Precedence Rule for Time-Shifting & Time-Scaling

$$y(t) = x(\underline{a}t - \underline{b})$$

✓ $y(0) = x(-b)$

✓ $x(0) = y\left(\frac{b}{a}\right)$

① Define an intermediate signal.

$$v(\underline{t}) = \underline{x}(\underline{t} - \underline{b})$$ <u>Shift!</u>

② $y(t) = v(at)$ <u>scale</u>

$$= x(at - b)$$

/* If we first scale and then shift:

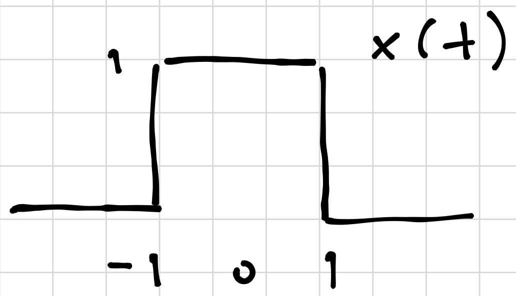
$$v(t) = x(at)$$

$$y(t) \neq v(t-b) = x[a(t-b)]$$

$$= x(at - ab)$$

Does not work! */

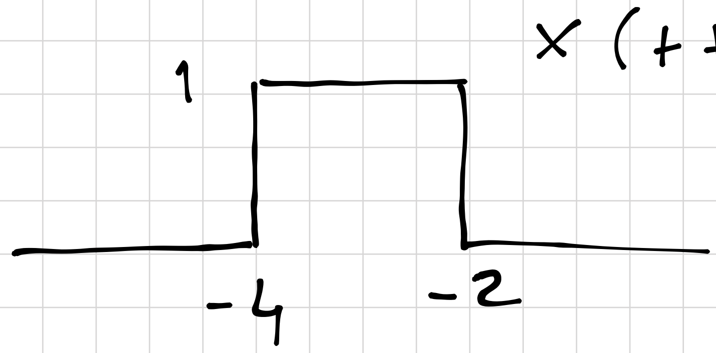
Ex



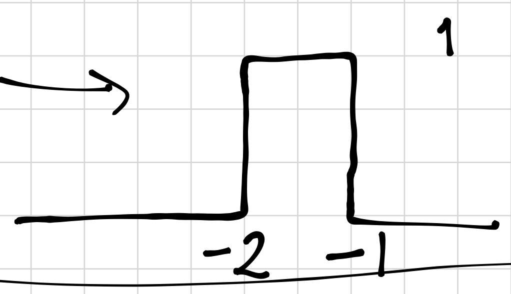
$x(2t+3) = ?$

$v(t) = x(t+3)$

$v(t) = x(t+3)$



$$y(t) = v(2t) = x(\underline{2}t + 3) \longrightarrow$$



Same remarks apply to DISCRETE-TIME Signals!!!

Elementary Signals

① Exponential Signals

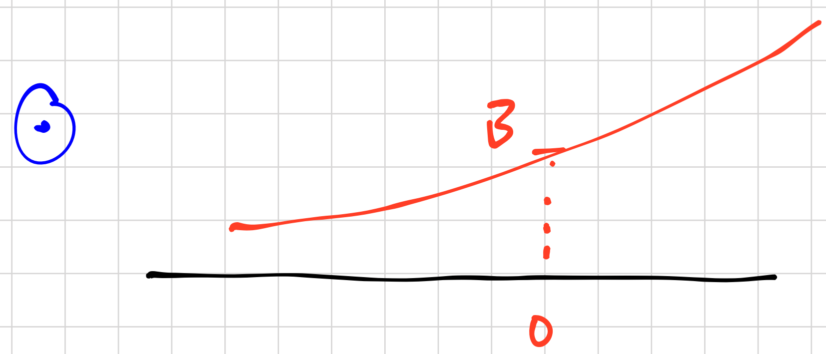
CT

$$x(t) = \underset{\substack{\uparrow \\ \text{amplitude}}}{B} \cdot e^{at} \quad ; \; B, a \in \mathbb{R}$$

- $a < 0$: Decaying exponential
- $a > 0$: Growing exponential.

