ELSEVIER

Economic Modelling 15 (1998) 543–573

Economic Modelling

The econometric critique of computable general equilibrium modeling: the role of functional forms

Ross R. McKitrick*

Department of Economics, The University of Guelph, Guelph, Ontario, Canada N1G 1M8

Accepted 31 January 1998

Abstract

Computable general equilibrium (CGE) models are among the most influential tools in applied economics. However, some serious questions have been raised about the empirical validity of these models. The core of the critique is that the parameter selection criteria are unsound and the use of first-order (CES class) functional forms imposes influential restrictions on the model's structure. A formal summary of the case against standard CGE modeling is presented, as is an alternative econometric-based modeling strategy which answers the critique. We then present a comparative CGE modeling experiment designed to assess the role of function forms. It is found that choice of functional forms affects not only industry-specific results, but aggregate results as well, even for small policy shocks. © 1998 Elsevier Science B.V. All rights reserved.

JEL classifications: C51, D58

Keywords: Computable general equilibrium; Econometric modeling; Flexible functional forms; Comparative modeling

1. Introduction

This paper revisits the debate over whether CGE models should be constructed using calibration or econometric methods. A compelling case has been made in favour of the latter (see review below), but even many years after the initial

* Tel.: +1 519 8244120, ext. 2532; e-mail: rmckit@css.uoguelph.ca

0264-9993/98/$19.00
PII S0264-9993(98)00028-5

literature, the calibration method is still almost always used. In some cases this can be attributed to constraints on data availability, but many calibrated CGE models are constructed for countries and regions where the data exist to support econometric CGE modeling. The question motivating this paper is whether the differences in technique actually matter, i.e. whether the deviations from standard econometric practice necessitated by the calibration method actually affect a model's performance. We focus here on the specific issue of functional form choice, and show that the answer is clearly in the affirmative. Considering the importance of CGE modeling in empirical applications and policy analysis, this result indicates that the econometric critique of the calibration method remains a challenge to practitioners which merits renewed attention.

The debate at hand is as follows. In the calibration method, some parameters are determined on the basis of a survey of empirical literature, some are chosen arbitrarily, and the remainder are set at values which force the model to replicate the data of a chosen benchmark year (Shoven and Whalley, 1992). This approach has been criticized by, among others, Jorgensen (1984), Lau (1984), Jorgensen et al. (1992), and Diewert and Lawrence (1994), on several grounds. First, researchers often use elasticities estimated for commodity and/or industry classifications which are inconsistent with those maintained in the model, and/or for countries other than the one(s) represented by the model, and/or obsolete estimates from past literature, not to mention outright guesses when no published figures are available. These expediencies detract from the ability of the model to represent the technology and tastes of the economy under study. Also, users of the simulation results have virtually no way to assess the evidence supporting the choice of most parameter values.

Second, the calibration procedure causes the quality of the model to be at least partly dependent on the quality of the data for an arbitrarily chosen benchmark year. Since there are always stochastic anomalies and extraordinary economic events associated with any one year of a time series, this will detract from the validity of generalizations drawn from the model. In addition, the data matrices often go through various scaling processes to force micro-consistency, introducing untraceable biases into the rows and columns. These errors and biases will directly influence the parameters of a calibrated model.

Third, the calibration approach tends to limit the researcher to the use of 'first order' functional forms (those in the Constant Elasticity of Substitution (CES) class), all of which embody restrictive assumptions about the structure of the industries being modeled, by imposing a single non-negative substitution elasticity across all pairs of goods in the aggregator. As Lau (1984) points out, a calibrated model can only determine an average of one unknown parameter per equation. A preferred alternative would be to use flexible functional forms, such as the translog or normalized quadratic, which have enough free parameters to provide a second-order approximation to any underlying preference or technology aggregator function, and consequently can represent all the relevant own- and cross-price elasticities derived from an arbitrary utility or profit function, without imposing prior constraints. Diewert and Wales (1987) show that the Generalized McFadden (or normalized quadratic) function retains this flexibility property even if restrictions must be imposed to ensure proper curvature. Use of flexible functional forms has recently been tried in the calibration approach (see Perroni and Rutherford, 1995), but implementation requires prior knowledge of all cross-price elasticities at the benchmark point, plus knowledge of how the elasticities evolve in the space surrounding the benchmark point, information which is generally unavailable.

