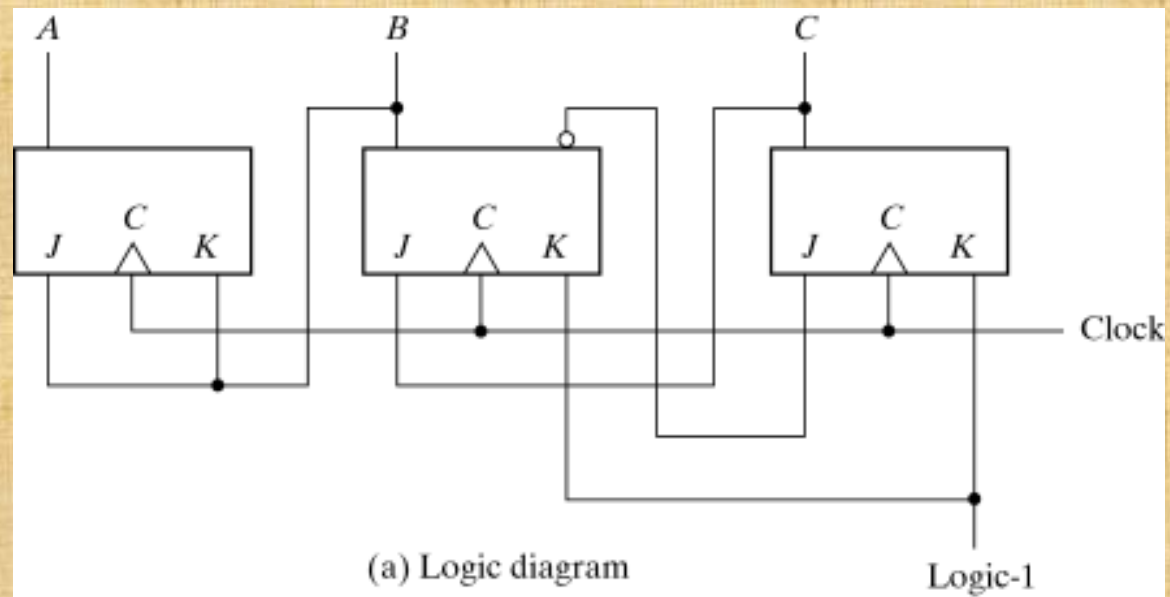
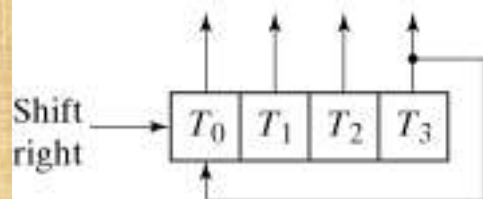
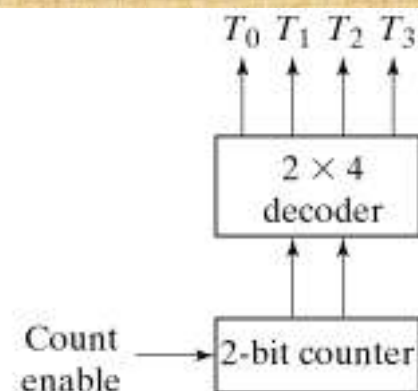


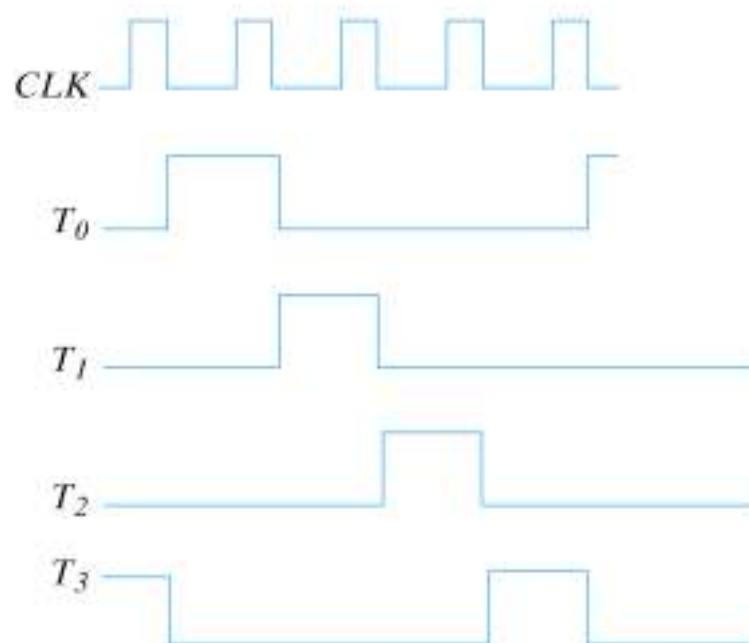
Fig. 6-16 Counter with Unused States



(a) Ring-counter (initial value = 1000)

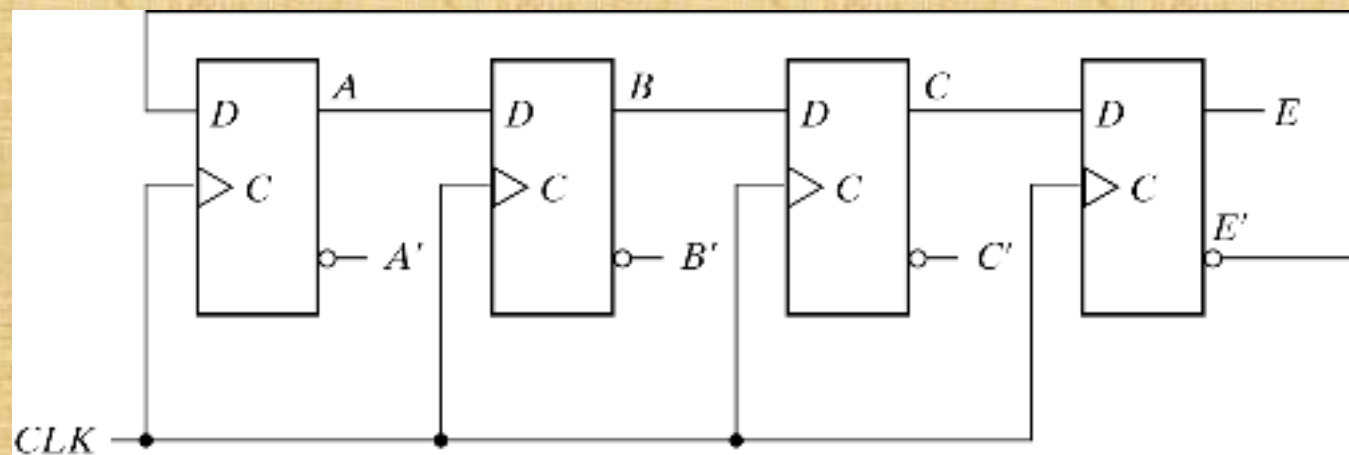


(b) Counter and decoder



(c) Sequence of four timing signals

Fig. 6-17 Generation of Timing Signals



(a) Four-stage switch-tail ring counter

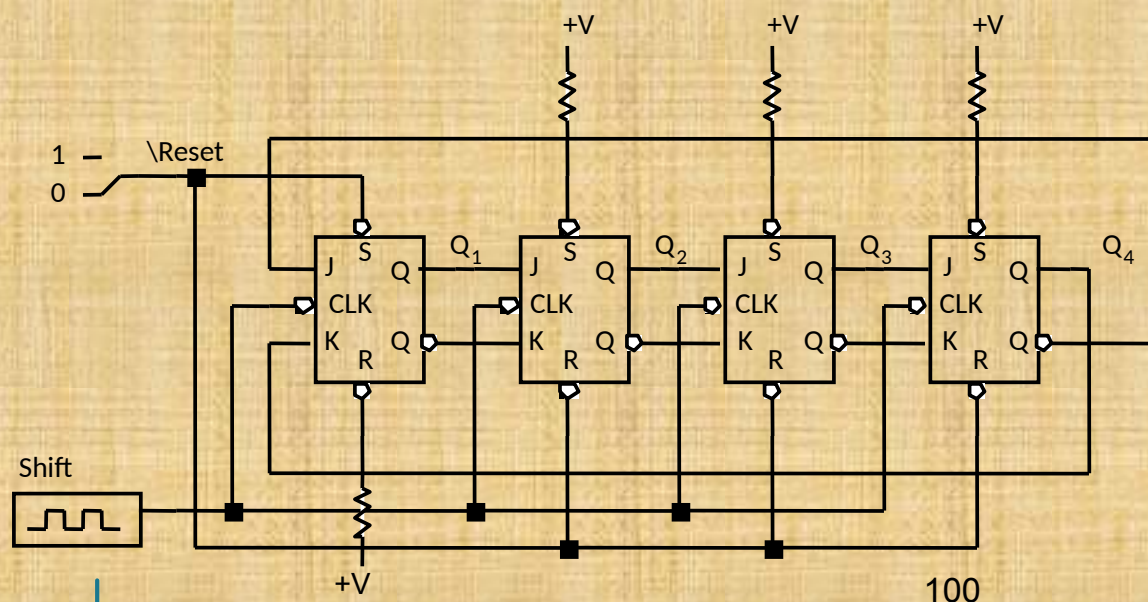
Sequence number	Flip-flop outputs				AND gate required for output
	A	B	C	E	
1	0	0	0	0	$A'E'$
2	1	0	0	0	$AB'$
3	1	1	0	0	$BC'$
4	1	1	1	0	$CE'$
5	1	1	1	1	$AE$
6	0	1	1	1	$A'B$
7	0	0	1	1	$B'C$
8	0	0	0	1	$C'E$

