Discrete Mathematics (26.12.2017)

1. Explain the terms Simple Graph, Path, Circuit, Euler Path, Euler Circuit, Tree, Branch and Link by giving an example for each term. **(30 pts)**.

2. Consider the boolean function $f(a,b,c) = abc + ab'c' + a'bc' + a'b'c'$. **(20 pts)**.

a) Implement $f(a,b,c)$ by using the logic gates without simplifying the function.

b) Implement $f(a,b,c)$ by using the logic gates in its simplest form.

3. a) Prove that a strictly increasing function from R to itself is one-to-one. **(05 pts)**.

b) Give an example of increasing function from R to itself that is not one-to-one. **(05 pts)**.

4. Let $a_n = -3a_{n-1} - 3a_{n-2} - a_{n-3} + n^2$ with $a_0 = 1$, $a_1 = 2$ and $a_2 = 3$. Find the solution of this recurrence relation. **(40 pts)**.

$a_3 = -3a_2 - 3a_1 - a_0 + 9$

$a_3 = -3 \cdot 3 - 3 \cdot 2 - 1 + 9 = -9 - 6 - 1 + 9 = -7$ $\boxed{a_3 = -7}$

Duration : 90 minutes.