$x(t)$ when $a < 0$

$x(t)$ when $a > 0$

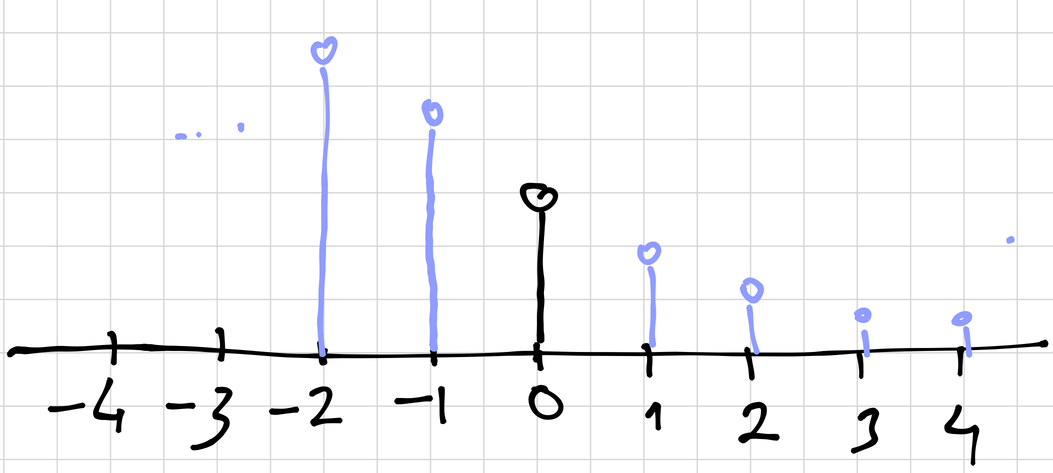


For DT signals:

$$x[n] = B \cdot r^n \qquad (* \; r = e^a) \qquad B, r \in \mathbb{R}$$

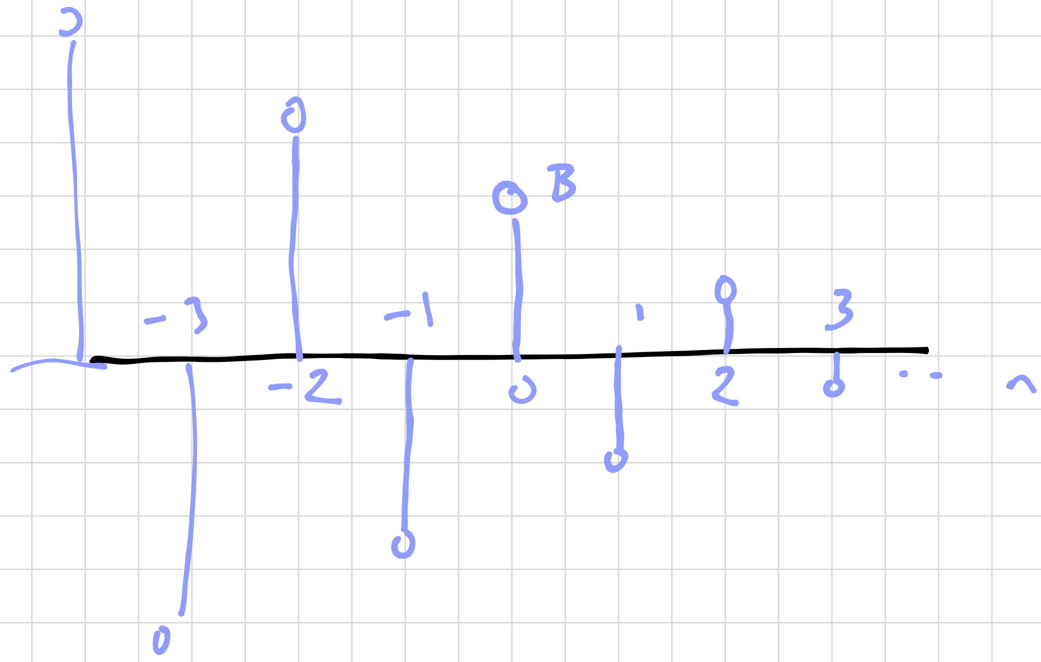
- when $-1 < r < 0$ or $0 < r < 1$ then $x[n]$ is a decaying exponential.
- $r > 1$ or $r < -1 \Rightarrow$ Growing Exponential.

