Random and Stochastic Processes 2014 Midterm Exam Solutions

Question # 1

Define event names :

G : The event that a defendant in fact guilty

J_n : The event that nth judge votes guilty

Given :

$P(J_n | G) = 0.7$

$\therefore P(J_n' | G) = 0.3$

$P(J_n | G') = 0.2$

$\therefore P(J_n' | G') = 0.8$

J_1, J_2 an J_3 are independent under the condition G !

Also given $\Big\} \rightarrow$ $\boxed{\begin{array}{l} P(G) = 0.7 \\ P(G') = 0.3 \end{array}}$

a) 20 pts

Asking for $P(J_3 | J_2 J_1)$

Using $\boxed{\text{multiplication rule}}$:

$$P(J_1 \cap J_2) = P(J_1 \cap J_2 | G)\, P(G) + P(J_1 \cap J_2 | G)\, P(G')$$

Since J_1 & J_2 are independent depending on guilt

$$P(J_1 \cap J_2) = P(J_1 | G)\, P(J_2 | G)\, P(G) + P(J_1 | G)\, P(J_2 | G)\, P(G')$$

$$= 0.7 \times 0.7 \times 0.7 + 0.2 \times 0.2 \times 0.3$$

$$= 0.355 \qquad (1)$$

Similarly

$$P(J_1 \cap J_2 \cap J_3) = P(J_1|6)\,P(J_2|6)\,P(J_3|6)\,P(6) + P(J_1|6')\,P(J_2|6')\,P(J_3|6')\,P(6')$$

$$= 0.7^3 \times 0.7 + 0.2^3 \times 0.2$$

$$= 0.2425 \quad (2)$$

Using (1) and (2)

$$P(J_3 \cap J_2 \cap J_1 \mid J_2 \cap J_1) = \frac{0.2425}{0.355} = \boxed{0.6831}$$

b) 20 pts

Between J_1 and J_2 either one could have casted the guilty vote. The question asks for the following:

$$P[J_3 \mid (J_1 \cap J_2') \cup (J_1' \cap J_2)]$$

which equals to $\dfrac{P\{J_3 \cap [(J_1 \cap J_2') \cup (J_1' \cap J_2)]\}}{P[(J_1 \cap J_2') \cup (J_1' \cap J_2)]}$

using de-Morgan and addition rule

$$= \frac{P(J_3 \cap J_1 \cap J_2') + P(J_1' \cap J_2 \cap J_3)}{P(J_1 \cap J_2') + P(J_1' \cap J_2)}$$

Conditional independence still applies

$$= P(J_1 \cap J_2' \cap J_3) = 0.7 \times (1-0.7) \times 0.7 \times 0.7 + 0.2 \times (1-0.2) \times 0.2 \times 0.3 = 0.1125 \quad (1)$$

(1) is equal to $P(J_1' \cap J_2 \cap J_3) = 0.1125$ (2)

Similarly

$$P(J_1 \cap J_2') = P(J_1' \cap J_2) = 0.7 \times (1-0.7) \times 0.7 + 0.2 \times (1-0.2) \times 0.3 = 0.195 \quad (3) \quad (4)$$

Using (1), (2), (3), (4)

The answer is $\dfrac{0.1125 + 0.1125}{0.195 + 0.195} = \boxed{0.5769}$ //

$\boxed{\text{3) 20 pts}}$

The question asks for $P(J_3 \mid J_1' \cap J_2') = \dfrac{P(J_3 \cap J_1' \cap J_2')}{P(J_1' \cap J_2')}$

Using the same formulae

$$P(J_3 \cap J_1' \cap J_2') = 0.7 \times (1-0.7) \times (1-0.7) \times \underbrace{0.7}_{p(6)} + 0.2 \times (1-0.2) \times (1-0.2) \times \underbrace{0.3}_{P(6')} = 0.0825 \quad (1)$$

$$P(J_1' \cap J_2') = (1-0.7) \times (1-0.7) \times \underbrace{0.7}_{P(6)} + (1-0.2) \times (1-0.2) \times 0.3 = 0.255 \quad (2)$$

Using (1) and (2) $P(J_3 \mid J_1' \cap J_2') = \dfrac{0.0825}{0.255} = \boxed{0.3235}$ //

Question 2

Define events

The event that a heart failure occurs due to:

N : Natural Occurences $\rightarrow P(N) = 0.87$

O : Outside factors $\rightarrow P(O) = P(N') = 0.13$

I : Induced substonces $P(I|O) = 0.73$

F : Foreign objets $P(F|O) = 0.27$

note that these are mutually exclusive and exhaustive events!

A : Arterial blockage $P(A|N) = 0.56$

D : Disease $P(D|N) = 0.27$

M : Infeection $P(M|N) = 0.17$

note that these are mutually exclusive and exhaustive events

a) 20 pts Asking for $P(I)$

Using addition rule

$P(I) = P(I|O)\,P(O) + P(I|N)\,P(N)$

$= 0.73 \times 0.13 + 0 \times 0.83$

$\boxed{P(I) = 0.0949}$ //

b) 20 pts Asking for $P(D \cup M)$ D and M are mutually exclusive so

$P(D \cup M) = P(D) + P(M)$

Since $P(D|O)$ and $P(M|O)$ are zero

$P(D \cup M) = P(D|N)\,P(N) + P(M|N)\,P(N)$

$= (0.27 + 0.17) \times 0.87$

$\boxed{P(D \cup M) = 0.3828}$ //