FEEDBACK NEURAL NETWORKS

Feedback NNs can have signals travelling in both directions by introducing loops in the network. Feedback networks are dynamic; their state is changing continuously until they reach an equilibrium point. They remain at the eq. point until the input changes and a new equilibrium needs to be found.

Feedback NNs are defined by the following differential equation:

$$\frac{dx_i}{dt} = -a_i x_i + \sum_{j=1}^{n} w_{ij} \cdot g_j(x_j) + I_i \quad , \quad i = 1, 2, \ldots, n$$

i: Neuron

n: number of neurons

$a_i > 0$: determines the convergence rate

w_{ij}: interconnection weights

I_i: Inputs

$g(\cdot)$: Activation function

x_i: State of neuron 'i'

Vector-Matrix Form:

$$\boxed{\dot{x} = -Ax + W.g(x) + I}$$

$x = [x_1 \;\; x_2 \; \dots \; x_n]^T$

$A = \text{diag}\{a_i > 0\}_{n\times n}$

$W = \{w_{ij}\}_{n\times n}$: Interconnection Matrix

$I = [I_1 \;\; I_2 \; \dots \; I_n]^T$

$g(x) = [g_1(x_1) \;\; g_2(x_2) \dots g_n(x_n)]^T$

$g(\cdot) \in S$ (Sigmoid Funct

x^* : Equilibrium Point

$$\boxed{0 = -Ax^* + Wg(x^*) + I}$$: Equilibrium Equation

- Equilibrium point might be either stable or unstable. This is determined by W-interconnection matrix.

LYAPUNOV STABILITY THEOREMS

Define information about the behaviour of the system without the need of solving the system.

1. First, we define Lyapunov Function belonging to the system. Function should satisfy the following properties:

 * $V(x) > 0$, $\forall x \neq 0$
 * $V(x) = 0$ only at $x = 0$.

2. Then, we analyze the derivative of the Lyapunov functio

If * $\dot{V}(x) \leq 0$, $\forall x \in R^n$
then $x=0$ is stable.

* $\dot{V}(x) < 0$, $\forall x \neq 0$
$\dot{V}(x) = 0$ only at $x=0$
then $x=0$ is asymptotically stable.

x^* : Ec

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(x^*): Equilibrium Point

$$\boxed{0 = -Ax^* + Wg(x^*) + I}$$: Equilibrium Equation

*Before applying Lyapunov Theorem, equilibrium point is shifted to origin.

$$\boxed{z = x - x^*}$$

$$\dot{z} = \dot{x}$$

$$\dot{z} = -A(z+x^*) + W.g(z+x^*) + Ax^* - Wg(x^*)$$

$$\dot{z} = -Az - \cancel{Ax^*} + W\underbrace{\left[g(z+x^*) - g(x^*)\right]}_{f(z)} + \cancel{Ax^*}$$

$$\boxed{\dot{z} = -Az + Wf(z)}$$ ⟹ The equilibrium point of this system is the origin.

1. The Lyapunov func. is determined for this system.
2. The derivative of this function is analysed to get the stability conditions.

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$\leq 0 \;, \forall x \in R^n$ (multiple eq. points)

$= 0$ is stable.

$< 0 \;, \forall x \neq 0$

$= 0$ only at $x = 0$

0 is asymptotically stable. (unique eq. point)

$$\dot{z} = -Az + Wf(z)$$

Stability Condition

$-W$: positive semi-definite.

Ex. $V(z) = \sum_{i=1}^{n} \int_0^{z_i} f(\phi)\, d\phi$

$V(z) > 0$

$V(z) = 0$, only at $z = 0$.

$$\dot{V}(z) = \sum_{i=1}^{n} f_i(z_i) \cdot \dot{z}_i$$

$$\dot{V}(z) = f^T(z) \cdot \dot{z}$$

$$\dot{V}(z) = f^T(z)\left[-Az + Wf(z)\right] = -Af^T(z) \cdot z + f^T(z) \cdot W \cdot f(z) = -Af^T(z)z - f^T(z)\underbrace{(-W)}_{-W \geq 0}f(z)$$

Vec

$\Rightarrow$

$X =$

$A =$

$W =$

$I =$

$g(x$

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tive
i-definite.

$-f^T(\tau)(-W)f(\tau)$

$-W \geq 0$

Vector-Matrix Form:

$$\boxed{\dot{x}(t) = -Ax(t) + Wg(x(t)) + I}$$

$$\boxed{\dot{x}(t) = -Ax(t) + Wg(x(t)-\tau) + I}$$

Pure-delayed Feedback NN

$$\boxed{\dot{x}(t) = -Ax(t) + Wg(x(t)) + W^2 g(x(t-\tau)) + I}$$

Hybrid

τ: Delay.