(b) Count sequence and required decoding

Fig. 6-18 Construction of a Johnson Counter

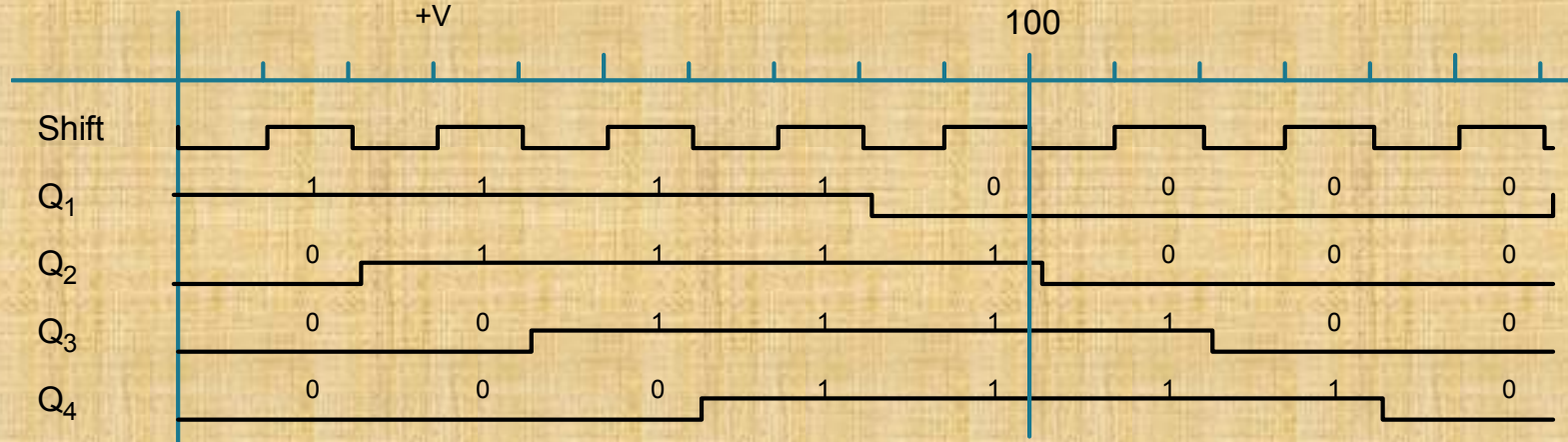


# Twisted Ring (Johnson, Mobius) Counter

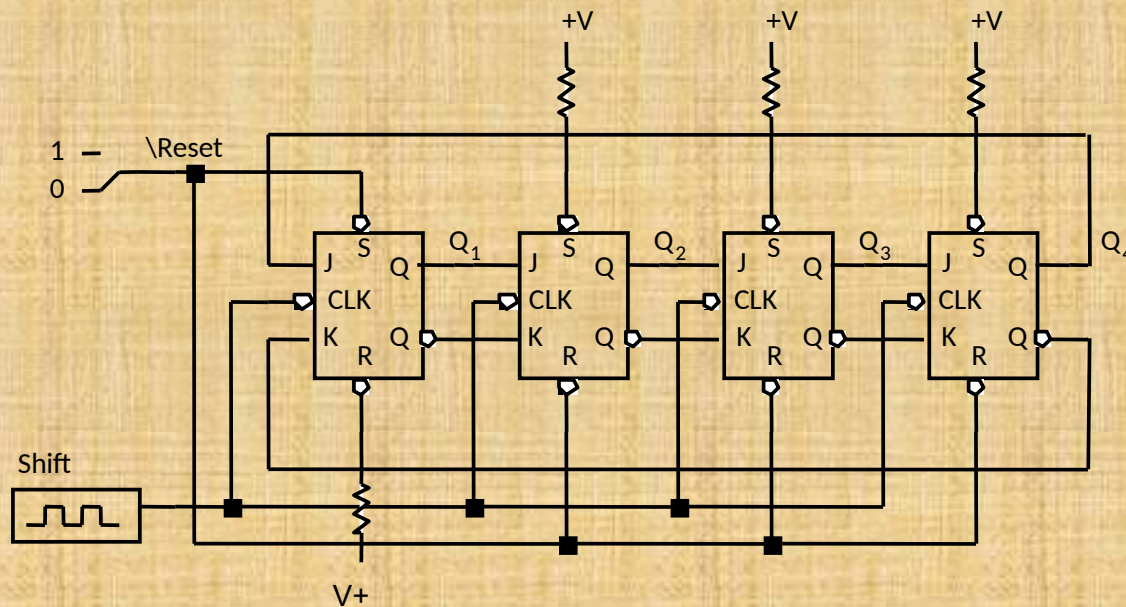


Inverted  
End-Around

8 possible states,  
single bit change  
per state, useful  
for avoiding  
glitches



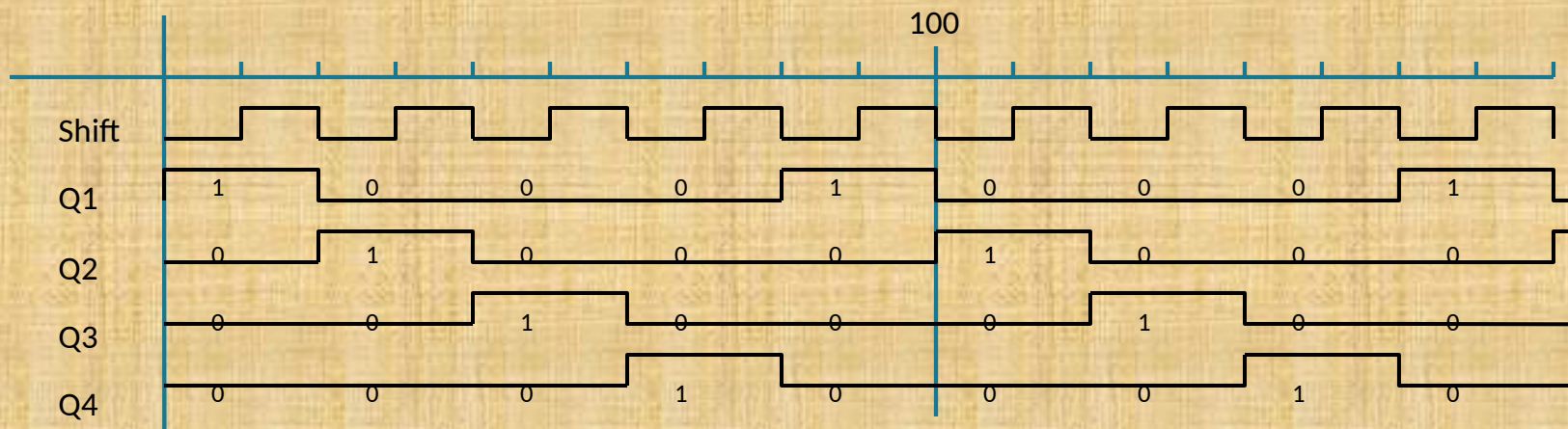
# Ring Counter



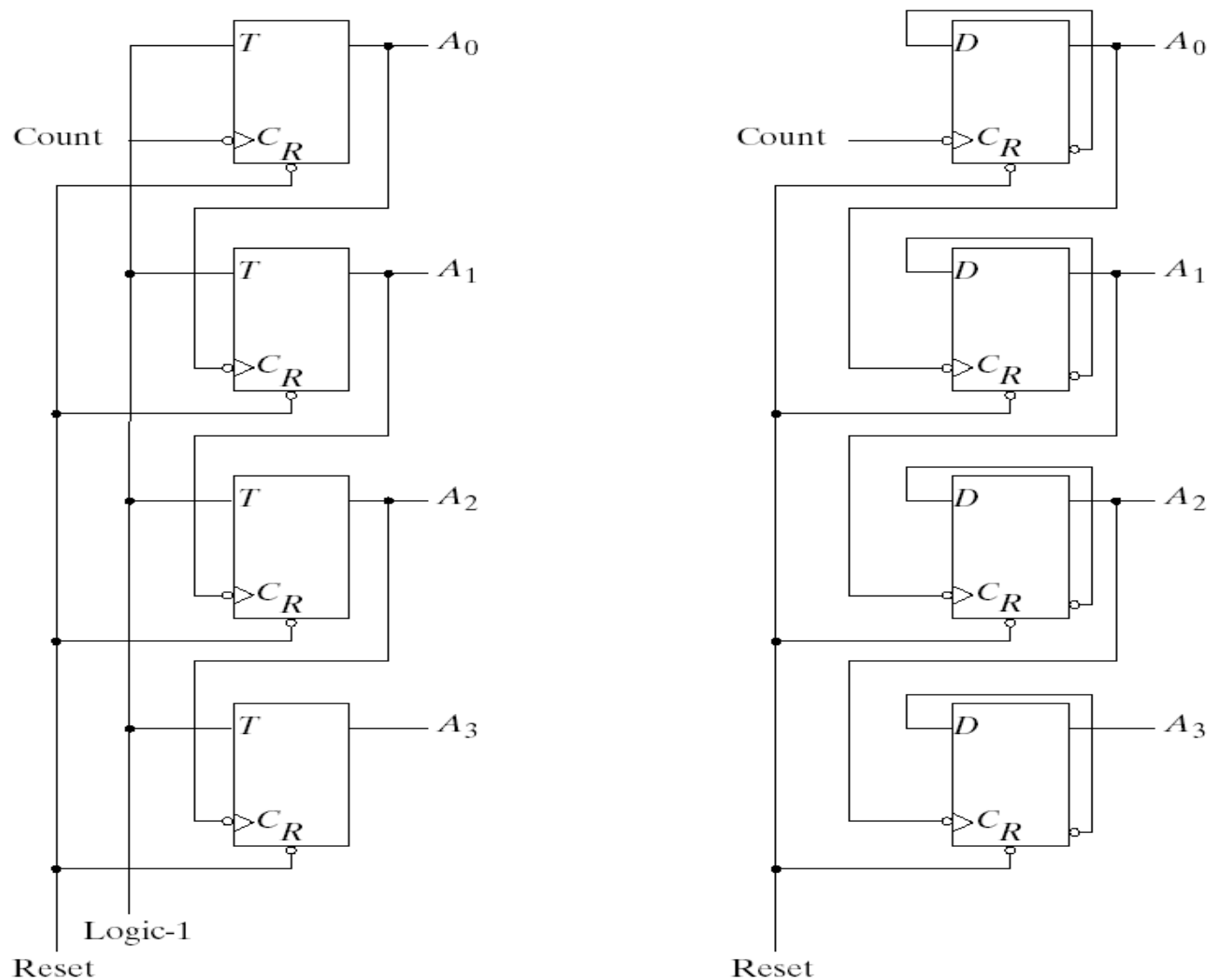
End-Around

4 possible states,  
single bit change per  
state, useful for  
avoiding glitches

Must be initialized



# 4-Bit Binary Ripple Counter



(a) With T flip-flops

(b) With D flip-flops

Fig. 6-8 4-Bit Binary Ripple Counter



# BCD Ripple Counter

- It is possible to build counters that go through any fixed sequence of binary numbers
- For example, a BCD ripple counter can be specified by the following state diagram:

