

A GENERAL EQUILIBRIUM CALCULATION OF THE EFFECTS OF DIFFERENTIAL TAXATION OF INCOME FROM CAPITAL IN THE U.S.*

John B. SHOVEN** and John WHALLEY**

Yale University, New Haven, Conn., U.S.A.

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1. Introduction

Economists have long been trying to measure the effects associated with various economic distortions from the perfectly competitive model. Both the efficiency costs and the incidence of market imperfections have received considerable attention. However, empirical estimates of the consequences of distortions (e.g. the corporate income tax) have been few, and those which have been presented have been subjected to criticism regarding the simplicity of the underlying model of the economy. Some modeling of the effects of either the imposition or the removal of the relevant distortion is necessary since in most cases it is impossible to observe the economy both in its presence and absence.

The model most commonly used in recent years for analyses of this type is the familiar static two sector – two factors of production general equilibrium model originally developed by Meade (1955) and Johnson (1956) for the study of international trade. Typically the economy is assumed to be perfectly competitive except for the single distortion being considered. Factors of production are assumed perfectly mobile between sectors but fixed in aggregate supply, and individuals are taken

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to have identical homothetic preference functions. Using this basic model, several studies have been presented (see, for example, Harberger, 1962, 1966 and Johnson and Mieszkowski, 1970) which compare the economy with and without the relevant distortion.

In general, there has been some dissatisfaction with the use of this model for this type of analysis, although no superior alternative has been suggested. One of the major criticisms of this approach has been that it is based on local analysis and approximations and as such is not well suited for the study of such distortions as the approximately fifty percent corporate income tax or the estimated fifteen percent wage differential between unionized and non-unionized labor. In addition, the following shortcomings have been evident in most studies:

- (1) The level of aggregation (two sectors, two factors) is too severe to capture the major impact of the market imperfection.
- (2) The assumption of fixed factor endowments is not realistic. This is particularly true when labor or capital return distortions are being considered.
- (3) Analyzing one distortion at a time can be misleading since the effect of two simultaneous distortions need not even approximate the sum of their individual effects.

The purposes of this paper are twofold. The first is to illustrate that with some modification an algorithmic approach due to Scarf (1967a, b, 1973) for the computation of general equilibrium prices is applicable to problems of the sort described above. The formulation is not based on differential calculus, requires no linearity assumptions, and is not as restrictive in either the number of sectors or the number of factors of production.¹ Several distinct consumers or consuming classes can be specified with differing tastes and initial endowments. It is possible to consider the effects of several distortions simultaneously, and the approach readily lends itself to dynamic extensions. Even in the static case the labor supply is free to vary in response to market distortions. In dynamic versions the capital stock could be augmented or depreciated through time.

The second purpose of this paper is to present an example of the technique by analyzing both the incidence effects and the efficiency

¹ The computation time rises rapidly with the number of factors plus the number of sectors. At the current stage of development, this sum can easily be as large as twenty without becoming prohibitively expensive.

costs associated with the differential taxation of income from capital in the U.S. economy. For purposes of simplicity of exposition and due to the specific characteristics of the problem being considered, several potential features of the approach will not be used in this application. However, the method of their inclusion will be made clear. This particular application seems appropriate for several reasons. Primary among them are the sheer size of the distortion, the fact that its effects have long been debated, and that it has been extensively studied by Harberger (1959, 1962, 1966), Rosenberg (1969) and Mieszkowski (1967) using the earlier technique mentioned above.

The economy considered here may be characterized by (1) a set of n market demand functions, n being the total number of commodities, both inputs and outputs (2) a description of the technological production possibilities of the economy through a listing of feasible activities, and (3) the economy's initial stock of commodities.

Total market demand is the sum of individual demands, each of which may be derived from utility maximization subject to a budget constraint. In this case, of course, information regarding the preferences and initial holdings of individuals is required. A competitive equilibrium for such an economy is characterized by two conditions: (1) supply equals demand in all markets and (2) all activities do no better than break even.

This general equilibrium model is modified to introduce differential taxation across sectors of income accruing to capital. To do this, a fictitious commodity, referred to as "capital tax tickets", is created. The firms in each taxed sector face a given ticket requirement for every unit of capital which they employ. Each consumer may be endowed with a given share of the total number of tickets which can be sold in an organized market. An individual's share of the proceeds equals his share of the tickets. If the revenue is to be equally distributed, then everyone is endowed with an equal number of tickets. These tickets do not enter directly into the individual's utility function and hence are all sold. The government can be given an initial endowment of tickets. Treating the government as a consumer, it can purchase goods and services with the proceeds derived from the sale of these tickets. This creation of an additional commodity offers a general method of dealing with the problem of how to handle the proceeds of a tax introduced into a general equilibrium framework.

2. The algorithm

The algorithm utilized in this paper is due to Scarf² with some modification by Hansen (1968). Its purpose is to compute a general equilibrium solution for a competitive economy, i.e. a price vector, the components of which are the prices of all outputs and inputs, which has the property that for commodities with a non-zero price supply equals demand and for free commodities supply is greater than or equal to demand. In addition, profit at this price vector is less than or equal to zero for all possible production techniques, being equal to zero for those techniques which are utilized. Constant returns to scale in production is assumed.

It is necessary for the working of the algorithm that the market demand functions satisfy Walras' law. This is guaranteed if the demand functions are derived from individual maximization of utility subject to a budget constraint. To show this, let x_{ij} be the demand for commodity i by individual j and let w_{ij} be his corresponding initial endowment of the i th commodity. Individual j 's budget constraint is simply

$$\sum_{i=1}^n P_i x_{ij} = \sum_{i=1}^n P_i w_{ij}, \quad (1)$$

where there are n commodities and P_i is the price of the i th good. Summing these budget constraints over the J individuals we get

$$\sum_{j=1}^J \sum_{i=1}^n P_i x_{ij} = \sum_{j=1}^J \sum_{i=1}^n P_i w_{ij}, \quad (2)$$

which can be written as

$$\sum_{i=1}^n P_i (X_i - W_i) = 0, \quad (3)$$

² For the reader unacquainted with the method, the article by Scarf in: *Essays in honor of Irving Fisher*, and his 1969 *AER* article offer lucid presentations. A more extensive elaboration will soon be available in his forthcoming book, *The computation of economic equilibria*. The exposition presented here is necessarily brief and sketchy but may be adequate for the reader primarily interested in the effects of distortionary taxation of capital income.

where

$$X_i = \sum_{j=1}^J x_{ij} = \text{market demand for commodity } i$$

and

$$W_i = \sum_{j=1}^J w_{ij} = \text{total initial endowment of commodity } i.$$

Eq. (3) is Walras' law, which states that the value of market excess demands is zero.

Production is described by an activity analysis matrix

$$A = \begin{bmatrix} -1 & 0 & \dots & 0 & a_{1,n+1} & \dots & a_{1,m} \\ 0 & -1 & \dots & 0 & a_{2,n+1} & & a_{2,m} \\ . & . & & . & & & . \\ . & . & & . & & & . \\ 0 & 0 & \dots & -1 & a_{n,n+1} & \dots & a_{n,m} \end{bmatrix} \quad (4)$$

in which each column represents a feasible activity. Outputs are given positive and inputs negative coefficients, and each activity can be operated at any non-negative level. The first n columns indicate the feasibility of free disposal of each commodity. It is assumed that the set of non-negative vectors y which satisfy $Ay + W \geq 0$ is bounded. This can be interpreted as implying that the production possibility frontier is finite in all dimensions.

In this notation a competitive equilibrium is defined by a price vector P^* and a vector of activity levels y^* such that

$$(1) \quad X_i(P^*) = W_i + \sum_{j=1}^m a_{ij}y_j^* \quad i = 1, \dots, n \quad (5)$$

$$(2) \quad \sum_{i=1}^n P_i a_{ij} \leq 0 \text{ for all } j, \text{ with equality if } y_j^* > 0.$$

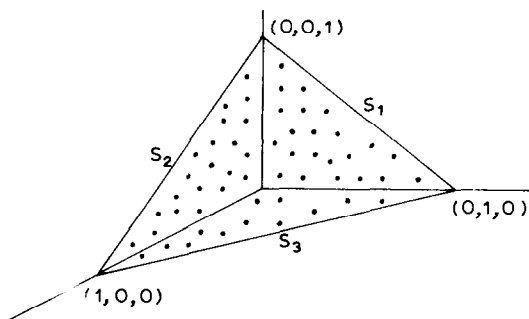


Fig. 1.

X and W are the vectors of market demands and initial holdings, respectively. The proof of existence of such an equilibrium has been thoroughly investigated (see, for example, McKenzie, 1959).

Since the demand functions are homogeneous of degree zero in terms of prices, we may arbitrarily normalize prices to sum to unity. That is, we will consider prices which are on the unit simplex

$$\sum_{i=1}^n P_i = 1 \quad P_i \geq 0. \quad (6)$$

The algorithm is essentially a search procedure on this unit simplex for an approximate equilibrium price vector. The objective is to find a P^* and associated y^* which approximately meet the two above conditions. For this purpose a fine grid of price vectors, P^{n+1}, \dots, P^k , is created on the unit simplex (see fig. 1). In practice k has been as large as 10^{30} . The vectors P^1, \dots, P^n , which represent the sides of the simplex s_1, \dots, s_n , are also created.

