$\int_0^2 \frac{dx}{\sqrt{|x-1|}}$ yakınsaklığını araşt.

$= \underbrace{\int_0^1 \frac{dx}{\sqrt{1-x}}}_{\text{II. tip}} + \underbrace{\int_1^2 \frac{dx}{\sqrt{x-1}}}_{\text{II. tip}} = \lim_{a\to 1^-} \int_0^a \frac{dx}{\sqrt{1-x}} + \lim_{b\to 1^+} \int_b^2 \frac{dx}{\sqrt{x-1}}$

$\int \frac{dx}{\sqrt{1-x}} \overset{1-x=u,\ -dx=du}{=} \int \frac{-du}{\sqrt{u}} = -2\sqrt{u} + c_1 = -2\sqrt{1-x} + c_1$

$\int \frac{dx}{\sqrt{x-1}} \overset{x-1=u}{=} \int \frac{du}{\sqrt{u}} = 2\sqrt{u} + c_2 = 2\sqrt{x-1} + c_2$

$= \lim_{a\to 1^-} -2\sqrt{1-x}\,\Big|_{x=0}^{a} + \lim_{b\to 1^+} 2\sqrt{x-1}\,\Big|_{x=b}^{2} = 4$ yakınsak

Örnek: $\int_{-\infty}^{\infty} e^{-|x|}dx = ? = \lim_{a\to -\infty} \int_a^0 e^{x}dx + \lim_{b\to \infty} \int_0^b e^{-x}dx$

$(a<0,\ b>0)$

$= \lim_{a\to -\infty} [e^{x}]_a^0 + \lim_{b\to\infty} [-e^{-x}]_0^b$

$= \lim_{a\to -\infty} (1-\cancel{e^{a}}) + \lim_{b\to\infty} (-\cancel{e^{-b}}+1) = 2$ //

Örnek: $\int_0^1 \frac{\ln x}{(1+x)^2}dx = ?$

$\left[\begin{array}{ll} \ln x = u & \frac{dx}{(1+x)^2} = dv \\ \frac{1}{x}dx = du & v = \frac{-1}{1+x} \end{array}\right.$

$\lim_{t\to 0^+} \int_t^1 \frac{\ln x}{(1+x)^2}dx \Rightarrow$

$\lim_{t\to 0^+} \left[-\frac{\ln x}{(1+x)}\Big|_t^1 + \underbrace{\int_t^1 \frac{dx}{x(1+x)}}\right]$

$\left(\frac{A}{x} + \frac{B}{x+1} \Rightarrow A+B=0,\ A=1,\ B=-1\right.$

$\left(\int \frac{1}{x} - \int \frac{1}{x+1}\right) = \ln x - \ln(x+1)$

$= \lim_{t\to 0^+} \left[-\frac{\ln x}{(1+x)} + \cancel{\ln x} - \ln(1+x)\right]_t^1$

$= \lim_{t\to 0} \left[-\ln 2 + \ln(1+t) - \frac{t\ln t}{1+t}\right] = \ln\frac{1}{2}$ //

Örnek: $\int_0^\infty \frac{\arctan x}{1+x^2}dx = ?$ 1. Geçit gene into

$= \lim_{t\to\infty} \int_0^t \frac{\arctan x}{1+x^2}dx = \lim_{t\to\infty} \frac{(\arctan x)^2}{2}\Big|_0^t \quad \left(\int \frac{1}{1+x^2}dx = \arctan x\right)$

$= \lim_{t\to\infty} \frac{(\arctan t)^2}{2} - \frac{\cancel{(\arctan 0)^2}}{2} = \frac{\pi^2}{8}$ //

$\arctan\infty = \frac{\pi}{2}$

Örnek: $\int_1^\infty \frac{\ln x}{x^2}dx = ?$

$\ln x = u \quad \frac{1}{x^2}dx = dv$

$\frac{1}{x}dx = du \quad \boxed{v = -\frac{1}{x}}$

$\int_1^t \left[-\frac{1}{x}\ln x - \int -\frac{1}{x}\cdot\frac{1}{x}dx\right] = \int_1^t \left[-\frac{1}{x}\ln x - \frac{1}{x}\right]$

$\lim_{t\to\infty} \left(\cancel{\frac{1}{t}\ln t} - \cancel{\frac{1}{t}} - (-\cancel{\ln 1} - 1)\right) = 1$ //

1) $\int_0^\infty e^{-3x}\cdot\cos x\,dx = ?$

2) $\int_0^\infty \frac{dx}{x^2+2x+1} = ?$

3) $\int_0^\infty \frac{dx}{x^4+1} = ?$

1) a) $P_1(1,1,1)$, $P_2(3,3,0)$

$|P_1P_2| = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$

$= \sqrt{(3-1)^2+(3-1)^2+(0-1)^2} = \sqrt{4+4+1} = 3$

b) $P_1(-1,1,5)$ $P_2(2,5,0)$

$|P_1P_2| = \sqrt{3^2+4^2+(-5)^2} = \sqrt{50} = 5\sqrt{2}$

Yarıçapı a ve merkezi (x_0, y_0, z_0) olan küreninki.