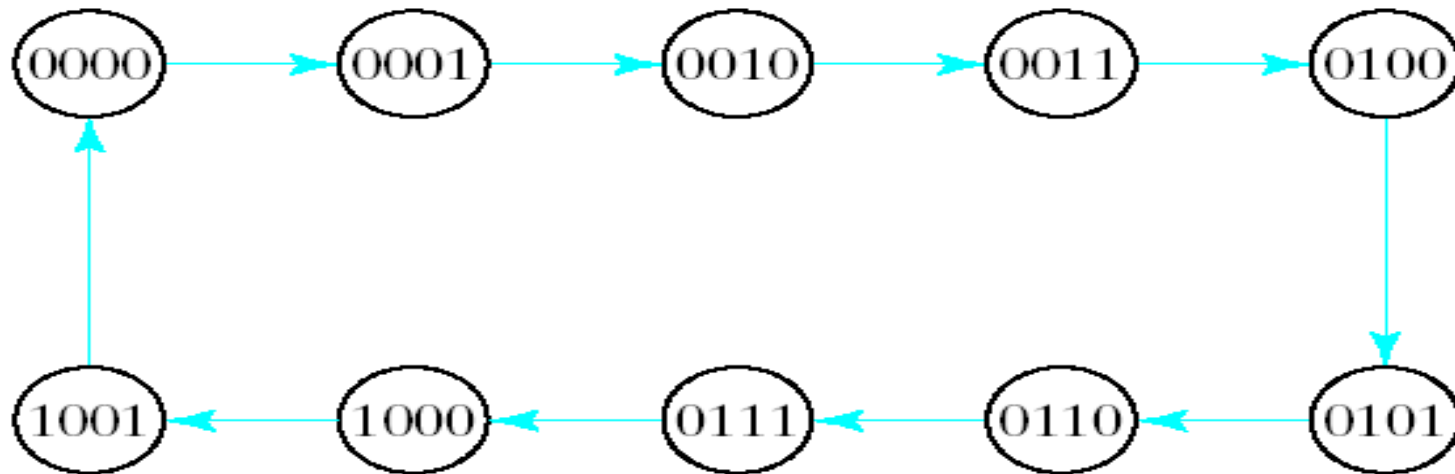
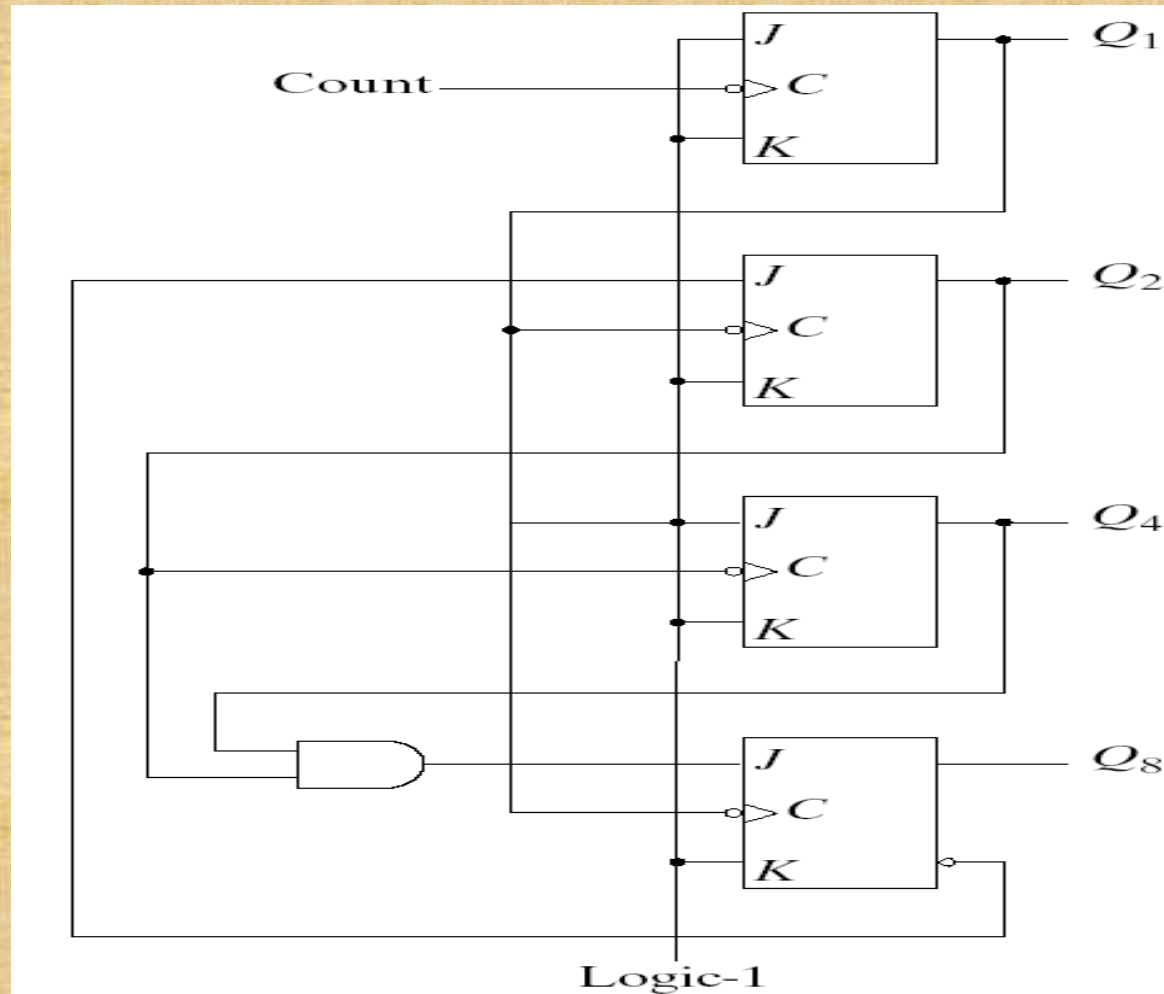


Fig. 6-9 State Diagram of a Decimal BCD-Counter

# 4-Bit BCD Ripple Counter



# 12-Bit BCD Counter

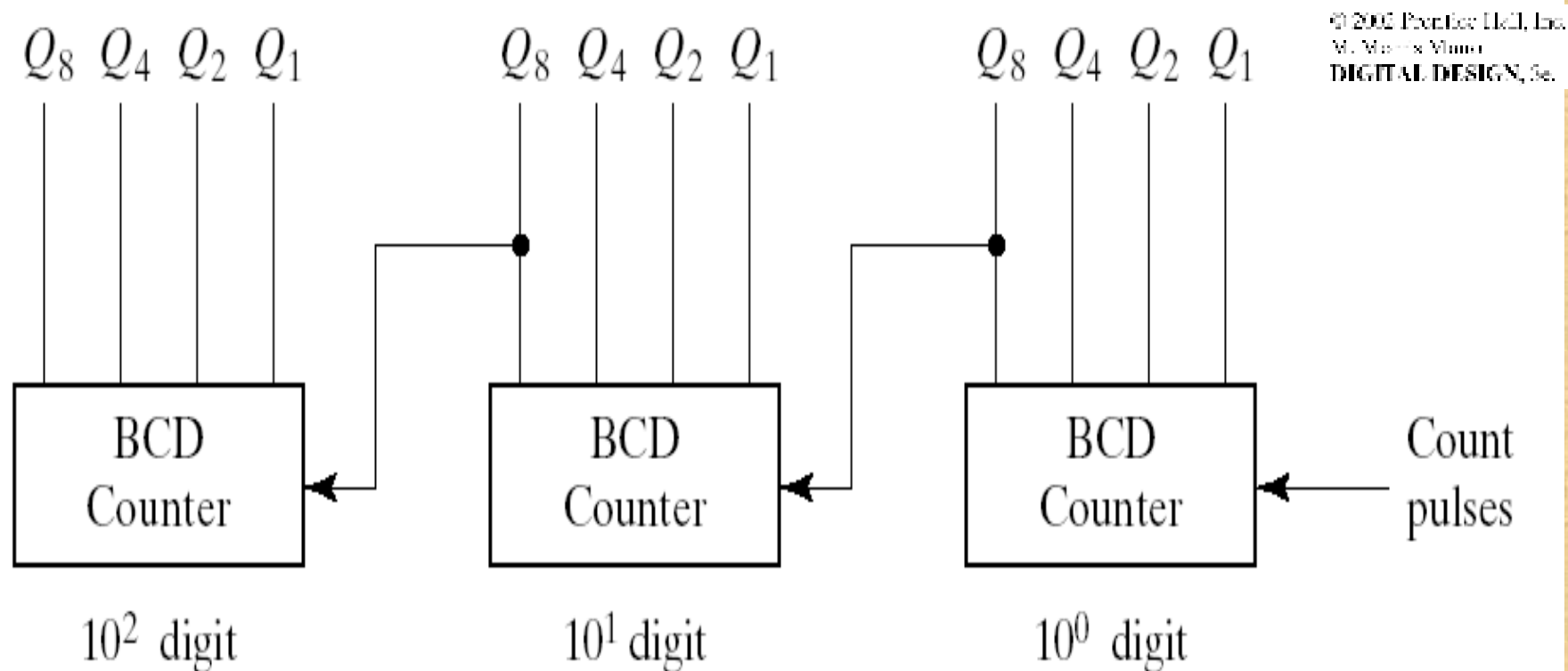


Fig. 6-11 Block Diagram of a Three-Decade Decimal BCD Counter



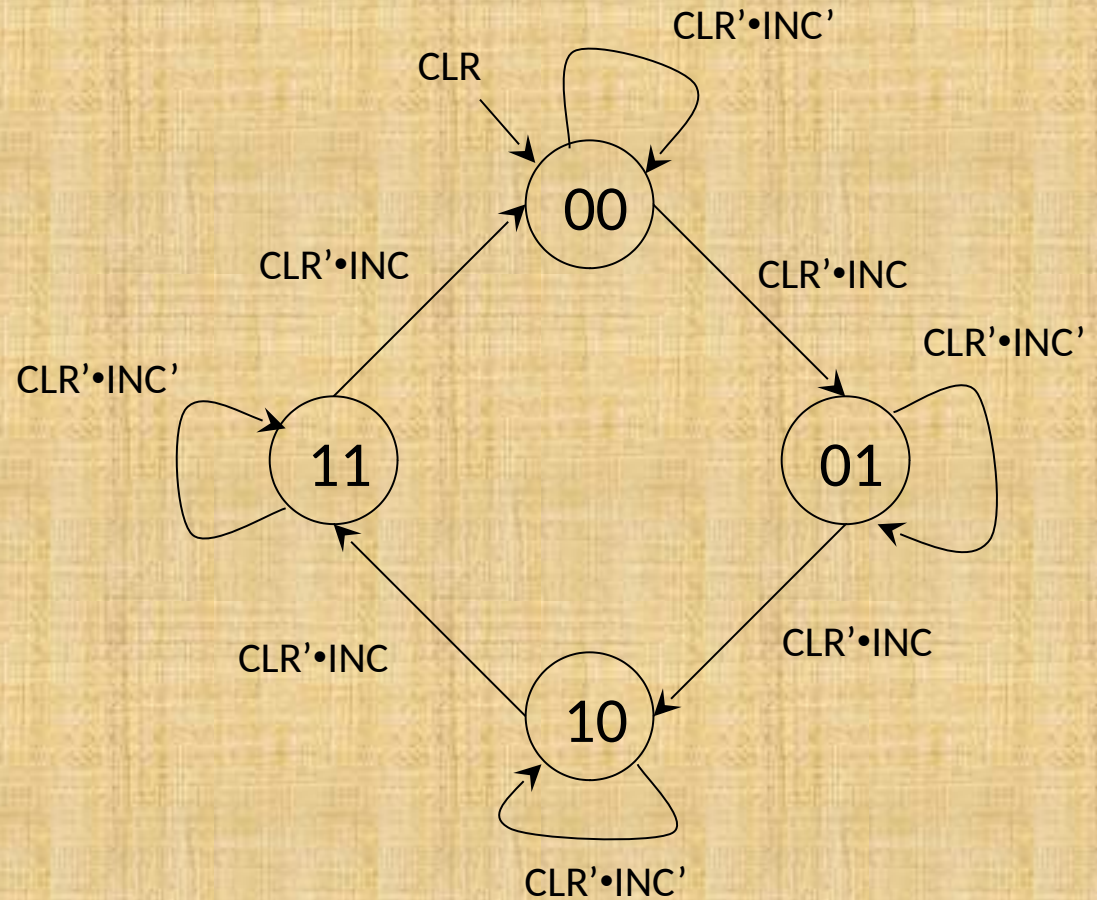
# Mod-N Counters

- Generally we are interested in counters that count up to specific count values
  - Not just powers of 2
- A mod-N counter has N states
  - Counts from 0 to N-1 then rolls over
  - Requires  $\lceil \log_2 N \rceil$  flip flops
- For example...
  - A 4-bit binary counter is a mod-16 counter
  - A counter that counts from 0-9 is a mod-10 counter

