A Multi-Sectoral Study of Economic Growth
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A Multi-Sectoral Study of Economic Growth[1]

By EDWARD ZABEL[2]

A study offered as a contribution to an important area of economic research and published under the ægis of a series noted for its contributions warrants a careful evaluation. In this review article such an evaluation is attempted of *A Multi-Sectoral Study of Economic Growth* by Professor Leif Johansen. The research reported in this book had its beginnings at the Institute of Economics, Oslo, in 1957, and was completed at the Department of Applied Economics, Cambridge University, in 1958–59. The study benefited, according to the author, from seminars on the research presented at the Institute of Economics (Oslo), the Department of Applied Economics (Cambridge) and the Netherlands Economic Institute (Rotterdam).

The major objective of the study is to develop a model to explain long-run growth of the sectors of an economy and to estimate the parameters of the model for a particular economy. To focus attention on the sectors, the growth of the economy as a whole is assumed to be determined primarily by exogenous variables. The study, as a consequence, has a limited objective and must be evaluated in this light.

On the empirical side the author uses Norwegian data to estimate parameters of the model, and from estimates of rates of growth of exogenous variables computes the relative rates of change of the sectors of the Norwegian economy with respect to the endogenous variables: labour employed, capital employed, prices and output.

In evaluating the study it is helpful to distinguish between the idea and its specific execution. The idea of the study I find to be a notable contribution to applied economics. I would hope that other economists are encouraged to follow Professor Johansen's lead. The execution, however, is open to some question. The model seems inappropriate in important respects and the peculiarities of the model impair the usefulness of the empirical results. In the following sections these reservations are examined in detail and suggestions for improving the model are offered as well.

I. THE IDEA OF THE STUDY AND THE MODEL OF PRODUCTION

The major idea of the study is to extend the static input-output model to a long-run growth model by introducing a time-dependent variable-coefficient production relation between labour and capital for each productive sector as well as time-dependent prices and demand variables. The appeal of this approach, and what I find to be its major

[1] *A Multi-Sectoral Study of Economic Growth*, by Leif Johansen, North-Holland Publishing Company, Amsterdam, 1960, pp. 172.

[2] The author is particularly indebted to his colleague Lionel McKenzie; he also acknowledges the valuable suggestions of his colleague Ronald Jones.

advantage vis-a-vis the Leontief dynamic input-output system, stems from the explicit introduction of prices, the possibility of substitution between labour and capital, and the admission of technological change in a model which shows promise of being amenable to empirical estimation. With these features and the specification of rules of behaviour for each sector, the model becomes a dynamic general equilibrium system the solution of which yields equilibrium relative rates of change of prices, output, labour and capital for each productive sector in terms of the relative rates of change of the exogenous variables. It is this possibility of empirical implementation of such a general equilibrium system which I judge to be the major contribution of Johansen's study. Since part of Johansen's own empirical work has suffered from defects of his particular model, the promise of his approach has yet to be adequately tested.

More specifically, Johansen's contribution to applied economics revolves about his treatment of production and his empirical work in estimating the structure of demand. Since the model of production concerns the idea of the study, it is examined in this section, the problem of demand estimation being deferred to a later section.

In the productive sectors inputs are classified as intermediate goods used for current production and labour and capital. For intermediate goods the postulates of an input-output model are assumed; specifically, production functions are linear homogeneous with fixed coefficients of production in these inputs. For labour and capital inputs production functions are Cobb-Douglas, with neutral shifts in productivity over time. To complete the model of production a constant proportion between buildings and plants and other fixed capital, as well as constant depreciation rates for capital, is assumed for each sector. With these assumptions all structural changes in production are confined to labour and capital inputs as the productivity and the relative prices of these inputs vary. The fact that substitution between labour and capital may occur without a time lag requires that the model be of a long-run character.

In terms of its application by Johansen to the Norwegian economy, the production model has 22 producing sectors. The first 19 sectors are domestic sectors producing goods as current inputs, domestic consumption, and exports. Sector 20 is the building and construction sector, while sector 21 represents the production of other investment goods. Sector 22 is a residual sector which is unspecified. Sectors 21 and 22 do not have labour and capital inputs since these sectors only collect and allocate goods which are produced by other sectors.

On empirical grounds a compelling advantage of the production model is the feasibility of estimating the parameters. The coefficients of production for intermediate goods are obtainable, directly or indirectly, without great expenditure of research resources, from available input-output data. To estimate coefficients of the Cobb-Douglas functions independent studies are more likely to be needed, but this task is clearly within reason. Varying degrees of sophistication might be used depending on the magnitude of a research undertaking. Similar remarks apply to the estimation of depreciation rates and capital proportions.

