Fourier Representations

	Discrete Time	CT
Periodic	DT Fourier Series DTFS	Fourier Series FS
Non periodic	DT Fourier Transform DTFT	Fourier Transform FT

① Periodic Signals (Fourier Series)

– Representing Periodic Signals as weighted superposition of complex sinusoids.

– Each sinusid in the representation must have the same period as the signal.

(Ex) $x(t) = x_1(t) + x_2(t)$ T : period, w : frequency

$x(t) = x_1(t + m \cdot T_1) + x_2(t + k \cdot T_2)$ $m, k \in \mathbb{Z}$

$T = m T_1 = k T_2 \Rightarrow \frac{m}{w_1} = \frac{k}{w_2} = \frac{1}{w}$ T

$m = \frac{w_1}{w}$: must be an integer

$k = \frac{w_2}{w}$: " " " "

∴ The fundamental frequency of $x_1(t)$ and $x_2(t)$ must be integer multiples of the fundamental frequency of $x(t)$

• If $x[n]$ is DT signal with period $\underline{N}$

We want to represent $x[n]$ by

$$\hat{x}[n] = \sum_{k} A[k] \cdot e^{jk\Omega_0 n} \quad , \; k \in \mathbb{Z}$$

Ω_0 : Fundamental frequency of $x[n]$

$k\Omega_0$ } Integer multiple of Ω_0, Frequency of the k^{th} sinusoid.

$A[k]$: Its weight | $e^{jk\Omega_0 t}$: k^{th} harmonic.

– Each of these sinusoids have the common period $\underline{N}$

CT $x(t)$, $w_0 = \frac{2\pi}{T}$: fundamental frequency

We can represent $x(t)$ by

$$\hat{x}(t) = \sum_{k} A[k] \; e^{jkw_0 t}$$

kw_0 : frequency of k^{th} sinusoid.

– $e^{jkw_0 t}$ is called the k^{th} $\boxed{\text{harmonic}}$ and $A[k]$ is its weight.

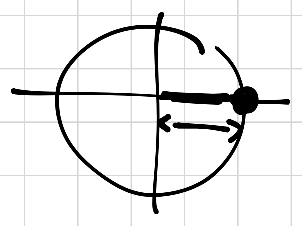
• How many harmonics should we use?

- DT Fourier Series

$e^{jk\Omega_0 n}$ is periodic if $\quad N = \frac{2\pi}{\Omega_0}$

$$e^{j(N+k)\Omega_0 n} = e^{jN\Omega_0 n} \cdot e^{jk\Omega_0 n}$$
$$= e^{j2\pi n} \cdot e^{jk\Omega_0 n}$$
$$= 1 \cdot e^{jk\Omega_0 n}$$

There are only N distinct complex sinusoids in the form of $e^{jk\Omega_0 n}$,



so we can only use N consecutive terms

, $k = 0 \ldots (N-1)$

$$\hat{x}[n] = \sum_{k=0}^{N-1} A[k] \cdot e^{jk\Omega_0 n}$$
$$= \sum_{k=\langle N\rangle} A[k] \cdot e^{jk\Omega_0 n}$$

The set of N consecutive terms is arbitrary, we can start from any value of k as long as we sum over $\underline{N}$ values.

$$k = \underbrace{(m) \ldots (N-1+m)}_{N} \qquad m \in [0..N]$$

For FS (CT)

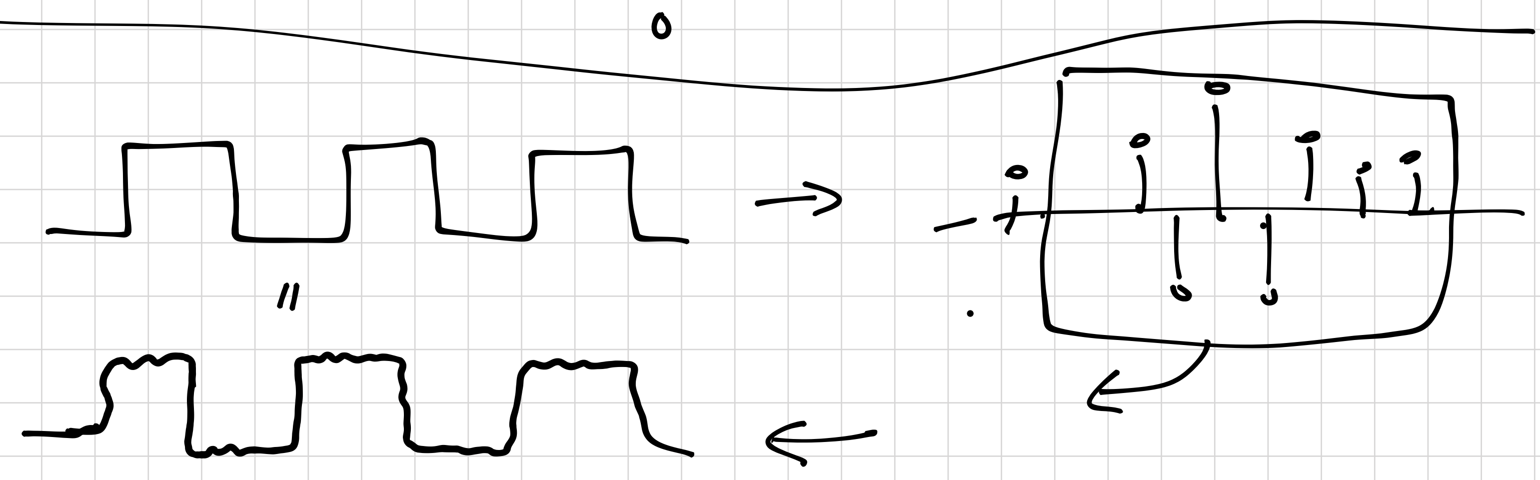
The CT complex sinusoids, $e^{jkw_0 t}$, with distinct frequencies $\underline{kw_0}$ are always distinct. So there could be an infinite number of distinct terms $\therefore$

$$\hat{x}(t) = \sum_{k=-\infty}^{+\infty} A[k] \cdot e^{jkw_0 t}$$

We want to find coefficients $A[k]$ such that $\hat{x}[n]$ and $\hat{x}(t)$ are good approximations to $x[n]$ and $x(t)$, respectively. We use a measure called MSE (mean-squared error) and we want to minimize this value.

DT $\qquad MSE = \frac{1}{N} \sum_{k=0}^{N-1} |x[n] - \hat{x}[n]|^2$

CT $\qquad MSE = \frac{1}{T} \int_0^T |x(t) - \hat{x}(t)|^2 \, dt$



Nonperiodic Signals : FT

- There are no restrictions on the period of the complex sinusoids.
  - The complex sinusoids will include a continuum of frequencies.

CT : $\quad \hat{x}(t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} X(jw) \, e^{jwt} \, dt$

DT : $\quad \hat{x}[n] = \frac{1}{2\pi} \int_{-\pi}^{\pi} X(e^{j\Omega}) \cdot e^{j\Omega t} \, dt$