Modelling MT

Question 1

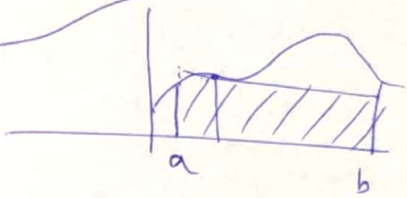
Given $I = \int_0^4 x^2 \, dx$, $u_1, u_2, u_3, u_4, u_5 \rightarrow$ uniforms $[0, 1]$

Calculate Integration using monte carlo

Sol:

From Lab: $F^n = (b-a) * \frac{1}{n} \sum_{i=1}^{n} f(x_i) = \frac{1}{n} \sum_{i=1}^{n} (b-a) f(x_i)$

sum area, area $= (b-a) *$ random value for $f(x)$



$\therefore \; I = \frac{1}{n} \sum_{i=1}^{n} I_i$

$I = \frac{1}{5} \sum_{i=1}^{n} (4-0) * (0 + 4U_i)^2$

$\overset{a}{0} + \overset{b-a}{4} U_i \rightarrow$ random value Uniform $[0, 4]$

$\cancel{a + (b-a) U_i}$

Remember:

$U_i \rightarrow [0, 1]$, but we want a value from $0 \rightarrow 4$ (integ. limits), So we need to transform it $[0,1] \rightarrow [a, b]$

$\cancel{a \rightarrow a}$ If $U_i = 0$

$\therefore \; a + (b-a) * 0 = a$

If $U_i = 1$

$\cancel{a + b}$

$\therefore \; a + (b-a) * 1 = b$

our range $[a, b]$ ✓✓✓

Note:-

ممكن ميدينا ش الـ U_i ويدينا

a, c, m, X_{seed}

ويقول انا نجيب uniforms

وبعدها نستعملها عادي

- مش متأكد: لوجبنا الـ uniform احنا ناخد الناتج الـ range ايه عشان نحوله للـ integ. limits !

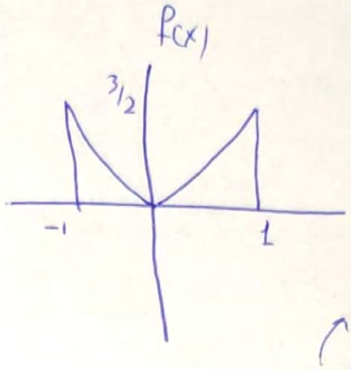
Recap: $I = \int_a^b x^2 dx$, $u_1 \dots u_5 \in [0, 1]$

$\therefore \; I = \frac{1}{5} \sum_{n=1}^{5} I_i$, $I_i = (b-a) * (a + (b-a) U_i)^2$

Question 2

Given $f(x) = \frac{3}{2} x^2$, $-1 \le x \le 1$

generate using acceptance-rejection a number from $f(x)$



given uniform values, if not uniform (or not given), we have to generate it

u_1	u_2
0.3	0.7
0.8	0.4

Sol:-

Start by generating a uniform RV from $-1 \to 1$

$g(x) \sim U(-1,1)$

$\underbrace{f(x)}_{\text{new pdf}} = \frac{3}{2} x^2$, $\quad \overbrace{g(x)}^{\text{old pdf}} = \frac{1}{b-a} = \frac{1}{2}$

2- $C = \max \frac{f(x)}{g(x)} = \frac{3/\cancel{2}}{1/\cancel{2}} = 3$ $\quad \boxed{\therefore C = 3}$

note: if it wasn't uniform we would have an expr. in terms of x, $\therefore$ ~~dx~~ diff. wrt x, get diff = 0 at x = ? use it (local maxima)

3- Y = uniform RV from -1 to 1 $\longrightarrow$ get it from u_1 — like in the slides

but $u_1 \to [0,1]$ $\quad \therefore$ $Y = a + (b-a)u$ $\quad \therefore Y = -1 + 2u_1$ (a = -1, b = 1)

$\boxed{Y = 2u_1 - 1}$

4- $u_1 = 0.3 \longrightarrow Y = -0.4$, $u_2 = 0.7$

Check: $u_2 \le \frac{f(Y)}{c\, g(y)} \longrightarrow 0.7 \le \frac{\cancel{\frac{3}{2}} \times (-0.4)^2}{\cancel{3 \times \frac{1}{2}}}$; $(-0.4)^2 \to 0.16$ $\quad \therefore$ reject

