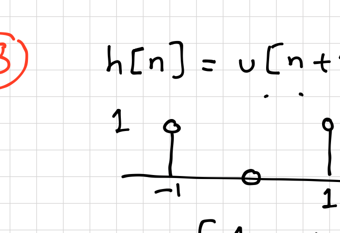
(8) $h[n] = u[n+1] - u[n-2] - \delta[n]$



$h[n] = \delta[n+1] + \delta[n-1]$

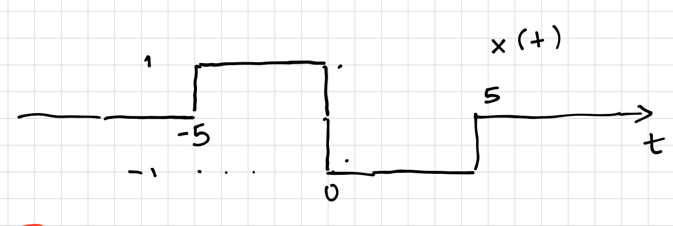
$$x[n] = \begin{cases} 1, & n = 1, 3 \\ -2, & n = 2 \\ 0, & \text{otherwise} \end{cases}$$

$$x[n] * h[n] = x[n] * \{\delta[n+1] + \delta[n-1]\}$$
$$= x[n+1] + x[n-1]$$

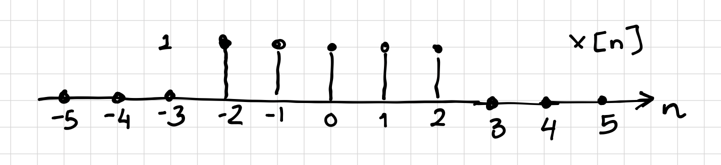
0	1	2	3	4	
1	-2	1	0	0	0
		1	-2	1	
1	-2	2	-2	1	

$$y[n] = \begin{cases} 1, & n = 0, 4 \\ -2, & n = 1, 3 \\ 2, & n = 2 \\ 0, & \text{otherwise} \end{cases}$$

(9) $x(t) = u(t+5) - 2u(t) + u(t-5)$



(10) $x[n] = u[n+2] - u[n-3]$



(11) $y(t) = 2\left|\frac{t}{t+1}\right| + x(t-1)$

- memoryless - no
- causal - yes
- $\mathcal{H}\{\alpha x(t)\} \overset{?}{=} \alpha \cdot \mathcal{H}\{x(t)\}$

$$2\left|\frac{t}{t+1}\right| + \alpha x(t-1) \neq \alpha\left\{2\left|\frac{t}{t+1}\right| + x(t-1)\right\}$$

not linear.

- time invariance

$$\mathcal{H}\{x(t-t_0)\} \overset{?}{=} y(t-t_0)$$
$$2\left|\frac{t}{t+1}\right| + x(t-t_0) \overset{?}{\neq} 2\left|\frac{t-t_0}{t-t_0-1}\right| + x(t-t_0)$$

not T. I.

- stability.

• Assume $x(t)$ is finite $x(t) \leq M_x < \infty$

$$y(t) \leq 2\left|\frac{t}{t+1}\right| + M_x$$

@ $t=-1$ $y(t)$ goes to infinity.

not BIBO STABLE !

(12) $h[n] = \left(\frac{1}{3}\right)^{n-3}\{u[n+4] - u[n-7]\}$

Express A and B in terms of n so that the following equation holds.

$$h[n-k] = \begin{cases} (1/3)^{n-k-3}, & A \leq k \leq B \\ 0, & \text{otherwise} \end{cases}$$

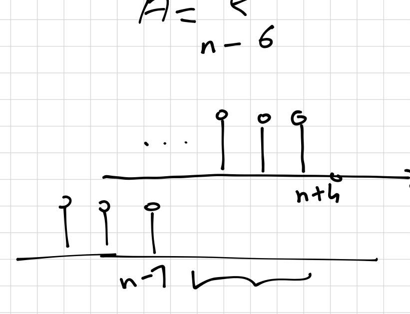
$$h[n-k] = (\quad)\{u[n-k+4] - u[n-k-7]\}$$

$k \leq n+4$ $\quad k > n-7$

$u(n-k+4) = u[(n+4) - k]$

$A = n-6$

$B = n+4$



Invertible Systems and Deconvolution.

We previously said that "a system is invertible only if the input of the system can be recovered from the output".