We define a B matrix whose columns correspond to the price vectors P^1, \dots, P^k ,

$$B = \begin{bmatrix} P^1 & P^2 & \dots & P^n & P^{n+1} & \dots & P^k \\ 1 & 0 & \dots & 0 & b_{1,n+1} & \dots & b_{1,k} \\ 0 & 1 & \dots & 0 & b_{2,n+1} & \dots & b_{2,k} \\ . & . & . & . & . & . & . \\ . & . & . & . & . & . & . \\ 0 & 0 & \dots & 1 & b_{n,n+1} & \dots & b_{n,k} \end{bmatrix} \quad (7)$$

Each price vector P^i in the list P^1, \dots, P^k is associated with the corresponding column vector b^i by the following rules:

- (1) If any of the elements of P^i are zero, the associated vector contains a 1 in place of the first zero price and 0's elsewhere.
- (2) The profitability of each activity in the technology matrix A is evaluated at prices P^i . Denoting the activity with the largest profit as a^* and its profit as π^* , then

- (i) if $\pi^* > 0$, $b^i = -a^*$
- (ii) if $\pi^* \leq 0$, $b^i = X(P^i)$.

The corresponding column is either a slack vector (the negative of the free disposal activity), the negative of an activity vector, or the column of market demands.

The algorithm always works with a subset of n of the price vectors in the list P^1, \dots, P^k which are referred to as a primitive set and which are "close to each other" in a particular sense. Let a_i be the smallest³ i th component among an arbitrary collection of n vectors, P^{j1}, \dots, P^{jn} , being considered. If there is no price vector P^j among the entire list P^1, \dots, P^k for which

$$P_i^j > a_i \text{ for all } i = 1, \dots, n, \quad (8)$$

then the vectors P^{j1}, \dots, P^{jn} form a primitive set. Thus, a primitive set can be thought of as an $n \times n$ matrix, PS , each column of which refers to a price vector and whose price columns have the above property. A three commodity example is

$$PS = \begin{pmatrix} 24/100 & 25/100 & 25/100 \\ 47/100 & 46/100 & 47/100 \\ 29/100 & 29/100 & 28/100 \end{pmatrix}. \quad (9)$$

The main theorem upon which the algorithm is based is the following:

³ In practice the list of price vectors P^{n+1}, \dots, P^k often consists of all price vectors which can be expressed as $(m_1/D, m_2/D, \dots, m_n/D)$, m_i and D being non-negative integers with $\sum_{i=1}^n m_i = D$. Numeric ties between two components can be broken in any of several consistent ways such as lexicographic ordering.

Theorem. There exists a primitive set P^{j_1}, \dots, P^{j_n} such that

$$By = W$$

has a non-negative solution where $y_j = 0$ for $j \neq j_1, \dots, j_n$.

For those familiar with the notion of a feasible basis from linear programming, this theorem can be restated even more briefly.

Theorem. There exists a primitive set P^{j_1}, \dots, P^{j_n} such that the columns j_1, \dots, j_n form a feasible basis for $By = W$.

For its proof see Scarf (1967a) or (1973). Also it is made clear in these references exactly in what sense the primitive set whose corresponding B columns form a feasible basis for $By = W$ defines an approximate competitive equilibrium. It is shown that at least one of the corresponding columns is a demand column (which indicates that at that particular price vector no production activity makes a profit) while others are the negative of activity columns (indicating that those activities earn positive profits). Since the price vectors of a primitive set are "close together", the profit of all utilized activities must be close to zero, with all unutilized activities having lower profits. The equations $By = W$ can be written as

$$-\sum_j a_{ij} Z_j + \sum_j X_i(P^j) y_j = W_i \text{ for } i = 1, \dots, n, \quad (10)$$

where those columns which are the negative of activities have been separated out and their corresponding weights are now indicated as Z 's. In the two Scarf references he shows that the sum of the weights corresponding to demand columns goes to unity as the grid size approaches infinity. Thus, in the limit (10) becomes

$$X_i(P) = W_i + \sum a_{ij} Z_j \text{ for } i = 1, \dots, n.$$

This, however, is the equilibrium condition that supply equals demand for all commodities with positive prices. Note that the only way the disposal of commodity j can occur is if $P_j \approx 0$.

The algorithm searches in a systematic way for this primitive set

whose corresponding B columns are a feasible basis for $By = W$, starting with a primitive set in one of the corners of the unit simplex and moving across the simplex surface according to fixed rules. The potential usefulness of the algorithm is due to the fact that empirically it finds a primitive set which approximates a competitive equilibrium very rapidly. While in many examples run there have been astronomical numbers of primitive sets defined on the unit simplex, rarely does the algorithm examine more than 1000 of them before terminating with the desired approximation.

As an example of the incorporation of taxes into the activity matrix which must be specified for the working of the algorithm, consider the following simplified illustration. Suppose there are two sectors each of which is adequately described by a Cobb–Douglas production function of the form

$$Q_i = \gamma_i K_i^{\alpha_i} L_i^{1-\alpha_i}; \quad 0 < \alpha_i < 1; \quad i = 1, 2. \quad (11)$$

Only the first sector faces the proposed tax on income accruing to capital. For the parameter values $\gamma_1 = 1.0$, $\gamma_2 = 2.0$, $\alpha_1 = 0.5$, $\alpha_2 = 0.25$ we may approximate each of the unit isoquants by three linear facets whose vertices represent capital–labor ratios of 1 and $1/2$. The A matrix is given by

$$A = \begin{bmatrix} -1 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & -1 & 0 & -1 & -1.414 & -0.5 & -0.842 \\ 0 & 0 & 0 & -1 & -1 & -0.707 & -0.5 & -0.421 \end{bmatrix} \begin{matrix} \text{output 1} \\ \text{output 2} \\ \text{labor} \\ \text{capital} \end{matrix} \quad (12)$$

Taxes are incorporated by introducing the additional commodity, tickets, and by requiring the purchase of one ticket per unit of capital used in the first sector. Thus, if sector one is the taxed sector, the activity matrix in the presence of the tax becomes

$$A = \begin{bmatrix} -1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & -1 & 0 & 0 & -1 & -1.414 & -0.5 & -0.842 \\ 0 & 0 & 0 & -1 & 0 & -1 & -0.707 & -0.5 & -0.421 \\ 0 & 0 & 0 & 0 & -1 & -1 & -0.707 & 0 & 0 \end{bmatrix} \begin{matrix} \text{output 1} \\ \text{output 2} \\ \text{labor} \\ \text{capital} \\ \text{tickets} \end{matrix} \quad (13)$$

Equilibrium prices can thus be determined by the algorithm both with and without tickets (i.e. taxes), and measures of the efficiency loss and incidence may be computed.

In the example used in this paper we have two sectors, the "heavily taxed" sector (predominately corporate) and the "lightly taxed" sector (agriculture, housing, crude oil and gas). The sectoral definitions correspond with those used by Harberger (1959, 1962, 1966) in his analysis of the distortion from the differential taxation of income to capital in the U.S. economy. Each sector's production possibilities are characterized by a constant elasticity of substitution (CES) production function

$$Q_i = \gamma_i [\alpha_i L_i^{-\rho_i} + (1 - \alpha_i) K_i^{-\rho_i}]^{-1/\rho_i} \quad i = 1, 2, \quad (14)$$

with two inputs, labor and capital services. By evaluating (14) for several different values of L_i with a fixed K_i value (or vice versa) given α_i , γ_i , and ρ_i , one could approximate the production function with a relatively small number of activities. One could then normalize the resultant output and inputs by dividing each of them by the output value. The resulting activities would approximate a unit isoquant.

However, when analyzing the effects of a market distortion, it is important to be able to measure such things as small changes in the input ratios of the various sectors. This requires either a very long list of feasible activities or an alteration in the algorithm to incorporate continuous production functions.

The optimal (i.e. cost minimizing) input ratio can be derived as an analytic function of input prices for CES production functions. Thus, for any price vector an optimal activity can be generated for each sector. In computing the corresponding commodity vector for a particular price vector, only the profitability of these optimal activities need be considered rather than a long list of feasible activities. In a two sector example, two cost minimizing unit production activities can be generated for any particular price vector. If the most profitable of these has a positive profit, then the B column associated with that price vector is the negative of the most profitable activity vector. Otherwise, the corresponding column is the column of demands evaluated at the price vector. If the price vector has a zero component, the same rule prevails as before.

Denoting the price of capital and labor as P_K and P_L respectively, the optimal inputs for a unit of output can be determined as

$$K = \left[\alpha \left[\frac{(1-\alpha)P_L}{\alpha P_K} \right]^{\rho/\rho+1} + (1-\alpha) \right]^{1/\rho} \gamma^{-1}. \quad (15)$$

$$L = \left[(1-\alpha) \left[\frac{\alpha P_K}{(1-\alpha)P_L} \right]^{\rho/\rho+1} + \alpha \right]^{1/\rho} \gamma^{-1}. \quad (16)$$

Given usual grid sizes on the unit price simplex, this method permits the consideration of as many as 10^{30} possible activities for each sector. The final approximation to an equilibrium price vector is further refined by a series of termination linear programs as described in the appendix in such a manner that the number of possible activities is limitless. That is, with the method described in this section in conjunction with the termination routine, truly continuous production functions can be handled.

3. Harberger's analysis of the taxation of income from capital

3.1. *The analytic model*

In recent years considerable attention has focused upon the distortions introduced into the operation of the price system in the U.S. economy by the differential tax rates applied to income from capital originating in various sectors of economic activity. Prominent in the literature have been three articles by Harberger (1959, 1962, 1966), in which he makes an empirical distinction between a heavily and lightly taxed sector. These sectors are sometimes referred to as the corporate and non-corporate sectors due to the major role played by the corporation income tax in causing the differential rates, although his sectoral division does not exactly correspond to the legal distinction between incorporated and unincorporated enterprises. He assumes that each sector employs two factors, capital services and labor, in the production of homogeneous outputs.

