Digital Communication

Assignment

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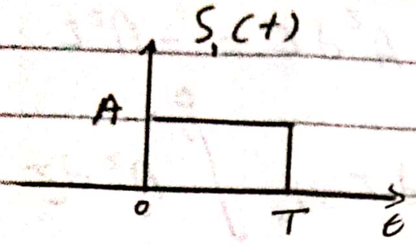
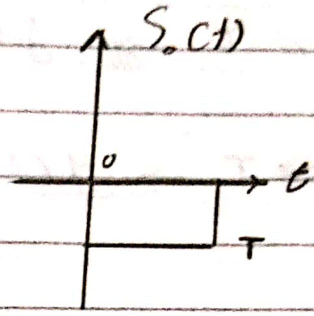
Date:
NO :

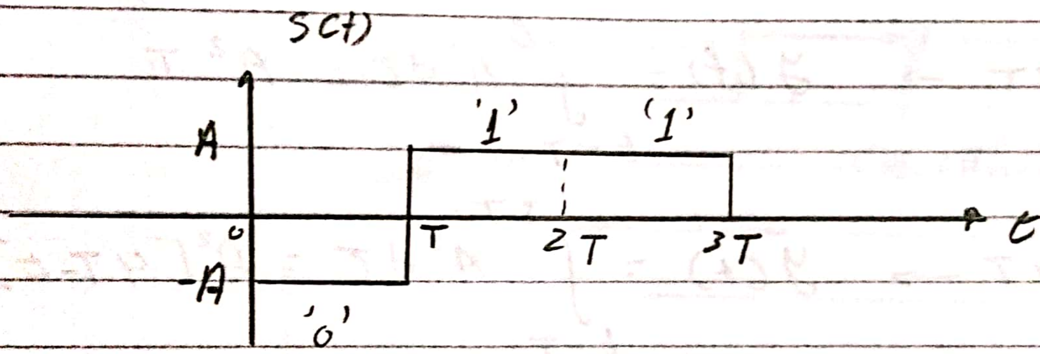
Question 1

$\boxed{1}$



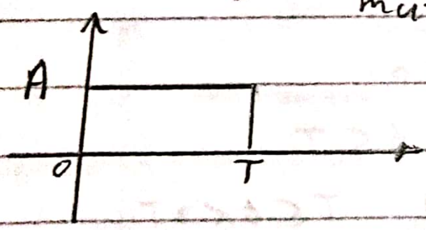


a)

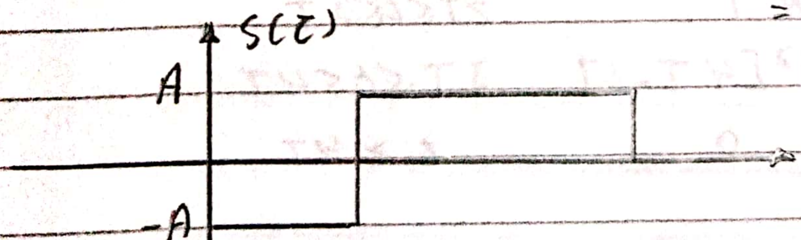


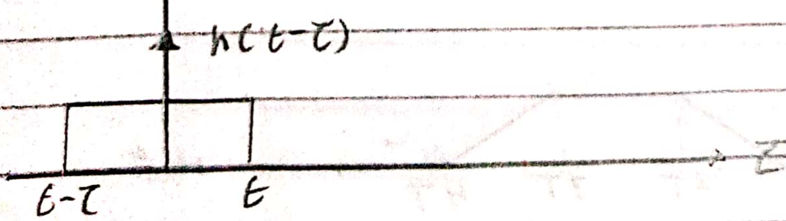
b)