On theoretical grounds the major advantages of the model are those noted above: substitution between labour and capital, and shifts in productivity are allowed. The major apparent drawbacks are the familiar objections to the fixed coefficient assumptions of input-output models which, I would judge, have even more force in a long-run equilibrium model such as the present one where the input-output coefficients of intermediate goods are independent of the employment of labour and capital and the relative prices of intermediate goods. Some objection might also be raised to the use of Cobb-Douglas production relations across the board for sectors producing widely differentiated goods and services and using highly disparate methods of production, as exemplified, say, by the wholesale and retail trade sector and the chemical products or basic metals sector. Nevertheless, given limitations on the magnitude of feasible research undertakings as well as shortcomings of data, I would argue that Johansen's production model could become a reasonable and useful approximation of reality, and it is from this base that I would encourage other economists to build upon his work.

II. The General Equilibrium Model

The general equilibrium system consists, mainly, of the production model, assumptions about demand, and rules of behaviour for the sectors. The first part of this section contains a brief exposition of this general equilibrium model, while a more detailed evaluation is given in the latter part of the section.

The final demand structure, the second major part of the model, has elements common to both the open and closed input-output models, i.e., final demand has both endogenous and exogenous components. Endogenous demand, consisting of private domestic demand for final goods, is described by demand functions for domestic goods and non-competitive imports. The demand functions depend on the prices of domestic goods, the price of non-competitive imports, the total consumption of all final goods in a base period, and a variable representing change in population. Exogenous demand equals government demand for domestic goods plus exports minus competitive imports.

In addition to production and demand variables, the remaining variables for each sector are the wage rate, the rate of return on capital including a risk premium, the total pure profit exclusive of the return to capital, and indirect taxes per unit of output. Neither direct taxes nor total income enters the model directly. The variables for the economy as a whole, not already explained, are total net domestic investment and the totals of the sector variables: labour, capital, the components of capital, and imports of non-competitive production goods. A final variable arises from an assumption that the sector rates of return on capital are in constant proportions at all points of time so that the sector rates are proportionate to a variable, defined as unity at a base point of time, serving as an index of the returns on capital. The differences in the proportions presumably reflect the riskiness of investment in the sectors.

Among the set of variables, those specified as exogenous functions of time are the exogenous demands, the shift variables in the Cobb-Douglas functions, the price of non-competitive imports, the total net domestic investment, the variable for change in population, and the economy totals for labour and capital. Variables given exogenously but assumed fixed are the wage rates and indirect taxes. The endogenous variables are listed as the index variable of the returns on capital, total consumption, and for each sector labour, capital, output and the price of output.

To complete the description of the model the rules of behaviour for the economy must be specified. Two rules are stated explicitly by Professor Johansen. The first is that the central government manages the policy parameters, e.g., the exogenous demands, to ensure full employment of labour and capital at all times.[1] The second is that producers behave as if they were maximising profits in conditions of constant input and output prices. The latter assumption means that each producing sector behaves in this manner, since profit-maximising equilibrium conditions for each sector are obtained by Johansen by differentiating a sector profit function with respect to labour and capital.

To obtain a dynamic general equilibrium solution, the next step in the study is to assume the existence of full-employment static equilibrium as the initial position and, by taking time-derivatives of the profit-maximising conditions, production functions, and other equilibrium conditions, to derive a dynamic system which may be solved for equilibrium relative rates of change of the endogenous variables in terms of the relative rates of change of exogenous variables given as functions of time. It is this final set of dynamic equations which Professor Johansen uses to estimate the multi-sectoral long-run growth of an economy. For the Norwegian economy the dynamic system comprises 86 linear equations in 86 endogenous variables and 46 exogenous variables. Matrix inversion of this linear system yields the solution giving endogenous variables in terms of the exogenous variables. (The problem of estimating the structure of the system is discussed in the next section.)

[1] As policy variables, the government demand components of exogenous demands require no extended explanation. Since exogenous demand also includes exports less competitive imports, it might be supposed, following Johansen's discussion, that these components are controlled abroad and domestically by direct import controls and/or tariff and subsidy policies. Of the remaining exogenous variables, the total labour supply is mainly determined by the total population; the total fixed capital and net private investment might be determined by government taxation and subsidy policies; the Cobb-Douglas shift variables could reasonably be assumed as given by the state of technology with or without taxation and subsidy incentives; while, finally, a given price of non-competitive imports would be appropriate for a country whose demands are relatively small on a world-wide scale.

In defence of his system Johansen has argued that the model is formally analagous, with a few exceptions (which are not specified), to a model of perfect competition, and that accordingly it is suitable for application to an economy in which firms are privately owned (hereafter called a free-enterprise economy).[1] In the remainder of this section I argue that, accepting Johansen's interpretation of his variables, the model violates at least one crucial postulate of long-run perfect competition and that, moreover, the type of system which is described by the model seems inadequate as an approximation of a free-enterprise economy such as Norway. In the following section I offer additional evidence to support the latter argument by examining empirical estimates of parameters.

As the basis for evaluating Johansen's model as a competitive system we use the static equilibrium model at the initial point of time. To avoid a tedious exposition we also adopt some simplifications but retain the essential features of the model. The first simplification is to present the model after many, but not all, of the substitutions and eliminations of variables performed by Johansen. Other simplifications are to combine the capital goods sectors, i.e., to assume only one kind of capital, and to eliminate the unspecified sector as well, leaving 20 sectors for the Norwegian model. Finally, while in Johansen's model the profit-maximising assumption is modified for the agriculture and fishing sectors to account for the mixture of wages and pure profits in the returns to owners, we assume the same profit function for each sector.