# A Mod-4 Counter

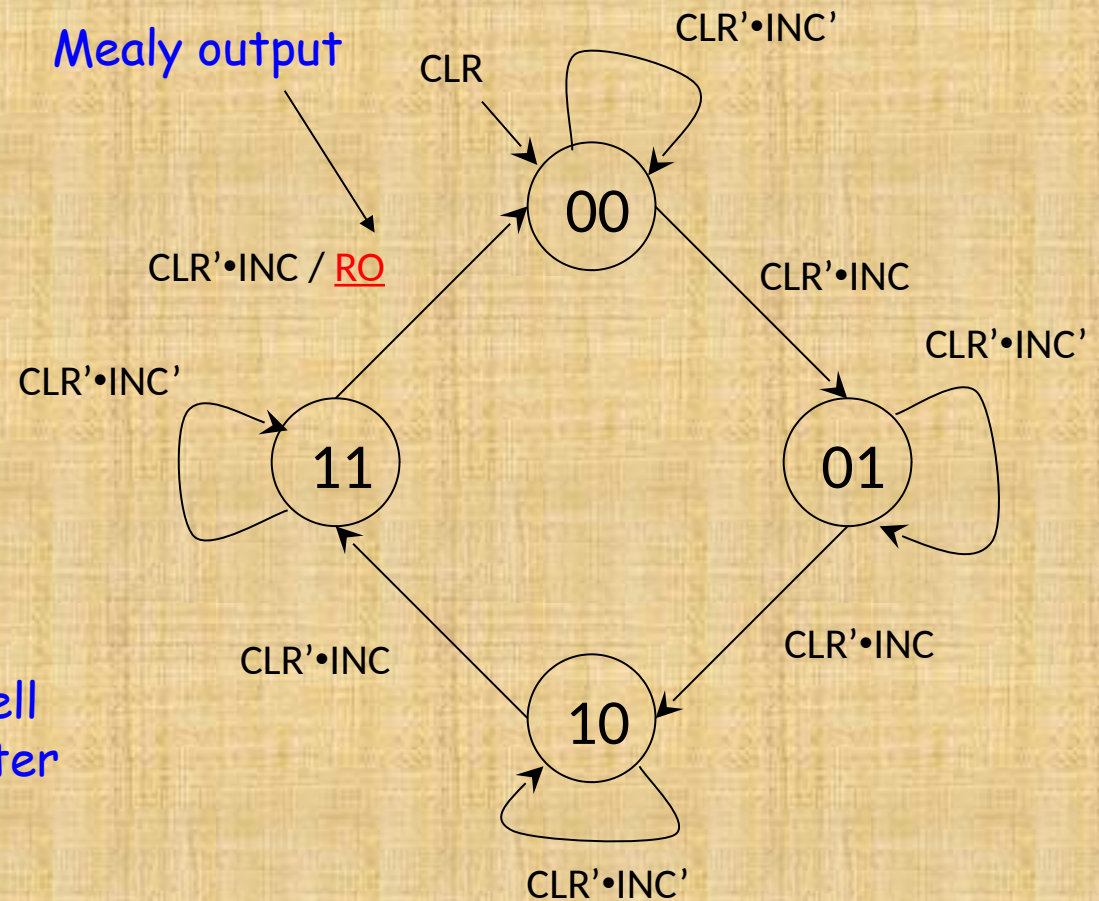
A.K.A. 2-bit counter

CLR	INC	Q1	Q0	N1	N0
0	0	0	0	0	0
0	0	0	1	0	1
0	0	1	0	1	0
0	0	1	1	1	1
0	1	0	0	0	1
0	1	0	1	1	0
0	1	1	0	1	1
0	1	1	1	0	0
1	-	-	-	0	0



# A Mod-4 Counter With Rollover Signal

CLR	INC	Q1	Q0	N1	N0	RO
0	0	0	0	0	0	0
0	0	0	1	0	1	0
0	0	1	0	1	0	0
0	0	1	1	1	1	0
0	1	0	0	0	1	0
0	1	0	1	1	0	0
0	1	1	0	1	1	0
0	1	1	1	0	0	1
1	-	-	-	0	0	0

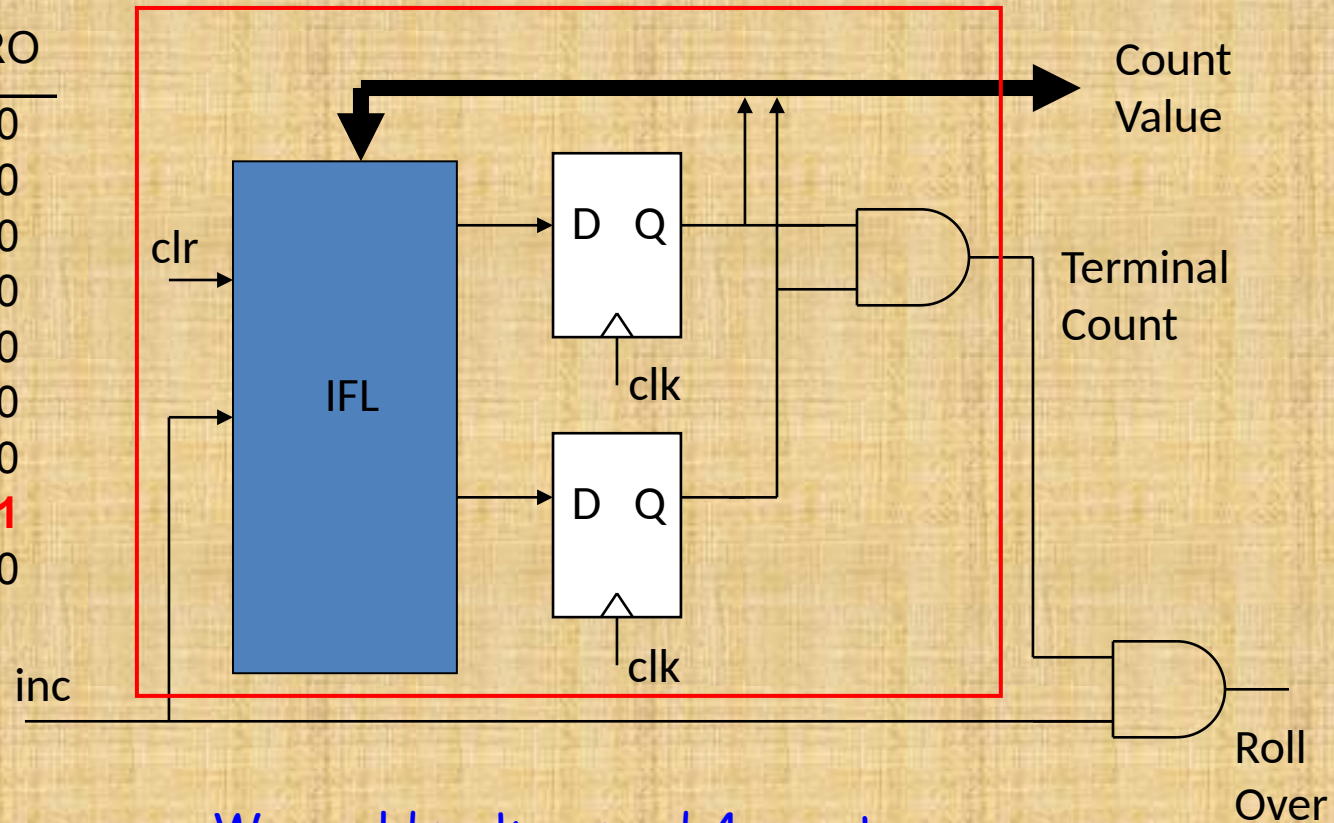


The ROLL signal is used to tell other circuitry that the counter is rolling over to all 0's.



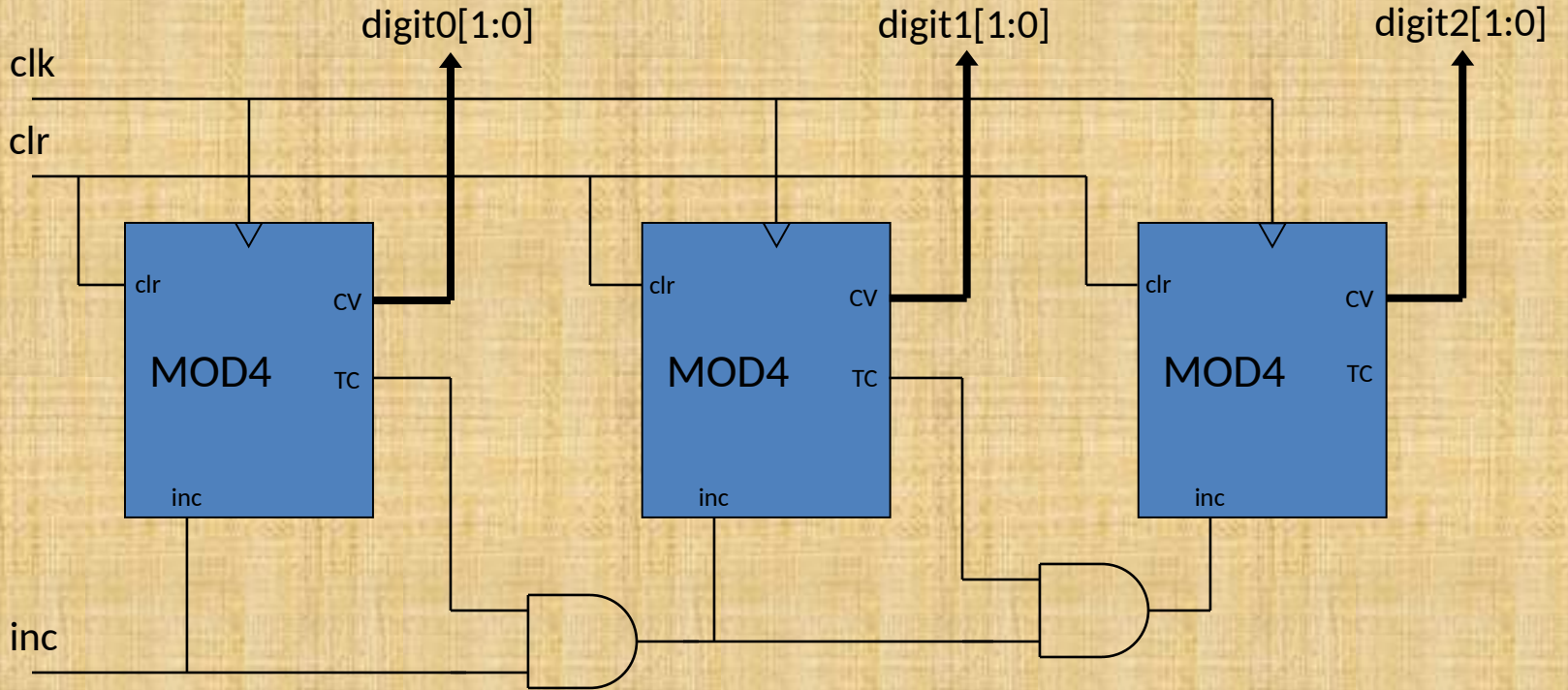
# A Mod-4 Counter

CLR	INC	Q1	Q0	N1	N0	RO
0	0	0	0	0	0	0
0	0	0	1	0	1	0
0	0	1	0	1	0	0
0	0	1	1	1	1	0
0	1	0	0	0	1	0
0	1	0	1	1	0	0
0	1	1	0	1	1	0
0	1	1	1	0	0	1
1	-	-	-	0	0	0