In order to estimate the efficiency loss due to the differential taxation of the return to capital, Harberger applies a form of welfare analysis in the tradition of Marshallian consumer surplus. His conclusion (Harberger, 1966) is that the efficiency loss from factor misallocations was in the range of 1.75-3.5 billion dollars a year for the period 1953-

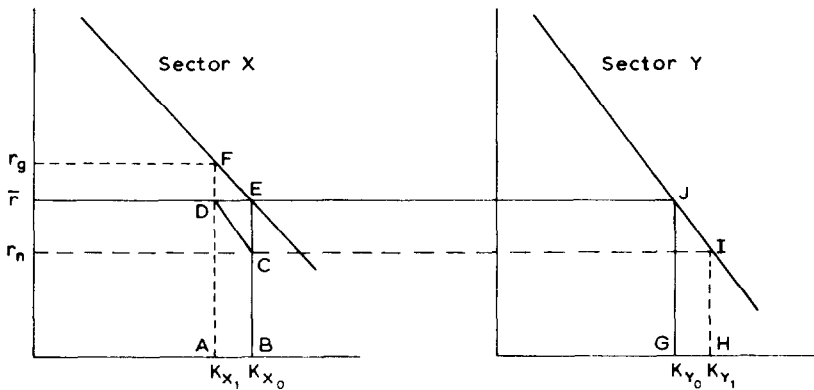


Fig. 2.

1959. In an earlier paper (Harberger, 1962) which focuses mainly on the incidence effects of the corporation income tax he concludes that for "plausible" values of production and demand function parameters capital bears the entire burden of the tax in that its gross share (and, therefore, also labor's share) are the same both in the presence and the absence of the tax.

In the computation of the first of these results, it is assumed that the marginal products of capital schedules for each sector are linear as drawn above. Output units are chosen so that both commodity prices are unity, and it is assumed that all prices other than the price of capital are unaffected by the presence of the differential taxation. Given these assumptions, the changes in the capital allocation can be used to generate a measure of the social waste imposed by the distortion. In the absence of any taxes, capital will allocate itself in a market economy such that the rate of return \bar{r} is equal for the two sectors and the capital endowment will be fully employed. Upon the imposition of a tax on capital income in sector X , the gross rate of return r_g in that sector must be such that the net rate of return r_n is equalized across the sectors and capital is again fully employed. The difference between r_g and r_n is, by definition, the tax T per unit of capital utilized in sector X .

In the graphs in fig. 2, the area $ABEF$ has the interpretation of the loss in output in sector X when K_X decreases from K_{X0} to K_{X1} upon the imposition of the tax. $GHIJ$, analogously, is the increase in output in sector Y . Since we know that capital is fully employed both in the

presence and absence of the tax, it must be true that $K_{X0} - K_{X1} = K_{Y1} - K_{Y0}$. The area *FECD* represents the social loss of the tax in the producer surplus sense (it is simply $ABEF - GHJ$) and is given by

$$\frac{1}{2}(r_g - \bar{r})(K_{X0} - K_{X1}) + \frac{1}{2}(\bar{r} - r_n)(K_{Y1} - K_{Y0}) = \frac{1}{2}T \Delta K_X. \quad (17)$$

The solution for ΔK_X is, in turn, computed by solving a system of equations corresponding to the description of the static two sector general equilibrium model due to Meade (1955) and Johnson (1956). A complete presentation and solution of the model used by Harberger is presented in appendix B. Utilizing the following notation from appendix B,

$$\begin{aligned} E &= \text{price elasticity of demand for } X, \\ S_X(S_Y) &= \text{elasticity of factor substitution in sector } X(Y), \\ f_K(g_K) &= \text{share of capital in sector } X(Y), \\ f_L(g_L) &= \text{share of labor in sector } X(Y), \end{aligned}$$

the solution of his model for the capital shift can be expressed as

$$\Delta K_X = K_X \cdot T \frac{-E \left[g_K S_X \frac{L_X}{L_Y} + f_K S_Y \right] - S_X S_Y f_L}{E(g_K - f_K) \left(\frac{K_X}{K_Y} - \frac{L_X}{L_Y} \right) - S_Y - S_X \left(\frac{f_L K_X}{K_Y} + \frac{f_K L_X}{L_Y} \right)}. \quad (18)$$

Similarly, the system of equations in the second appendix can be solved for the change in the price of capital, ΔP_K , giving

$$\Delta P_K = \frac{E f_K \left(\frac{K_X}{K_Y} - \frac{L_X}{L_Y} \right) + S_X \left(\frac{f_L K_X}{K_Y} + \frac{f_K L_X}{L_Y} \right)}{E(g_K - f_K) \left(\frac{K_X}{K_Y} - \frac{L_X}{L_Y} \right) - S_Y - S_X \left(\frac{f_L K_X}{K_Y} + \frac{f_K L_X}{L_Y} \right)} \cdot T. \quad (19)$$

Harberger uses the solution to (19) to answer the question of the incidence of the distortion. If ΔP_K is equal to $-TK_X/(K_X + K_Y)$, capital is said to bear the full burden of the tax since the gross return to capital is the same in both the presence and the absence of the tax. If $\Delta P_K = 0$, capital and labor are said to bear the burden of the tax equally in the

sense that relative shares are unchanged, since by assumption the price of labor is normalized as unity.

Before examining the empirical work which has been done for the purpose of evaluating (18) and (19) for the U.S. economy, the point emphasized in the introduction regarding the use of local approximations for the analysis of very large distortions should be reiterated. The assumptions of linear marginal product of capital schedules and constant returns to scale production functions are inconsistent for a large distortion such as being considered. The linear marginal product schedules imply a term quadratic in capital services in the production functions.

Also, the explanation of the lack of an income term in the demand functions (that the government spends the tax proceeds in the same manner as the taxpayers would have done) seems somewhat unsatisfactory (for a large distortion). This ignores the efficiency loss of the tax and hence the set of demand functions is not derivable from utility maximization. In addition, Harberger looks at incidence as the effect of the distortion on the functional distribution of income. While this is of interest, in the U.S. economy capitalists work, laborers save, and both to a limited extent exercise a work leisure choice. Thus, at the least the incidence of both the taxation and the expenditure side on the personal distribution would be an additional interesting aspect of the distortion. One defense, of course, of not handling the expenditure side is to argue that what is being compared is the current tax situation to a situation in which capital is taxed in a non-distortionary manner (this is theoretically feasible due to the assumption of a fixed endowment of capital).

The corporation income tax is a tax on the return to equity capital in the corporate sector. In analyzing the effects of this tax, Harberger assumes that its removal would cause no change in the industrial division between the corporate and non-corporate forms of business. In addition, he assumes that the presense of the tax does not affect a sector's debt-equity ratio. These assumptions, which are also necessary for the formulation presented in this paper, are, of course, open to question. One argument in support of the first of these assumptions is the fact that there are few large firms which are closely held and which therefore could easily unincorporate. A partial defense of the second is made possible by appealing to data presented by Tambini (1969). His data shows that over the period 1925–1955 the debt-equity ratio of the corporate sector was relatively stable.

3.2. Empirical application of the model

A number of problems must be dealt with in order to empirically apply the model developed above. A natural sectoral division into a "heavily" and "lightly" taxed sector must be obtained. For this sectoral division some computation must be made of the effective tax rates faced by the two sectors on income from capital. In addition, the other parameters of the system (e.g. elasticities of substitution, price elasticities) must be chosen in such a way as to achieve reasonable consistency with empirical estimates which have been obtained elsewhere.

It may be noted that the institutional workings of any particular tax are always far more complex than can be conveniently incorporated into a general equilibrium framework. Moreover, the simple characterization of taxes as *ad valorem*, constant marginal rate taxes may be questioned for the case examined here. There remains a question as to whether, for instance, the property tax should be viewed as a tax on income from capital or a tax on site rents, or whether the corporation tax may be better treated as a factor tax or tax on residual income accruing to entrepreneurs. These problems while being recognized will not be dealt with here.

The question of the appropriate sectoral division may be considered by examining the approach adopted by Harberger. In his 1959 article, which focuses solely upon the corporation income tax, he asserts that a clear distinction can be drawn between the corporate and the non-corporate sectors. Disaggregating fifty classes of output for 1953–1955 averaged data, the mean figure for total corporation income tax payments as a percentage of the total return to capital is observed as twenty-nine percent. With the exclusion of (1) farms; (2) agricultural services, forestry, and fisheries; (3) real estate; and (4) miscellaneous repair services, the average figure for the remaining 46 sectors rises to 45%, while for the excluded sectors the percentage of their total return to capital going to corporation income tax payments is 1.2, 3.5, 2.3, and 3.6, respectively. The next smallest figure is 19.6% for crude petroleum and natural gas.

In his 1966 paper, focusing on all taxation of income from capital, Harberger produces the table which is here reproduced as table 1. Columns (1) and (2) are drawn from a disaggregated study of Rosenberg (1969). Column (3) is meant to reflect the impact of the personal income tax on income from capital. Appealing to columns (4) and (5), Harberger notes that total taxes on net income in the "corporate" and

Table 1
Taxes on income from capital, by major sectors (annual averages, 1953–1959,
in millions of dollars).