The Cobb-Douglas production assumptions may now be written as:

$$X_i = A_i N_i^{\gamma_i} K_i^{\beta_i} e^{\epsilon_i t} \; (i = 1, \dots, 20) \quad \dots\dots\dots\dots\dots\dots (1)$$

where for sector i gross output is X_i, N_i is employment, K_i is the stock of fixed capital, the sum of the constants γ_i and β_i is the scale coefficient of the homogeneous Cobb-Douglas function, the constant ϵ_i is the neutral shift in productivity, A_i is a constant, and t is time.

The profit function for each sector is:

$$\Pi_i = P_i^* X_i - W_i N_i - Q_i K_i \; (i = 1, \dots, 20)$$

with total profit as Π_i, W_i as the fixed wage rate, $Q_i = P_{20}(\delta_i + \zeta_i R)$ as the cost of using capital for a period where P_{20} is the price of capital, R is the index of the returns on capital, δ_i is the constant rate of depreciation and (since R is unity at a base point of time), ζ_i is the rate of return on capital in sector i at the base point of time, and

$$P_i^* = P_i - \sum_{j=1}^{20} P_j \alpha_{ji} - P_0 u_i - \theta_i$$

is the "net price" to sector i after subtracting from the price P_i the

[1] The passages supporting this view most directly are given in *op. cit.*, pp. 27 and 170–2. Johansen has also argued that the model may be applicable to a centrally-planned economy with optimal allocation of resources as an objective. Since the argument is not developed at length, and since the study is nearly exclusively concerned with application to a free-enterprise economy, I do not comment directly on this proposition.

cost of intermediate goods and indirect taxes per unit of output. The constants α_{ji} and u_i are the fixed coefficients of production, respectively, for domestic intermediate goods and non-competitive imports used for production, P_0 is the price of non-competitive imports and θ_i measures the constant amount of indirect taxes per unit of output. Under the assumption of constant input and output prices to each producing sector, P_i^* and Q_i are constant to sector i as well as W_i.

We now differentiate the profit function with respect to labour and capital and equate the derivatives with zero to obtain as the sector equilibrium conditions:

$$\gamma_i P_i^* X_i = W_i N_i \quad (i = 1, \ldots, 20) \quad \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

$$\beta_i P_i^* X_i = Q_i K_i \quad (i = 1, \ldots, 20) \quad \ldots\ldots\ldots\ldots\ldots\ldots (3)$$

Since the quantities of each good consumed times the respective prices must sum to the total consumption expenditure on all goods, and the variable representing change in population equals unity at the base point of time, we obtain next:

$$P_0 g_0 + P_1 g_1 + \ldots + P_{19} g_{19} = Y \ldots\ldots\ldots\ldots\ldots\ldots (4)$$

where at the base point of time Y is the total consumption expenditure, $g_0 = g_0(P_0, P_1, \ldots P_{19}, Y)$ is the demand function for non-competitive imports, and $g_i = g_i(P_0, P_1, \ldots P_{19}, Y)$ for $i = 1, \ldots, 19$ is the demand function for the ith domestic good. At any point of time the demand is, respectively, Vg_0 and Vg_i, where V equals population divided by the initial population.

In the static system we also have the following equations, called bookkeeping relations by Johansen:

$$X_i = \sum_{j=1}^{20} \alpha_{ij} X_j + V g_i + Z_i \quad (i = 1, \ldots, 19) \quad \ldots\ldots\ldots\ldots (5)$$

$$X_{20} = \sum_{j=1}^{20} \delta_i K_i + \dot{K} + Z_{20} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (6)$$

where in sector i the exogenous demand is Z_i and in sector 20, the capital goods sector, $\dot{K}$ is total net domestic investment, and Z_{20} is exogenous demand for capital goods.

Finally, we have two equations, called identities:

$$\sum_{j=1}^{20} N_i = N \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (7)$$

$$\sum_{i=1}^{20} K_i = K \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (8)$$

with N and K as the total amounts of labour and capital available in the economy.

Before interpreting the static equilibrium system given in equations (1) to (8), we perform a final elimination of a variable to point out a purely mathematical lapse of Johansen's which obscures the economic meaning of his solution. We may eliminate the variable Y by removing the equation in (4) and substituting for Y in the demand functions g_i in (5). Johansen removed the same equation but neglected to eliminate a variable, thereby retaining Y as endogenous and requiring an otherwise endogenous variable in his model to be exogenous. To replace Y we assume that net investment $\dot{K}$ is endogenous.

The endogenous variables of the remaining 82 equations are the X_i, the K_i, the N_i, R, and $\dot{K}$; the exogenous variables as functions of time are the Z_i, the ϵ_i, N, K, V, and P_0. (By combining the capital goods sectors and removing the unspecified sector we have eliminated four equations and four endogenous variables: X_{21}, X_{22}, P_{21}, and P_{22}.)