$0 < r < 1$

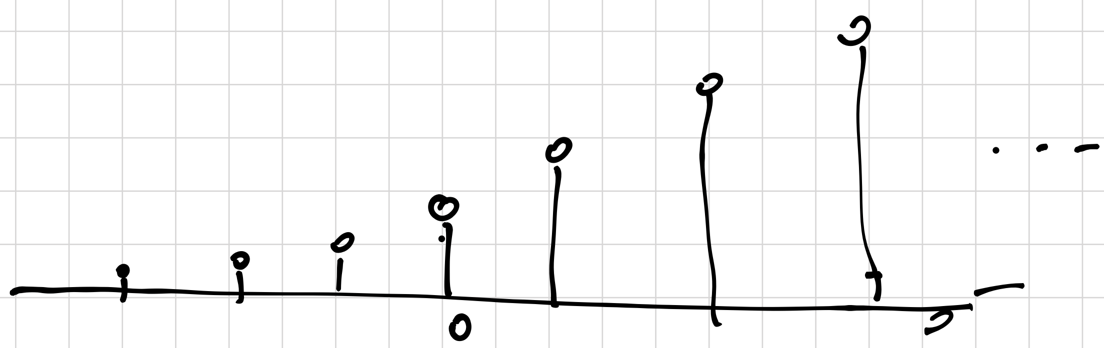


$-1 < r < 0$

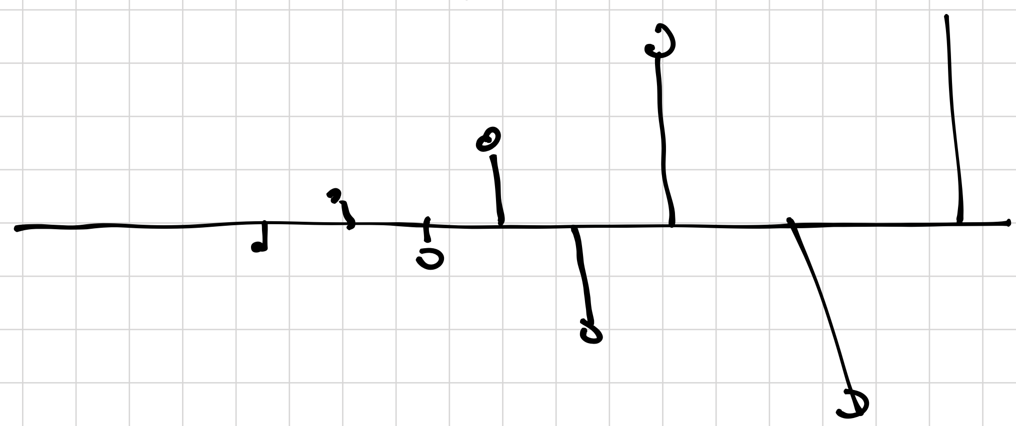
$x[n] = B \cdot r^n$



$r > 1$



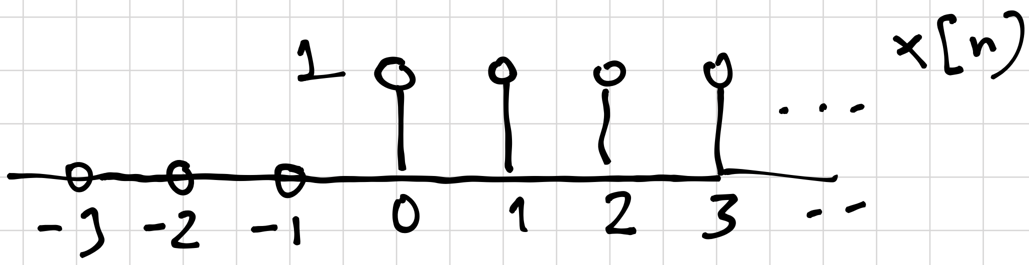
$r < -1$



Step Function

DT The discrete-time unit step function is defined by:

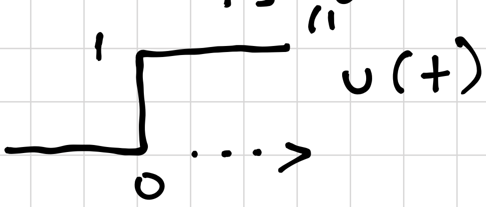
$$\underline{u}[n] = \begin{cases} 1, & n \geq 0 \\ 0, & n < 0 \end{cases}$$



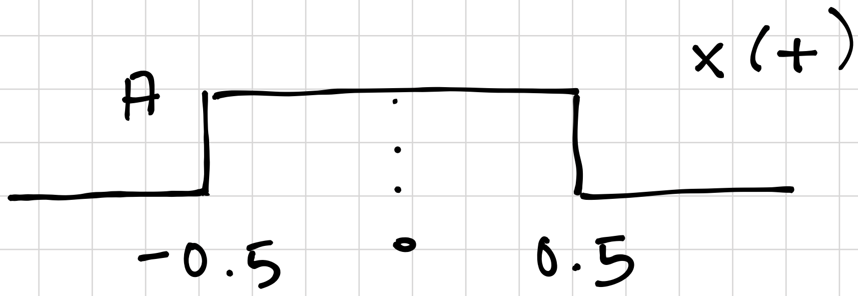
CT Continuous-Time Unit-Step Function

$$u(t) = \begin{cases} 1, & t > 0 \\ 0, & t < 0 \end{cases}$$

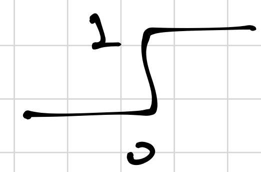
at $t=0$ unit step function is "undefined"



Ex

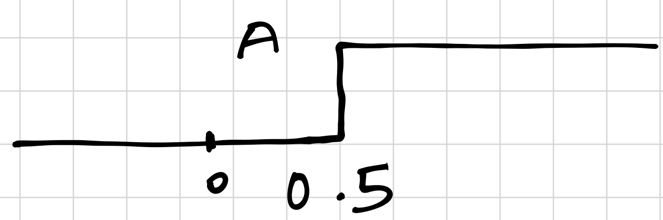


Express $x(t)$ as a weighted sum of two step functions.