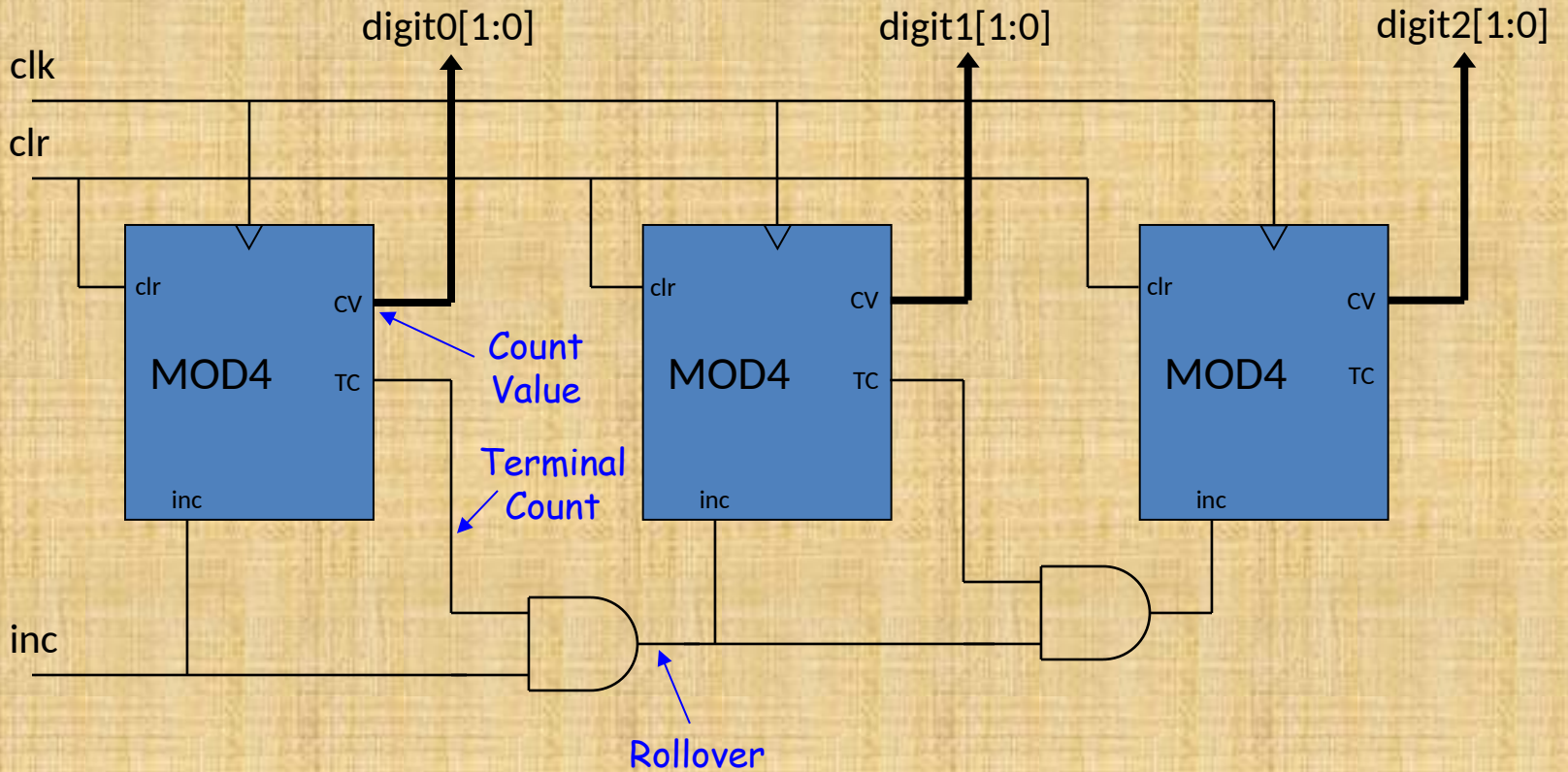


We could make a mod-4 counter from the block shown in red.

# Cascaded Counters

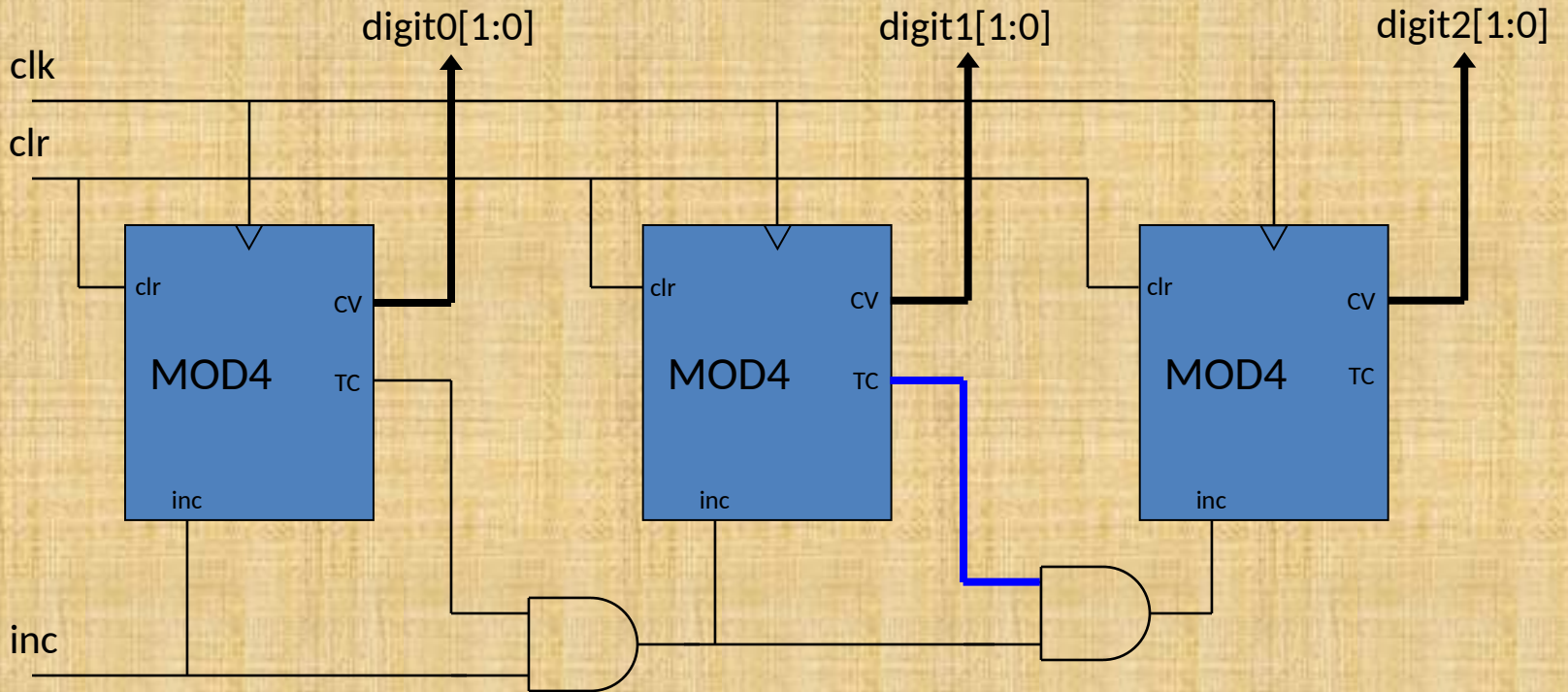


# Cascaded Counters



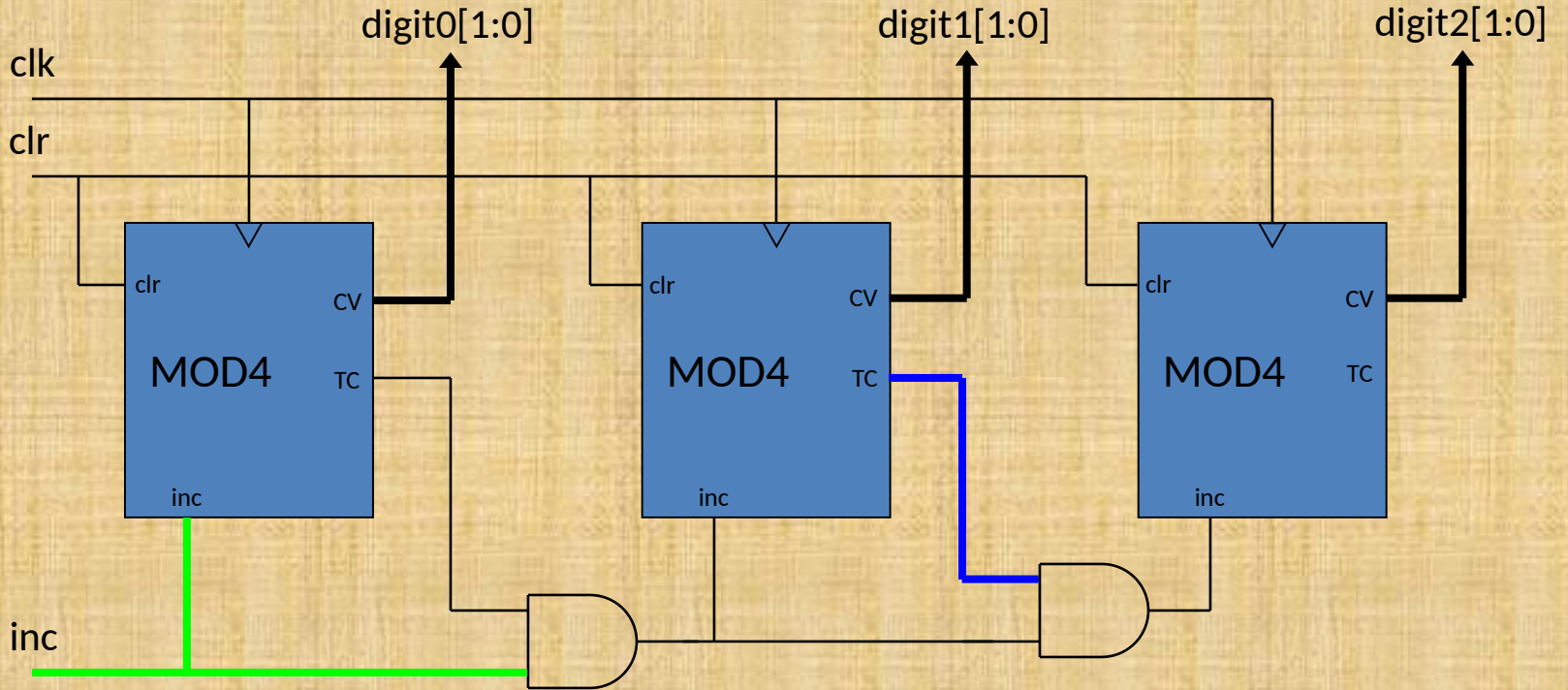


# Cascaded Counters

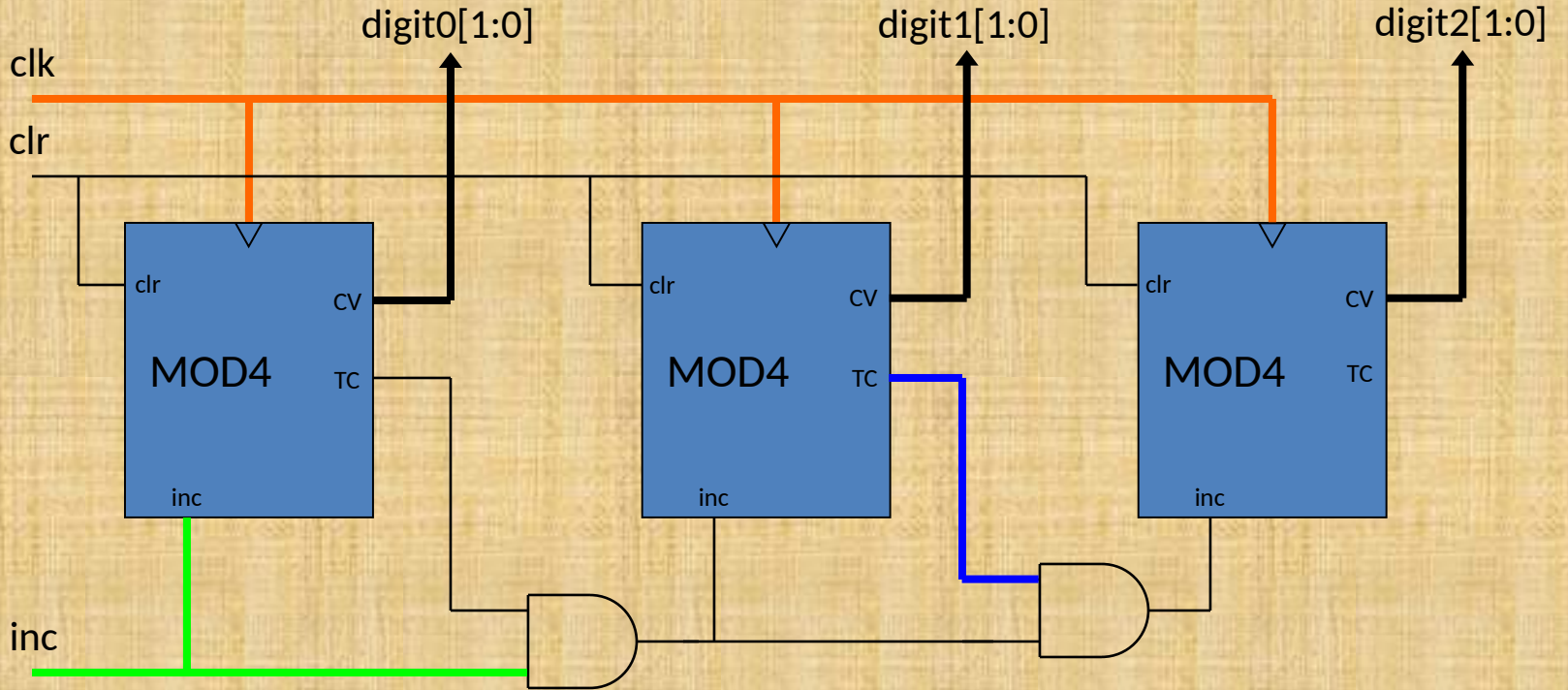


Assume that the second timer is already at the terminal count.