To qualify as a long-run competitive system a model should satisfy appropriate necessary and sufficient conditions. One necessary condition is that, for any set of permissible values of parameters of the model, pure profits be zero for any sector producing positive output in long-run equilibrium. In Johansen's model, in particular, the condition must be satisfied for any permissible values of γ_i and β_i of the Cobb-Douglas functions.[1] Suppose for some i the scale coefficient $c_i = \gamma_i + \beta_i$ does not equal unity, and in equilibrium $X_i > 0$ which implies $P_i^* X_i > 0$. Then from the profit function we obtain:

$$1 = \Pi_i / P_i^* X_i + W_i N_i / P_i^* X_i + Q_i K_i / P_i^* X_i$$

and from the sector equilibrium conditions in (2) and (3):

$$c_i = W_i N_i / P_i^* X_i + Q_i K_i / P_i^* X_i$$

so that if c_i does not equal unity, Π_i does not equal zero, violating the necessary condition for a competitive system.[2] In a later section we note that if all $c_i = 1$, then the solution to the model is identical to the solution to a long-run competitive system.

The failure of the model to satisfy competitive conditions is not, of course, sufficient evidence of its failure as a reasonable approximation of reality. Since " reasonable " has many facets, we do not propose to offer such sufficient evidence but instead raise arguments which we think cast considerable doubt on its usefulness. The argument given in this section, using economic interpretations of the equations (1) to (8), is that the model seems inadequate as an approximation of the free-enterprise system. Two other arguments are given, respectively, in

[1] The ranges of γ_i and β_i specified in the model are $0 < \gamma_i < 1$ and $0 < \beta_i < 1$. (*Op. cit.*, p. 34.) We take for granted the existence and uniqueness of non-zero equilibrium outputs for any $0 < \gamma_i < 1$ and $0 < \beta_i < 1$, as does Johansen.

[2] A proviso to this conclusion is that if the sector uses specific factors limited in amount, the variable Π_i may be interpreted as the return to the specific factors with competitive pressures forcing Π_i to the maximum amount subject to firms just making zero profits. Thus, if the coincidence of limited specific factors whenever c_i does not equal unity always obtains, the necessary condition may be satisfied. A revision of the model suggested in Section IV uses a similar notion, allowing but not requiring factors to be specific.

sections III and IV: the first concerns the empirical results of the study, and the second a revision of the model which, we speculate, promises to be more satisfactory.

Economic interpretations of the model hinge on the meaning of the behavioural assumptions. We now give two economic systems consistent with these assumptions. While the model may no doubt satisfy sufficient conditions for still other economic systems, we feel that the interpretations of this section offer evidence that the model departs significantly from a long-run competitive system and, correspondingly, does not appear suitable for a free-enterprise system.

In the first economic system suppose that the sector profit assumption implies that the firms in each sector behave jointly as a profit-maximising monopolist. The equations (1), (2), and (3) then determine profit-maximising labour, capital and output for monopolists for given prices, wage rates and rates of return on capital. Suppose also that the assumption of constant input and output prices to the sectors implies that prices are determined directly by some external agency. In particular, assume that prices are fixed by the central government to clear the markets. The equations (5) and (6) are then the market-clearing equations which, in conjunction with (1), (2), and (3), determine labour, capital, output, and prices for the sectors for given exogenous demands, net investment, and the index variable of the returns on capital and assuming no restrictions on the economy totals of labour and capital. We may view (7) and (8) as the full-employment equations which, with the preceding equations, determine full-employment equilibrium values of net investment $\dot{K}$ and the index R along with the sector variables. We need not belabour the point that such a system is likely to be a poor approximation of a free-enterprise economy.

In the second economic system suppose that the sector profit assumption and the assumption of constant input and output prices imply that each firm in a sector behaves competitively but that the number of firms is given. We may then interpret the profit conditions in (2) and (3) as representing for each sector the summation of firm demands for labour and capital to yield sector demand for labour and capital. On substitution for labour and capital in (1) from (2) and (3), we view (1) as representing the summation of firm marginal costs to give sector marginal cost. From the assumptions that firms behave competitively and that the number of firms is fixed, the sector marginal cost also represents the sector supply function. The equations in (5) and (6) may be interpreted as the economy demand functions for the first 19 commodities and the capital produced at the initial point of time. Equations (7) and (8) may again be seen as full-employment equations. A major difficulty of this system as an approximation of a free-enterprise economy is the absence of entry and exit of firms as a long-run adjusting mechanism. Other difficulties arise for particular values of c_i. Note that in this system the sum c_i depends only on returns to scale for firms. Thus, if $c_i > 1$, some firms at least have increasing returns to scale and, since input prices to firms are fixed, these firms have decreasing long-run average costs which is inconsistent with the assumption of competitive firms.[1]

III. Estimation of Parameters

In this section we examine empirical results of the study, on the one hand pointing to peculiarities of estimating equations and empirical conclusions, and on the other hand arguing that still other estimating procedures may prove to be highly useful in obtaining parameters of a variety of general equilibrium models.