$$x_1(t) = \underline{A}\, u(t+0.5)$$

$x_2(t)$



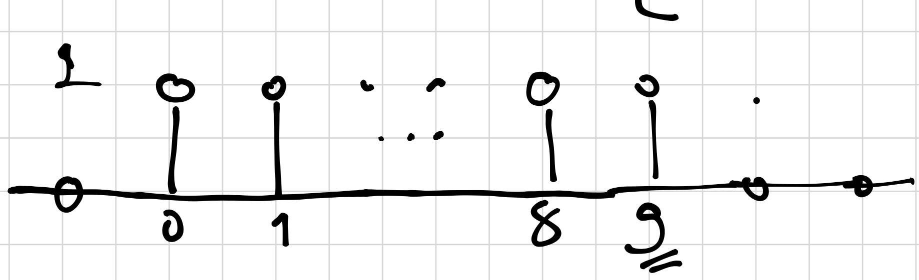
$$x_2(t) = A\, u(t-0.5)$$

$$x(t) = x_1(t) - x_2(t)$$

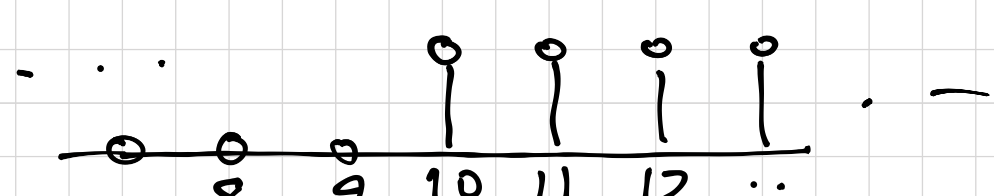
$$x(t) = A\, u(t+0.5) - A\, u(t-0.5)$$

Ex

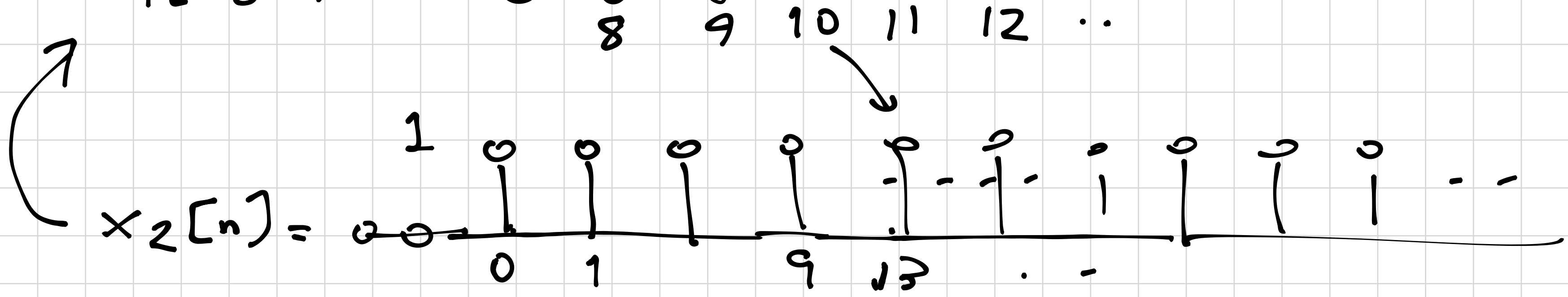
$$x[n] = \begin{cases} 1, & 0 \leq n \leq 9 \\ 0, & \text{otherwise} \end{cases}$$



$x_1[n] \Rightarrow$



$x_2[n] =$



$$x_2[n] = u[n] \qquad x_1[n] = u[n-10]$$

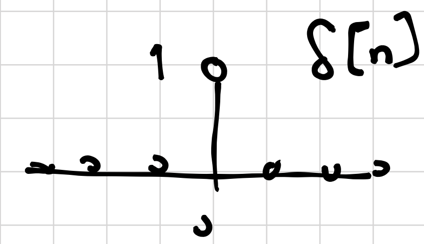
$$x[n] = x_2[n] - x_1[n]$$

$$= u[n] - u[n-10]$$

Impulse Function (Dirac-Delta Function)

DT Discrete-time unit impulse function

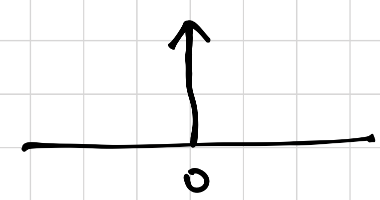
$$\delta[n] = \begin{cases} 1, & n = 0 \\ 0, & \text{otherwise} \end{cases}$$



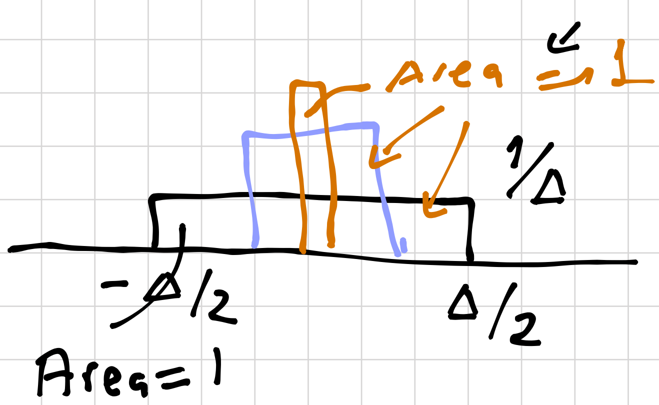
CT Continuous-Time unit impulse function is define by the following two relations :

① $\delta(t) = 0, \quad t \neq 0$

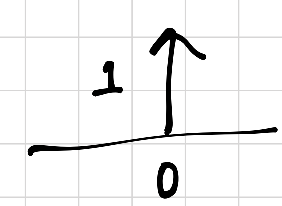
② $\displaystyle\int_{-\infty}^{+\infty} \delta(t)\, dt = \underline{\underline{1}}$ → Strength of $\delta(t)$



