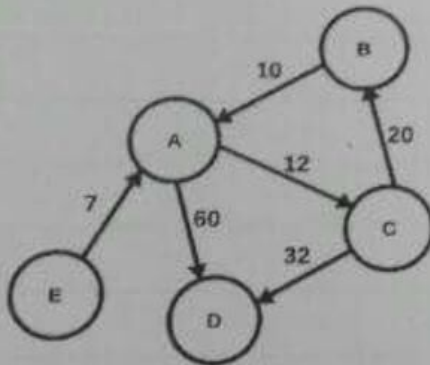


- Write your answers in English. Duration: 75 minutes

Questions

- 1) (15 pts) Calculate efficiency classes (Big-theta notation) of the following using Master theorem:
 - a) $T(n) = 3T(n/2) + n$
 - b) $T(n) = 5T(n/5) + n/2$
 - c) $T(n) = 4T(n/2) + n / \log n$
- 2) (15 pts) Insert elements of array $A = \{12, 6, 8, 2, 5, 1, 4, 9\}$ in a 2-3 balanced tree. Show all steps.
- 3) (20 pts) Give complete pseudo-code of an algorithm to find K-length cycles in an undirected connected graph G represented by adjacency matrix A .
- 4) (20 pts) Apply Floyd's algorithm on the directed graph given below. Calculate and show all $D^{(k)}$ matrices ($k = 0, 1, 2, 3, 4, 5$).



ALGORITHM $Floyd(W[1..n, 1..n])$
 //Input: The weight matrix W of a graph with no negative-length cycle
 //Output: The distance matrix of the shortest paths' lengths
 $D \leftarrow W$ //is not necessary if W can be overwritten
 for $k \leftarrow 1$ to n do
 for $i \leftarrow 1$ to n do
 for $j \leftarrow 1$ to n do
 $D[i, j] \leftarrow \min[D[i, j], D[i, k] + D[k, j]]$
 return D

- 5) (30 pts) Ali is running a web-hosting company. He needs a total bandwidth of 30 Gb/s at least (NOT exactly) and wants to pay lowest price possible for this bandwidth need. He can get service from five different Internet Service Providers (ISPs) listed below:

ISP	Monthly Cost (c_i)	Provided Bandwidth (b_i)
Telecom 1	3	5 Gb/s
Telecom 2	2	10 Gb/s
Telecom 3	4	15 Gb/s
Telecom 4	6	20 Gb/s
Telecom 5	5	25 Gb/s

Due to state regulations, Ali can sign only one contract per ISP. He asks for your help to choose the right subset of ISPs with minimum cost.

- a) Give the recurrence rule ($C(i, j) = \dots$) of dynamic programming to get at least required bandwidth with lowest price. Show cost of i th ISP by c_i and provided bandwidth by b_i . (10 pts)
- b) Form the dynamic programming table (ISPs vs. total bandwidth) and apply backtracking in order to determine ISPs to be chosen. (15 pts)
- c) If Ali uses the greedy approach instead of asking for your help, which ISPs will he choose and how much will he pay in total? (5pts)

**WRITE ALL YOUR ANSWERS ON ANSWER SHEET.
SOLUTIONS WRITTEN ON THIS SHEET WILL NOT BE EVALUATED!!!**