	(1) Total income* from capital	(2) Property and cor- porate income taxes	(3) Other tax adjust- ments	(4) Total tax on income from capital	(5) Net income from capital
"Non-corporate" sector	26,873	6,639	1,724	8,363	18,510
Agriculture	7,481	1,302	927 ^a	2,229	5,252
Housing	18,429	5,140	797 ^b	5,937	12,492
Crude oil and gas	963	197	— ^c	197	766
"Corporate" sector	53,339	22,907	9,945 ^d	32,852	19,547
Total	79,272	29,546	11,669	41,215	38,057

* "Income" (Rosenburg, 1969, p. 125) is defined as income from capital for non-financial industry and includes

(1) Corporate sector net income before corporate profits tax liability and property tax payments.

(2) For the unincorporated sector, the portion of the total income of the unincorporated enterprise that is a return on equity capital, plus property tax payments.

(3) Net monetary interest paid by businesses on borrowed capital in the form of debt obligations.

(4) Net rent paid by an industry to persons for the use of physical capital.

(5) Net realized capital gains by the corporate sector that are considered as income to an industry.

^a Assumes a 15% effective tax on income from capital in agriculture after payment of property and corporate income taxes.

^b Assumes that 70% of income from capital in the housing sector is generated by owner-occupied housing, on which no personal income tax liability is incurred. It is assumed that the remaining 30% of capital income from housing is subjected to a 20% income tax rate after the deduction of property and corporate income taxes incurred.

^c Assumes personal tax offsets on account of oil depletion allowances and similar privileges offsets any taxes on dividends and capital gains in this sector.

^d Assumes a 50% dividend distribution rate, and a "typical" effective tax rate of 40% on dividend income.

(Source: Harberger, 1966, p. 110.)

"non-corporate" sectors average respectively 168% and 45%. Thus, the taxation of income from capital in the United States during this period may be approximated by a general tax of 45% on all net income from capital and an 85% surtax on the net income from capital originating in the heavily taxed sector ($1.45 \times 1.85 = 2.68$).

Upon close examination of the Rosenberg averaged data for the

period 1953–59, the proper division between the heavily taxed sector and the highly taxed sector is less than clear-cut. Taking corporate income tax plus property tax as a percentage of total return to capital (that is, the column (2) entry as a percentage of the column (1) entry) for the industries Harberger includes in the lightly taxed sector gives the following:

Farms	17.19
Agricultural services, forestry and fisheries	28.37
Crude petroleum and gas	20.35
Real estate	27.89

The corresponding figures for some industries included in the heavily taxed sector are

Lumber and wood products	28.69
Petroleum and coal products	25.43
Personal services	27.75
Business services not elsewhere classified	25.10

It would seem that on the basis of the Rosenberg data alternative formulations of the “heavy” and “light” taxed sectors could be justified. If the four sectors above are transferred from the heavily taxed sector, the effective surtax rate increases from 85% to 92.1%. This figure was calculated using Harberger’s assumptions regarding the effects of the personal income tax on the return to capital for these sectors. It seems conceivable that reasonable redefinitions of the two sectors would significantly change Harberger’s estimate of the efficiency cost of the distortion.

While the figures concerning the corporation income tax by industry may be viewed as reliable, being derived from corporate income tax returns, Rosenberg encounters severe difficulties in the determination of both the total return to capital and the property tax payments by sector. Adequate data on some components of the total return to capital such as unrealized capital gains and capital gains in the unincorporated sector are unavailable. An apportionment of the income of a proprietor of an unincorporated enterprise between return to capital and return to labor must, of necessity, be somewhat arbitrary. These and other problems force Rosenberg’s data for column (1) to be only a good approximation of the desired information. Determining the prop-

er assignment of property tax liability by industry is also very difficult and poses a more serious problem. Aggregate property tax payments averaged 11.24 billion dollars per year for this period, and, as such, were of the same order of magnitude as the corporate income tax payments, which averaged 18.306 billion dollars annually.

A number of procedures which lessen the reliability of the assignment are forced upon Rosenberg due to the lack of more appropriate data. Use is made of 1957 census data on assessed valuations to compute property tax revenues by types of property, but the assumption is required that statewide general property tax rates exist. Moreover, for the majority of property tax estimates it is assumed that the average property tax rate for 1957 is applicable to either the entire 1953–1959 period or that it can be used for the 1953–1957 period with a simple adjustment of the 1961 census estimate providing the data for 1958 and 1959. In that property tax revenues increased from 9.4 billion dollars a year in 1953 to 15.0 billion in 1959 and to 18.0 billion in 1961, the assumption of an unchanging rate structure is questionable. One further problem with the property tax is that it is a tax on the value of an asset rather than the flow of returns generated by that asset. Thus, for two assets with similar values at a point in time but with different lifetimes and return streams (assumed constant for the life of the asset), the one with the longer life but lower annual return will bear a higher tax rate on the income from capital. In addition, to the extent that future property tax payments are capitalized into the asset values, the tax may not fall fully on returns to capital generated by these assets.

While columns (1) and (2) of table 1 are derived from detailed statistical considerations of the available data, column (3) is not. The effects of the personal income tax treatment of the income from capital of the various sectors is an important determinant of the distortions in the tax treatment of income from capital, and, therefore, the arbitrariness in the derivation of column (3) is of some concern. Certainly both Harberger and Rosenberg are aware of most, if not all, of these qualifications concerning their data. Many of their assumptions are required by the lack of available information.

For the purpose of choosing an appropriate parameterization of the model Harberger takes as a unit of capital that amount which generates one dollar of net income in either sector. With this definition, column (5) of table 1 states that there were on average 18,510 million units of

capital in the "non-corporate" sector during the 1953–1959 period, while the corresponding figure for the "corporate" sector was 19,547 million units. Thus, $K_Y = 18.51$ billion; $K_X = 19.547$ billion. Likewise, his definition of a unit of labor is that amount which generates one dollar of return to labor (thus, the amount of labor in a sector is the same as the wage bill for that sector). With this definition, Harberger states that the labor allocation for the 1953–1959 period approximated 200 billion units in the heavily taxed sector and 20 billion units for the lightly taxed sector. Thus, $L_Y = 20$ billion units; $L_X = 200$ billion units. Noting that the gross return to capital in the "corporate" sector was about 50 billion dollars out of a total revenue product of approximately 250 billion dollars annually, he takes f_K , capital's share in the "corporate" sector, as 0.2, and, correspondingly, $f_L = 0.8$. In the lightly taxed sector the gross return to capital is about 27 billion dollars per year out of a total revenue product of some 50 billion dollars. Thus, $g_K = 0.54$.

Given that T is taken to be the surtax rate (that is, 0.85) the only additional parametric values needed in order to evaluate the change in the capital allocation given by (18) are E , S_X , and S_Y . In his 1962 article, Harberger assumes that the demand elasticity for the "corporate" sector is $-1/7$, while the elasticity of demand for the "non-corporate" sector is $-6/7$. The figure of $E_X = -1/7$ is also used in his 1966 paper. These figures are derived by using expenditure shares and a value of unity for a term V , referred to as the elasticity of substitution between X and Y .

$$V = \frac{\partial X}{\partial P_X} \frac{P_X}{X} \left[\frac{P_Y Y}{P_X X + P_Y Y} \right].$$

The conclusion that the demand for the product of the "non-corporate" sector (agriculture, housing, and crude oil and gas) is six times as price elastic as all other products on average seems counter-intuitive at best. A much more acceptable assumption (which Harberger makes in his 1959 article) seems to be to take as unity the price elasticity of demand for all products.

Many different estimates have been made of the elasticities of substitution between labor and capital for various sectors, often concluding with contradictory results. Most of the work based on cross section

data suggests that for most two-digit manufacturing industries S is not significantly different from one (see, for example, Solow, 1964 and Minasian, 1961). On the other hand, time series studies yield estimates significantly less than one (see Lucas, 1969). Given this situation, Harberger looks at several combinations of S_X and S_Y .

Using the above data and parameter values, Harberger generates the following table of results concerning the efficiency loss question (Harberger, 1966):

S_X	S_Y	ΔK_X (billions of units)	$-\frac{1}{2}T\Delta K_X$ (\$ billions)
1	1	- 6.9	2.9
1	0.5	- 5.9	2.5
0.5	1	- 5.2	2.2
0.5	0.5	- 4.8	2.0
1	0	- 4.7	2.0
0.5	0	- 3.9	1.7

In addition, he makes some calculations for the assumptions that $V = 1/2$, and therefore $E = -1/14$.

From the above data, the conclusion is drawn that the efficiency loss due to the differential taxation of income from capital was in the range 1.75–3.5 billion dollars per year for the period 1953–1959. Using similar data and parameter considerations, Harberger finds that ΔP_K from equation (19) approximates $-TK_X/(K_X + K_Y)$ and, therefore, that capital bears the full burden of the tax.

4. Results

Recomputations of the efficiency loss and the incidence effects of the differential taxation of income from capital have been performed using the algorithmic approach described above. This method allows a complete comparison of the equilibria both in the presence and the absence of the surtax. Wherever feasible, Harberger's numerical values have been adopted in order to give a reasonable correspondence between the two approaches. Thus, to a limited extent, the validity of the type of local assumptions he uses may be gauged for this particular example. However, as will be made clear, we deviate from his assump-

tions in several respects. Perhaps most importantly, two classes of consumers are considered. This permits to some degree the evaluation of usage effects (the differential impact of the surtax across individuals due to diverse tastes) of the surtax as well as its effect on the personal distribution of income. In several of the examples presented here, the individuals have the ability to exercise a labor-leisure choice. This, too, offers a greater generality than allowed for by Harberger.