$(x-x_0)^2+(y-y_0)^2+(z-z_0)^2 = a^2$

a) $(-2,0,2)$ $r=\sqrt{8}=2\sqrt{2}$

b) $\left(x+\frac{1}{2}\right)^2 + \left(y+\frac{1}{2}\right)^2 + \left(z+\frac{1}{2}\right)^2 = \frac{21}{4}$ $r = \sqrt{\frac{21}{4}} = \frac{\sqrt{21}}{2}$

merkezi $\left(-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}\right)$

3) a) $M(1,2,3)$; $r=\sqrt{14} \Rightarrow \boxed{(x-1)^2+(y-2)^2+(z-3)^2=14}$

b) $M(0,-1,5)$ $r=2 \Rightarrow x^2+(y+1)^2+(z-5)^2=4$

4) a) $x^2+y^2+z^2+4x-4z=0$

$x^2+4x+8-8+y^2+z^2-4z=0$

$\boxed{(x+2)^2+y^2+(z-2)^2=8}$ $M(-2,0,2)$ $r=2\sqrt{2}$

b) $x^2+y^2+z^2-6y+8z=0$

$x^2+(y-3)^2+(z+4)^2-25=0$

$M(0,3,-4)$ $r=5$

5) $P_1(5,7,-1)$, $P_2(2,9,-2) \Rightarrow \overrightarrow{P_1P_2} = ?$

$P_1 = 5i+7j-k$, $P_2 = 2i+9j-2k$

$\overrightarrow{P_1P_2} = (2-5)i+(9-7)j+(-2-(-1))k = -3i+2j-k$

f) $u(-1,0,2)$, $v=(1,1,1) \Rightarrow -2u+3v = ?$

$u = -i+2k$, $v = i+j+k$

$\left.\begin{array}{l} -2u = 2i-4k \\ 3v = 3i+3j+3k \end{array}\right\} \Rightarrow 5i+3j-k$

6) uzunluğunun ve yönünün çarpımı olarak ifade etmek demek

$\boxed{v = |v|\cdot\frac{v}{|v|}}$

a) $v = 2i+j-2k \Rightarrow |v| = \sqrt{2^2+1^2+(-2)^2} = \sqrt{9} = 3$

$v = \underbrace{3}_{\text{vektörün uzunluğu}} \cdot \underbrace{\left[\frac{2}{3}i+\frac{1}{3}j-\frac{2}{3}k\right]}_{\text{vektörün yönü}}$

9) a) $v = 2i-4j+\sqrt{5}k$, $u = -2i+4j-\sqrt{5}k$

$v\cdot u = 2\cdot(-2)+(-4)\cdot 4+\sqrt{5}\cdot(-\sqrt{5}) = -4-16-5 = -25$

$|v| = \sqrt{2^2+(-4)^2+(\sqrt{5})^2} = \sqrt{4+16+5} = 5$

$|u| = \sqrt{(-2)^2+4^2+(-\sqrt{5})^2} = 5$

$\theta = \arccos\left(\frac{u\cdot v}{|u|\cdot|v|}\right) = \arccos\left(\frac{-25}{5\cdot 5}\right) = \pi$ //

$\cos\theta = \cos\pi = -1$ //

$\text{proj}_v u = \left(\frac{u\cdot v}{|v|^2}\right)\cdot v = \frac{-25}{25}\cdot(2i-4j+\sqrt{5}k)$

$= -2i+4j-\sqrt{5}k$ //

$\boxed{\text{skaler bileşen; } |u|\cdot\cos\theta = \frac{u\cdot v}{|v|}}$

$= (-2i+4j-\sqrt{5}k)\left(\frac{2}{5}i-\frac{4}{5}j+\frac{\sqrt{5}}{5}k\right)$

$= \left(-2\cdot\frac{2}{5}\right)+\left(4\cdot\left(-\frac{4}{5}\right)\right)+\left(-\sqrt{5}\cdot\frac{\sqrt{5}}{5}\right) = -5$ //