* For the following NN:

$\Rightarrow \dot{u}(t) = -Au(t) + Wf(u(t)) + W^2 f(u(t-\tau)) + I$

a) Define the equilibrium equation

b) Shift the equilibrium point to the origin

$$\dot{z}(t) = \underbrace{-Az(t) + Wg(z(t)) + W^2 g(z(t-\tau))}$$

The equilibrium point of this system is the origin.

a) u^* : Equilibrium point

$$0 = -Au^* + Wf(u^*) + W^2 f(u^*) + I \quad : \text{Equilibrium Equation}$$

b) $z(t) = u(t) - u^*$

$$\dot{z}(t) = \dot{u}(t)$$

$$\dot{z}(t) = -A(z(t)+u^*) + Wf(z(t)+u^*) + W^2 f(z(t-\tau)+u^*) + Au^* - Wf(u^*) - W^2 f(u^*)$$

$$\dot{z}(t) = -Az(t) - \cancel{Au^*} + W\Big[\underbrace{f(z(t)+u^*) - f(u^*)}_{g(z(t))}\Big] + W^2\Big[\underbrace{f(z(t-\tau)+u^*) - f(u^*)}_{g(z(t-\tau))}\Big] + \cancel{Au^*}$$

* For the following hybrid BAM NN:

$$\dot{u}(t) = -Au(t) + Wf(z(t)) + W^2 f(z(t-\tau)) + I$$

$$\dot{z}(t) = -Bz(t) + Vf(u(t)) + V^2 f(u(t)-\sigma) + J$$

a) Define the equilibrium equation of the system

b) Shift the equilibrium point of the system to the origin

u^*, z^* : equilibrium points

$$\left.\begin{aligned} 0 &= -Au^* + Wf(z^*) + W^2 f(z^*) + I \\ 0 &= -Bz^* + Vf(u^*) + W^2 f(u^*) + J \end{aligned}\right\} \text{Equilibrium Equation}$$

b)

$$\left.\begin{aligned} x(t) &= u(t) - u^* \\ y(t) &= z(t) - z^* \end{aligned}\right\} \quad \begin{aligned} \dot{x}(t) &= \dot{u}(t) \\ \dot{y}(t) &= \dot{z}(t) \end{aligned}$$

$$(z(t-\tau)) + I \longrightarrow \dot{x}(t) = -A(x(t)+u^*) + Wf(\quad y(t)+z^*) + W^2 f(y(t-\tau)+z^*) + Au^* - Wf(z^*) - W^2 f(z^*)$$

$$(u(t)-\sigma) + J \longrightarrow \dot{y}(t) = -B(y(t)+z^*) + Vf(\quad x(t)+u^*) + V^2 f(x(t-\sigma)+u^*) + Bz^* - Vf(u^*) - V^2 f(u^*)$$

$$\dot{x}(t) = -Ax(t) - \cancel{Au^*} + W[\underbrace{f(y(t)+z^*) - f(z^*)}_{g(y(t))}] + W^2[\underbrace{f(y(t-\tau)+z^*) - f(z^*)}_{g(y(t-\tau))}] + \cancel{Au^*}$$

he system

tem to the origin

$$\dot{y}(t) = -By(t) - \cancel{Bz^*} + V[\underbrace{f(x(t)+u^*) - f(u^*)}_{g(x(t))}] + V^2[\underbrace{f(x(t-\sigma)+u^*) - f(u^*)}_{g(x(t-\sigma))}] + \cancel{Bz^*}$$

Equilibrium Equation

$$\boxed{\begin{aligned} \dot{x}(t) &= -Ax(t) + Wg(y(t)) + W^2 g(y(t-\tau)) \\ \dot{y}(t) &= -By(t) + Vg(x(t)) + V^2 g(x(t-\sigma)) \end{aligned}}$$

Equilibrium point of this sys. is the origin.