$u_1 = 0.8 \to Y = 0.6$

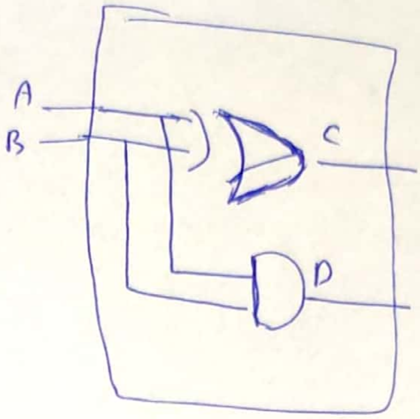
$\underbrace{0.4}_{u_2} \le \frac{\cancel{\frac{3}{2}} \times (0.6)^2}{\cancel{3 \times \frac{1}{2}}}$; $(0.6)^2 \to 0.36$ $\quad \therefore$ reject

iterate until we accept

$\therefore X = Y_{last}$

Question 3:

a) For the following circuit (half adder) calculate delay and simulate for input (1, 0, 0)



Assume delay for XOR = 4.2 ns

" " " OR, And = 2.4 ns

wire delay = 0

Sol:

- First event is And(0, 0) at

$0 + 2.4 = 2.4$ ns

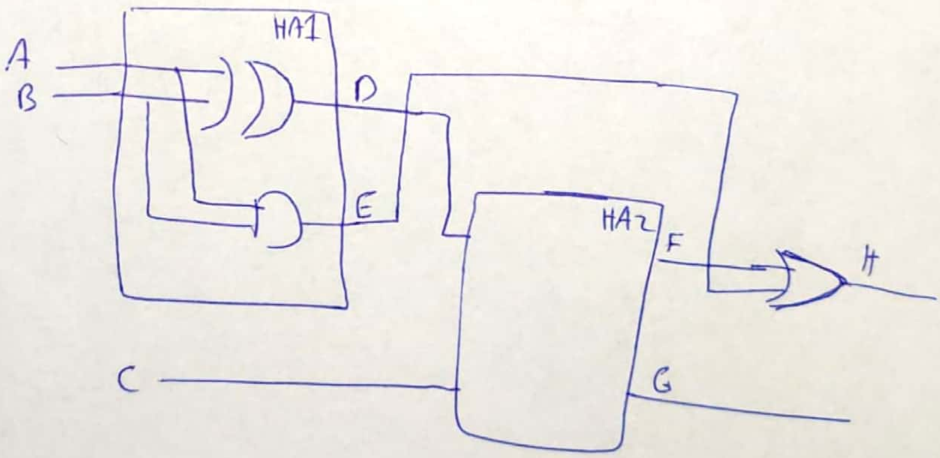
- second event is XOR(0,0) at

$0 + 4.2 = 4.2$ns

time	A	B	C	D
0	1	0	X	X
2.4	1	0	X	0
4.2	1	0	1	0

∴ output for H.A for input (1, 0) = (1, 0) after 4.2 ns

b) For the Full adder circuit consisting of 2 half adders, calc. delay and simulate for input (1, 0, 0)



- First event at

$0 + 4.2 = 4.2$ (HA_1)

- second event

at $4.2 + 4.2 = 8.4$ (HA_2)

- Third event

at $8.4 + 2.4 = 10.8$

t	A	B	C	D	E	F	G	H
0	10	0	0	x	x	x	x	x
4.2	10	0	0	1	0	x	x	x
8.4	10	0	0	1	0	1	0	x
10.8	10	0	0	1	0	1	0	1
	10	0	0	1	0	1	0	1
	10	0	0	1	0	1	0	1

output = (1,0) after 10.8ns.

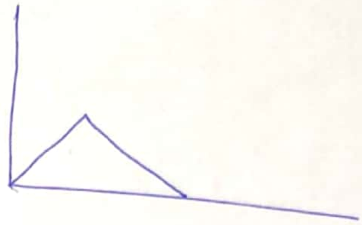
Q4 **Probability** → حاجة CS1

1- Interarrival follows exponential with rate 3

$P(X>5)=?$ $X \sim \exp(3)$

$e^{-5/3} \Rightarrow ??$

q2- pdf: $\begin{cases} x & 0 \le x \le 1 \\ 2-x & 1 \le x \le 2 \end{cases}$

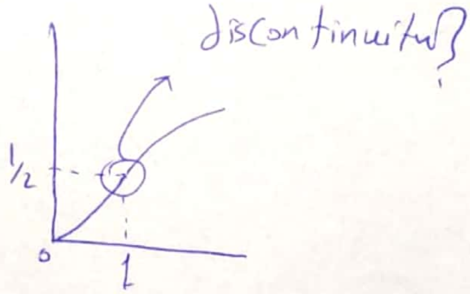


get inverse transform given some uniform function

given $u=0.3$

Sol:

get CDF:



- $\int x = \frac{x^2}{2}$ $0 \to 1$

- $\int 2-x \, dx = 2x - \frac{x^2}{2}$ $1 \to 2$

if $u < 0.5$ → subs. in $\frac{x^2}{2} = 0.3$ $\therefore x = \sqrt{0.6}$

$2x - \frac{x^2}{2} = u$ if $u > 0.5$