PERCEPTRON L.R. $\Longrightarrow$ Only works for threshold activation function.
(Supervised)

(sgn(·) or step(·))

x_i : Inputs

w_i : Weights

$u_i = \langle w_i, x_i \rangle$: Net inputs

$y_i = f(u_i)$: Outputs

c : Learning Rate

r : Learning Signal

d_i : Desired Outputs

$$\boxed{r = d - y}$$

$$\boxed{\Delta w^k = c \cdot r \cdot x^k}$$

$$\Longrightarrow w^{k+1} = w^k + \Delta w^k$$

$$\boxed{w^{k+1} = w^k + c \cdot (d^k - y^k) \cdot x^k}$$

tion.

Ex.

No	Inputs	Desired Outputs
1	$x^1 = [1 \ \ 0 \ \ 1]^T$	-1
2	$x^2 = [0 \ \ -1 \ \ -1]^T$	1
3	$x^3 = [-1 \ \ -0.5 \ \ -1]^T$	1

$f(\cdot) = \mathrm{sgn}(\cdot)$

$w^1 = [1 \ \ -1 \ \ 0]^T$

$c = 0{,}1$

Train the system.

$\underline{\underline{k=1}}$

$$y^1 = f(u^1) = \operatorname{sgn}\big(\underbrace{\langle w^1, x^1\rangle}\big) = 1 \neq d^1 \quad (d^1 = -1)$$

$$\begin{bmatrix} 1 & -1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} = 1$$

$$\Delta w^1 = c \cdot (d^1 - y^1) \cdot x^1 = 0{,}1 \cdot (-1-1) \cdot \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -0{,}2 \\ 0 \\ -0{,}2 \end{bmatrix}$$

$$w^2 = w^1 + \Delta w^1 = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} + \begin{bmatrix} -0{,}2 \\ 0 \\ -0{,}2 \end{bmatrix} = \begin{bmatrix} 0{,}8 \\ -1 \\ -0{,}2 \end{bmatrix}$$

$$\ldots gn(\underbrace{\langle w^1, x^1\rangle}) = 1 \neq d^1 \quad (d^1 = -1)$$

$$\ldots] = 1$$

$$\ldots - y^1)\cdot x^1 = 0{,}1\cdot(-1-1)\cdot\begin{bmatrix}1\\0\\1\end{bmatrix} = \begin{bmatrix}-0{,}2\\0\\-0{,}2\end{bmatrix}$$

$$\begin{bmatrix}1\\-1\\0\end{bmatrix} + \begin{bmatrix}-0{,}2\\0\\-0{,}2\end{bmatrix} = \begin{bmatrix}0{,}8\\-1\\-0{,}2\end{bmatrix}$$

$$f = sgn(x) \Rightarrow \begin{array}{l} x \geq 0 \rightarrow f(\cdot) = 1 \\ x < 0 \rightarrow f(\cdot) = -1 \end{array}$$

$k=2$

$$y^2 = f(u^2) = sgn(\underbrace{\langle w^2, x^2\rangle}) = 1 = d^2$$

$\Longrightarrow$ only works for threshold activation (sgn(·) ... ·))

$$\begin{bmatrix}0{,}8 & -1 & -0{,}2\end{bmatrix}\begin{bmatrix}0\\-1\\-1\end{bmatrix} = 1{,}2$$

$$\Delta w^2 = 0$$

$$w^3 = w^2$$

$f = \text{sgn}(x) \Rightarrow$ $x \geq 0 \rightarrow f(\cdot) = 1$

$x < 0 \rightarrow f(\cdot) = -1$

$\underline{k=2}$

$y^2 = f(u^2) = \text{sgn}(\langle w^2, x^2 \rangle) = 1 = d^2$

$\begin{bmatrix} 0{,}8 & -1 & -0{,}2 \end{bmatrix} \begin{bmatrix} 0 \\ -1 \\ -1 \end{bmatrix} = 1{,}2$

$\Delta w^2 = 0$

$w^3 = w^2$

$\underline{k=3}$

$y^3 = f(u^3) = \text{sgn}(\langle w^3, x^3 \rangle) = -1 \neq d^3$

$u^3 = \begin{bmatrix} 0{,}8 & -1 & -0{,}2 \end{bmatrix} \begin{bmatrix} -1 \\ -0{,}5 \\ -1 \end{bmatrix} = (-0{,}1)$

$\Delta w^3 = c.(d^3 - y^3).x^3 = 0{,}1\,(1-(-1)).\begin{bmatrix} -1 \\ -0{,}5 \\ -1 \end{bmatrix} = \begin{bmatrix} -0{,}2 \\ -0{,}1 \\ -0{,}2 \end{bmatrix}$

$w^4 = w^3 + \Delta w^3 = \begin{bmatrix} 0{,}8 \\ -1 \\ -0{,}2 \end{bmatrix} + \begin{bmatrix} -0{,}2 \\ -0{,}1 \\ -0{,}2 \end{bmatrix} = \begin{bmatrix} 0{,}6 \\ -1{,}1 \\ -0{,}4 \end{bmatrix}$

Ex.

