

1) Lagrange ✓

2) Newton →

3) Newton ilerl

4) Newton Geri

5) Newton Orta

6) Hermite

7) n=1 Trapez

8) n=2 Simpson

9) n=3 Simpson 3/8

10) n=4 Boole

11) Euler

12) Heun

13) Taylor

14) Runge Kutta

15) LU Ayrıştırması (Soru Gelebilir) ★

16) Newton

17) Sabit Nokta (1)

18) Sabit Nokta (2)

$$P_3(x) = a_0 + a_1(x-x_0) + a_2(x-x_0)(x-x_1) + a_3(x-x_0)(x-x_1)(x-x_2)$$

$$P_3(x) = f(x_0) + sh f(x_0, x_1) + s(s-1) \cdot h^2 f(x_0, x_1, x_2) + s(s-1)(s-2) \cdot h^3 f(x_0, x_1, x_2, x_3) \quad (s = \frac{x-x_0}{h})$$

$$P_3(x) = f(x_0) + sh \left(\frac{f(x_1, x_0) + f(x_0, x_1)}{2} \right) + s^2 h^2 f(x_1, x_0, x_1) \quad (s = \frac{x-x_0}{h}) \quad (h = x_1 - x_0)$$

mesela aralık 1 ve 1,6 arasında;

$$\frac{1}{2} \cdot h \cdot \begin{pmatrix} 1 & 1 \\ 1 & 1,6 \end{pmatrix} \quad \frac{1}{3} \cdot h \cdot \begin{pmatrix} 1 & 4 & 1 \\ 1 & 1,3 & 1,6 \end{pmatrix} \quad \frac{3}{8} \cdot h \cdot \begin{pmatrix} 1 & 3 & 3 & 1 \\ 1 & 1,2 & 1,4 & 1,6 \end{pmatrix} \quad \frac{2}{45} \cdot h \cdot \begin{pmatrix} 7 & 32 & 12 & 32 & 7 \\ 1 & 1,15 & 1,30 & 1,45 & 1,60 \end{pmatrix}$$

Yani;

$$\frac{2}{45} \cdot h \cdot (7 \cdot f(1) + 32 \cdot f(1,15) + 12 \cdot f(1,30) + 32 \cdot f(1,45) + 7 \cdot f(1,6)) \text{ olur.}$$

h değerleri:

- n=1 Trapez $\frac{b-a}{1}$
- n=2 Simpson $\frac{1}{3}$
- n=3 Simpson $\frac{3}{8}$
- n=4 Boole $\frac{b-a}{4}$

örnek:

$$y' = \frac{t-y}{2} \quad [0,3] \quad y(0)=1 \quad h=1/4$$

$$y_1 = 1 + \frac{1}{4} \cdot \left(\frac{0-1}{2} \right) = 0,875$$

$$y_2 = 0,875 + \frac{1}{4} \cdot \left(\frac{0,25-0,875}{2} \right) = 0,79 \dots$$

Euler: $y(t_1) = y(t_0) + hf(t_0, y(t_0))$

Heun: $p_{k+1} = y_k + hf(t_k, y_k)$

$$y_{k+1} = y_k + \frac{h}{2} \cdot (f(t_k, y_k) + f(t_{k+1}, p_{k+1}))$$

ör) $y' = \frac{t-y}{2} \quad [0,3] \quad y(0)=1 \quad h=1/4$

$$p_1 = 1 + \frac{1}{4} \cdot \left(\frac{0-1}{2} \right) = 0,875$$

$$y_1 = 1 + \frac{1}{8} \cdot \left(\left(\frac{0-1}{2} \right) + \left(\frac{0,25-0,875}{2} \right) \right) = 0,89 \dots$$

$p_2 = \dots$
 $y_2 = \dots$
 $p_3 = \dots$
 $y_3 = \dots$

Taylor: $y_{k+1} = y_k + d_1 h + \frac{d_2 h^2}{2!} + \frac{d_3 h^3}{3!} + \frac{d_4 h^4}{4!}$

ör) $y' = \frac{t-y}{2} \quad [0,3] \quad y(0)=1 \quad h=1/4$

$$y'(t) = \frac{t-y}{2} \quad d_1 = y'(0) = \frac{0-1}{2} = (-0,5)$$

$$y''(t) = \frac{1-y'}{2} = \frac{1 - \frac{t-y}{2}}{2} = \frac{2-t+y}{4} \quad d_2 = y''(0) = 0,75$$

$$y'''(t) = \frac{0-1+y'}{4} = \frac{0-1 + \frac{t-y}{2}}{4} = \frac{-2+t-y}{8} \quad d_3 = y'''(0) = -0,375$$

$$y^{(4)}(t) = \frac{-2-t+y}{16} \quad d_4 = 0,1875$$

$$\Rightarrow y_1 = 1 + \frac{1}{4} \cdot (-0,5) + \left(\frac{1}{4} \right)^2 \cdot \frac{0,75}{2!} + \left(\frac{1}{4} \right)^3 \cdot \frac{-0,375}{3!} + \left(\frac{1}{4} \right)^4 \cdot \frac{0,1875}{4!} = 0,89 \dots$$

Runge Kutta: $y_{k+1} = y_k + h(f_1 + 2f_2 + 2f_3 + f_4)$

$$f_1 = f(t_k, y_k)$$

$$f_2 = f\left(t_k + \frac{h}{2}, y_k + \frac{h}{2} f_1\right)$$

$$f_3 = f\left(t_k + \frac{h}{2}, y_k + \frac{h}{2} f_2\right)$$

$$f_4 = f(t_k + h, y_k + h \cdot f_3)$$

ör) $y' = \frac{t-y}{2}$, $[0,3]$, $y(0)=1$, $h=1/4$ (Runge-kutta)

Çöz: $y_{k+1} = y_k + h \cdot \frac{f_1 + 2f_2 + 2f_3 + f_4}{6}$

$f_1 = f(t_k, y_k) = \frac{0-1}{2} = -0,5$

$f_2 = f(t_k + \frac{h}{2}, y_k + \frac{h}{2} \cdot f_1) = \frac{0,125 - 1 + 0,125 \cdot -0,5}{2} = -0,46875$

$f_3 = f(t_k + \frac{h}{2}, y_k + \frac{h}{2} \cdot f_2) = \frac{0,125 - 1 + 0,125 \cdot -0,46875}{2} = -0,466796875$

$f_4 = f(t_k + h, y_k + h \cdot f_3) = \frac{0,25 - 1 + 0,25 \cdot -0,466796875}{2} = -0,433390625$

$y_1 = 1 + \frac{1}{4} \left(\right)$

$= 0,88315$

LU: $(A = L \cdot U)$

$A \cdot x = b$
 $L \cdot U \cdot x = b$
 $\quad \quad \quad \underline{y}$
 $Ly = b$
 $Ux = y$

$R(x) = x - J^{-1} \cdot f(x)$

Newton's: $Scale = \text{initial value} - J^{-1} \cdot f(x)$

örün

2×2

2×1