$h(t) = S_1(T-t)$ matched filter



$$y(t) = s(t) * h(t)$$
$$= \int_{\tau} s(\tau)\, h(t-\tau)\, d\tau$$





@ $t<0 \rightarrow y(t)=0$

@ $0<t<T \rightarrow y(t) = \int_0^t -A^2 d\tau = -A^2 t$

@ $T<t<2T \rightarrow y(t) = \int_{t-T}^{T} -A^2 d\tau + \int_T^t A^2 d\tau$

$= -A^2[2T-t] + A^2(t-T)$

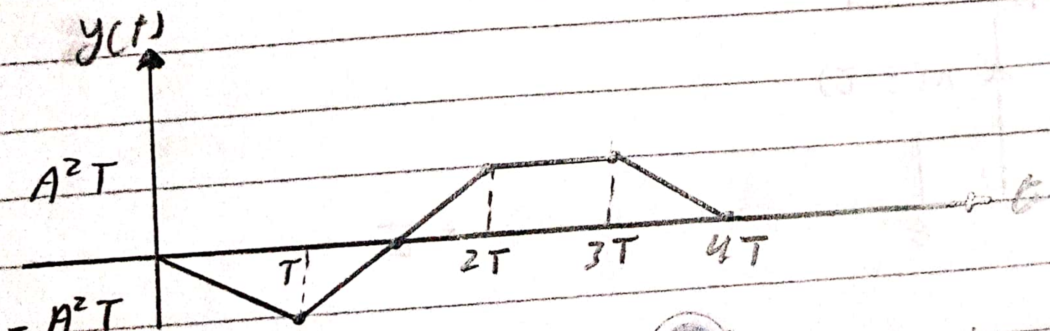
$= -3A^2T + 2A^2t = A^2(2t-3T)$

@ $2T<t<3T \rightarrow y(t) = \int_{t-T}^{t} A^2 d\tau = A^2 T$

@ $3T<t<4T \rightarrow y(t) = \int_{t-T}^{3T} A^2 d\tau = A^2[4T-t]$

@ $t>4T \rightarrow y(t)=0$

$$y = \begin{cases} 0 & t\leq 0 \\ -A^2 t & 0\leq t\leq T \\ A^2(2t-3T) & T\leq t\leq 2T \\ A^2 T & 2T\leq t\leq 3T \\ A^2[4T-t] & 3T\leq t\leq 4T \\ 0 & t\geq 4T \end{cases}$$



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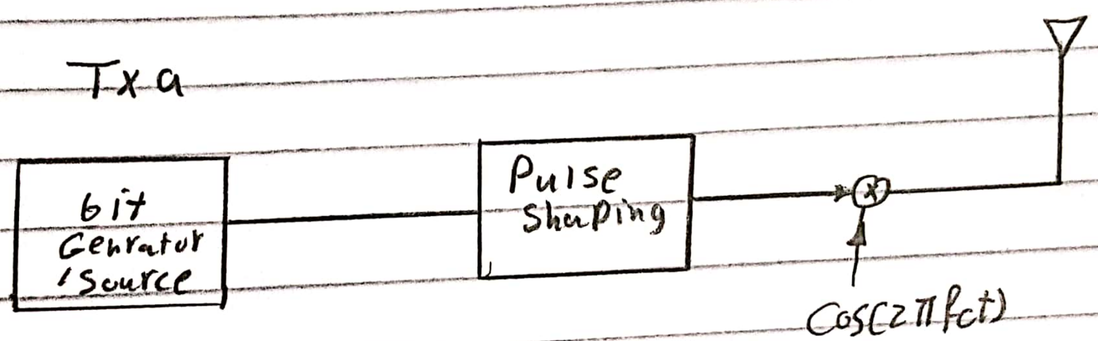
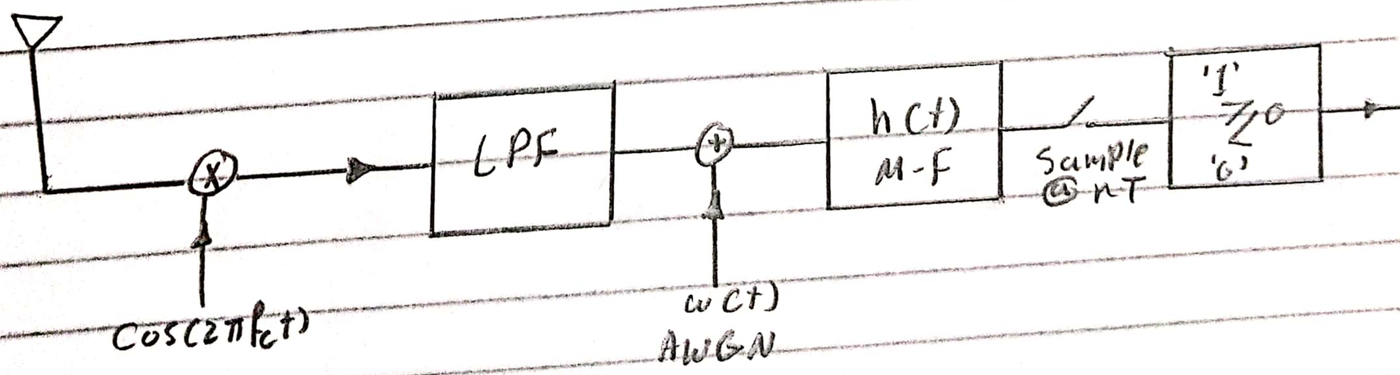
c) Sample at $t = T$ for b_0
Sample at $t = 2T$ for b_1
Sample at $t = 3T$ for b_2