I call this literature the 'econometric critique' of CGE modeling. It can be summarized with a simple taxonomy. An empirical economic model, such as a CGE model, embodies three types of information: analytical, functional and numerical. The analytical structure is the background theoretical material which identifies the variables of interest and posits their causal relations. The functional structure is the mathematical representation of the analytical material, and consists of the algebraic equations which make up the actual model. The numerical structure consists of the signs and magnitudes of the coefficients in the equations which form the functional structure. The econometric critique of CGE modeling is not directed at the analytical structure of these models, which is, in the main, the neoclassical canon. But it calls into serious question the functional and numerical structures of calibrated CGE models.

The question needing to be answered now is: does the Walrasian general equilibrium framework impose enough constraints on the range of possible results from a CGE policy experiment to render the choice of functional forms and parameter values irrelevant? Unless this is so, the econometric critique must be considered a serious challenge to practitioners in the field, as it claims there are strong prior reasons for doubting the validity of the functional and numerical structures of many CGE models in current use.

This paper provides the first systematic attempt to address this question, focusing specifically on the issue of functional form choice.[1] We compare the performance of two CGE models whose differences reflect the innovations in functional structures called for by the econometric critique. Other authors have compared the performance of various independently-constructed models to an identical policy shock (e.g. the carbon tax survey of Dean and Hoeller, 1992), but the aim has been to increase the information base on which to assess the economic response to a policy option, and the differences in such models go well beyond functional forms and parameter estimation criteria. On the role of the numerical structure, most CGE simulations report sensitivity analyses to gauge the robustness of the results to certain key parameter values. This has obvious importance in increasing the reader's understanding of the robustness of the results, although no more than two or three parameters (out of possibly hundreds) are typically varied, and the

[1] Moreover, we focus on the classical Scarf-Shoven-Whalley static CGE framework, although the econometric critique asks pertinent questions about the feasibility of dynamic modeling in calibration settings as well. It should also be noted that the present exercise considers the role of econometric techniques only in the area of model construction, rather than testing, on which see Hansen and Heckman (1996). I am grateful to a referee for this point.

calibration procedure often leaves one with little guidance as to what would be the most preferred parameter value in cases of non-robustness.

As far as the author is aware, there has never been an attempt to compare the results from the same model on the same policy shock after being revised along the grounds called for by the econometric critique, to assess the role of the entire functional structure.[2] It will be demonstrated that moving from CES-class functions to flexible functional forms changes the performance of a CGE model quite noticeably: in fact the two models seem to be entirely distinct descriptions of the economy. Hence it can be concluded that the preference among econometricians for flexible forms has ramifications for CGE modeling which to date have not been adequately reflected in practice.

The paper proceeds as follows. Section 2 briefly contrasts the calibration and econometric approaches to CGE modeling. Section 3 outlines the general model structure. Section 4 outlines the sectoral equations for the non-flexible version of the model, and Section 5 outlines those for the version of the model based on flexible functional forms. Section 6 presents the policy simulations and explores the similarities and differences in models' predictions. Section 7 presents conclusions.

2. The calibration and econometric approaches to CGE modelling[3]

Consider a CGE model consisting of n net demand equations for each of h sectors, written as the system:

$$F^{ij}(\mathbf{p}_t, \mathbf{X}_t, \beta^j) = q_t^{ij} \tag{2.1}$$

where $i = 1,\ldots,n$ commodities (including labour); $j = 1,\ldots,h$ producing and consuming sectors; $t \in \{1,\ldots,Z\}$, Z is the length of the time-series of available data: $\mathbf{p}_t$ is an n-vector of prices at time t; $\mathbf{X}_t$ is an $n \times k$ matrix of exogenous data, including capital stocks, tax rates, predetermined quantities, and other economic variables known at time t, β^j is an m-vector of unknown parameters, where $m > n$ and q_t^{ij} is the net demand for quantity i in sector j at time t. The calibration approach can be described as follows. First, choose a single year $T \in \{1,\ldots,Z\}$ as the modeling base. Second, adjust the quantities, q_t^{ij} using a numerical algorithm such as row-and-column scaling to yield a matrix Q_t' such that

$$\mathbf{Q}_t{}'\iota = \mathbf{0} \tag{2.2}$$

[2]Robinson et al. (1992) compared a model using CES and AIDS Import demand functions and found substantial differences in responses to tariff policy changes. The United States International Trade Commission have recently begun revising a CES-based model to have an AIDS consumer model. This work is at an early stage (see Pogany, 1996). Mansur and Whalley (1984) present a comparison of a small (one consumer, two sector) CGE model where one version is calibrated and the other is estimated but each uses CES-class functional forms; estimates of tax-induced welfare losses nevertheless diverge considerably for some periods.