* For the following hybrid BAM NN:

$$\dot{u}(t) = -Au(t) + Wf(z(t)) + W^2 f(z(t-\tau)) + I \longrightarrow \dot{x}(t) = -A(x(t)+u^*) + Wf(y(t)+z^*) + W^2 f(y(t-\tau)+z^*) + Au^* - Wf(z^*) - W^2 f(z^*)$$

$$\dot{z}(t) = -Bz(t) + Vf(u(t)) + V^2 f(u(t-\sigma)) + J \longrightarrow \dot{y}(t) = -B(y(t)+z^*) + Vf(x(t)+u^*) + V^2 f(x(t-\sigma)+u^*) + Bz^* - Vf(u^*) - V^2 f(u^*)$$

$$\dot{x}(t) = -Ax(t) - \cancel{Au^*} + W[\underbrace{f(y(t)+z^*) - f(z^*)}_{g(y(t))}] + W^2[\underbrace{f(y(t-\tau)+z^*) - f(z^*)}_{g(y(t-\tau))}] + \cancel{Au^*}$$

$$\dot{y}(t) = -By(t) - \cancel{Bz^*} + V[\underbrace{f(x(t)+u^*) - f(u^*)}_{g(x(t))}] + V^2[\underbrace{f(x(t-\sigma)+u^*) - f(u^*)}_{g(x(t-\sigma))}] + \cancel{Bz^*}$$

a) Define the equilibrium equation of the system

b) Shift the equilibrium point of the system to the origin

u^*, z^* : equilibrium points

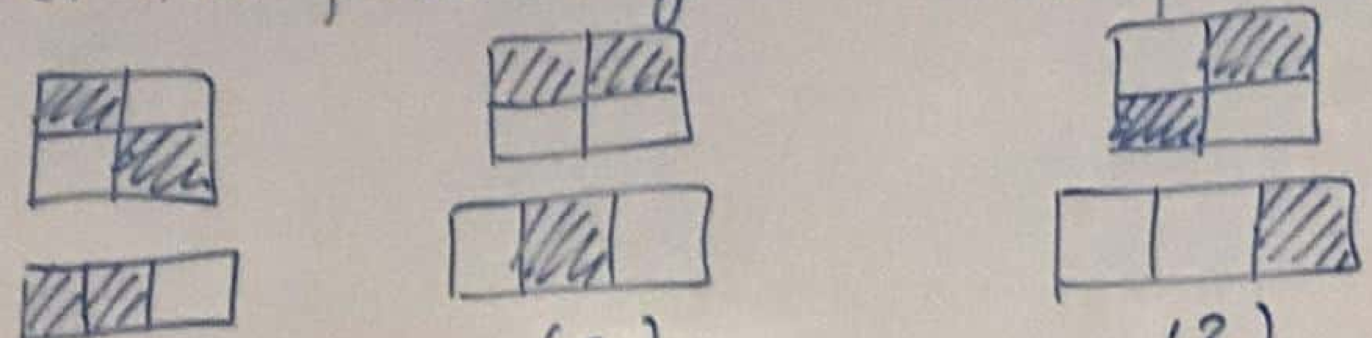
$$\left.\begin{aligned} 0 &= -Au^* + Wf(z^*) + W^2 f(z^*) + I \\ 0 &= -Bz^* + Vf(u^*) + W^2 f(u^*) + J \end{aligned}\right\} \text{Equilibrium Equation}$$

$$\boxed{\begin{aligned} \dot{x}(t) &= -Ax(t) + Wg(y(t)) + W^2 g(y(t-\tau)) \\ \dot{y}(t) &= -By(t) + Vg(x(t)) + V^2 g(x(t-\sigma)) \end{aligned}}$$

Equilibrium point of this sys. is the origin.

b) $$\left.\begin{aligned} x(t) &= u(t) - u^* \\ y(t) &= z(t) - z^* \end{aligned}\right\} \begin{aligned} \dot{x}(t) &= \dot{u}(t) \\ \dot{y}(t) &= \dot{z}(t) \end{aligned}$$

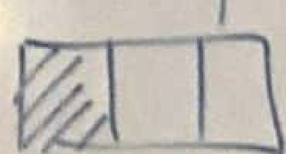
* For the following set of patterns :



(1) (2) (3)

a) Give the general architectural graph.

b) Find memory matrix (Process storage phase)