d) Tx a



e)



Question 2

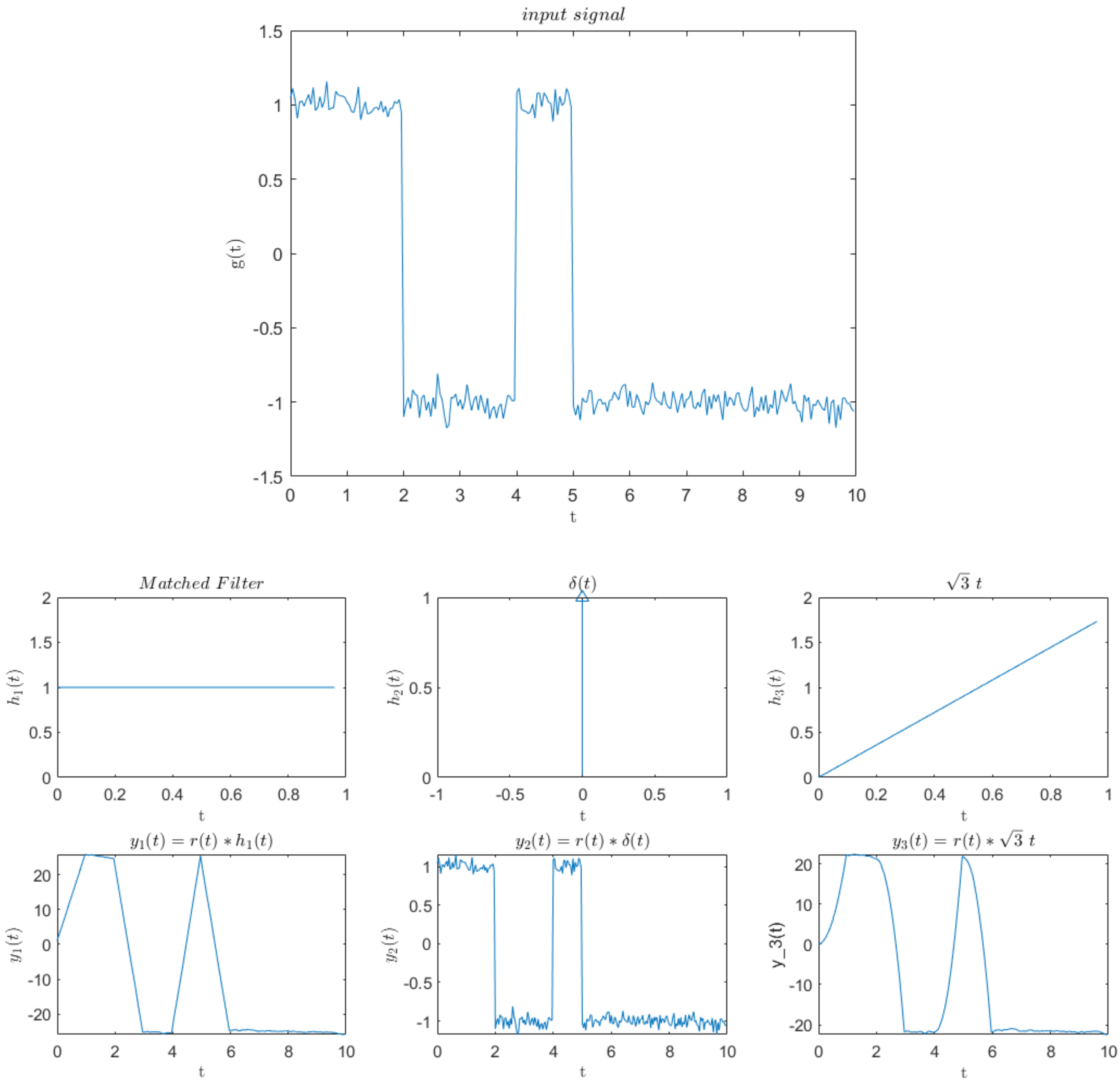


Figure 1 impulse response of filters and output with $E/N_0 = 21$ before thresholding