[3]See Mansur and Whalley (1984) and Lau (1984) for more detailed comparison of the two methods.

where ι and $\mathbf{0}$ are n-vectors of 1s and 0s, respectively. Note that the $'$ symbol denotes the base case data following row-and-column scaling; matrix transposition will be denoted $\top$. Third, partition β^j into an n-vector θ^j and an $(m-n)$ vector σ^j. Usually σ^j will contain all the elasticity and income share parameters and θ^j will contain the remaining scale or shift parameters.

Determine the values of the elements σ^j by literature surveys and other a priori methods. Determine each element θ^j by inverting (2.1) to yield:

$$\theta^{ij} = F^{ij^{-1}}(\mathbf{p}_t, \mathbf{X}_t, \sigma^j, q_t{}'^{ij}) \tag{2.3}$$

Substituting (2.3) into (2.1) defines the base-case solution of the model:

$$F^{ij}(\mathbf{p}_t, \mathbf{X}_t, \sigma^j) = q_t{}'^{ij} \tag{2.4}$$

As long as Walras' law holds on the system of equations F^{ij}, (2.2) ensures that this is a valid general equilibrium. A counterfactual equilibrium is computed by introducing a new set of exogenous variables $\mathbf{X}_T^c$ in place of $\mathbf{X}_T$, for instance by changing some of the tax rates, and searching for a counterfactual price vector $\mathbf{p}_t^c$ (subject to 2.2) to yield the solution:

$$F^{ij}(\mathbf{p}_t^c, \mathbf{X}_T^c, \sigma^j) = (q_t^{ij})^c \tag{2.5}$$

Note that the c superscript identifies the counterfactual solution, which is contrasted against the base case data, denoted with the $'$ symbol. In this case, $n \cdot h$ new right-hand side quantities and $n-1$ new left-hand side relative prices must be determined simultaneously. (2.2) and (2.5) yield n and $n \cdot h$ equations, respectively, and Walras' law implies that one is not independent of the rest. So the solution is computable, as long as existence conditions are satisfied.[4]

The econometric approach, proposed by Jorgensen (1984), begins by retaining (or obtaining) the full time series of Z observations, yielding, for each of the $n \cdot h$ quantities, an econometric model:

$$q_t^{ij} = F^{ij}(\mathbf{p}_t, \mathbf{X}_t, \beta^j) + e_t^{ij}, \quad t = 1, \ldots, Z \tag{2.6}$$

Statistical estimation yields $\hat{\beta}^j$. The base-case solution for some period T consists of the price vector $\mathbf{p}_T{}'$ at which (2.2) is satisfied, yielding:[5]

[4]See Varian (1984 ch. 5) for a discussion of sufficient conditions on market demand functions for an equilibrium to exist.

[5]Even if T is one of the years in the sample and (2.2) holds throughout the data series, the observed prices in the base year will not generally yield an equilibrium, because $\hat{\beta}^j$ is not defined so as to eliminate the residuals in any one year, but to minimize the sum of squared residuals across a span of years. Furthermore, (2.2) will probably not hold exactly in each sample year, because of problems of missing and suppressed data. This means that the benchmark equilibrium itself must be computed by the model. Alternatively, Lau (1984) points out that restrictions can be imposed on (2.6) to ensure that the estimated parameters satisfy (2.2) for a particular benchmark year. The parameters would of course be sensitive to the choice of benchmark year, as they are in the calibration approach. Use of the unrestricted estimator implies that the 'benchmark' is the entire span of years in the database.

$$q'^{ij} = F^{ij}\left(\mathbf{p}_T{}',\mathbf{X}_T,\hat{\beta}^j\right) \tag{2.7}$$

A counterfactual equilibrium can then be generated as above by replacing $\mathbf{X}_t$ with $\mathbf{X}_t^c$ and searching for a counterfactual price vector $\mathbf{p}_t^c$ such that

$$F^{ij}\left(\mathbf{p}_t^c,\mathbf{X}_t^c,\hat{\beta}^j\right) = \left(q_t^{ij}\right)^c \tag{2.8}$$