No	Inputs
1	$x^1 = [1$ …
2	$x^2 = [0$ …
3	$x^3 = [-1$ …

$f(\cdot) = \text{sgn}(\cdot)$

$w^1 = [1 \;\; -1 \;\; 0]^T$

$c = 0{,}1$

Train the syst…

$x, y \in R^n$

$\langle x, y \rangle = x^T \cdot y$

$\underline{k=4}$

$$y^1 = f(u^1) = \mathrm{sgn}(\underbrace{\langle w^4, x^1 \rangle}) = 1 \stackrel{?}{=} d^1$$

$$\begin{bmatrix} 0,6 & -1,1 & -0,4 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} = 0,2$$

$$\Delta w^4 = 0,1 \cdot (-1-1) \cdot \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -0,2 \\ 0 \\ -0,2 \end{bmatrix}$$

$$w^5 = w^4 + \Delta w^4 = \begin{bmatrix} 0,6 \\ -1,1 \\ -0,4 \end{bmatrix} + \begin{bmatrix} -0,2 \\ 0 \\ -0,2 \end{bmatrix} = \begin{bmatrix} 0,4 \\ -1,1 \\ -0,6 \end{bmatrix}$$

$\rightarrow f(\cdot) = 1$

$\rightarrow f(\cdot) = -1$

$1 = d^2$

k=6

$$y^3 = f(u^3) = \mathrm{sgn}(\underbrace{\langle w^6, x^3 \rangle}) = 1 = d^3$$

$$\begin{bmatrix} 0{,}4 & -1{,}1 & -0{,}6 \end{bmatrix} \begin{bmatrix} -1 \\ -0{,}5 \\ -1 \end{bmatrix} = 0{,}75$$

$$\Delta w^6 = 0$$

$$\boxed{w^7 = w^6 = w^5}$$

L.R

$$\Delta w = c \cdot r \cdot x$$

k=7

$$y^1 = f(u^1) = \mathrm{sgn}(\underbrace{\langle w^7, x^1 \rangle}) = d^1$$

$$\begin{bmatrix} 0{,}4 & -1{,}1 & -0{,}6 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} = -0{,}2$$

$$\Delta w^7 = 0$$

$$\boxed{w = \begin{bmatrix} 0{,}4 & -1{,}1 & -0{,}6 \end{bmatrix}^T}$$

Ex.

No	Inputs	Desired Outputs
1	$x^1 = [1 \ 0 \ 1]^T$	-1
2	$x^2 = [0 \ -1 \ -1]^T$	1
3	$x^3 = [-1 \ -0{,}5 \ -1]^T$	1

$f(\cdot) = \mathrm{sgn}(\cdot)$

$w^1 = [1 \ -1 \ 0]^T$

$c = 0{,}1$

Train the system.

Scanned by CamScanner

$$f = \mathrm{sgn}(x) \Rightarrow \begin{cases} x \geq 0 \rightarrow f(\cdot) = 1 \\ x < 0 \rightarrow f(\cdot) = -1 \end{cases}$$

DELTA LEARNING RULE
(Supervised)

$$\boxed{r = (d - y) \cdot y'}$$

$$y = f(\langle w, x \rangle)$$

$$y' = \frac{df(x)}{dx}$$

WIDROW-HOFF L.R.
(Supervised)

$$\boxed{r = (d - w.x)}$$

CORRELATION L.R.

$$\boxed{r = d}$$

$x, y \in \mathbb{R}^n$

$$\langle x, y \rangle = x^T \cdot y$$

$k = 4$

$$y^1 = f(u^1) = \operatorname{sgn}(\underbrace{\langle w^4, x^1 \rangle}) = 1 \stackrel{?}{=} d^1$$

$$\begin{bmatrix} 0{,}6 & -1{,}1 & -0{,}4 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} = 0{,}2$$

$$\Delta w^4 = 0{,}1 \cdot (-1-1) \cdot \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -0{,}2 \\ 0 \\ -0{,}2 \end{bmatrix}$$

$$w^5 = w^4 + \Delta w^4 = \begin{bmatrix} 0{,}6 \\ -1{,}1 \\ -0{,}4 \end{bmatrix} + \begin{bmatrix} -0{,}2 \\ 0 \\ -0{,}2 \end{bmatrix} = \begin{bmatrix} 0{,}4 \\ -1{,}1 \\ -0{,}6 \end{bmatrix}$$

$x, y \in R^n$

$\langle x, y \rangle = x^T . y$

<u>k=4</u>

$y^1 = f(u^1) = sgn(\underbrace{\langle w^4, x^1 \rangle}) = 1 \stackrel{?}{=} d^1$

$[0,6 \quad -1,1 \quad -0,4] \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} = 0,2$

$\Delta w^4 = 0,1 . (-1-1) . \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -0,2 \\ 0 \\ -0,2 \end{bmatrix}$

$w^5 = w^4 + \Delta w^4 = \begin{bmatrix} 0,6 \\ -1,1 \\ -0,4 \end{bmatrix} + \begin{bmatrix} -0,2 \\ 0 \\ -0,2 \end{bmatrix} = \begin{bmatrix} 0,4 \\ -1,1 \\ -0,6 \end{bmatrix}$

$f = sgn(x) \Rightarrow \begin{matrix} x \geq 0 \rightarrow f(\cdot) = 1 \\ x < 0 \rightarrow f(\cdot) = -1 \end{matrix}$

DELTA LEARNING RULE
(Supervised)

$\boxed{r = (d - y) . y'}$

$y = f(\langle w, x \rangle)$

$y' = \frac{df(x)}{dx}$

WIDROW_HOFF L.R.
(Supervised)

$\boxed{r = (d - w.x)}$

CORRELATION L.R.

$\boxed{r = d}$

HEBBIAN L.R.
(Unsupervised)

$\boxed{r = f(w.x)}$

$\boxed{r = y}$

<u>k=7</u>

$y^1 = f(u^1) = sgn(\underbrace{\langle w^7, x^1 \rangle}) = d^1$

$[0,4 \quad -1,1 \quad -0,6] \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} = -0,2$

$\Delta w^7 = 0$

$\boxed{w = [0.4 \quad -1,1 \quad -0,6]^T}$

Ex.

f(.

w