On the production side of the economy, each of the two sector's technological production possibilities is characterized by a CES production function such as

$$Q_i = \gamma_i [\alpha_i L_i^{-\rho_i} + (1 - \alpha_i) K_i^{-\rho_i}]^{-1/\rho_i}. \quad (20)$$

Noting the discussion in the previous section of plausible estimates of the elasticity of substitution (in this case $S_i = 1/(1 + \rho_i)$), six cases were considered: two in line with Harberger, and four others. These may be listed as

Case	S_X	S_Y
(1)	1	1
(2)	1	0.5
(3)	1	0.25
(4)	0.75	0.25
(5)	0.5	0.25
(6)	0.25	0.25

Rather than alter the program for the cases where $S_i = 1$ (and, therefore, $\rho_i = 0$) to handle Cobb-Douglas production functions, a CES formulation was used with $\rho_i = 0.01$. The α_i , γ_i were determined by appealing to the relative shares of labor and capital in the two sectors in a manner which will be described later.

Two consumers are considered; the first heuristically represents the upper ten percent of the income recipients, while the second represents the lower ninety percent. The first is endowed with approximately 23% of the economy's labor (corresponding to the observed share of labor income going to the top decile of income receivers) and 40% of total stock of capital. This latter figure roughly corresponds with the share of capital income going to the top 10% of the income receivers although it is much lower than the share of capital income going to the top ten

percent of wealth holders (Projector and Weiss, 1966). Endowing the high income receivers with more than ten percent of the labor appeals to an equal endowment of labor in natural units but a disproportionate endowment in efficiency units.

Each consumer's demand functions are of the form

$$x_{ij} = \frac{a_{ij}I_j}{P_i} \quad i = 1, \dots, n; j = 1, 2, \quad (21)$$

where x_{ij} is consumer j 's demand for commodity i , a_{ij} measures the intensity of his desire for commodity i , P_i is the price of the i th good, and I_j is individual j 's income, given by

$$I_j = \sum_{i=1}^n P_i w_{ij}, \quad (22)$$

where the w_{ij} are his initial holdings.

Such demand functions have price and income elasticities of minus one. This, of course, deviates from Harberger's assumption (commented on in section 3) of price elasticities of $-1/7$ in the "corporate" sector and $-6/7$ in the "non-corporate" sector.

Using the demand functions (21) permits one to impose the observed aggregate 5.35 to 1.0 expenditure ratio for the outputs of the heavily and lightly taxed sectors. Harberger's data reveal that approximately 252 billion dollars a year was spent on "corporate" products, while only 47 billion was spent on the output of the "non-corporate" sector. If each individual's tastes were such that

$$a_{1j} = 5.35a_{2j} \quad j = 1, 2,$$

where "corporate" product is labelled commodity 1 and the "non-corporate" product commodity 2, the expenditure ratios of the model would exactly correspond to the 5.35 to 1 figure. However, in the examples investigated here lower income individuals are taken to place a relatively higher weight on the output of the lightly taxed sector than higher income individuals. This is consistent with observed higher budget shares allocated to food expenditures (agricultural output) by lower income people. The ratios a_{1j}/a_{2j} used are 7.00 for the higher income consumer and 4.90 for the lower income consumer.

The total endowments of the economy are assumed to correspond with Harberger's figures. That is, in the absence of a labor-leisure choice, the labor endowment is taken to be 220 billion units while the capital service endowment is assumed as 38 billion units. In the presence of a labor-leisure choice each individual's endowment of labor is increased by a factor of seven to four. This represents the possibility of working up to a seventy-hour week instead of the more common forty hours. The demand for leisure is then imposed in such a way that each individual supplies the same amount of labor in the presence of the surtax as in the fixed labor supply case. This artificial construction is then used to generate different solutions for cases where distortionary taxes are absent. In the presence of the surtax, 19.5 billion units of the 38 billion units of capital are allocated to the "corporate" sector (from table 1, section 3). The initial endowment of "capital tax tickets" is chosen in such a way that with equal capital and ticket prices the amount of tickets required by the "corporate" sector is 1.68 times the amount of capital in that sector (19.5 billion units). Similarly, the amount of tickets required by the "non-corporate" sector is 0.45 of the amount of capital in that sector. Thus, the observed tax rates of 168 and 45% can be imposed. To focus solely on effects of the tax, individuals are endowed with tickets and capital in the same proportions.

Each of the six sets of elasticity of substitution assumptions was examined with and without a labor-leisure choice, for a total of twelve cases. The procedural method for each case was to first consider the distortionary situation. In this case, Harberger states that the return to labor was approximately ten times the return to capital in the "corporate" sector (\$200 billion vs. \$20 billion) while the two returns were approximately equal in the "non-corporate" sector (\$20 billion for each) for the 1953-1959 period. The aggregate wage bill-net capital return ratio was 5.5. The parameters α_X , γ_X and α_Y , γ_Y were chosen so as to match these observations and such that all equilibrium prices are equal. This equality of prices in the distortionary situation is required for consistency with measurement units. Once these parameters are determined, the distortionary tax (Harberger's 85% surtax in the "corporate" sector) is removed (that is, capital income from each sector is taxed at a flat 45% rate rather than at the two rates 45 and 168%) and the two equilibria are compared.

Table 2 contains a partial description of the distortionary and non-distortionary equilibria for the fixed labor supply case with $S_X = S_Y =$

Table 2
 Sample results; fixed labor supply.
 Assumptions: $S_X = 1$ $S_Y = 1$ $E = 1$.

	Non-distortionary				Distortionary			
(1) Prices	0.9486	1.2299	1.0000	1.4338	1.0000	1.0000	1.0000	1.0000
(2) Tax rate (%)		45.0				168.0		
(3) $(K/Q)_X$		0.0944				0.0774		
(4) $(L/Q)_X$		0.7523				0.7926		
(5) $(K/Q)_Y$		0.3399				0.3731		
(6) $(L/Q)_Y$		0.5231				0.4262		
(7) Total demand for X		265.898				252.307		
(8) Total demand for Y		38.145				46.926		
(9) $P_L L / P_K K$		4.030				5.778		
(10) $P_L L_X / P_K K_X$		5.557				10.255		
(11) $P_L L_Y / P_K K_Y$		1.073				1.077		
(12) $P_X / (P_X X + P_Y Y)$		0.843				0.843		
(13) Individual 1's demands	75.32	8.30	0.00	0.00	71.48	10.21	0.00	0.00
(14) Individual 2's demands	190.58	29.85	0.00	0.00	180.83	36.71	0.00	0.00
(15) Individual 1's rel. share		0.273				0.273		
(16) Individual 2's rel. share		0.727				0.727		
(17) GNP at non-distortionary prices		299.148				297.056		
(18) GNP at distortionary prices		304.044				299.235		
(19) Laspeyres real income index	= 0.99301							
(20) Paasche real income index	= 0.98418							
(21) Shift factor	= 0.0036							

1. When vectors are presented, the components refer to commodities in the order (1) "corporate" output, (2) "non-corporate" output, (3) labor, and (4) capital services. The first row gives commodity prices in the two tax regimes, normalized such that the price of labor is unity. As one would expect, the price of "corporate" output increases while the price of the "non-corporate" output and the net price of capital services decrease with the imposition of the distortionary surtax. This lowering of the price of the lightly taxed output benefits the lower income consumer relatively more than the higher income consumer,

since he spends a larger fraction of his budget on these items. The factor substitution effects of the surtax are given in rows (3)–(6), while rows (7) and (8) show the change in aggregate output of the two sectors. Row (9) gives the aggregate relative share of labor, and, as such, is useful in determining the incidence of the distortion on the functional distribution of income. Rows (10) and (11) give the sectoral disaggregation of labor's relative share. The figure in row (12) represents the fraction of national income spent on the output of the heavily taxed sector and corresponds closely to Harberger's observation of 5/6. The individual relative share information presented in rows (15) and (16) is useful in estimating the impact of the surtax on the personal distribution of income.⁴ Rows (17)–(20), on the other hand, provide statistics concerning the efficiency cost of the distortion. If we let X_1 be the total market demand for "corporate" output and X_2 the total market demand for "non-corporate" output, then row (17) evaluates GNP (i.e. $P_1X_1 + P_2X_2$) at non-distortionary prices (row 1), while row (18) utilizes the distortionary prices. Row (19) gives the Laspeyres real income index of the ratio of real income with the surtax to real income in its absence, while row (20) contains the Paasche index for this ratio. While the true social loss due to the surtax cannot be determined since we do not have or know the social utility function defined over aggregate outputs, it is easily shown (given "reasonable" assumptions) that the Laspeyres index, based on non-distortionary or "initial" prices, provides a lower bound for welfare losses while the Paasche index, based on distortionary or "final" prices, provides an upper bound. Row (21), the "shift factor", is calculated to provide further insight into the incidence of the surtax. The definition of this term is

$$\text{shift factor} = 1 + \frac{\Delta P_K K}{\text{govt. revenue from surtax}} = 1 + \frac{\Delta P_K K}{\Delta(P_T T)}, \quad (23)$$

where ΔP_K is the change in the net price of capital, T is the total endowment of tickets, and P_T is the price of tickets. A shift factor of one implies that $\Delta P_K = 0$ and, given Harberger's definition, capital and labor may be said to bear the burden of the surtax equally. If the shift

⁴ It may be noted that utility evaluations may be calculated both before and after the tax for the purpose of gaining further insight into the incidence questions. However, since the utility function assumed is determinate only up to a monotonic transformation, such welfare comparisons are to a degree arbitrary.

factor is zero, the decrease in the total return to capital, $-\Delta P_K K$, is equal to the government revenue from the surtax, and, in this sense, capital bears the full burden. A negative value of the shift factor would indicate that capital bears more than 100% of the burden of the surtax.

