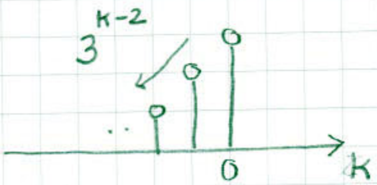
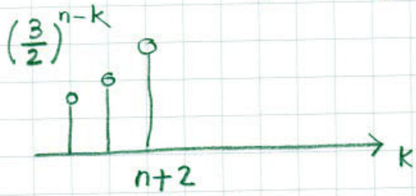
Q1 20p



$x[k]$



$h[n-k]$

① $n+2<0 \rightarrow n<-2$

$$y[n] = \sum_{k=-\infty}^{n+2} 3^{k-2}\left(\frac{3}{2}\right)^{n-k} = \left(\frac{3}{2}\right)^n \cdot 3^{-2} \sum_{k=-\infty}^{n+2} 3^k \cdot \left(\frac{2}{3}\right)^k$$

$$= \left(\frac{3}{2}\right)^n 3^{-2} \sum_{k=-\infty}^{n+2} 2^k = \left(\frac{3}{2}\right) 3^{-2} \cdot 2^{n+2}\left(\frac{2}{2-1}\right)$$

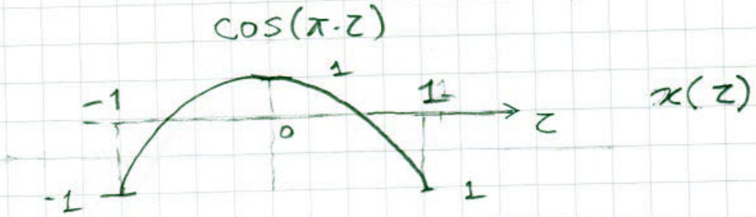
$$= \left(\frac{3}{2}\right)^n \cdot 3^{-2} \cdot 2^{n+3}$$

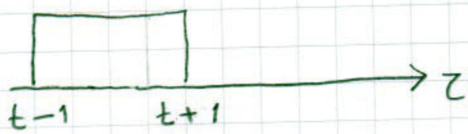
② $n \geqslant -2$

$$y[n] = \left(\frac{3}{2}\right)^n 3^{-2} \sum_{k=-\infty}^{0} 2^k = \left(\frac{3}{2}\right)^n 3^{-2} \cdot 2$$

$$y[n] = \begin{cases} \left(\frac{3}{2}\right)^n \cdot 3^{-2} \cdot 2^{n+3}, & n<-2 \\ \left(\frac{3}{2}\right)^n \cdot 3^{-2} \cdot 2, & n \geqslant -2 \end{cases}$$

Q2
20p





① $t+1 \leqslant -1 \rightarrow t \leq -2 \qquad y(t) = 0$

② $-1 \leqslant t+1 < 1 \rightarrow -2 \leqslant t < 0$

$$y(t) = \int_{-1}^{t+1} \cos(\pi \cdot z)\, dz = \frac{1}{\pi} \sin(\pi z) \Big|_{-1}^{t+1}$$

$$= \frac{1}{\pi} \left[\sin(\pi(t+1)) - \sin(-\pi) \right]$$

$$= \frac{1}{\pi} \sin[\pi(t+1)]$$

③ $t \geqslant 0 \rightarrow t-1 < 1 \quad t < 2$

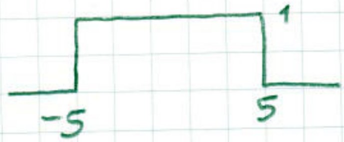
$$y(t) = \int_{t-1}^{1} \cos(\pi z)\, dz = \frac{1}{\pi} \sin(\pi z) \Big|_{t-1}^{1} = -\frac{1}{\pi} \sin[\pi(t-1)]$$

④ $t \geqslant -2 \qquad \underline{\underline{y(t) = 0}}$

$$y(t) = \begin{cases} \frac{1}{\pi} \sin[\pi(t+1)], & -2 < t < 0 \\ -\frac{1}{\pi} \sin[\pi(t-1)], & 0 \leqslant t < 2 \\ 0, & \text{elsewhere} \end{cases}$$

3a / 10p

$$h(t) = \int_{-5}^{5} \delta(t-\tau)\, d\tau = u[t+5] - u[t-5]$$



3b / 10p

$t < -5$ $\quad s(t) = 0$

$-5 \leq t < 5$ $\quad s(t) = \int_{-5}^{t} 1 . d\tau = t-5$

$5 \leq t$ $\quad s(t) = \int_{-5}^{5} 1 . d\tau = 10$

$$s(t) = \begin{cases} t-5 , & -5 \leq t < 5 \\ 10 , & t \geq 5 \\ 0 , & t < -5 \end{cases}$$

4a / 3p not memoryless

4b / 3p not causal

4c / 4p

$$\sum_{k=-2}^{\infty} \left(\frac{3}{2}\right)^k = \infty \quad \therefore \boxed{\text{not stable}}$$

5a / 3p not memoryless

5b / 3p not causal

5c / 4p

$$\int_{-5}^{5} 1\, d\tau = 10 < \infty \quad \therefore \boxed{\text{stable}}$$

Q6a
9.10

Homogenity

$$a\, e^{-|t|} x(t) = a\, y(t)$$

Superposition

$$e^{-|t|}\left(x_1(t) + x_2(t)\right) = \underbrace{e^{-|t|} x_1(t)}_{y_1(t)} + \underbrace{e^{-|t|} x_2(t)}_{y_2(t)}$$

This system is Linear

Q6b
10P

$$e^{-|t|} x(t-t_0) \neq e^{-|t-t_0|} x(t-t_0)$$

not T.I.