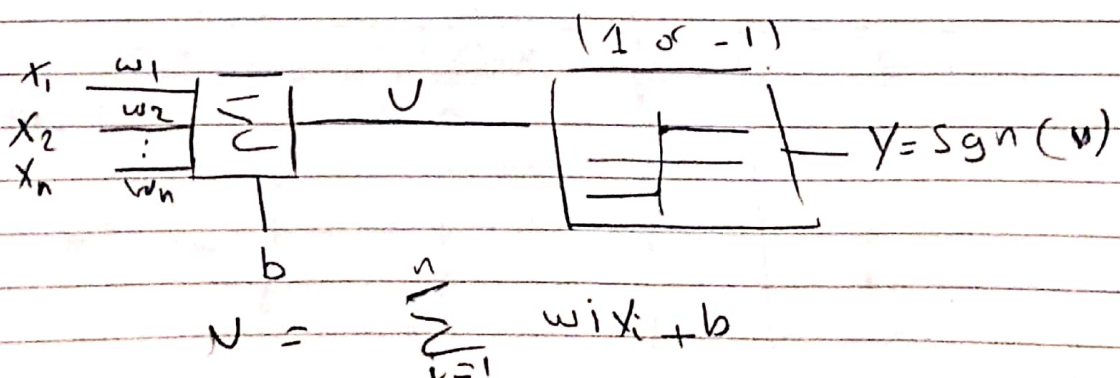


The perceptron

- * the perceptron is a simplified representation of the biological neuron in the brain
- * The perceptron model was proposed by McCulloch and Pitts in 1943
- * The perceptron is the simplest form of a neural network for patterns that are linearly separable.
- * The structure of a perceptron consists of a single neuron with adjustable synaptic weights and bias.
- * The weights are adjusted during the training phase, as training data is presented to it
- * The model consists of a linear combiner followed by a hard limiter (performing the signum function).
- * Also incorporates an externally applied bias.



Linearity Separable?

* perceptrons can only classify linearly separable cases.

* let's say we want to classify a set of data into Group (A) (G_A)
either

or group B (G_B).

* if G_A and G_B are linearly separable there exist a separating hyperplane between the two groups which is linear in nature.

* in simple terms there is a straight line dividing between G_A and G_B .

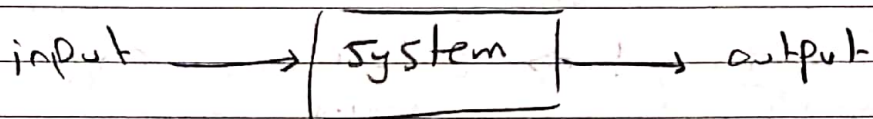
EX: consider a case of AND and OR.

Input (A)/(B)		Output (AND)	
0	0	0	} G_A
0	1	0	
1	0	0	} G_B
1	1	1	

(A)	(B)	Output (OR)	
0	0	0	} G_A
0	1	1	
1	0	1	} $\rightarrow G_B$
1	1	1	

* Mathematical Review for Neural networks.

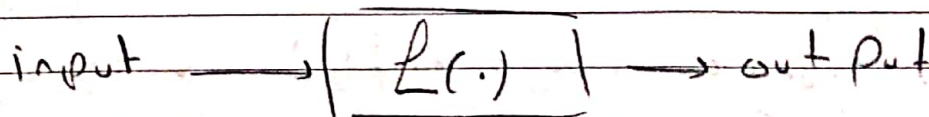
System : is a collection of elements or components that are organized for a common purpose.



- Systems have structure, defined by components / elements and their composition.
- Systems have behaviour, which involves inputs, processing and outputs of material, energy, information or data.
- Systems have interconnectivity, the various parts have structural or functional relationships to each other.

linear system :

linear system is a mathematical model of a system based on the use of a linear operator



linear operator.

- linear system satisfies the properties of Superposition, additivity and homogeneity

Superposition principle :

homogeneity:

$$\left. \begin{array}{l} L(u_1) = y_1 \\ L(u_2) = y_2 \end{array} \right\} \Rightarrow \begin{array}{l} L(au_1) = ay_1 \\ L(au_2) = ay_2 \end{array}$$

additivity :

$$\left. \begin{array}{l} L(u_1) = y_1 \\ L(u_2) = y_2 \end{array} \right\} \Rightarrow L(u_1 + u_2) = L(u_1) + L(u_2) = y_1 + y_2$$

$$\underline{\alpha_1 u_1 + \alpha_2 u_2} \quad \left| \quad \underline{L(\cdot)} \quad \right| \quad \underline{\alpha_1 y_1 + \alpha_2 y_2}$$

→ Non linear System :

a non linear system is a mathematical model of a system that does not satisfy the superposition principle. The output of a non linear sys is not directly proportional to its input.

$$\text{Ex 8 } \int (\alpha_1 y_1 + \alpha_2 y_2) dy \underset{\text{linear}}{=} \alpha_1 \int y_1 dy + \alpha_2 \int y_2 dy$$

$$\cos(\alpha_1 x_1 + \alpha_2 x_2) \neq \alpha_1 \cos x_1 + \alpha_2 \cos x_2$$

Non linear

State Space :

To provide a convenient way to model and analyze sys with multiple inputs and outputs, one needs to represent the state space of a system and can be represented as a vector within that space.

The space is a domain that determines the conditions of the sys.

$$\frac{dx(t)}{dt} = A \cdot x(t) + u \rightarrow \text{Time Invariant system matrix}$$

(numerical values of physical components of a system)

(No time dependence)

$$\frac{dx(t)}{dt} = A(t) \cdot x(t) + u \rightarrow \text{Time Variant}$$

(time dependence)

متغير مع الزمن
متغير بالزمن

→ functions :

* Continuous :

$$\lim_{x \rightarrow c} f(x) = f(c)$$

مستمر

Continuous

منقطع

discontinuous

* increasing functions.

→ if $y > x$ then $f(y) \geq f(x)$

$f(x)$ monotonically increasing.
(non-decreasing).

→ if $y > x$ then $f(y) > f(x)$

$f(x)$: strictly increasing.

* decreasing :

if $y > x \Rightarrow f(y) \leq f(x)$

$f(x)$: monotonically decreasing.
(non increasing)

if $y > x \rightarrow f(y) < f(x)$

$f(x)$: strictly decreasing.

* Derivative fun :

if $\frac{df(x)}{dx} > 0$, $\forall x \Rightarrow f(x)$ strictly increasing.

if $\frac{df(x)}{dx} < 0$, $\forall x \Rightarrow f(x)$ strictly decreasing.

if $\frac{df(x)}{dx} = 0$, $\forall x \Rightarrow$ Stationary point
global max
global min
local max
local min

$\hookrightarrow d'(f) \geq 0 \Rightarrow (\text{local/global}) \min$

$d'(f) < 0 \Rightarrow (\text{local/global}) \max$

ss pinvles "x" \Rightarrow y: local
N: Global

* Bounded fun is

if there exist a real number $M < \infty$
such that

$|f(x)| \leq M, |x| \rightarrow \infty \rightarrow f(x): \text{Bounded.}$