The starting point for the empirical work is the selection of a base point of time as the initial position of general equilibrium with full employment. For the Norwegian economy 1950 is chosen because, in brief, it was a year of nearly full employment and also a year for which an input-output table exists. In estimating the parameters of the production model the coefficients for intermediate goods are obtained by aggregating, interpolating, and otherwise rearranging the input-output data to conform to the framework of the model. The major remaining tasks are to estimate the parameters of the Cobb-Douglas functions, the private domestic demand functions and the rates of return on capital. Attention will be confined to these tasks and some consequences.

For the Cobb-Douglas functions the coefficients γ_i and β_i for sector i are obtained in two steps. One step is to estimate the sum of the coefficients as a measure of the returns to scale of the Cobb-Douglas functions. The second step is to estimate γ_i by the wage-share in sector i in 1950, with β_i obtained by simple subtraction.

Owing to the scarcity of special studies on Cobb-Douglas functions, Professor Johansen felt obliged (except in one case) to rely on intuition in estimating the scale coefficients c_i. For sectors not directly exploiting scarce natural resources, comprising 16 of the 20 sectors using labour and capital, he assumes constant returns to scale. In the remaining four sectors the estimates vary from ·9 to ·66, with only the estimate of ·66 for agriculture, based on a Norwegian study of dairy farms and arable crops farms, not being intuitive.[2] The justification of a wage-share method in estimating γ_i stems from the profit assumption for producing sectors. On solving (2) for γ_i:

$$\gamma_i = W_i N_i / P_i^* X_i$$

[1] Alternatively, following the argument in footnote 2, p. 290, we might assume that each sector uses specific factors fixed in amount and develop a similar interpretation of equations (1) to (8), also subject to similar reservations.

[2] In the Norwegian study of farms labour, capital, and cultivated land were the inputs, and for the two types of farms the estimates of the scale coefficients were 1·06 and ·99. The estimate of ·66 for agriculture was obtained by reducing the scale coefficients by the coefficients for land and taking a weighted average of the remainders. (*Op. cit.*, p. 74.) According to Johansen this procedure requires that cultivated land be fixed in amount or changes in the amount of land somehow be reflected in the productivity coefficient ϵ_i. In either event land is fixed at each moment of time and, consequently, it is assumed to be a limited specific factor. This assumption, however, is arbitrary and may not be warranted.

so that γ_i equals the wage share in sector i. The net output $P_i^*X_i$ is estimated as the sum of wages, non-wages, and depreciation items in the input-output table, and W_iN_i is identified as the wage item after adjusting for imputed wages of owners. Finally, β_i is computed as $\beta_i = c_i - \gamma_i$.

Such a method of obtaining γ_i and β_i seems innocent enough; but taking a final step, not explicitly noted by Johansen, the method also implies, from (2) and (3) and the profit function,

$$1 - c_i = \Pi_i / P_i^* X_i$$

so that in long-run equilibrium in sector i the profit-share equals one minus the scale coefficient of the Cobb-Douglas function. This result, arising from the fact that units of labour and capital are paid the value of their marginal products with the remainder of net output $P_i^*X_i$ going to profits (or to specific factors), influences the computation of other coefficients as well as γ_i and β_i, and while such a measure of profits might be reasonable for a competitive firm with a homogeneous production function, it would seem questionable in general for sectors in long-run equilibrium in a free-enterprise system.[1] While it would be difficult to test the reasonableness of the estimates of γ_i and β_i, a seemingly anomalous consequence of computing profits by this method may be noted in the estimation of ζ_i.

To aid in evaluating the estimates of ζ_i, recall that ζ_i is used in computing Q_i, the cost per period of using capital in sector i. At the base point of time $Q_i = P_{20}(\delta_i + \zeta_i)$ where, again, P_{20} is the price of capital, δ_i is the rate of depreciation of capital, and (to be meaningful in the cost equation), ζ_i is the opportunity-cost rate of return on capital per period reflecting earnings foregone by investing in sector i.

To compute ζ_i Johansen identifies the non-wage item F_i in the input-output table, after adjusting for wages of owners, as $F_i = \Pi_i + \zeta_i RK_i$. Since R is assumed equal to one at the initial point of time:

$$\zeta_i = \frac{F_i - \Pi_i}{K_i} = \frac{F_i - (1 - c_i) P_i^* X_i}{K_i}.$$

Some peculiarities of this estimating procedure are apparent immediately. For a sector with strong decreasing returns to scale the estimate of ζ_i may well be negative, or, if returns to scale are increasing, the numerator exceeds the non-wage item F_i with the possibility of a very large ζ_i. Since, however, the estimates of c_i for the Norwegian model cluster around one, and since constant returns to scale is perhaps the most favourable case for the computing method, we confine discussion of empirical estimates of ζ_i to sectors with c_i equal to one.

