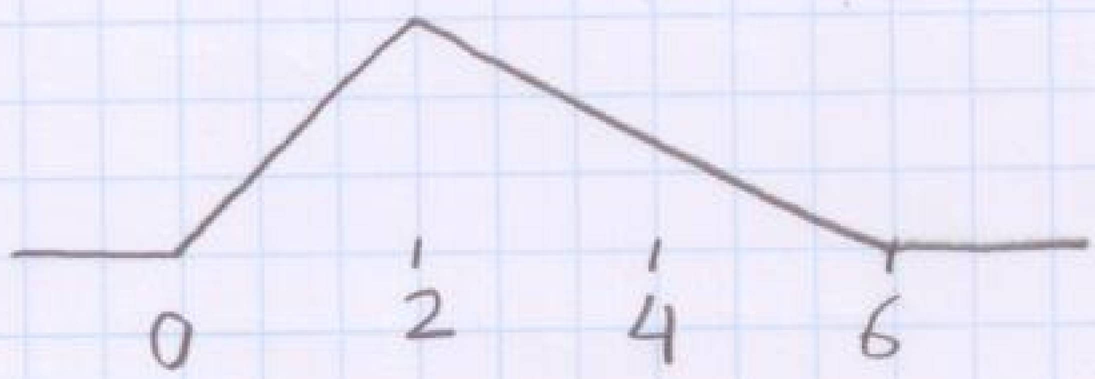
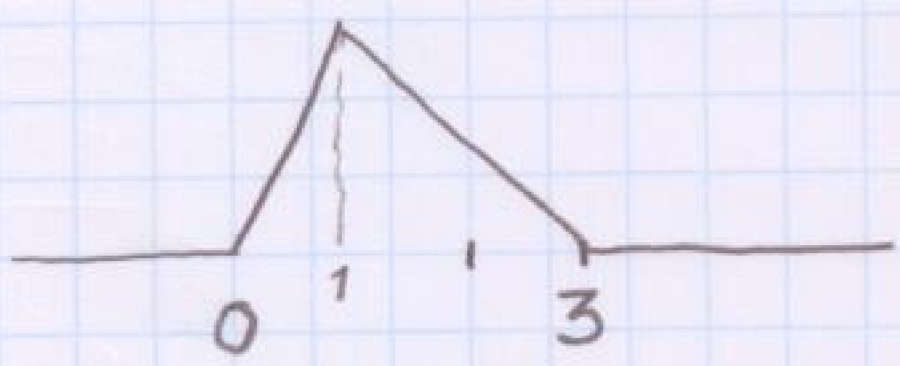
Signal Processing Örgün Midterm-Mazeret 2015
Solutions

(1)

a-15



$x(t-2)$



$x(2t-2)$

b-15

$x(t/2+2)$

c-15

$$x(t) = \begin{cases} t+2, & -2<t<0 \\ 2-t/2, & 0<t<4 \\ 0 & \text{otherwise} \end{cases}$$

$$E = \int_{-\infty}^{+\infty} x^2(t)\,dt = \int_{-2}^{0} (t+2)^2\,dt + \int_{0}^{4} (2-t/2)^2\,dt$$

$$= \int_{-2}^{0} (t^2+2t+2)\,dt + \int_{0}^{4} (4-2t+t^2/4)\,dt$$

$$= \left[\frac{t^3}{3} + t^2 + 2t\right]_{-2}^{0} + \left[4t - t^2 + \frac{t^3}{12}\right]_{0}^{4}$$

$$= (0) - \left(\frac{-8}{3} + \cancel{4} - \cancel{4}\right) + \left[\underbrace{16-16}_{0} + \frac{64}{12} - 0\right]^4$$

$$E = \frac{64}{12} + \frac{8}{3} = 8$$

$\boxed{x(t) \text{ is an energy signal!}}$

(2)

$$y[n] = x[n] \sum_{k=-\infty}^{+\infty} \delta[n-2k]$$

$$= x[n] \cdot (\dots + \delta[n+4] + \delta[n+2] + \delta[n] + \delta[n-2] + \delta[n-4] \dots)$$

$$= \dots + x[n+4] + x[n+2] + x[n] + x[n-2] + \dots$$

i-3 not memoryless

ii-3 not causal

iii-3 $|x[n]| \leq M_x$

$$y[n] \leq \sum_{k=-\infty}^{+\infty} M_x$$

$y[n]$ is not bounded.