Table 3 contains a summary of the results of the twelve cases examined in this study. The figures in parentheses are those of Harberger for the two cases most comparable to those shown in his 1966 article. Column (4) indicates that the distortionary treatment of income from capital may change the relative share of the top ten percent of the income recipients by a small degree in either direction depending on the assumptions regarding the elasticities of input substitution. The results shown in column (5) indicate the capital shift due to the presence of the surtax and, for the two comparable cases, our estimates of this transfer are somewhat smaller than Harberger's. Columns 6 through 9 indicate the shifts of labor between sectors in response to the tax change. For the cases allowing a labor-leisure choice, changes in the aggregate supply of labor services are observed. Column 10 of table 3 is of interest in that, in six of our twelve cases capital bears more than the full burden of the surtax, while in the remaining six cases, labor shares in the burden. These results contrast strikingly with the conclusion of Krzyzaniak and Musgrave (1963) that the corporate income tax is more than 100% shifted and, hence, capital bears none of the burden. Their analysis, in contrast with that undertaken here, is concerned with short-run shifting in which movement of capital between sectors is excluded. The final three columns of table 3 contain estimates of the efficiency cost of the distortionary tax. The first of these, column (11), is the change in GNP evaluated at non-distortionary prices. Column (12) similarly is the GNP change evaluated at distortionary prices. For the cases where a labor-leisure choice is present, the value of consumption of leisure is excluded from the computation of GNP as is conventional in national income accounting procedures. Thus, in these cases, the welfare interpretation of the changes in this index are tenuous at best. The negative sign in case six, column (11) can be explained by this exclusion of leisure valuation. In this particular case the consumption of leisure decreases by 1.42 million units with the imposition of the surtax. If evaluated at the price of labor this implies an efficiency cost of at least \$1.082 billion for this case. For the two cases for which a comparison is possible, Harberger's point estimates are well within the two extreme values given here. However, it should be noted that in all

Table 3
Summary of results.

Case	(1) S_X	(2) S_Y	(3)	(4) Relative share of rich D/ND	(5) $(K_X)^{ND}$ $-(K_X)^D$	(6) $(L_X)^D$	(7) $(L_Y)^D$	(8) $(L_X)^{ND}$	(9) $(L_Y)^{ND}$	(10) Shift factor	(11) ΔGNP ND prices \$ billion	(12) ΔGNP D prices \$ billion	(13) Harberger's estimate of ΔGNP \$ billion
1	1.0	1.0	Fixed L	0.273/0.273	5.601 (6.9)	200.0	20.0	200.04	19.96	0.0036	2.029	4.809	2.9
2	1.0	1.0	Labor/leisure	0.273/0.273	5.600	200.0	20.0	200.05	19.95	0.0031	2.104	4.823	
3	1.0	0.5	Fixed L	0.274/0.273	4.928	200.0	20.0	202.07	17.93	-0.1227	1.438	4.639	2.5
4	1.0	0.5	Labor/leisure	0.274/0.273	4.934	200.0	20.0	201.22	17.88	-0.1074	0.562	3.714	
5	1.0	0.25	Fixed L	0.275/0.273	4.561	200.0	20.0	203.24	16.75	-0.1958	1.034	4.553	
6	1.0	0.25	Labor/leisure	0.274/0.273	4.576	200.0	20.0	201.88	16.70	-0.1699	-0.338	3.102	
7	0.75	0.25	Fixed L	0.273/0.273	4.249	200.0	20.0	203.06	16.93	-0.0368	1.038	4.068	
8	0.75	0.25	Labor/leisure	0.273/0.273	4.250	200.0	20.0	202.81	16.92	-0.0313	0.795	3.707	
9	0.50	0.25	Fixed L	0.271/0.273	3.710	200.0	20.0	202.77	17.23	0.2138	1.054	3.352	
10	0.50	0.25	Labor/leisure	0.272/0.273	3.726	200.0	20.0	204.20	17.26	0.1746	2.461	4.903	
11	0.25	0.25	Fixed L	0.266/0.273	2.643	200.0	20.0	202.19	17.81	0.6277	0.906	2.122	
12	0.25	0.25	Labor/leisure	0.269/0.273	2.807	200.0	20.0	206.25	17.85	0.4907	4.870	6.604	

ND = non-distortionary case.

D = distortionary case.

cases other than $S_x = S_y = 1.0$ the addition of the possibility of a labor–leisure choice substantially changes the loss estimates.

5. Conclusion

The algorithmic method presented for evaluating the effects of economic distortions seems to offer a great deal of added generality and flexibility over previous methods. For the example presented in this paper, it has proven to be a practical, usable technique. In particular, it would seem that distortions which due to their size or multiplicity can only be inadequately dealt with by the techniques of differential calculus now can be more satisfactorily analyzed at an empirical level. Moreover, in the example presented, while a similarity of results between these general equilibrium calculations and earlier findings is obtained in the specific case of an elasticity of substitution in the neighborhood of unity in each sector, this similarity weakens when other formulations of the production side of the economy are considered. When the added generality of a labor–leisure choice is introduced, even more divergence from earlier calculations is obtained. It would seem that in those areas where policy judgments are to be made on the basis of calculations of distortionary impacts, major attention should be focused upon analyzing the effects with general equilibrium computation techniques such as presented here.

Appendix A⁵

A.1. *Final termination routine*

The algorithm terminates with a final primitive set of n price vectors which are close to each other and approximate an equilibrium price vector. As a first point estimate of an equilibrium vector we can take the center (i.e. the mean) of the n primitive set price vectors. This first approximation may be improved upon by solving a series of linear programming problems.

⁵ While the other sections are a product of joint work, the material of this appendix was developed by John Shoven with valuable guidance from Herbert Scarf and reference to T. Hansen (1968).

Let the final primitive set of the algorithm be given by

$$PS^* = \begin{pmatrix} P_1^{*1} & \dots & P_1^{*n} \\ \vdots & & \vdots \\ P_n^{*1} & \dots & P_n^{*n} \end{pmatrix}, \quad (\text{A.1})$$

where subscripts identify commodities and superscripts price vectors. Given this notation, the first approximation for a competitive equilibrium price vector is given by

$$\begin{aligned} P_1^* &= \sum_{j=1}^n P_1^{*j}/n \\ &\vdots \\ P_n^* &= \sum_{j=1}^n P_n^{*j}/n. \end{aligned} \quad (\text{A.2})$$

With P^* as a point estimate we generate a set of n price vectors surrounding it which together define the region of search for a better approximation. These are given by

$$\begin{aligned} P_1^1 &= \frac{P_1^* - \Delta P_1^*}{D_1} & \dots & P_1^n = \frac{P_1^*}{D_n} \\ P_2^1 &= \frac{P_2^*}{D_1} & \dots & P_2^n = \frac{P_2^*}{D_n} \\ &\vdots & & \vdots \\ P_n^1 &= \frac{P_n^*}{D_1} & \dots & P_n^n = \frac{P_n^* - \Delta P_n^*}{D_n} \end{aligned} \quad (\text{A.3})$$

where Δ is a small number (typically, say, 0.02) and

$$D_j = 1 - \Delta P_j^* / \sum_{i=1}^n P_i^* \quad j = 1, \dots, n. \quad (\text{A.4})$$

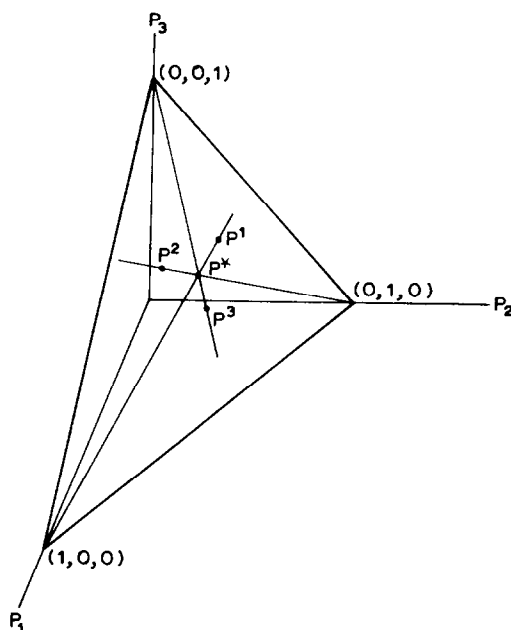


Fig. 3.

P^i can be thought of as a vector arrived at by a movement along the ray connecting P^* and the i th vertex in a direction away from that vertex as is shown for $n = 3$ in fig. 3.

Assuming an interior solution (i.e. no free commodities), we desire a price vector for which demand is equal (or very nearly so) to supply for all commodities and for which the profitability of each sector is zero when it utilizes its optimal technology. Let $f(P) = X(P) - S(P)$ be the vector of excess demands and let $\pi^i(P)$ be the per unit profit of the i th sector when it operates with cost minimizing input proportions. We shall make use of the linearization assumptions

$$f\left(\sum_{j=1}^n \alpha_j P^j\right) \cong \sum_{j=1}^n \alpha_j f(P^j) \quad (\text{A.5})$$

and

$$\pi^i\left(\sum_{j=1}^n \alpha_j P^j\right) \cong \sum_{j=1}^n \alpha_j \pi^i(P^j) \quad \text{for all } i \quad \sum_{j=1}^n \alpha_j = 1 \quad \alpha_j \geq -\delta. \quad (\text{A.6})$$

That is, the excess demand vector for some weighted sum of the price vectors P^j is approximately the weighted sum of the excess demands at those prices, and similarly for the profitability of each sector.