$x(t) \to \boxed{\mathcal{H}} \to y(t) \to \boxed{\mathcal{H}^{inv}} \to x(t)$

If $\mathcal{H}^{inv}$ exists then $\mathcal{H}$ is "invertible".

• If an LTI system is invertible then it must have an LTI inverse.

$x(t) \to \boxed{h(t)} \to y(t) \to \boxed{h^{inv}(t)} \to x(t)$

$$\{x(t) * h(t)\} * h^{inv}(t) = x(t)$$
$$x(t) * \underbrace{\{h(t) * h^{inv}(t)\}}_{= \delta(t)} = x(t)$$

CT $\quad h(t) * h^{inv}(t) = \delta(t)$

DT $\quad h[n] * h^{inv}[n] = \delta[n]$

Example

$$y[n] = \mathcal{H}\{x[n]\} = x[n] + \alpha x[n-1]$$

Find a causal inverse system of $\mathcal{H}$.

- The impulse response of $\mathcal{H}$ is :

$$\mathcal{H}\{\delta[n]\} = h[n] = \delta[n] + \alpha\,\delta[n-1]$$

- The following must be satisfied:

$$h[n] * h^{inv}[n] = \delta[n]$$

$$h^{inv}[n] + \alpha h^{inv}[n-1] = \delta[n]$$

- Since the inverse system needs to be causal

(1) - for $n<0$ $\quad h^{inv}[n] = 0$ —

(2) - at $n=0$, $\delta[n]=1$ $\quad h^{inv}[0] + \alpha \underbrace{h^{inv}[-1]}_{0} = 1$

$$h^{inv}[0] = 1$$

(3) - $n>0$ $\Rightarrow$ $\delta[n] = 0$

$$h^{inv}[n] + \alpha h^{inv}[n-1] = 0$$
$$h^{inv}[n] = -\alpha h^{inv}[n-1]$$

$$h^{inv}[1] = -\alpha$$
$$h^{inv}[2] = -\alpha(-\alpha) = (-\alpha)^2$$
$$h^{inv}[3] = -\alpha(-\alpha)^2 = (-\alpha)^3$$
$$\vdots$$
$$h^{inv}[n] = (-\alpha)^n$$

$$\boxed{h^{inv}[n] = (-\alpha)^n u[n]}$$

for stability : $\sum_{n=0}^{\infty}(-\alpha)^n$ must be finite

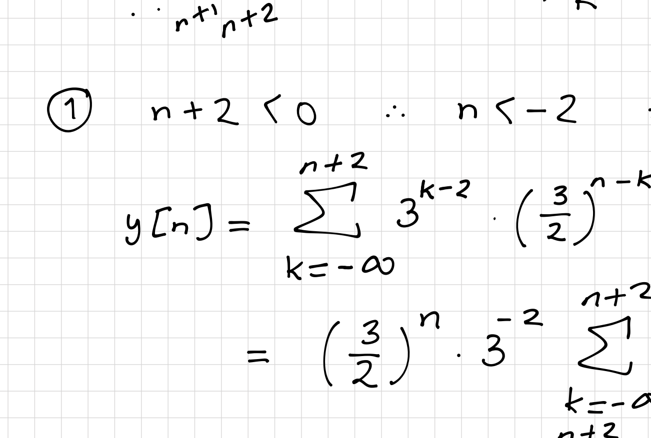
if $|\alpha| < 1$ $\Rightarrow$ H^{inv}_{stable}

otherwise H^{inv} is unstable.

General Examples

Ex. Find the DT convolution of the following two signals.

$$x[n] = 3^{n-2} \cdot u[-n]$$
$$h[n] = \left(\frac{3}{2}\right)^n \cdot u[n+2]$$



(1) $n+2 < 0 \quad \therefore \quad n < -2 \Rightarrow$

$$y[n] = \sum_{k=-\infty}^{n+2} 3^{k-2} \cdot \left(\frac{3}{2}\right)^{n-k}$$
$$= \left(\frac{3}{2}\right)^n \cdot 3^{-2} \sum_{k=-\infty}^{n+2} 3^k \cdot \left(\frac{2}{3}\right)^k$$
$$= \left(\frac{3}{2}\right)^n 3^{-2} \sum_{k=-\infty}^{n+2} 2^k$$
$$= \left(\frac{3}{2}\right)^n 3^{-2}\, 2^{n+2}\left(\frac{2}{2-1}\right)$$
$$= \left(\frac{3}{2}\right)^n \cdot \frac{1}{9} \cdot 2^{n+3}$$
$$= \frac{3^n}{2^n} \cdot \frac{8}{9} \cdot 2^n = 8 \cdot 3^{n-2}$$