# Cascaded Counters

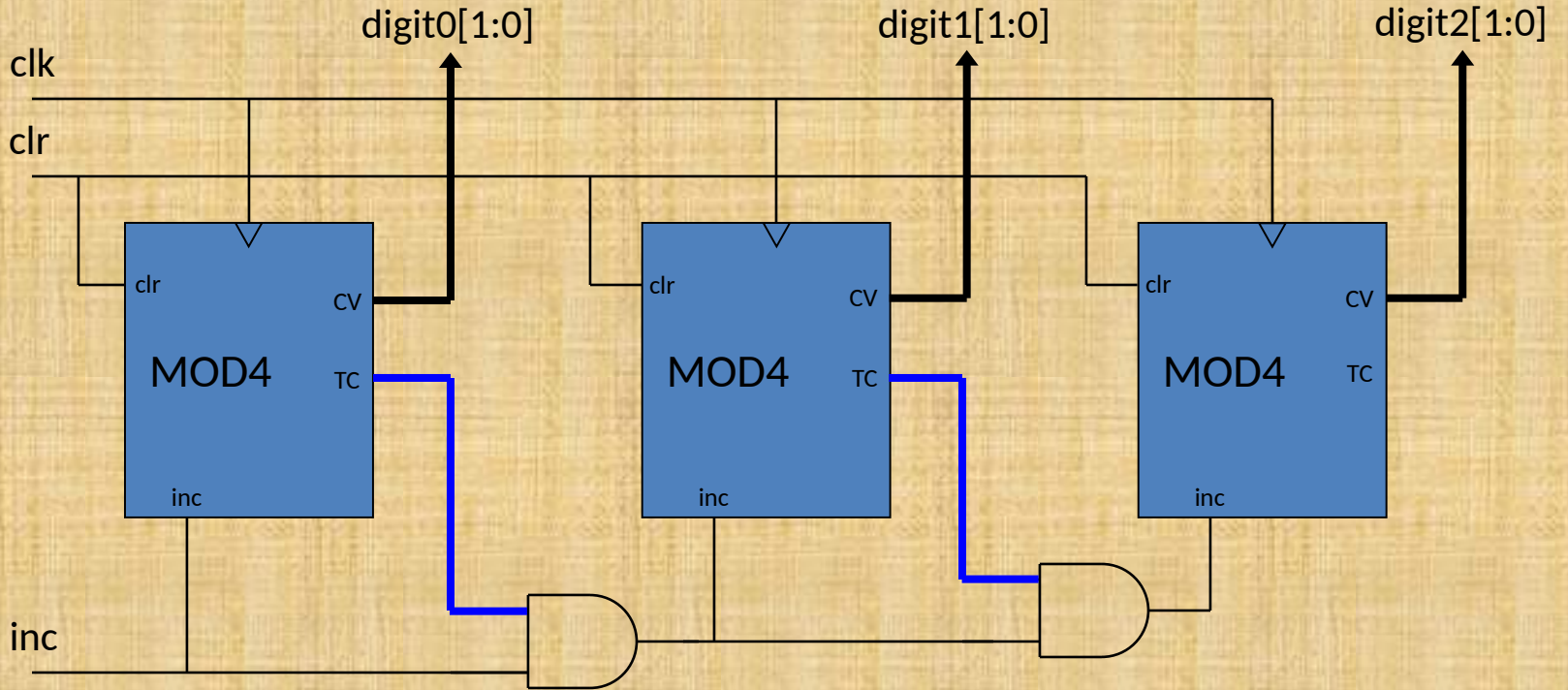


# Cascaded Counters

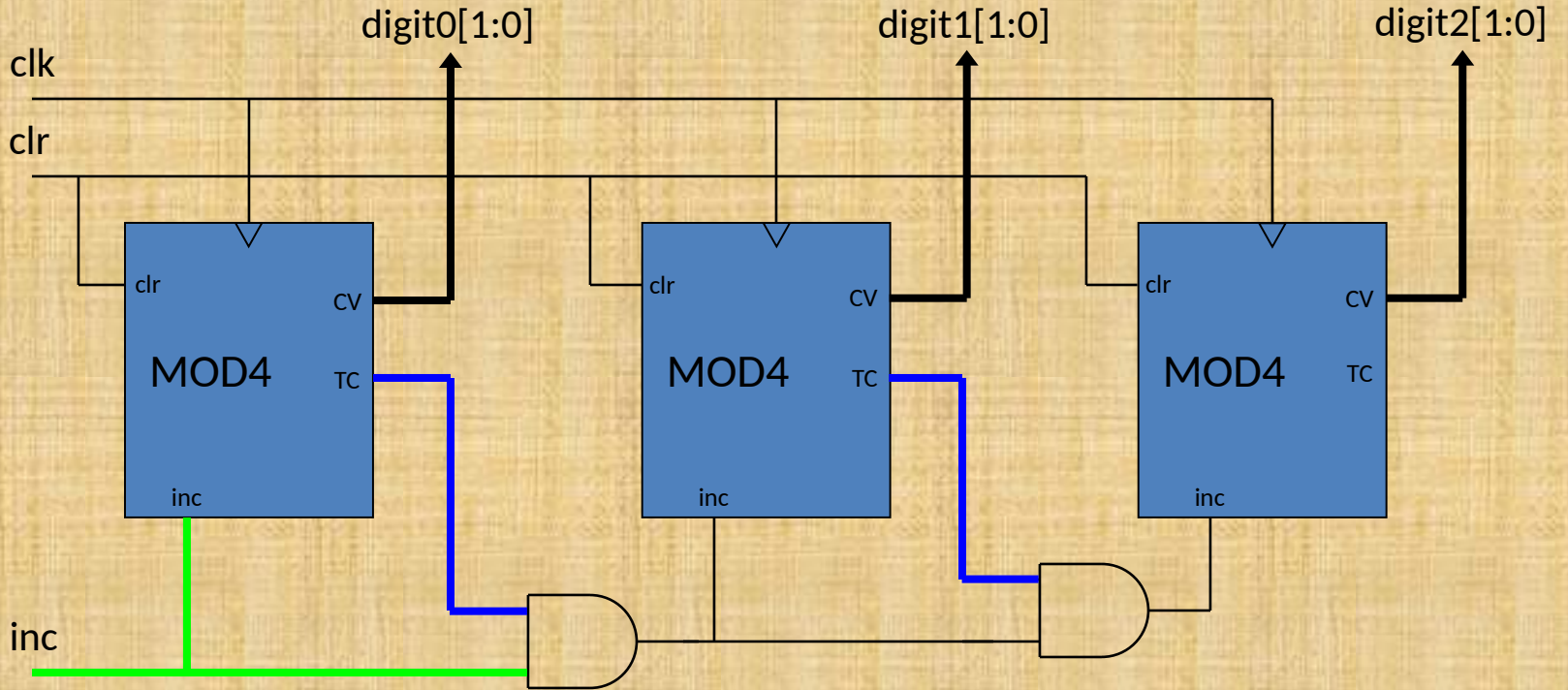




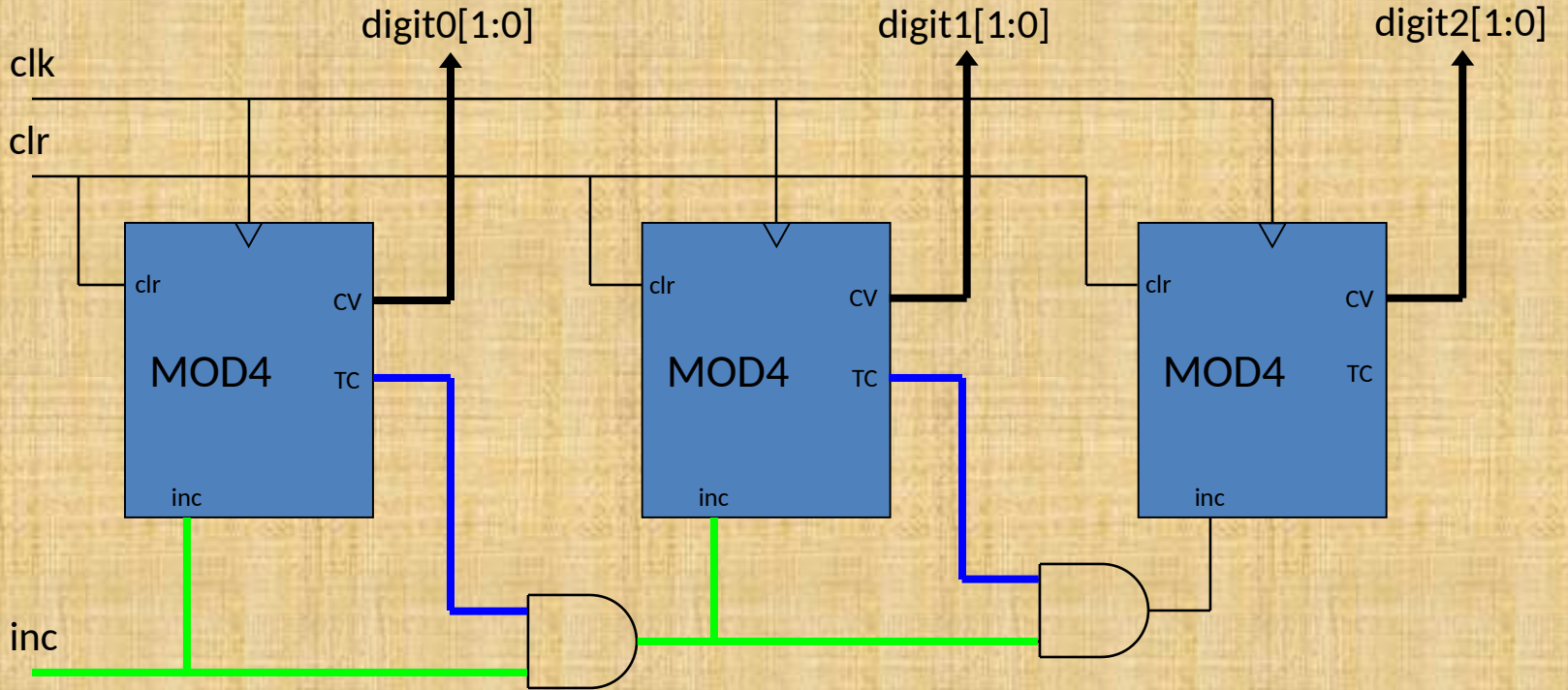
# Cascaded Counters



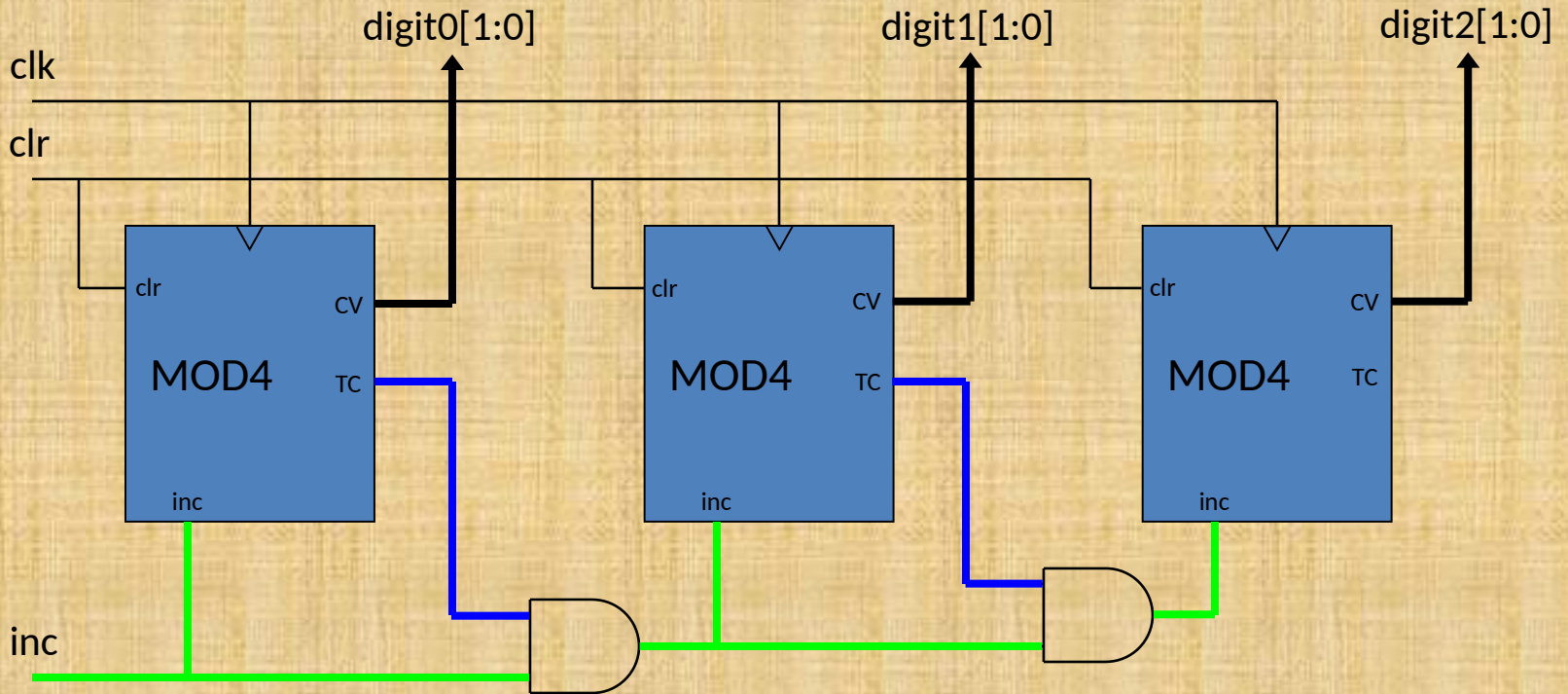
# Cascaded Counters



# Cascaded Counters



# Cascaded Counters

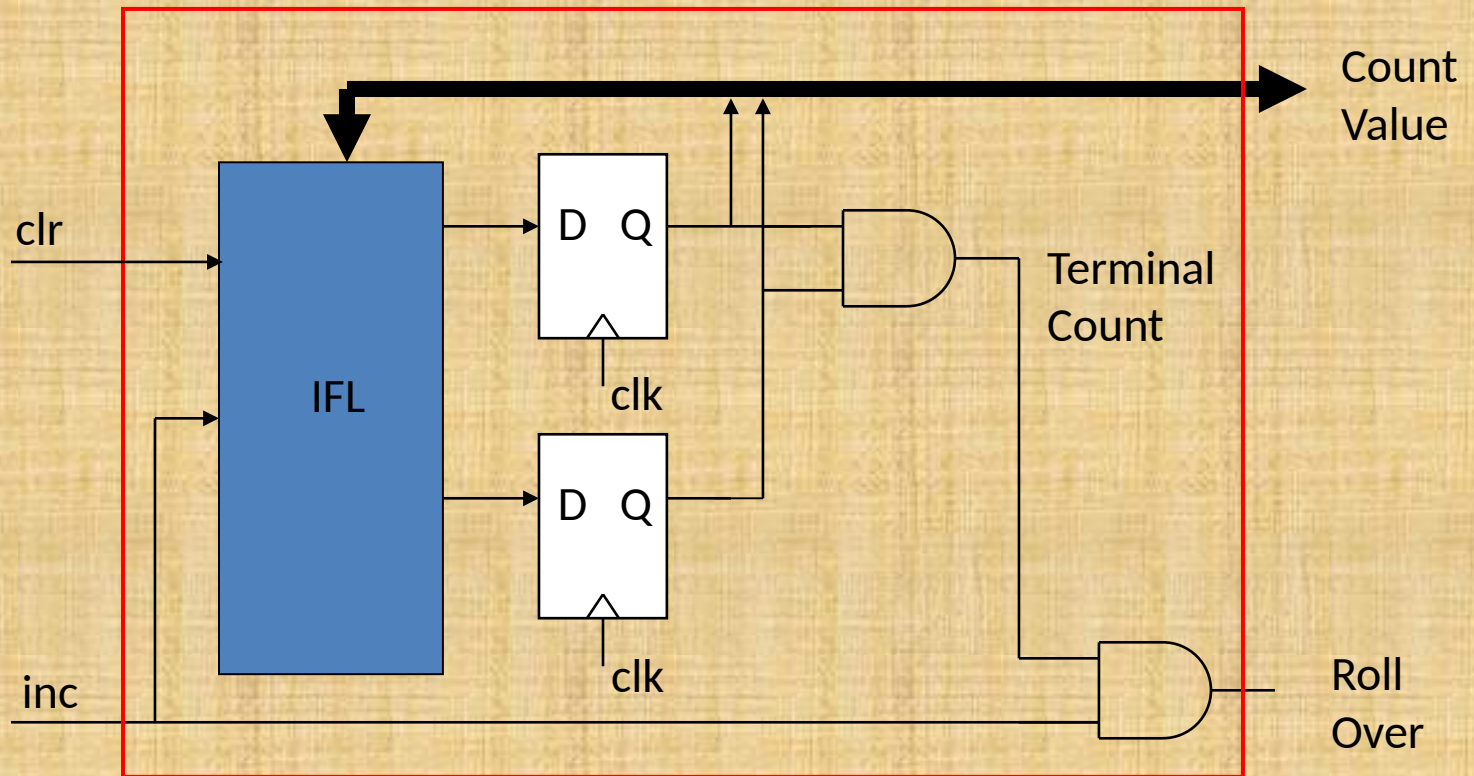


It looks like the inc signal ripples from counter to counter.  