The market demand functions $X(P)$ are assumed continuous and uniquely defined for all positive price vectors P . However, since we have assumed constant returns to scale (CRTS) production technology, the supply response $S(P)$ is less well defined. As is well known, there is a scale indeterminacy with CRTS when profits are zero. One possible supply response would give all of the capital services (a fixed factor) to the sector whose optimal activity is most profitable if that profit is non-negative. If none of the sectors can break even, there would be no production, and the supply, $S(P)$, would simply be the vector of initial holdings, W . This provides such a discontinuous supply response that it and other similar formulations were found unworkable. Linearizing such a response is simply not a good approximation.

We know that the absolute value of the profitability of each sector is quite small in the vicinity of the final primitive set since at least one of the corresponding commodity (B) vectors is a column of demands (i.e. none of the sectors can make a non-negative profit) while at the same time others are the negative of activity vectors, indicating a positive profitability. Therefore, a reasonable supply response operates each of the sectors with its optimal technology and scales their activity levels so as to meet output demand as nearly as possible given the fixed supply of capital services. That is, given a price vector P , the vector of market demands $X(P)$ can be determined. If we let k_i = the optimal capital service input per unit of output in sector i , then the total requirement for capital services is

$$K^* = \sum_{i=1}^{n\text{sect}} X_i k_i \quad \text{where } n\text{sect} = \text{the number of sectors.} \quad (\text{A.7})$$

Each sector i may be allocated capital services in the following manner:

$$K_i = \frac{X_i k_i}{K^*} K, \quad (\text{A.8})$$

where K is the economy's endowment of capital services. This then determines the scale of each sector and results in a continuous supply

response which can reasonably be linearized. Certainly there are other supply responses which may work as well as this one.

In the supply response described above, all of the capital services available are used. That is, the net supply of capital services is zero. Since in this model only outputs and leisure give utility, the demand for capital services is also zero. The excess demand for capital services is by definition zero, and hence we are concerned only with the excess demands of the remaining $n-1$ commodities being small in absolute value. In the linear programming problems referred to earlier we also impose the constraints that the absolute value of the profitability of each sector be "small" in a sense which will be made clear.

The linear programming problem can be formulated as

$$\min \epsilon$$

subject to

$$\begin{aligned} \sum_{j=1}^n \alpha_j f_i^j(P^j) &\leq \epsilon & \text{for } i = 1, \dots, n-1 \\ \sum_{j=1}^n \alpha_j \pi^i(P^j) &\leq \epsilon & \text{for } i = 1, n\text{sect} \\ \sum_{j=1}^n -\alpha_j \pi^i(P^j) &\leq \epsilon & \text{for } i = 1, n\text{sect} \\ \sum_{j=1}^n \alpha_j &= 1, \quad \alpha_j \geq -\delta. \end{aligned} \tag{A.9}$$

That is, the α_j , $j = 1, \dots, n$, are desired which minimize the largest of the linearized excess demands for the $n-1$ commodities other than capital services and the absolute value of the per unit profitability of each sector. This can be transformed into a more manageable problem as will be shown.

Consider the first $n-1$ constraints. Add to each side $\delta \sum_{j=1}^n f_i^j(P^j)$ and then divide by $\epsilon + M$ where M is a positive constant. This gives

$$\sum_{j=1}^n \frac{(\alpha_j + \delta)}{\epsilon + M} f_i^j(P^j) \leq \frac{\epsilon + \delta \sum_{j=1}^n f_i^j(P^j)}{\epsilon + M} \quad i = 1, \dots, n-1. \tag{A.10}$$

Let

$$y_j = \frac{\alpha_j + \delta}{\epsilon + M} \quad (\text{A.11})$$

and note that

$$\sum_{j=1}^n y_j = \frac{1 + n\delta}{\epsilon + M}. \quad (\text{A.12})$$

Since M , δ , and n are given parameters, maximizing $\sum y_j$ is equivalent to minimizing ϵ . The first $n-1$ constraints can be written as

$$\sum_{j=1}^n y_j f_i(P^j) \leq 1 + \frac{\delta \sum_{j=1}^n f_i(P^j) - M}{\epsilon + M} \quad i = 1, \dots, n-1. \quad (\text{A.13})$$

From (A.12) above we know that

$$\frac{1}{\epsilon + M} = \sum_{j=1}^n y_j / (1 + n\delta), \quad (\text{A.14})$$

and thus (A.13) can be written as

$$\sum_{j=1}^n y_j f_i(P^j) \leq 1 + \left(\frac{\delta \sum_{j=1}^n f_i(P^j) - M}{1 + n\delta} \right) \sum_{j=1}^n y_j \quad i = 1, \dots, n-1. \quad (\text{A.15})$$

Let

$$C_i = \frac{\delta \sum_{j=1}^n f_i(P^j) - M}{1 + n\delta} \quad (\text{A.16})$$

and subtract $C_i \sum_{j=1}^n y_j$ from each side of (A.15). The first $n-1$ constraints are then given by

$$\sum_{j=1}^n (f_i(P^j) - C_i) y_j \leq 1 \quad \text{for } i = 1, \dots, n-1. \quad (\text{A.17})$$

The remaining four constraints can be similarly transformed giving

$$\sum_{j=1}^n (\pi^i(P^j) - D_i)y_j \leq 1 \quad \text{for } i = 1, nsect \quad (A.18)$$

and

$$\sum_{j=1}^n (-\pi^i(P^j) - E_i)y_j \leq 1 \quad \text{for } i = 1, nsect \quad (A.19)$$

where

$$D_i = \frac{\delta \sum_{j=1}^n \pi^i(P^j) - M}{1 + n\delta} \quad (A.20)$$

and

$$E_i = \frac{-\delta \sum_{j=1}^n \pi^i(P^j) - M}{1 + n\delta} . \quad (A.21)$$

With the above manipulations, the problem (A.9) has been transformed to

$$\begin{aligned} & \max \quad \sum_{j=1}^n y_j \\ & \text{subject to} \quad \sum_{j=1}^n (f_i(P^j) - C_i)y_j \leq 1 \quad \text{for } i = 1, \dots, n-1 \\ & \quad \quad \quad \sum_{j=1}^n (\pi^i(P^j) - D_i)y_j \leq 1 \quad \text{for } i = 1, nsect \\ & \quad \quad \quad \sum_{j=1}^n (-\pi^i(P^j) - E_i)y_j \leq 1 \quad \text{for } i = 1, nsect \\ & \quad \quad \quad y_j \geq 0 . \end{aligned} \quad (A.22)$$

Given a solution to this linear programming problem, the optimal α 's and ϵ for the original one (A.9) are

$$\hat{\epsilon} = \frac{1 + n\delta}{\sum_{j=1}^n \hat{y}_j} - M$$

and

$$\hat{\alpha}_j = (\hat{\epsilon} + M)\hat{y}_j - \delta \quad j = 1, \dots, n.$$

The new point approximation of an equilibrium price vector is given by

$$P^{**} = \sum_{j=1}^n \hat{\alpha}_j P^j.$$

At P^{**} the excess demands and the profitabilities of the sectors are computed and if they are too large in absolute value a new linear programming problem is set up by defining a set of n price vectors P^j which center around P^{**} . The area of search, defined by Δ in (A.3), is systematically reduced, and as this occurs, the linearity assumptions become more valid. Another estimate of an equilibrium price vector is generated and, again, the process may be repeated. The computational experience we have had indicates that $\hat{\epsilon}$, and, indeed, actual excess demands, rapidly converges towards zero.

Appendix B

B.1. Harberger's model of partial capital taxation

Working with the two sector model of the economy described in section 3, Harberger determines the change in the capital allocation and in the equilibrium price of capital resulting from the imposition of a surtax on the return to capital in one sector (X). He assumes that the demand for each product X and Y depends upon the level of consumer income and on relative prices. However, since Harberger makes the assumption that the government spends the tax revenue in the same manner as consumers would when faced with the existing prices, only relative commodity prices affect aggregate demand (this is a local approximation which ignores the income loss due to the inefficiency of the taxation). Working then with the assumption that the quantity of X demanded depends only on P_X/P_Y , he differentiates this function obtaining

$$\frac{dX}{X} = E \frac{d(P_X/P_Y)}{(P_X/P_Y)}, \quad (\text{B.1})$$

where E is the price elasticity of demand for X . Given that $P_X \equiv P_Y \equiv 1$, a local approximation of (B.1) gives

$$\frac{dX}{X} = E(dP_X - dP_Y). \quad (\text{B.2})$$

The production function of sector X

$$X = F(K_X, L_X) \quad (\text{B.3})$$

is assumed to be continuous, differentiable, and homogeneous of degree one. Taking a total derivative through (B.3) one gets

$$dX = \frac{\partial F(K_X, L_X)}{\partial K_X} dK_X + \frac{\partial F(K_X, L_X)}{\partial L_X} dL_X. \quad (\text{B.4})$$

By dividing both sides by X , this can be written as

$$\frac{dX}{X} = \frac{\frac{\partial F(K_X, L_X)}{\partial K_X} K_X}{X} \frac{dK_X}{K_X} + \frac{\frac{\partial F(K_X, L_X)}{\partial L_X} L_X}{X} \frac{dL_X}{L_X} \quad (\text{B.5})$$

or

$$\frac{dX}{X} = f_K \frac{dK_X}{K_X} + f_L \frac{dL_X}{L_X}, \quad (\text{B.6})$$

where f_K, f_L may be interpreted as the relative factor shares in sector X .