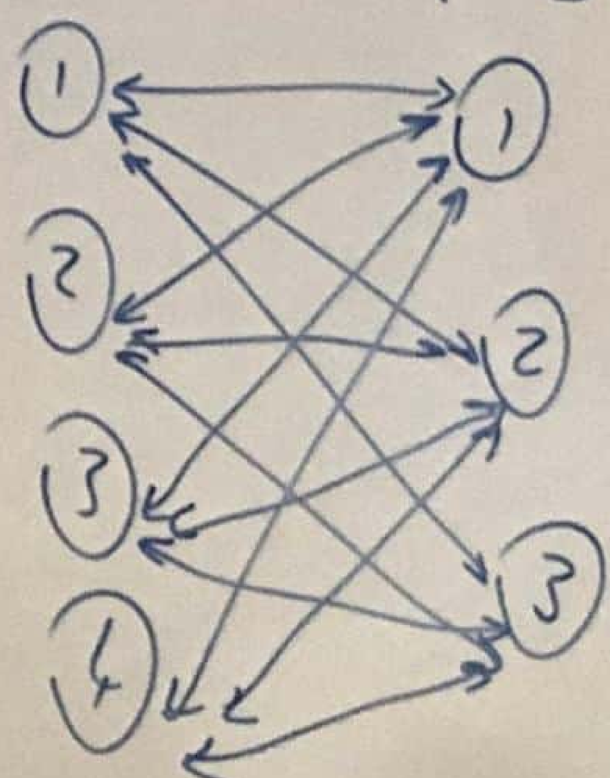
c) Process retrieval phase for the following key pattern : 

$$x^1 = [1 \ \ -1 \ \ -1 \ \ 1]^T$$
$$y^1 = [1 \ \ 1 \ \ -1]^T$$

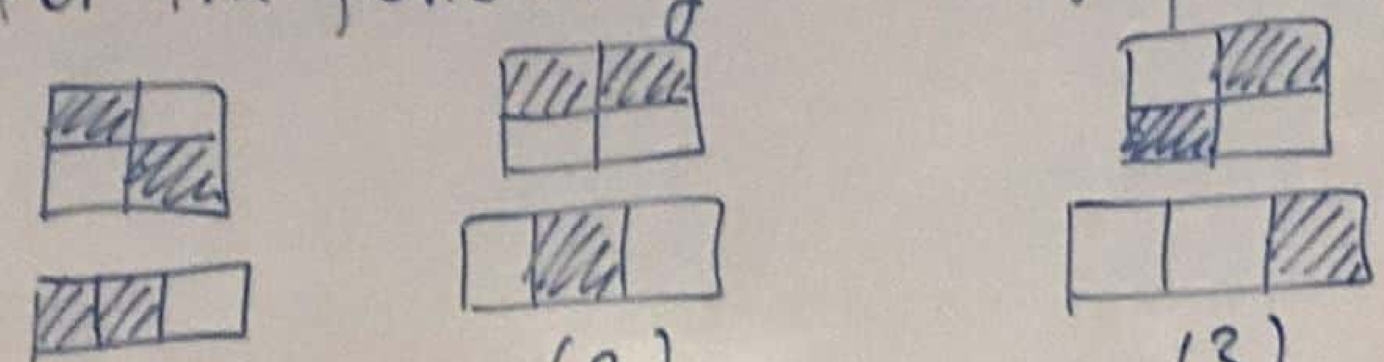
$$x^2 = [1 \ \ 1 \ \ -1 \ \ -1]^T$$
$$y^2 = [-1 \ \ 1 \ \ -1]^T$$

$$x^3 = [-1 \ \ 1 \ \ 1 \ \ -1]^T$$
$$y^3 = [-1 \ \ -1 \ \ 1]^T$$

Layer A Layer B

a) 

* For the following set of patterns :



(1) (2) (3)