There are some clear differences between the two approaches. The calibration approach makes use of very little data, which is an advantage in cases where little is available, but is an unnecessary loss of information in contexts where time-series data can actually be obtained. Furthermore, the n parameters in θ^j are determined in part by the quantity matrix $\mathbf{Q}_t{}'$, which is constructed from national accounts data by eliminating row and column discrepancies. Row-and-column scaling (RAS) disperses discrepancies across a data matrix in a way that is largely untraceable, introducing further sources of bias into the parameter magnitudes. By contrast, in the econometric approach, the benchmark data can be computed by the general equilibrium model itself. This is possible because the parameters are all available before the base case data set is specified. When the CGE model itself is used to clear the discrepancies, it does so based on each sector's demand and supply characteristics, rather than on the basis of a numerical algorithm. And, as explained in footnote 5, this method implicitly treats the entire sample as the benchmark, rather than one arbitrarily chosen year.

It would be possible, in principle, to estimate the $h \cdot n$ Eq. (2.7) simultaneously. If done in this way, market-clearing restrictions would allow n demand equations to be dropped, and application of sectoral budget constraints would allow h equations to be dropped, so (2.7) would only require estimation of $(h-1)\cdot(n-1)$ equations. In fact, because multi-stage nesting (via cost minimizing share functions) will be required for both the estimated models, there are considerably more than $h \cdot n$ equations in the model. Neither of the sectoral models estimated below could be treated with one behavioural aggregator function. In the case of the CES model, nesting is required to increase flexibility. In the case of the normalized quadratic equation model, nesting is required to decrease the needed degrees of freedom. For estimation purposes, seeming unrelatedness is assumed for all systems which do not have independent variables (prices) in common, or which are in separate sectors. While this may reduce the efficiency of the estimates, it is a necessary expedient given the length of the time-series available and the non-linearity of the equations.[6]

Despite the arguments in its support, the econometric approach seems not to

[6]Shoven and Whalley (1992, p. 106) criticize the econometric approach on the grounds that these restrictions are 'overly severe', while their relaxation makes estimation impossible. In response it should be pointed out that while calibration does not require such identifying restrictions for solving (2.3), this only yields some of the needed parameters. The remainder, sometimes the majority, and usually the most influential, are at best taken from preexisting econometric studies on which such 'overly severe' restrictions were implicitly in force; moreover, actual modeling exercises rarely achieve this best-practice standard.

have been adopted widely in general equilibrium modeling. As far as the author is aware, outside of Jorgensen's (and his co-authors') own work, Clements (1980), Mansur and Whalley (1984), Hazilla and Kopp (1990) and Diewert and Lawrence (1994) are the only authors who have applied it.[7]

3. The outline of the models and the supporting data base

Two short-run CGE models will be constructed for this paper. One will be based on CES functional forms, and hence will be called the CES model. The other will be based on normalized quadratic functions and hence will be called the NQ model. In both, all equations are econometrically estimated on a single 29-year time series data base specially constructed for the present exercise. Most of the data were taken from Statistics Canada's Canadian Socio-Economic Information Management System (CANSIM), and show inter-industry and final demand transactions for each year, linked to National Accounts measures of government revenue and expenditure, and to National Balance Sheet measures of changes in financial and real asset stocks. Price and quantity indices are constructed from input-output data for the specific commodity and industry aggregates used in the model. Further details are in Appendix E of McKitrick (1996).

The Canadian economy is divided into four sectors: households, firms, the government and the rest-of-the-world. Table 1 lists some of the details of each sector, as well as the assumptions needed to compute an equilibrium. Table 2 lists the sectors, industries, factors and commodities, as well as their identifying codes.

Production is exhaustively partitioned into six industries: agriculture, mining, refining, utilities, manufacturing and services. The technology of each industry is summarized by a short run profit function with constant returns to capital. Each firm is assumed to choose a dividend level which fully disburses profits after taxes and capital consumption allowances. The capital consumption allowance is computed using estimated sector-specific depreciation rates applied to current real capital, valued at the market price of durable goods.

Ten domestic commodities plus two factors, labour and capital, are specified. An industry is not limited to producing only one commodity, and one commodity can be produced by more than one industry. Each domestic commodity can be sold as an export and competes with imports. A separate category of non-competing imports is also identified.