In sector Y we have by the definition of the elasticity of substitution between labor and capital in sector Y , S_Y , that

$$\frac{d(K_Y/L_Y)}{(K_Y/L_Y)} = S_Y \frac{d(P_K/P_L)}{(P_K/P_L)}. \quad (\text{B.7})$$

A local approximation of (B.7) gives

$$\frac{dK_Y}{K_Y} - \frac{dL_Y}{L_Y} = S_Y (dP_K - dP_L) . \quad (\text{B.8})$$

In the above expression (B.8) dP_K is the change in the price of capital relevant for production decisions in sector Y . That is, it is the change in the price of capital net of the tax. For sector X , the relevant change in the price of capital is the gross change, $dP_K + T$. Thus, the equation analogous to (B.8) for sector X is given by

$$\frac{dK_X}{K_X} - \frac{dL_X}{L_X} = S_X (dP_K + T - dP_L) . \quad (\text{B.9})$$

The price of labor is taken to be the *numeraire*, the price in terms of which other prices are expressed, and, as such, is taken to be unity both in the presence and absence of the tax.

$$dP_L = 0 . \quad (\text{B.10})$$

By the assumption of full employment of all factors, the relations

$$dK_Y = -dK_X \quad (\text{B.11})$$

$$dL_Y = -dL_X \quad (\text{B.12})$$

are obtained.

The production function of sector Y

$$Y = G(K_Y, L_Y) \quad (\text{B.13})$$

is also assumed continuous, differentiable, and homogeneous of the first degree. These properties, along with competition in the factor markets, guarantee that factor payments just exhaust revenue, or

$$P_Y Y = P_L L_Y + P_K K_Y . \quad (\text{B.14})$$

Taking a total derivative of each side of (B.14) and appealing to a local approximation gives

$$P_Y dY + Y dP_Y = P_L dL_Y + L_Y dP_L + P_K dK_Y + K_Y dP_K . \quad (\text{B.15})$$

The equation analogous to (B.4) for sector Y is

$$dY = \frac{\partial G(K_Y, L_Y)}{\partial L_Y} dL_Y + \frac{\partial G(K_Y, L_Y)}{\partial K_Y} dK_Y . \quad (\text{B.16})$$

Noting that competition implies that the marginal product of labor in Y is (P_L/P_Y) and that of capital is (P_K/P_Y) , (B.16) may be written as

$$dY = \frac{P_L}{P_Y} dL_Y + \frac{P_K}{P_Y} dK_Y$$

or

$$P_Y dY = P_L dL_Y + P_K dK_Y . \quad (\text{B.17})$$

Subtracting this result from (B.15) gives

$$Y dP_Y = L_Y dP_L + K_Y dP_K . \quad (\text{B.18})$$

Dividing both sides by Y and recalling that the initial prices of both factors and outputs are assumed to be unity, one gets

$$dP_Y = g_L dP_L + g_K dP_K \quad (\text{B.19})$$

where g_L , g_K are the relative factor shares in sector Y . Performing a similar procedure for sector X results in the relation

$$dP_X = f_L dP_L + f_K (dP_K + T) . \quad (\text{B.20})$$

Equations (B.20), (B.19) and (B.10) can be substituted into equation (B.2) giving

$$\frac{dX}{X} = E[f_K (dP_K + T) - g_K dP_K] . \quad (\text{B.21})$$

By similarly substituting (B.20), (B.11) and (B.12) into (B.8) and (B.12) into (B.9) one obtains

$$\frac{K_X(-dK_X)}{K_Y K_X} - \frac{L_X(-dL_X)}{L_Y L_X} = S_Y dP_K \quad (\text{B.22})$$

and

$$\frac{dK_X}{K_X} - \frac{dL_X}{L_X} = S_X (dP_K + T). \quad (\text{B.23})$$

By equating the right hand sides of equations (B.21) and (B.6) and rearranging terms in (B.22) and (B.23) the following system of three equations is derived:

$$\begin{aligned} Ef_K T &= E(g_K - f_K) dP_K + f_L \frac{dL_X}{L_X} + f_K \frac{dK_X}{K_X} \\ 0 &= S_Y \cdot dP_K - \frac{L_X}{L_Y} \frac{dL_X}{L_X} + \frac{K_X}{K_Y} \frac{dK_X}{K_X} \\ S_X T &= -S_X \cdot dP_K - \frac{dL_X}{L_X} + \frac{dK_X}{K_X} \end{aligned} \quad (\text{B.24})$$

The solution for dK_X , which is required in order to evaluate the efficiency loss of the capital income taxation distortion as expressed in the text can be achieved by applying Cramer's rule to (B.24). That is,

$$dK_X = K_X \cdot T \cdot \frac{\begin{vmatrix} E(g_K - f_K) & f_L & Ef_K \\ S_Y & -\frac{L_X}{L_Y} & 0 \\ -S_X & -1 & S_X \end{vmatrix}}{\begin{vmatrix} E(g_K - f_K) & f_L & f_K \\ S_Y & -\frac{L_X}{L_Y} & \frac{K_X}{K_Y} \\ -S_X & -1 & 1 \end{vmatrix}} \quad (\text{B.25})$$

or

$$dK_X = K_X \cdot T \frac{-E \left[g_K S_X \frac{L_X}{L_Y} + f_K S_Y \right] - S_X S_Y f_L}{E(g_K - f_K) \left(\frac{K_X}{K_Y} - \frac{L_X}{L_Y} \right) - S_Y - S_X \left(\frac{f_L K_X}{K_Y} + \frac{f_K L_X}{L_Y} \right)} \quad (\text{B.26})$$

Similarly, the system of equations (B.24) can be solved for dP_K giving

$$dP_K = \frac{Ef_K \left(\frac{K_X}{K_Y} - \frac{L_X}{L_Y} \right) + S_X \left(\frac{f_L K_X}{K_Y} + \frac{f_K L_X}{L_Y} \right)}{E(g_K - f_K) \left(\frac{K_X}{K_Y} - \frac{L_X}{L_Y} \right) - S_Y - S_X \left(\frac{f_L K_X}{K_Y} + \frac{f_K L_X}{L_Y} \right)} T. \quad (\text{B.27})$$

References

- Hansen, T., 1968, On the approximation of a competitive equilibrium (Ph.D. Thesis, Yale University).
- Harberger, A.C., 1959, The corporation income tax: an empirical appraisal. In: Tax revision compendium, (House Committee on Ways and Means, 86 Congress 1 sess., Vol. 1), pp. 231–240.
- Harberger, A.C., 1962, The incidence of the corporation income tax, *Journal of Political Economy* 70, 215–240.
- Harberger, A.C., 1964, The measurement of waste, *American Economic Review Papers and Proceedings*, 54, 58–76.
- Harberger, A.C., 1966, Efficiency Effects of Taxes on Income from Capital. In: M. Kryzaniak (ed.), *Effects of corporation income tax* (Detroit).
- Johnson, H.G., 1956, General equilibrium analysis of excise taxes: comment, *American Economic Review* 46, 151–156.
- Johnson, H.G., 1966, Factor market distortions and the shape of the transformation curve, *Econometrica* 34, 686–695.
- Johnson, H.G. and P.M. Mieszkowski, 1970, The effects of unionization on the distribution of income: a general equilibrium approach, *Quarterly Journal of Economics* 84.
- Krzyzaniak, M. and R.A. Musgrave, 1963, *The shifting of the corporation income tax* (Baltimore, Md., Johns Hopkins Press).
- Lucas, R.E., J.r., 1969, Labor–capital substitution in U.S. manufacturing. In: Arnold C. Harberger and Martin J. Bailey (eds.), *The taxation of income from capital* (Washington, D.C., The Brookings Institution).
- McKenzie, L.W., 1959, On the existence of general equilibrium for a competitive market, *Econometrica* 27, 54–71.
- Meade, J., 1955, *Mathematical supplement to trade and welfare* (Oxford, Oxford University Press).

- Mieszkowski, P.M., 1967, On the theory of tax incidence, *Journal of Political Economy* 75, 250–262.
- Minasian, J.R., 1961, Elasticities of substitution and constant–output demand curves for labor, *Journal of Political Economy* 69, 261–270.
- Projector, D.S. and G.S. Weiss, 1966, Survey of financial characteristics of consumers (Washington, D.C., Federal Reserve Technical Papers).
- Rosenburg, L.G., 1969, Taxation of income from capital, by industry group. In: Arnold C. Harberger and Martin J. Bailey (eds.), *The taxation of income from capital* (Washington, D.C., The Brookings Institution).
- Scarf, H.E., 1967, On the computation of equilibrium prices. In: *Ten economic studies in the tradition of Irving Fisher* (New York, John Wiley and Sons, Inc.).
- Scarf, H.E., 1969, An example of an algorithm for calculating general equilibrium prices, *The American Economic Review* 59, 669–677.
- Scarf, H.E., 1973, The computation of economic equilibria, forthcoming.
- Solow, R.M., 1964, Capital, labor, and income in manufacturing. In: *The behavior of income shares* (NBER, 1964, Princeton, Princeton University Press).
- Tambini, L., 1969, Financial policy and the corporation income tax. In: Arnold C. Harberger and Martin J. Bailey *The taxation of income from capital* (eds.), (Washington, D.C., The Brookings Institution).