$x^1 = [1 \ \ -1 \ \ -1 \ \ 1]^T$

$y^1 = [1 \ \ 1 \ \ -1]^T$

$x^2 = [1 \ \ 1 \ \ -1 \ \ -1]^T$

$y^2 = [-1 \ \ 1 \ \ -1]^T$

$x^3 = [-1 \ \ 1 \ \ 1 \ \ -1]^T$

$y^3 = [-1 \ \ -1 \ \ 1]^T$

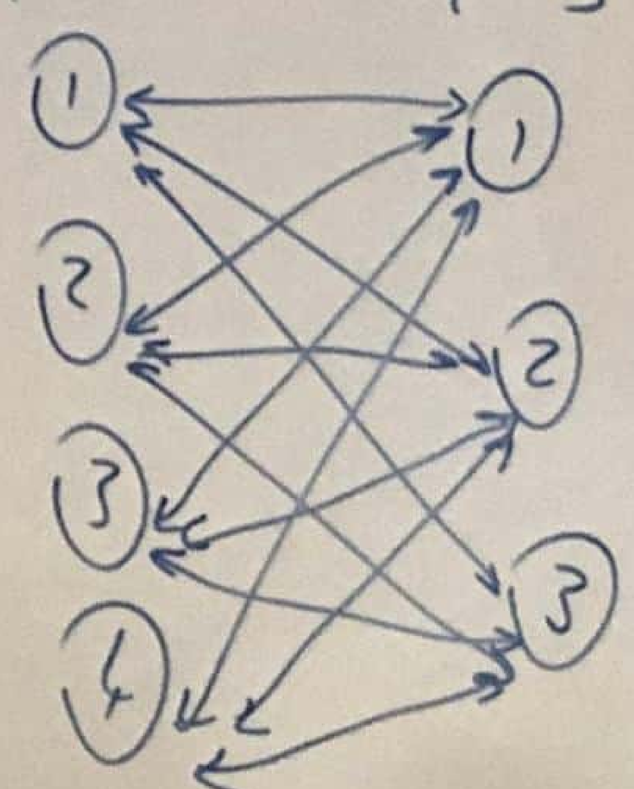
a) Give the general architectural graph.

b) Find memory matrix (Process storage phase)

c) Process retrieval phase for the following key pattern : 

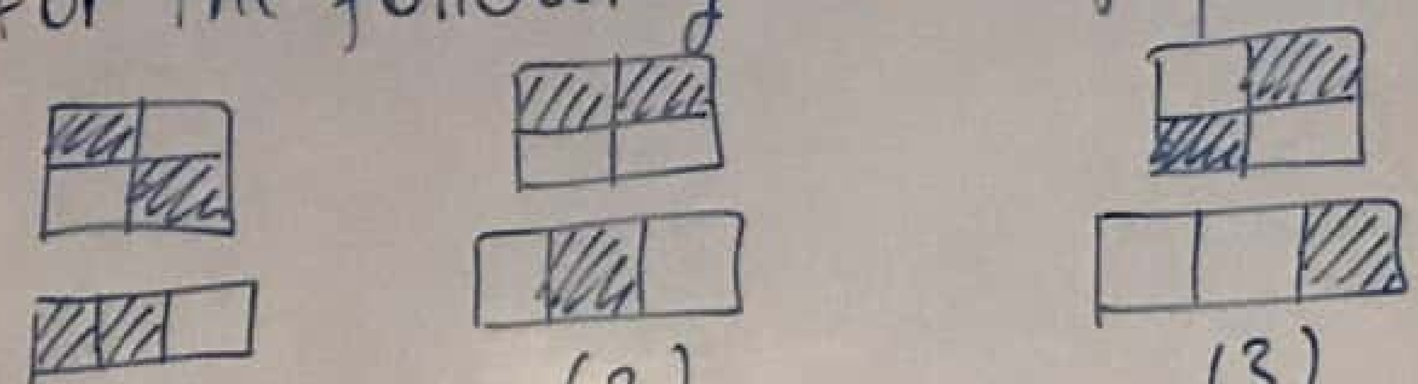
Layer A Layer B

a)



$$M = \sum_{i=1}^{3} x_i \cdot y_i^T$$

* For the following set of patterns :



(1) (2) (3)