Of the 16 sectors with constant returns to scale, 10 sectors have a computed rate of return of over 20 per cent., 6 sectors of over 30 per

[1] As a peculiarity of the system note also that if $c_i > 1$, which is possible in applications even though in the Norwegian model all $c_i \leq 1$, then $\Pi_i < 0$. It is easy to see that negative pure profits in this case arise since the labour and capital obtained from (2) and (3) for any positive P_i^*, Q_i and W_i violate higher-order conditions for profit-maximisation.

cent., 4 sectors of over 40 per cent., the forestry sector has a computed rate of return of 80 per cent. and the wholesale and retail trade sector of 220 per cent. Of the remaining sectors with constant returns to scale the land transport sector has a rate of return of -2 per cent.[1] Such estimates demand explanation, and Johansen offers some good arguments to explain the figures, one being that they reflect interest on working capital (a component which is neglected in the measure of capital, causing a tendency to underestimate the denominator of the estimating equation). More generally, since the entire non-wage item F_i, after the adjustment noted above, is imputed as the return to capital, the estimates reflect all payments in this category such as short-run profits, long-run profits in non-competitive sectors, rents and quasi-rents. It would seem that attempts to use the method relentlessly could lead to gross distortions in measuring relative rates of growth of the sectors.[2]

Turning to the estimation of the demand structure, it is here that possibly useful estimating procedures are to be found. Since the major objections to the procedures are readily apparent and, with one exception, well recognised by Johansen, comments on this part of the empirical work are limited, mainly, to brief remarks of an expository nature. To complete the estimation of the parameters of the model, partial derivatives of the demand functions with respect to total consumption expenditure and prices are needed. Such a problem might appear insoluble for a research programme of limited resources, such as Johansen's since the available data are limited mainly to the effects of changes in income and of changes in own-prices for some goods. Fortunately, a computing procedure devised by Ragnar Frisch is available to accomplish this task, although highly restrictive assumptions about consumer behaviour are implied by his method.[3]

One major simplifying assumption, noted by Johansen, is that of independent utilities, i.e., the utility of a good is independent of the consumption of other goods. A second major assumption, overlooked by Johansen, is that of the additivity of individual utilities, or the validity of interpersonal comparisons of utility. With these assumptions and with data on the partial derivatives with respect to total expenditure and one other variable, say, the own-price, the computing procedure yields the partial derivatives with respect to price by simple formulæ.

[1] *Op. cit.*, p. 81, table 5·3; 5.

[2] In the experiment performed by Johansen comparing computed relative rates of growth of sectors with observed growth, it appears that in general the results are neither very good nor very bad. (*Op. cit.*, pp. 151–8.) In interpreting these empirical findings it should be recalled that (1) the total consumption Y has erroneously been included as an endogenous variable and (2) since most estimates of c_i are unity, the solution otherwise is close to a perfectly competitive one. Finally, I would judge that if the rent variable suggested in the following section had been part of the model, the estimates of ζ_i in particular would have been more reasonable and the computed rates of growth closer to the observed ones.

[3] Ragnar Frisch, " A Complete Scheme for Computing All Direct and Cross Demand Elasticities in a Model with Many Sectors ", *Econometrica*, April, 1959, pp. 177–96. Johansen's application is the first attempt, after modifications required by his study, to use this method in empirical work.

On the basis of independent checks performed by Johansen, the computed results appear rather good, despite the simple assumptions about consumer behaviour; and I would judge the method to be a promising one in lieu of more direct and more complex estimates of coefficients of demand functions.[1]

IV. A Revision of the Model

The objective of this section is to suggest a revised model which satisfies conditions of perfect competition and provides computing procedures free from the criticisms advanced in section III. Only the simplest revisions consistent with this objective are proposed. In particular, since the behavioural assumptions are the locus of objection to Johansen's model, modification of these assumptions is the basis for suggested changes.

Assume that a sector long-run equilibrium condition is the equality of market price and long-run minimum average cost. In lieu of the profit assumption and constant input and output prices to the sectors, suppose input prices only are constant to the sectors and each sector minimises the total cost of output. To account for factor payments in each sector which cannot properly be imputed to labour, capital or intermediate goods, introduce a variable, which we shall call *rent*, and designate T_i as the total amount of rent in sector i at the base point of time. Rent is then a composite variable, and other revisions might well be suggested in which components of T_i, such as short-run profits reflected in the data or rents to specific factors, are introduced separately. As noted later, however, even to estimate the composite variable T_i requires data not contained in input-output tables, and attempts to decompose T_i would necessitate still more intensive investigation of individual sectors. We do not propose to discuss these problems in detail here.

To derive minimum costs we need an hypothesis for rent. The composite nature of rent makes it difficult to suggest a reasonable hypothesis, and we adopt the expedient of assuming, simply, that total rent is a linear function of output, say, $T_i = t_i X_i$ for sector i.