Let define $x_\Delta(t)$

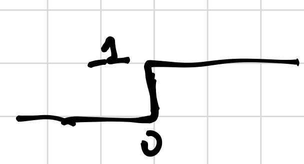


$$\delta(t) = \lim_{\Delta \to 0} x_\Delta(t)$$



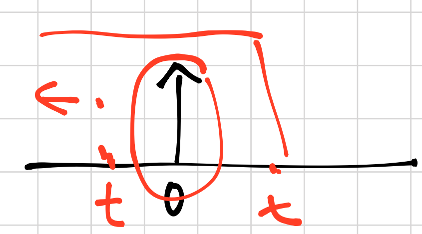
/* $3\,\delta(t)$ has a strength of 3 */

● $\delta(t)$ and $u(t)$ are related to each other



$$\delta(t) = \frac{d}{dt} u(t)$$

$$u(t) = \int_{-\infty}^{t} \delta(\tau)\, d\tau$$



● $u[n]$ and $\delta[n]$ are related to each other

$$\delta[n] = u[n] - u[n-1]$$

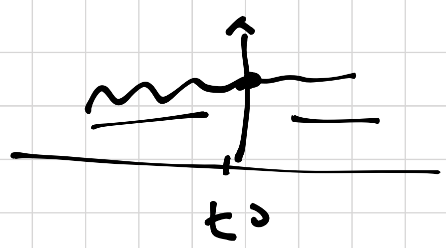
$$u[n] = \sum_{k=-\infty}^{n} \delta[k]$$

* $\left.\begin{matrix}\delta(t)\\ \delta[n]\end{matrix}\right\}$ is an even function

$$\delta(t) = \delta(-t)$$

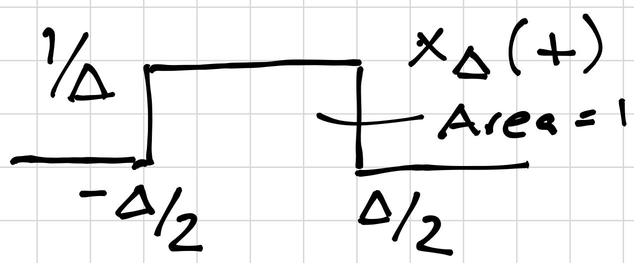
* Sifting property

$$\int_{-\infty}^{+\infty} x(t)\cdot \delta(t-t_0)\, dt = \underline{x(t_0)}$$



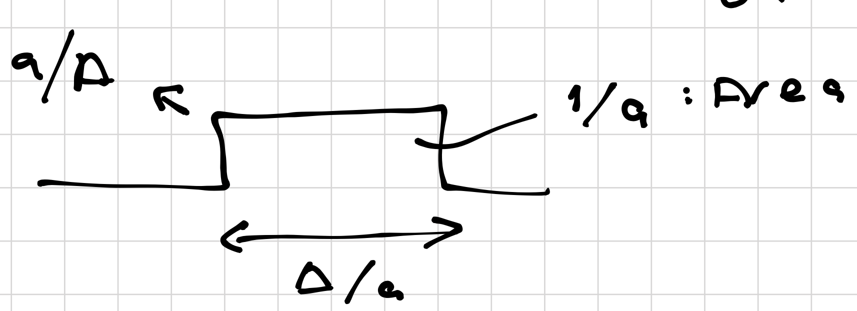
* Time-scaling Property

$$\delta(a\cdot t) = \frac{1}{a}\delta(t) \quad , \quad a>0$$



$$\lim_{\Delta\to 0} x_\Delta(t) = \delta(t)$$

$$\delta(at) = \lim_{\Delta\to 0} x_\Delta(a.t) = \frac{1}{a}\delta(t)$$

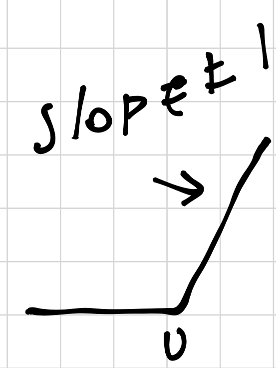


Ramp Function

CT

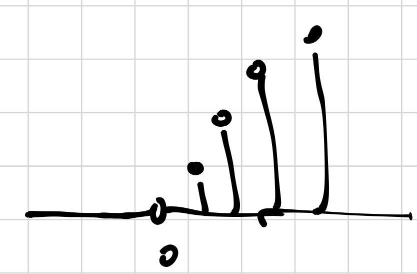
Unit ramp fn.