How is this different from the ripple counter examples?



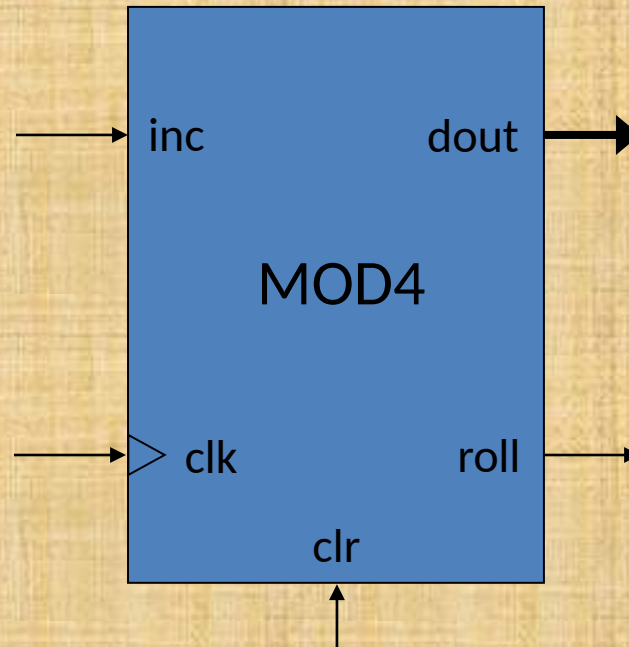
# A Mod-4 Counter

With consolidated rollover logic



A good mod-4 counter includes the logic within the red block.

# A Mod-4 Counter



# Cascading two Mod-4 Counters

Count Sequence:

digit1 digit0

↓ ↓

00

01

02

03

10

11

12

13

20

21

22

23

30

31

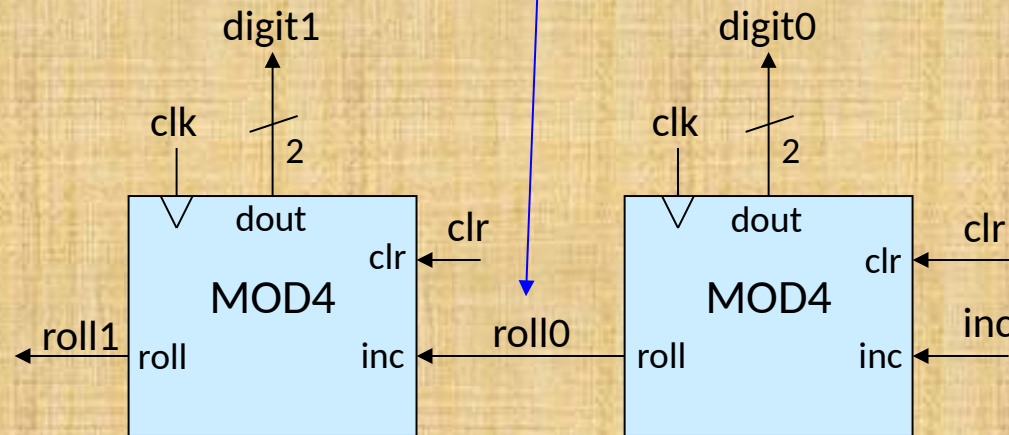
32

33

00

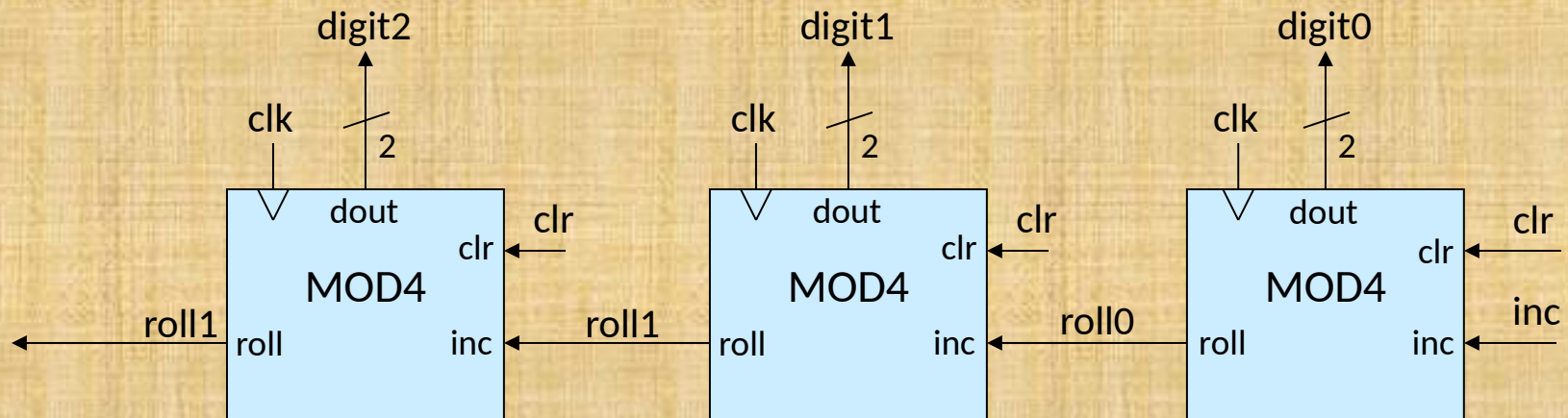
...