The total cost for sector i may now be written as:

$$TC_i = W_i N_i + Q_i K_i + T_i + P_i^{**} X_i \quad (i = 1, \ldots, 20) \ldots\ldots (9)$$

where

$$P_i^{**} = \sum_{j=1}^{20} P_j \alpha_{ji} + P_0 u_i + \theta.$$

Minimising (9) subject to an output constraint specified by the production function in (1), obtain as the minimum cost "expansion path" for labour and capital inputs:

$$\frac{W_i N_i}{Q_i K_i} - \frac{\gamma_i}{\beta_i} = 0 \quad (i = 1, \ldots, 20) \ldots\ldots\ldots\ldots\ldots\ldots\ldots (10)$$

[1] The genesis of the computing method dates back to the 1930's. For a critical discussion of this early work, cf. Paul A. Samuelson, *Foundations of Economic Analysis*, Cambridge, Mass., 1947, pp. 173–83.

To obtain minimum total cost, substitute for N_i and K_i in (9) from (1) and (10) so that:

$$\min TC_i = c_i\left(\frac{W_i}{\gamma_i}\right)^{\gamma_i/c_i}\left(\frac{Q_i}{\beta_i}\right)^{B_i/c_i}\left(\frac{X_i}{A_i e^{\epsilon_i t}}\right)^{1/c_i} + T_i + P_i^{**}X_i \ldots (11)$$

Finally, dividing (11) by X_i to derive minimum average cost and substituting $P_i^* = P_i - P_i^{**}$, obtain as a sector long-run equilibrium condition, which is also a zero profit condition:

$$P_i^* = c_i\left(\frac{W_i}{\gamma_i}\right)^{\gamma_i/c_i}\left(\frac{Q_i}{\beta_i}\right)^{\beta_i/c_i}\left(\frac{X_i^{1-c_i}}{A_i e^{\epsilon_i t}}\right)^{1/c_i} + t_i \quad (i = 1, \ldots, 20) \ldots (12)$$

Equations (10) and (12) now replace (2) and (3) as sector equilibrium conditions.

Retaining other parts of Johansen's model, the competitive equilibrium model consists of the equations in (1), (10), (12), (5), (6), (7), and (8). We may view (1) and (10) as cost-minimising conditions which determine labour and capital for given output, wage rates, price of capital and index of the returns on capital. The equations in (12) we may represent as the supply functions and the equations in (5) and (6) as the demand functions of the sectors. The equations (7) and (8) we may again interpret as full-employment conditions.[1]

In comparing the models, observe also that conditions (10) may be obtained by dividing conditions (2) by (3). Recalling that, in minimisation, (10) is actually obtained by a similar process of division which serves to eliminate Lagrangean multipliers, and that division of (2) by (3) eliminates the net prices P_i^*, we may interpret (2) and (3) as representing the conditions (10) and also equating the net prices with the Lagrangean multipliers, which may be similarly viewed as net marginal costs. Neglecting rents, we then see that the two models yield the same solutions if and only if marginal cost equals average cost in each sector or, in other words, the sectors all have constant returns to scale.

The revised model implies modifications of computing equations as well. In estimating γ_i and β_i a two-step procedure may be used, but a wage-share method for γ_i is not available in general. To estimate γ_i use (10) and $c_i = \gamma_i + \beta_i$ to obtain:

$$\gamma_i = c_i W_i N_i / (Q_i K_i + W_i N_i) \quad (i = 1, \ldots, 20) \quad \ldots\ldots\ldots (13)$$

From (9) and the zero profit equilibrium condition, the total return, or net product, of sector i is:

$$P_i^* X_i = W_i N_i + Q_i K_i + T_i \, (i = 1, \ldots, 20) \ldots\ldots\ldots\ldots (14)$$

From (13) and (14) observe that a wage-share method for γ_i is correct only in sectors where the scale coefficient c_i is unity and total rent is zero. An estimate of γ_i is readily achieved also in any sector having total rent of zero. I would judge, however, that while an estimate of

[1] Note that in the revised model the returns to scale of the Cobb-Douglas functions are to be explained by non-pecuniary external economies or diseconomies.

constant returns to scale might occur frequently, as in the Norwegian model, an estimate of zero rent is unlikely since the rent variable reflects not only rents or quasi-rents but short-run profits imbedded in the data and monopolistic returns in sectors where the assumption of perfect competition is not wholly satisfactory. In the absence of zero rents and as the non-wage item in the input-output data combines rent items and returns to capital, an independent estimate of either Q_iK_i or T_i must be made to obtain γ_i from (13).

To estimate the rate of return on capital ζ_i let $F_i = T_i + \zeta_i RK_i$ and $R = 1$ at the base point of time. From these two equations:

$$\zeta_i = (F_i - T_i)/K_i \qquad (i = 1, \ldots, 20) \ldots\ldots\ldots\ldots\ldots\ldots (15)$$

The remarks above apply here as well: an independent estimate of either Q_iK_i or T_i is required to estimate ζ_i from (15).

Since the structure of demand is unchanged, the procedures proposed by Johansen to estimate demand coefficients may also prove useful for the revised model. The modifications of the dynamic model implied by the revised static model are obtained in a straightforward but tedious manner and are detailed in the Appendix.