$$r(t) = \begin{cases} t, & t \geq 0 \\ 0, & t<0 \end{cases}$$



DT

$$r[n] = \begin{cases} n, & n \geq 0 \\ 0, & n<0 \end{cases}$$



- $r(t) = t\cdot u(t)$
- $r[n] = n\cdot u[n]$

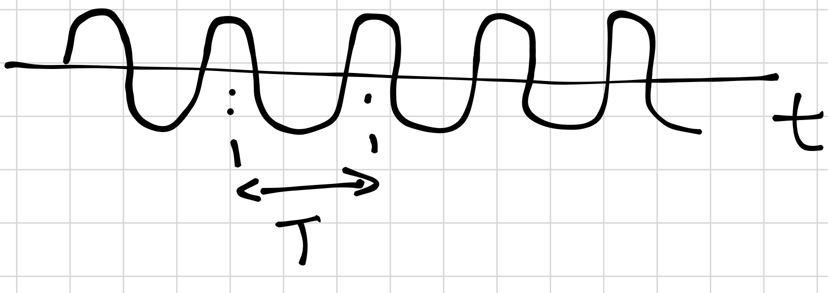
Sinusoidal Signals

CT

$$x(t) = \underbrace{A}_{\text{Amplitude}} \cos(\underbrace{\omega}_{\substack{\text{Frequency} \\ \text{rad/sec.}}} t + \underbrace{\phi}_{\substack{\text{Phase angle} \\ \text{(radians)}}})$$

CT sinusoidal signals are PERIODIC

Period $\quad T = \dfrac{2\pi}{\omega}$



DT

$\cos(a)$
$= \cos(a + 2\pi m)$

$$x[n] = A \cdot \cos[\Omega n + \phi]$$

DT sinusoidal signals MAY or MAY NOT be periodic.

In order for $x[n]$ to be periodic there must be an integer N that satisfy the following for all n.

$$x[n] = x[n+N]$$

$$x[n+N] = A \cdot \cos[\Omega n + \underline{\Omega N} + \phi]$$

So,

$$\Omega \underline{\underline{N}} = 2\pi \underline{\underline{m}} \quad , \quad m \in \mathbb{Z}$$

should be satisfied!

$$\Omega = \frac{2\pi m}{N} \text{ radians / cycle}$$

m, N must be integers.

Ex.

$$x[n] = \sin[5\pi n]$$

$$\Omega = 5\pi = 2\pi \cdot \frac{m}{N}$$

$$\frac{m}{N} = \frac{5}{2} \quad \text{so for} \quad \begin{matrix} m = 5 \\ N = 2 \end{matrix}$$

the equation is satisfied.

$\therefore$ Period is $\underline{\underline{2}}$

Ex

$$x[n] = \underline{\underline{\sin}}[2n]$$

$$\Omega = 2 \qquad 2 = 2\pi \cdot \frac{m}{N}$$

no inter (m, N) pair exists, therefor $x[n]$ is not periodic.

$j = \sqrt{-1}$

Relation Between Sinusoidal and Complex Exponential Signals.

Euler's identity $\quad e^{j\theta} = \cos\theta + j\sin\theta$

$$x(t) = A \cdot e^{j\omega t}$$
$$= A\left[\cos(\omega t) + j\sin(\omega t)\right]$$

$$A \cdot \cos(\omega t) = Re\{x(t)\}$$

$$A \sin(\omega t) = Im\{x(t)\}$$

$e^{j\omega t}$

For DT signals

$$x[n] = A\, e^{j\Omega n}$$

$$A \cos(\Omega n) = Re\{x[n]\}$$

$$A \sin(\Omega n) = Im\{x[n]\}$$

Exponentially Damped Sinusoidal Signals.

DT $\quad x(t) = A \cdot e^{-\alpha t} \cdot \sin(\omega t + \phi), \quad \alpha > 0$