(2) $n \geq -2$

$$y[n] = \left(\frac{3^{n-2}}{2^n}\right) \cdot \underbrace{\sum_{k=-\infty}^{0} 2^k}_{= 2}$$
$$= \frac{3^{n-2}}{2^{n-1}}$$

$$y[n] = \begin{cases} 8 \cdot 3^{n-2}, & n < -2 \\ \frac{3^{n-2}}{2^{n-1}}, & n \geq -2 \end{cases}$$

Ex. The step response of a CT system is given as :

$$s(t) = (1 - e^{-2t})\, u(t)$$

Find the impulse response.

$$h(t) = \frac{d}{dt} s(t)$$

$$h(t) = \frac{d}{dt}\left\{(1 - e^{-2t})\, u(t)\right\}$$
$$= \left\{\frac{d}{dt}(1 - e^{-2t})\right\} \cdot u(t) + \left\{\frac{d}{dt} u(t)\right\} \cdot (1 - e^{-2t})$$
$$= 2e^{-2t} u(t) + \underbrace{(1 - e^{-2t})}_{0}\,\delta(t)$$

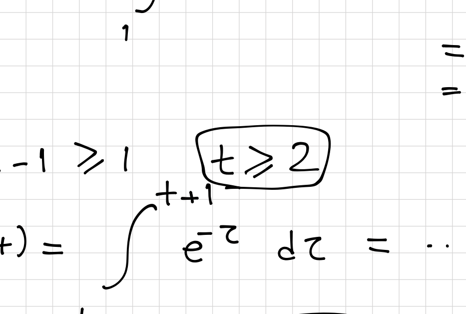
$$x(t) \cdot \delta(t) = x(0) \cdot \delta(t)$$

$$\boxed{h(t) = 2 \cdot e^{-2t} u(t)}$$

Ex.

$$h(t) = u(t+1) - u(t-1)$$
$$x(t) = e^{-t}\, u(t-1)$$
$$y(t) = x(t) * h(t) = ?$$

$$y(t) = \int_{-\infty}^{+\infty} h(\tau) \cdot x(t-\tau)\, d\tau$$
$$= \int_{-\infty}^{+\infty} x(\tau) \cdot h(t-\tau)\, d\tau$$



(1) $t+1 < 1 \quad t < 0 \quad y(t) = 0$

(2) $t+1 \geq 1$, $t-1 < 1 \quad 0 \leq t < 2$

$$y(t) = \int_1^{t+1} e^{-\tau} \cdot d\tau = \left[-e^{-\tau}\right]_1^{t+1}$$
$$= e^{-1} - e^{-(1+t)}$$
$$= e^{-1}(1 - e^{-t})$$

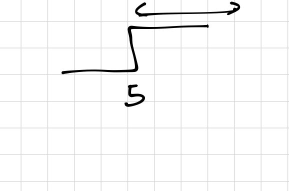
(3) $t-1 \geq 1$ $\quad \boxed{t \geq 2}$

$$y(t) = \int_{t-1}^{t+1} e^{-\tau}\, d\tau = \ldots = \boxed{e^{-t}(e - e^{-1})}$$

$$y(t) = \begin{cases} 0, & t < 0 \\ e^{-1}(1 - e^{-t}), & 0 \leq t < 2 \\ e^{-t}(e - e^{-1}) & t \geq 2 \end{cases}$$

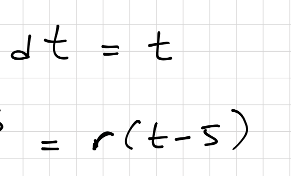
Ex.

$$y(t) = \int_{-\infty}^{t} x(\tau - 5)\, d\tau$$



$$\mathcal{H}\{\delta(t)\} = \int_{-\infty}^{t} \delta(\tau - 5)\, d\tau = u(t-5)$$

$$h(t) = u(t-5)$$



- step function

$t < 5 \quad s(t) = 0$

$t > 5 \quad s(t) = \int^{t} 1 \cdot dt = t$

$$s(t) = \begin{cases} t, & t \geq 5 \\ 0, & t < 5 \end{cases} = r(t-5)$$
$$= t \cdot u(t-5)$$