$\boxed{\mathcal{H} \text{ is not stable}}$

iv-3

Superposition

$$\mathcal{H}\{x_1[n] + x_2[n]\} = \sum_{k=-\infty}^{+\infty} (x_1[n-2k] + x_2[n-2k])$$

$$= y_1[n] + y_2[n] \checkmark$$

Homogenity

$$\mathcal{H}\{a\, x[n]\} = \sum_{k=-\infty}^{+\infty} a\, x[n] = a\, y[n] \checkmark$$

$\boxed{\mathcal{H} \text{ is LINEAR}}$

v-3

$$H\{x[n-n_0]\} = \sum_{k=-\infty}^{+\infty} x[n-n_0-2k] = y[n-n_0]$$

$\boxed{\text{Time invariant}}$

$$y(t) = x(2-t)$$

i-3 Not memoryless

ii-3 Not causal (e.g. $t=-5 \rightarrow y(-5) = x(7)$ Future value!)

iii-3 $|x(t)| \leq M_x$

$|y(t)| \leq M_x \rightarrow \boxed{\text{stable}}$

iv-3

Superposition

$$\mathcal{H}\{x_1(t) + x_2(t)\} = \underbrace{x_1(2-t)}_{y_1(t)} + \underbrace{x_2(2-t)}_{y_2(t)} \checkmark$$

Homogenity

$$\mathcal{H}\{a\,x(t)\} = a\,x(2-t) = a\,y(t) \checkmark$$

$\boxed{\text{LINEAR}}$

(v)-3

$$\mathcal{H}\{x(t-t_0)\} = x(2-(t-t_0)) = y(t-t_0) \checkmark$$

$\boxed{\text{Time-invariant}}$

3

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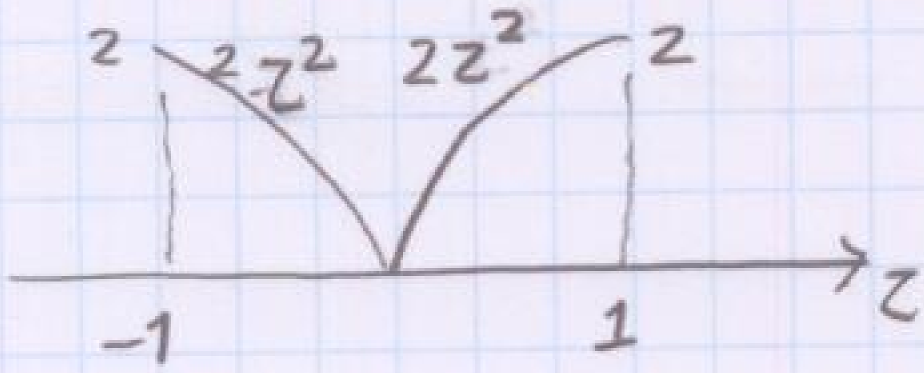
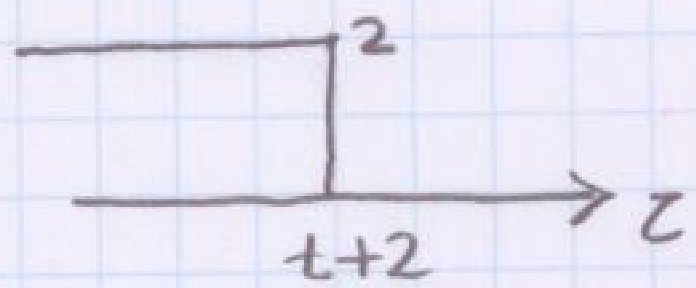
$$x(t) = 2t^2\,[u(t+1) - u(t-1)]$$

$$h(t) = 2\,u(t+2)$$



$x[\tau)$



$h(t-\tau)$

① $t+2 < -1 \rightarrow t < -3 \qquad y(t) = 0$

② $-1 \leq t+2 < 1 \rightarrow -3 \leq t < -1$

$$y(t) = \int_{-1}^{t+2} 2\tau^2 \cdot 2 \; d\tau = \frac{4}{3}\,\tau^3 \Big|_{-1}^{t+2}$$

$$= \frac{4}{3}(t+2)^3 + \frac{4}{3}$$

③ $t \geq -1$

$$y(t) = 4 \int_{-1}^{1} \tau^2\, d\tau = 4\,\frac{\tau^3}{3}\Big|_{-1}^{1}$$

$$= \frac{4}{3} + \frac{4}{3} = \frac{8}{3}\ /\!/$$

$$\boxed{y(t) = \begin{cases} \frac{4}{3}(t+2)^2 + \frac{4}{3} & , \quad -3 \leq t < -1 \\ \frac{8}{3} & , \quad t \geq -1 \\ 0 & , \quad \text{otherwise} \end{cases}}$$