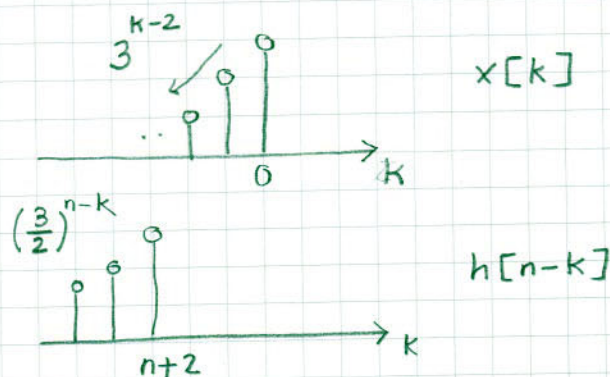


Q1
20p

$$\textcircled{1} \quad n+2 < 0 \rightarrow n < -2$$

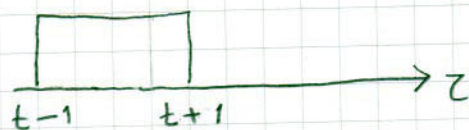
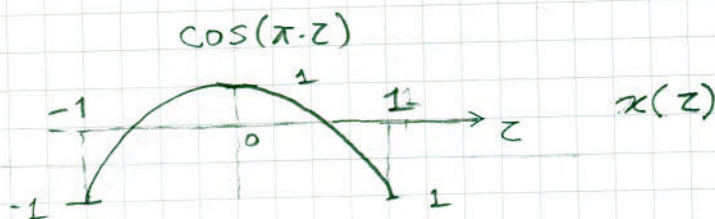
$$\begin{aligned} y[n] &= \sum_{k=-\infty}^{n+2} 3^{k-2} \left(\frac{3}{2}\right)^{n-k} = \left(\frac{3}{2}\right)^n \cdot 3^{-2} \sum_{k=-\infty}^{n+2} 3^k \cdot \left(\frac{2}{3}\right)^k \\ &= \left(\frac{3}{2}\right)^n \cdot 3^{-2} \sum_{k=-\infty}^{n+2} 2^k = \left(\frac{3}{2}\right)^n \cdot 3^{-2} \cdot 2^{n+2} \left(\frac{2}{2-1}\right) \\ &= \left(\frac{3}{2}\right)^n \cdot 3^{-2} \cdot 2^{n+3} \end{aligned}$$

$$\textcircled{2} \quad n \geq -2$$

$$y[n] = \left(\frac{3}{2}\right)^n \cdot 3^{-2} \sum_{k=-\infty}^0 2^k = \left(\frac{3}{2}\right)^n \cdot 3^{-2} \cdot 2$$

$$y[n] = \begin{cases} \left(\frac{3}{2}\right)^n \cdot 3^{-2} \cdot 2^{n+3} & , \quad n < -2 \\ \left(\frac{3}{2}\right)^n \cdot 3^{-2} \cdot 2 & , \quad n \geq -2 \end{cases}$$

Q2
20p



① $t+1 < -1 \rightarrow t < -2 \quad y(t) = 0$

② $-1 \leq t+1 < 1 \rightarrow -2 \leq t < 0$

$$y(t) = \int_{-1}^{t+1} \cos(\pi z) dz = \frac{1}{\pi} \sin(\pi z) \Big|_{-1}^{t+1}$$

$$= \frac{1}{\pi} [\sin(\pi(t+1)) - \sin(-\pi)]$$

$$= \frac{1}{\pi} \sin[\pi(t+1)]$$

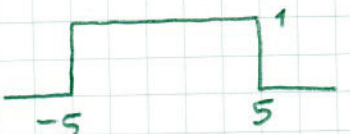
③ $t \geq 0 \rightarrow t-1 < 1 \quad t < 2$

$$y(t) = \int_{t-1}^1 \cos(\pi z) dz = \frac{1}{\pi} \sin(\pi z) \Big|_{t-1}^1 = -\frac{1}{\pi} \sin[\pi(t-1)]$$

④ $t \geq -2 \quad y(t) = 0$

$$y(t) = \begin{cases} \frac{1}{\pi} \sin[\pi(t+1)], & -2 \leq t < 0 \\ -\frac{1}{\pi} \sin[\pi(t-1)], & 0 \leq t < 2 \\ 0, & \text{elsewhere} \end{cases}$$

3a
10p

$$h(t) = \int_{-5}^5 \delta(t-z) dz = u[t+5] - u[t-5]$$


The graph shows a rectangular pulse function $h(t)$ that is 1 for $-5 \leq t \leq 5$ and 0 elsewhere. The pulse is labeled $h(t)$ and has a height of 1.

3b
10p

$$\begin{aligned} t < -5 & \quad s(t) = 0 \\ -5 \leq t < 5 & \quad s(t) = \int_{-5}^t 1 \cdot dz = t - 5 \\ t \geq 5 & \quad s(t) = \int_{-5}^5 1 \cdot dz = 10 \end{aligned}$$

$$s(t) = \begin{cases} t-5, & -5 \leq t < 5 \\ 10, & t \geq 5 \\ 0, & t < -5 \end{cases}$$

4a
3p

not memoryless

4b
3p

not causal

4c
4p

$$\sum_{k=-2}^{\infty} \left(\frac{3}{2}\right)^k = \infty$$

\therefore not stable

5a
3p

not memoryless

5b
3p

not causal

5c
4p

$$\int_{-5}^5 1 dz = 10 < \infty$$

\therefore stable

Q6a
Q.10

Homogeneity

$$a e^{-|t|} x(t) = a y(t) \quad \checkmark$$

Superposition

$$e^{-|t|} (x_1(t) + x_2(t)) = \underbrace{e^{-|t|} x_1(t)}_{y_1(t)} + \underbrace{e^{-|t|} x_2(t)}_{y_2(t)} \quad \checkmark$$

This system is Linear

Q6b
10P

$$e^{-|t|} x(t-t_0) \neq e^{-|t-t_0|} x(t-t_0)$$

not T.I.