



Cairo University - Faculty of Engineering  
Computer Engineering Department  
Machine Intelligence CMP 402B – Spring 2022



## Assignment 2

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# Problem 2.24

$$(a) \quad E_{in}(g) = \sum_{i=1}^2 (f(x_i) - h(x_i))^2 = \sum_{i=1}^2 (x_i^2 - (ax_i + b))^2$$

$$(1) \quad \frac{\partial E_{in}(g)}{\partial a} = -2 \sum_{i=1}^2 x_i (x_i^2 - ax_i - b) = 0$$

$$(2) \quad \frac{\partial E_{in}(g)}{\partial b} = -2 \sum_{i=1}^2 (x_i^2 - ax_i - b) = 0$$

$$x_1 (1) - x_2 (2)$$

$$x_1^2 - ax_1 - b = 0$$

$$x_2^2 - ax_2 - b = 0$$

$$a = x_1 + x_2$$

$$b = -x_1 x_2$$

$$\text{So } g^D(x) = (x_1 + x_2)x - x_1 x_2$$

$$\tilde{g}(x) = E_D(g^D(x)) = E_D((x_1 + x_2)x - x_1 x_2)$$

$$= E_D[x_1]x + E_D[x_2]x - E_D[x_1]E_D[x_2]$$

due to independence of  $x_1, x_2$

b)

1. get  $\bar{g}(x)$

- fix  $x$
- for a number of times, e.g. 1000
  - Sample two data points from  $[-1, 1]$
  - Compute  $g^p(x)$  using  $a, b$  derived in last question
- ~~Take the average~~ Take the average value of  $g^p(x)$  so we get  $\bar{g}(x)$  at  $x$

2. get Variance and Entropy

- for a number of times, e.g. 5000
  - Sample  $x$  from  $[-1, 1]$
  - follow the procedure to get  $\bar{g}(x)$  to generate an array of values of function  $g^p(x)$  evaluated at that  $x$
  - Compute the variance  $E_D[(g^p(x) - \bar{g}(x))^2]$
  - we will use  $\bar{g}(x)$  to compute  $[(\bar{g}(x) - f(x))^2]$  at each  $x$
  - we use the array of values to compute an array  $[(g^p(x) - \bar{g}(x))^2]$ . take the average of the resulting array. we get  $E_D[(g^p(x) - f(x))^2]$
- now we take the average of above calculated  $E_D[(g^p(x) - \bar{g}(x))^2]$ ,  $[(\bar{g}(x) - f(x))^2]$ ,  $E_D[(g^p(x) - f(x))^2]$  and get the expected values of var, bias, Entropy
  - $E_x[E_D[(g^p(x) - \bar{g}(x))^2]]$ ,  $E_x[(\bar{g}(x) - f(x))^2]$ ,  $E_x[E_D[(g^p(x) - f(x))^2]]$

### Exercise 3.7

we take derivative of  $E_{in}(w)$  with respect to  $w$ ,  $E_{in}(w) = \frac{1}{N} \sum_1^N \ln(1 + e^{-y_n w^T x_n})$

$$\begin{aligned}\nabla E_{in}(w) &= -\frac{1}{N} \sum_n^N \frac{y_n x_n e^{-y_n w^T x_n}}{1 + e^{-y_n w^T x_n}} \\ &= \frac{1}{N} \sum_1^N -y_n x_n \sigma(-y_n w^T x_n)\end{aligned}$$

when a sample is misclassified

$y_n w^T x_n < 0$ ,  $\sigma(-y_n w^T x_n) > 0.5$  and  
when a sample is correctly classified  
 $\sigma(-y_n w^T x_n) < 0.5$ , so the contribution  
of misclassified example is more to  
the gradient than a correctly classified  
one.