RSP iö Final 2015 Çözümler

(1)

$$c\int_e^{e^3}\int_e^{x}\frac{1}{xy}\,dy\,dx = 1$$

$$c\int_e^{e^3}\frac{1}{x}\left(\int_e^{x}\frac{1}{y}\,dy\right)dx = 1$$

$$c\int_e^{e^3}\frac{1}{x}\,\ln y\Big|_e^{x}\,dx = 1$$

$$c\int_e^{e^3}\frac{1}{x}\,(\ln x - 1)\,dx = 1$$

$$\frac{1}{2}\left(\ln^2 x - \ln x\right)\Big|_e^{e^3} = 1/c$$

$$\left(\frac{9}{2}-\frac{3}{2}\right)-\left(\frac{3}{2}-\frac{1}{2}\right) = \frac{1}{c}$$

$$3-1=\frac{1}{c} \Rightarrow \boxed{c=\frac{1}{2}}\ //$$

(2)

$$f_X(x) = c\int_e^{x}\frac{1}{xy}\,dy = c\frac{1}{x}\ln(y)\Big|_e^{x} = c\frac{1}{x}(\ln x - 1)$$

(3)

$$f_{Y|X=e^2}(y) = \frac{f(e^2,y)}{f_X(e^2)} = \frac{c\frac{1}{e^2 y}}{c\frac{1}{e^2}(\ln e^2 - 1)} = \frac{1}{y}$$

(4)

$$M_{Y|X=e^2} = \int_e^{e^2} y\cdot\frac{1}{y}\cdot dy = y\Big|_e^{e^2} = e(e-1)$$

(5)

$$E(T) = \int_e^{e^3}\int_e^{e}\int_e^{x}\ln(x)\ln(y)\,\frac{c}{xy}\,dy\,dx = c\int_e^{e^3}\frac{\ln(x)}{x}\int_e^{e}\frac{\ln(y)}{y}\,dy\,dx$$

$$= \frac{c/}{0.5}\int_e^{e^3}\frac{\ln(x)}{x}\left[\ln^2(y)\right]_e^{x}\,dy\,dx$$

$$= \int_e^{e^3}\frac{\ln(x)}{x}\left[\ln^2(x)-\frac{1}{x}\right]dx\ // = \boxed{4}$$

Note:

$$\int\frac{\ln^{\alpha}(x)}{x}\,dx = \frac{\ln^{\alpha+1}(x)}{\alpha+1}$$