Finally, I wish to emphasise again that the revised model is not meant to be optimal or best in any sense: the intention was merely to satisfy minimally the major objections to Johansen's model raised in previous sections. In particular, the assumption that rents are linear functions of output is questionable and other ways of introducing rents might well be more satisfactory. Some reservations noted earlier, for example, those concerning the production assumptions, also apply equally to the revised model, and yet other objections to both models are reasonably evident and require no extended discussion. For example, the decision to omit income and some of its components causes the submergence of possibly important relationships in a study of sector growth. An explicit and perhaps peculiar manifestation of this omission is that demands depend on consumption rather than income. Still another reservation arises in the treatment of international trade. Important ways in which trade effects growth, for example, exchange-rate policy and balance-of-payments constraints, are simply neglected. It is possible that the model may be extended in several directions without vastly increasing the task of estimating coefficients.

V. Conclusion

Though a number of criticisms of Johansen's study have been suggested in this review, it is important to recognise that his work is a valuable contribution to applied economics. The shortcomings of his particular model are not at all damaging to the idea of the study and some of his estimating procedures have value in themselves. A revision of the model, such as the one suggested in section IV, may well lead to a realisation of his objectives.

E

APPENDIX

The purpose of the Appendix is to present the dynamic equations for the revised model. These dynamic equations differ from those of the Johansen study for three reasons: (*a*) the simplifications incorporated into the revised model for expository purposes; (*b*) the substitution of conditions (10) and (12) for (2) and (3); and (*c*) the introduction of net investment $\dot{K}$ as an endogenous variable.

The dynamic equations are obtained by differentiation with respect to time of the equations of the revised model and various simplifications and substitutions of the type used by Johansen. We present only the final equations: in most cases derivations which are not evident may be followed in the Johansen study.

From equations (1) we obtain:

$$\gamma_i n_i + \beta_i k_i - x_i = -\epsilon_i \quad (i=1, \ldots, 20) \ldots\ldots\ldots\ldots (1a)$$

where n_i, k_i, and x_i are the relative rates of change of N_i, K_i, and X_i.

From equations (10) and assuming P_{20}, as well as R, equals unity at the base point of time:

$$n_i - k_i - \left(\frac{\delta_i + \zeta_i}{Q_i}\right) p_{20} - \frac{\zeta_i}{Q_i} r = 0 \quad (i=1, \ldots, 20) \ldots\ldots\ldots (10a)$$

with p_{20} and r as the relative rates of change of P_{20} and R.

From equations (12) and assuming P_0 and all P_i equal unity at the base point of time:

$$-\frac{(1-c_i)}{c_i} x_i + \sum_{j=1}^{20} \frac{A_{ji}}{(P_i^* - t_i)} p_j = -\frac{\epsilon_i}{c_i} + \frac{u_i}{(P_i^* - t_i)} p_0 \quad (i=1, \ldots, 20) \ldots (12a)$$

where p_j and p_0 are the relative rates of change of P_j and P_0 and $A_{ij} = e_{ij} - \alpha_{ij}$, for $e_{ij}=1$ if $i=j$, and $e_{ij}=0$ if $i \neq j$.

Since equation (4) cannot actually be solved for Y unless we know the explicit form of the demand functions, we reintroduce (4) into the dynamic system and retain Y as an endogenous variable to obtain:

$$\left[1 - \sum_{j=0}^{19} G_j\right] Yy - \sum_{j=1}^{19} \left[g_j + \sum_{i=0}^{19} g_{ij}\right] p_j = \left[g_0 + \sum_{i=0}^{19} g_{i0}\right] p_0 \ldots (4a)$$

where y is the relative rate of change of Y, G_j is the partial derivative of the jth demand function with respect to consumption Y, and g_{ij} is the partial derivative of the ith demand function with respect to the jth price.

From equations (5) we derive:

$$X_i x_i - \sum_{j=1}^{20} X_{ij} x_j - \sum_{j=1}^{19} g_{ij} p_j - G_i Y y = g_i v + g_{i0} p_0 + z_i \quad (i=1, \ldots, 19) \ldots\ldots (5a)$$

where $X_{ij} = \alpha_{ij} X_j$, the variable v is the relative rate of change of V, and z_i is the rate of change of Z_i.

From equation (6) we obtain a similar equation:

$$X_{20} x_{20} - \sum_{j=1}^{20} \delta_i K_i k_i - \ddot{K} = z_{20} \ldots\ldots\ldots\ldots (6a)$$

where $\ddot{K}$ is the rate of change of net investment K and z_{20} is the rate of change of Z_{20}.

From equations (7) and (8):

$$\sum_{j=1}^{20} \frac{N_j}{N} n_j = n \qquad (7a)$$

$$\sum_{j=1}^{20} \frac{K_j}{K} k_j = k \qquad (8a)$$

with n and k as the relative rates of change of N and K.

Apart from simplifications, the changes in the dynamic system are confined to (12a) which replaces a corresponding set of equations in the Johansen model and (4a) which is an added equation. Also, in (6a) the variable $\ddot{K}$ is to be interpreted as endogenous rather than exogenous. The set of 83 endogenous variables for the dynamic system consists of the p_i, the x_i, the n_i, the k_i, r, y, and $\ddot{K}$.

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