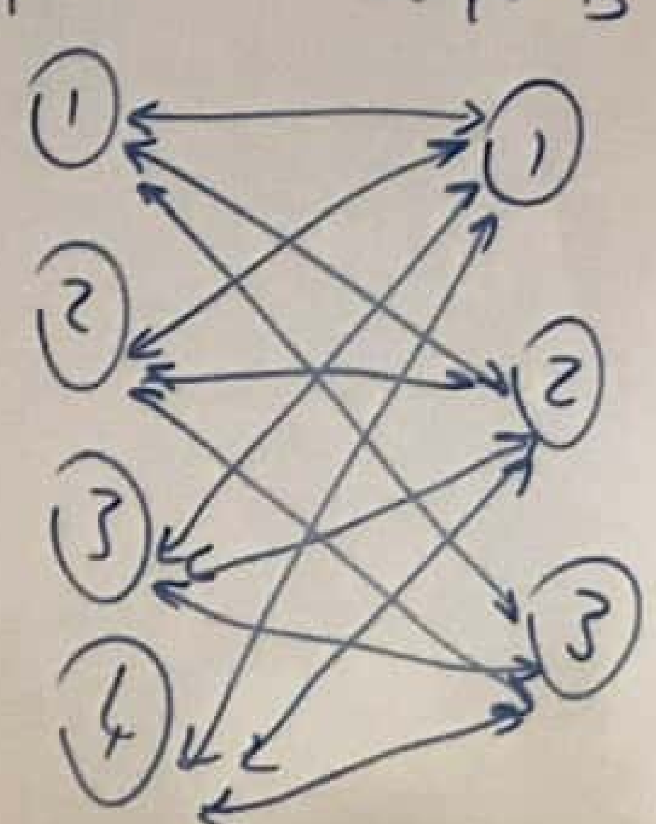
$$x^1 = [1\ \ -1\ \ -1\ \ 1]^T$$
$$y^1 = [1\ \ 1\ \ -1]^T$$

$$x^2 = [1\ \ 1\ \ -1\ \ -1]^T$$
$$y^2 = [-1\ \ 1\ \ -1]^T$$

$$x^3 = [-1\ \ 1\ \ 1\ \ -1]^T$$
$$y^3 = [-1\ \ -1\ \ 1]^T$$

a) Give the general architectural graph.
b) Find memory matrix (Process storage phase)
c) Process retrieval phase for the following key pattern :

a)



Memory Matrix

$$\begin{array}{c} \begin{array}{ccc} x^1 & x^2 & x^3 \\ \downarrow & \downarrow & \downarrow \end{array} \\ \begin{bmatrix} 1 & 1 & -1 \\ -1 & 1 & 1 \\ -1 & -1 & 1 \\ 1 & -1 & -1 \end{bmatrix} \end{array} \begin{array}{c} y^1 \rightarrow \\ y^2 \rightarrow \\ y^3 \rightarrow \end{array} \begin{bmatrix} 1 & 1 & -1 \\ -1 & 1 & -1 \\ -1 & -1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 3 & -3 \\ -3 & -1 & 1 \\ -1 & -3 & 3 \\ 3 & 1 & -1 \end{bmatrix}_{4\times 3}$$

Storage phase

$$x^0 = \begin{bmatrix} -1 & -1 & -1 & -1 \end{bmatrix}^T$$

$$y^0 = \begin{bmatrix} 1 & -1 & -1 \end{bmatrix}^T$$

$$x^1 = \Gamma\left[M . y^0\right] = \Gamma \begin{bmatrix} 1 & 3 & -3 \\ -3 & -1 & 1 \\ -1 & -3 & 3 \\ 3 & 1 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \\ -1 \end{bmatrix} = \Gamma \begin{bmatrix} 1 \\ -3 \\ -1 \\ 3 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ -1 \\ 1 \end{bmatrix}$$

$$y^2 = \Gamma\left[M^T . x^1\right] = \Gamma \begin{bmatrix} 1 & -3 & -1 & 3 \\ 3 & -1 & -3 & 1 \\ -3 & 1 & 3 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \\ -1 \\ 1 \end{bmatrix} = \Gamma \begin{bmatrix} 8 \\ 8 \\ -8 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$$

$$x^3 = \Gamma\left[M . y^2\right] = \Gamma \begin{bmatrix} 1 & 3 & -3 \\ -3 & -1 & 1 \\ -1 & -3 & 3 \\ 3 & 1 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix} = \Gamma \begin{bmatrix} 7 \\ -5 \\ -7 \\ 5 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ -1 \\ 1 \end{bmatrix}$$

$$x^1 = x^3 \longrightarrow y^2 = y^4$$

$$(x^1, y^2) \Longrightarrow (x^1, y^1) \Rightarrow \left(\begin{bmatrix} 1 & -1 & -1 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 1 & -1 \end{bmatrix}\right)$$

Retrieval Phase