Increment higher digit's  
counter when lower digit's  
counter is rolling over



# Three-digit Mod-4 Counter

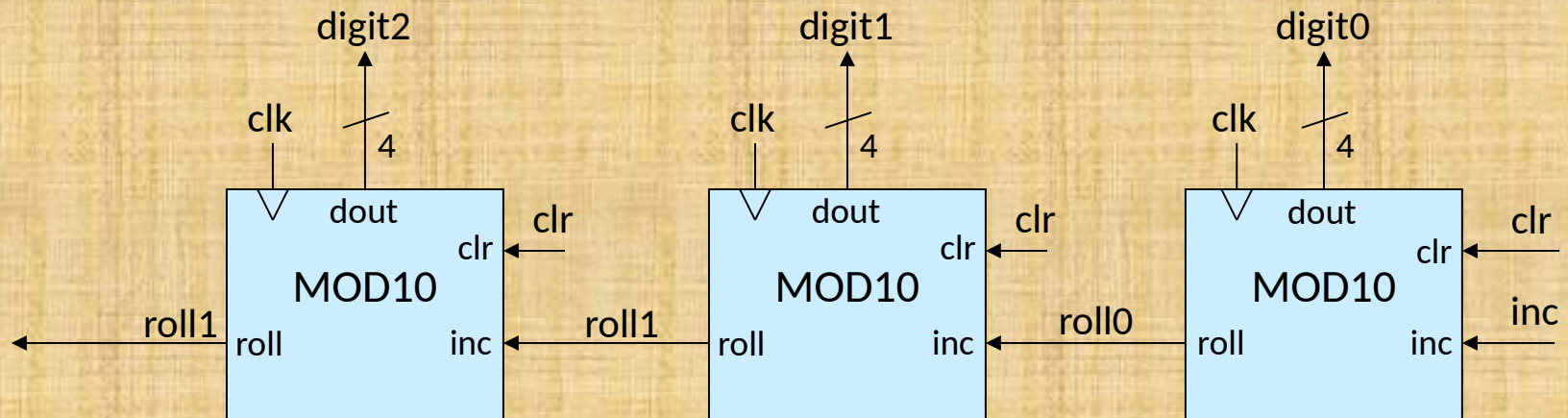
- Can combine any counters that have a rollover signal to make larger counters
  - Combine two 16-bit counters to make a 32-bit counter
  - Combine three mod-4 counters to make a three-digit mod-4 counter



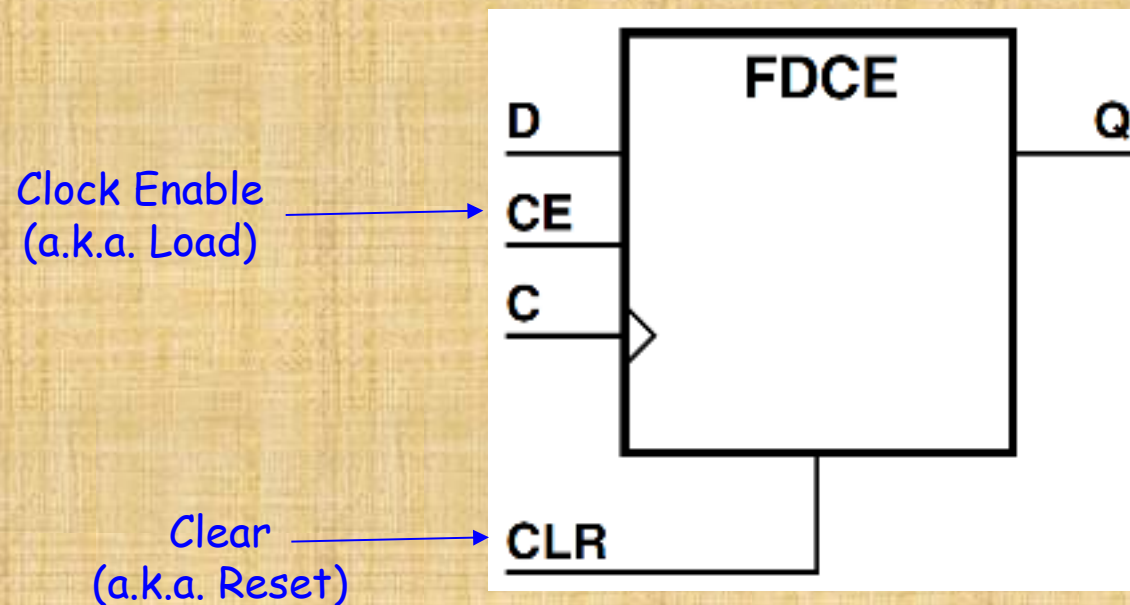


# BCD Counter

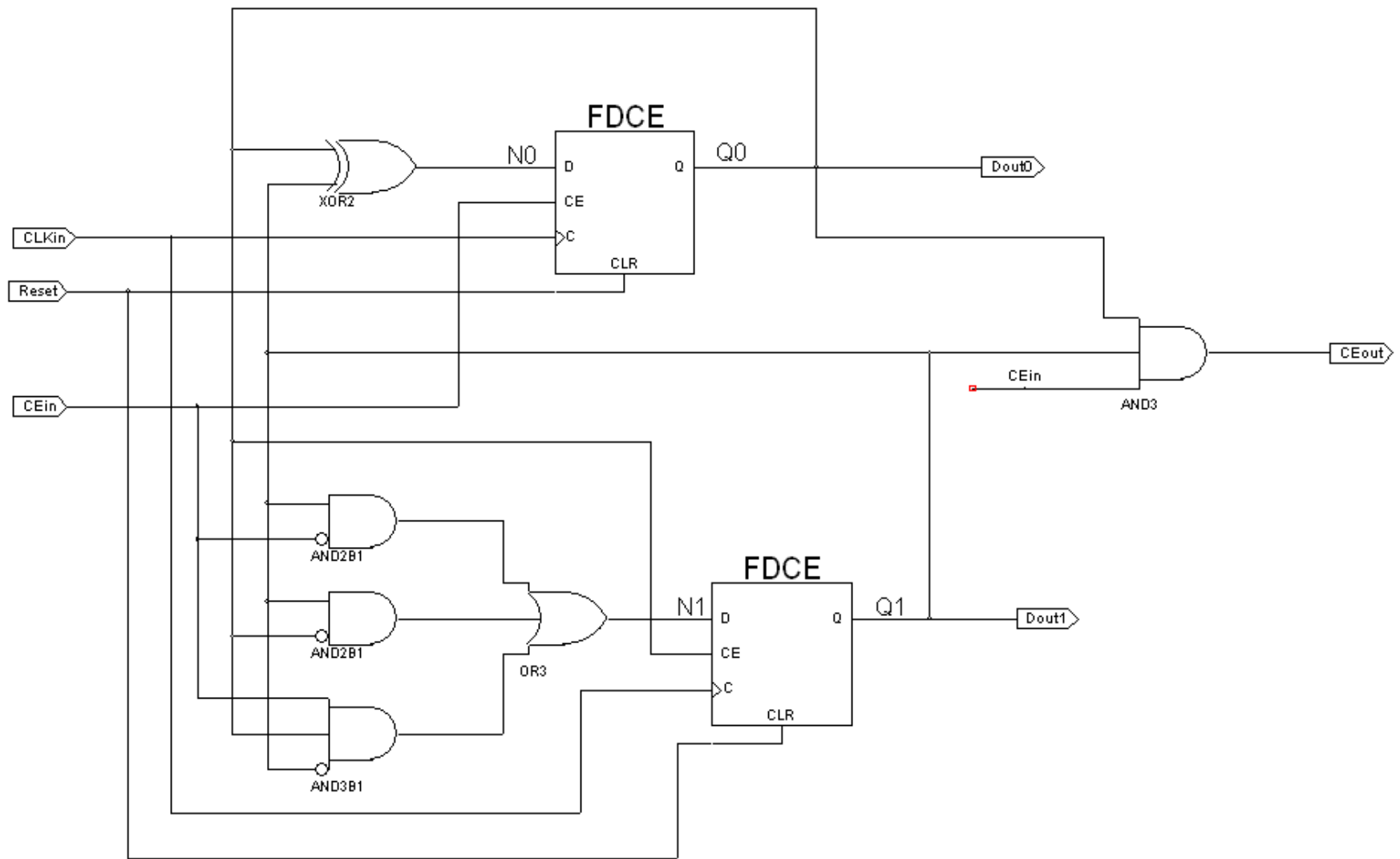
- Combine to create non-binary counters
  - BCD counter



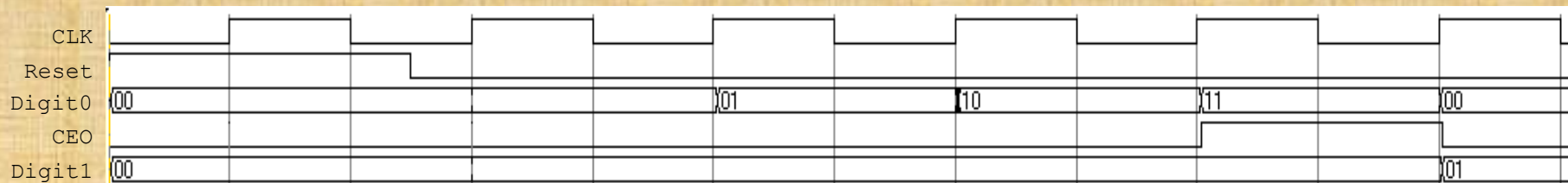
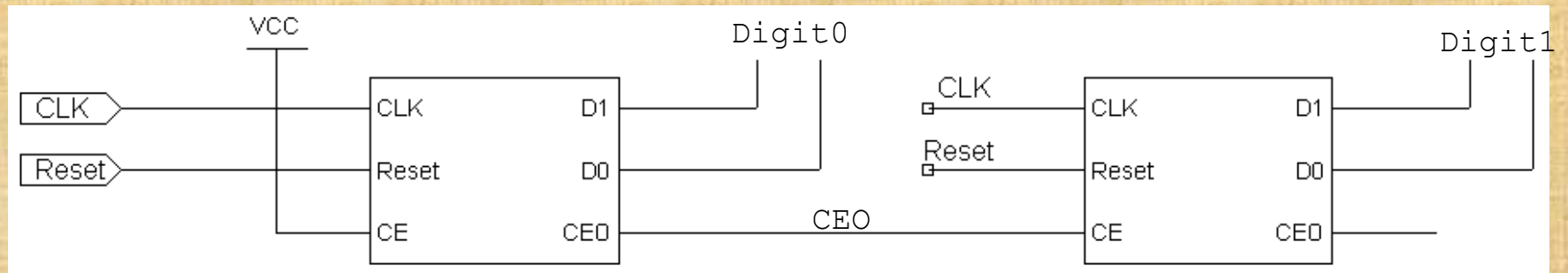
# D Flip Flop with Asynchronous Clear and Clock Enable



# Mod-4 Counter



# Cascaded Synchronous Counter





# Hybrid Counters

- Can combine different kinds of mod counters
  - Combine an 8-bit counter with a 16-bit counter to create a 24-bit counter
  - Combine mod-24 and mod-60 counters to create a digital H:M:S clock