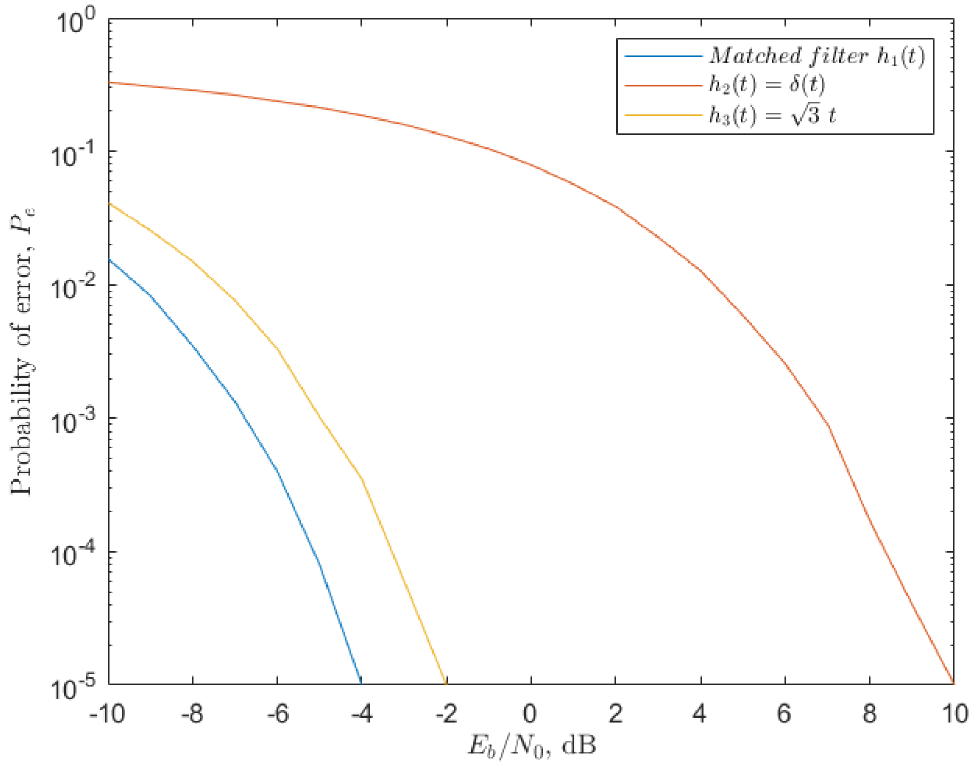


Figure 2 bit error rate while sending 10000 bits

(4) as shown in Figure 2 BER is a decreasing function of E_b/N_0 as when E_b/N_0 decreases the N_0 increases which increases the noise power but SNR which is the signal power to the noise power decreases which will result in a higher probability of flipping the bit from -A to higher than zero and A to less than zero as signal after noise.

$$r(t) = \begin{cases} +A + w(t), & \text{for bit '1'}, \quad 0 \le t \le T_b \\ -A + w(t), & \text{for bit '0'}, \quad 0 \le t \le T_b \end{cases}$$

(5) Matched filter is the lowest BER. As it is the optimum receiving filter having the max SNR with impulse response $h(t) = kg(T - t)$ which achieves $\eta_{max} = \frac{E}{N_0/2}$ and increasing SNR decreases BER as described earlier.

Using Matlab R2018a

```
close all;
  % bits to be transmitted
  bitsNumber = 100000;

% frequency of the signal 1/T
signalFreq = 1;

samplingFreq = 25;

% generate random bits with bits number
bits = randi([0 1],bitsNumber,1);
PNRZ = [];
ber1 = [];
ber2 = [];
ber3 = [];
E_N0 = [];
for snr = -10:21
    N0 = 10^(-snr/10);

    % create filters
    h1 = ones(1, samplingFreq);
    h2 = ones(1, 1);
    h3 = sqrt(3) * linspace(0, 1 / signalFreq ,samplingFreq);

    %pulse shaping
    bipolarValue = bits * 2 - 1;
    waveForm = repelem(bipolarValue, samplingFreq);
    waveForm = waveForm + sqrt(N0/2)*randn(size(waveForm));
    plot((0:10*samplingFreq-1)/samplingFreq, waveForm(1:10*samplingFreq));
    title({'$input\ signal$'},'Interpreter','latex');
    ylabel({'g(t)'},'Interpreter','latex');xlabel({'t'},'Interpreter','latex');

    %apply convolution with different receivers
    y1 = conv(waveForm, h1) ;
    y2 = conv(waveForm, h2) ;
    y3 = conv(waveForm, h3) ;

    % plot the filters and the output of the filters
    if(snr == 21)
        displaySignals(h1, h3, samplingFreq, y1, y2, y3)
    end

    % sampling to get the output signals
    y1 = y1(samplingFreq - 1: samplingFreq:bitsNumber * samplingFreq);
    y2 = y2(samplingFreq - 1: samplingFreq:bitsNumber * samplingFreq);
    y3 = y3(samplingFreq - 1: samplingFreq:bitsNumber * samplingFreq);

    %thresholding with lambda = 0
    y1(y1 <= 0 ) = 0;
    y1(y1 > 0 ) = 1;
    y2(y2 <= 0 ) = 0;
    y2(y2 > 0 ) = 1;
    y3(y3 <= 0 ) = 0;
    y3(y3 > 0 ) = 1;

    % calculate the bit error rate (probability of error) for SNR value
    ber1(end+1) = sum(abs(y1 - bits) / length(bits));
    ber2(end+1) = sum(abs(y2 - bits) / length(bits));
    ber3(end+1) = sum(abs(y3 - bits) / length(bits));
    E_N0(end+1) = snr;
end
```

```
figure;
semilogy(E_N0, ber1, E_N0, ber2, E_N0, ber3);
set(legend('$Matched\ filter\ h_{1}(t)$','$h_{2}(t)=\delta(t)$','$h_{3}(t)=\sqrt{3}\
t$'),'Interpreter','latex');
xlabel({'\(E_b/N_0\), dB'},'Interpreter','latex');
ylabel({'Probability of error, \(P_e\)'},'Interpreter','latex');


% function to display the filters and output of receiver
function a = displaySignals(h1, h3, samplingFreq, output1, output2, output3)
    figure;
    set(gcf,'position',[100 100 1000 400])
    subplot(2,3,1);
    plot((0:samplingFreq-1)/samplingFreq,h1);
    title({'$Matched\ Filter$'},'Interpreter','latex');
    ylabel({'$h_{1}(t)$'},'Interpreter','latex');xlabel({'t'},'Interpreter','latex');

    subplot(2,3,2);
    stem(0,1, '^');
    title({'$\delta(t)$'},'Interpreter','latex');
    ylabel({'$h_{2}(t)$'},'Interpreter','latex');xlabel({'t'},'Interpreter','latex');

    subplot(2,3,3);
    plot((0:samplingFreq-1)/samplingFreq,h3);
    title({'$\sqrt{3}\ t$'},'Interpreter','latex')
    ylabel({'$h_{3}(t)$'},'Interpreter','latex');xlabel({'t'},'Interpreter','latex');

    subplot(2,3,4);
    plot((0:10*samplingFreq-1)/samplingFreq, output1(1:10*samplingFreq));
    title({'$$y_1(t)=r(t)*h_1(t)$'},'Interpreter','latex');
    ylabel({'$y_1(t)$'},'Interpreter','latex');xlabel({'t'},'Interpreter','latex');

    subplot(2,3,5);
    plot((0:10*samplingFreq-1)/samplingFreq, output2(1:10*samplingFreq));
    title({'$y_2(t)=r(t)*\delta(t)$'},'Interpreter','latex');
    ylabel({'$y_2(t)$'},'Interpreter','latex');xlabel({'t'},'Interpreter','latex');

    subplot(2,3,6);
    plot((0:10*samplingFreq-1)/samplingFreq, output3(1:10*samplingFreq));
    title({'$y_3(t)=r(t)*\sqrt{3}\ t$'},'Interpreter','latex');
    ylabel({'$y_3(t)$'},'Interpreter','latex');xlabel({'t'},'Interpreter','latex');
end
```