Since the input-output tables do not identify from which sectors exports come, and to which sectors imports go, these were arbitrarily assigned. This is in contrast to the usual Armington approach, in which a stand-alone technology aggregates domestic and foreign goods into a composite commodity. The approach taken here ensures that imports and exports flow through sectors to which an explicit budget

[7] Diewert and Lawrence used the parameters from a New Zealand GNP function and consumer demand system to estimate marginal excess burdens of various taxes, but did not compute an equilibrium or a counterfactual scenario.

Table 1
Summary of the four sectors and the computation of the equilibrium in econometric models

Consumers			Firms
•	Single aggregate household represented by a nested utility function	•	Technology represented by a nested profi function with capital fixed each period
•	Three allocations: labour/leisure, current/future consumption, current consumption shares	•	Multiple-outputs permitted
•	Sole supplier of labour, co-owner of capital with government and foreign sector	•	Variable inputs and outputs chosen basec current net prices
•	Subject to income, retail, payroll taxes as well as GST	•	All firms assumed to be price-takers
•	Gross substitutability not imposed	•	Investment demands based on profits, prices and interest rates
		•	All imports flow through Mfg and Service
		•	Subject to retail, payroll, profits taxes, GS
Government			Rest-of-the-world
•	One level	•	Imports supplied elastically, exports dema elastically
•	Revenues: taxes, deficit, investments	•	Clears savings market at world interest ra (US prime)
•	Expenses: transfers to households (domestic and foreign), subsidies to businesses, consumption subsidies, labour demand, interest on debt; rest is allocated to goods and services using estimated budget share equations	•	Importing sector pays tariffs
		•	Current and capital accounts balance eac
Computation of equilibrium			
•	Exchange rate is the numéraire, current and capital account balance ensured by Walras' Law		
•	Base-case data set generated by the model based on forecasts of exogenous variables		
•	Values of exogenous variables fixed across policy simulations, as were savings and investment estimates		

Table 2
Aggregate sectors, industries, factors and commodities for the econometric models

Code	Definition		Outputs and factors supplied
H	Households	HL	Labour
		K^H	Capital (share owned by households)
F	FIRMS, consisting of:		
A	Agriculture	MN	Non-durable goods
X	Extraction (mines, quarries and oil wells)	XG	Natural gas
		XC	Coal
		XO	Other minerals
R	Refining	RF	Refined fuels
		RO	Other refinery outputs
U	Utilities	UE	Electricity
		UO	Other utilities
M	Manufacturing	MD	Durable goods
		NM	Non-durable goods
S	Services	SV	Services
J	GOVERNMENT	K^J	Capital (share owned by government)
W	REST-OF-WORLD	WI	Non-competing imports
			All other imported goods
		K^W	Capital (share owned by foreigners)

constraint applies. It also avoids imposing the assumption that commodities are differentiated on the basis of country of origin. Such an approach conflicts with the treatment in the input-output tables, in which imports combine linearly with domestic production for the purpose of determining total domestic output. The present approach assumes only that imports are differentiated from domestic goods based on relative price differences, with the degree of actual substitution determined by econometric estimation. Almost all imports are assumed to be purchased by the manufacturing sector, except non-competing imports (for which the purchasing sectors are specified), manufactured durables and non-durables. These latter two are assumed to be imported by the service sector. Exports were assumed to come from sectors which are the major net producers of the commodity.

Final demand categories include all six industries (investment demands), households, government, and the rest-of-the-world. Each industry has a sector-specific capital stock which is augmented by annual purchases of investment goods, net of a sector-specific rate of depreciation.

There is a single aggregate household which provides labour and capital to firms. It receives income from wages, interest, dividends and government transfers. It chooses its labour supply, savings rate and consumption shares to optimize a quasi-concave utility function.

There is a single government sector which collects revenues in several ways.

Direct taxes are applied to corporate profits and household earnings. A portion of corporate taxation is treated as lump-sum, to reflect the existence of minimum taxes payable even in years when no profits are earned. Indirect taxes are applied to purchases of intermediate and final demand goods, and tariffs are applied to imports. The Goods and Services Tax (GST) is applied to value-added within each production sector. Profits and investment income accruing to foreigners are subject to withholding taxes. Indirect taxes are applied to all commodities. Payroll taxes include Canada/Quebec Pension Plan contributions, Unemployment Insurance and Workers' Compensation premiums and provincial payroll levies. Separate rates are identified for the employer and employee portions (see Appendix E of McKitrick (1996) for details). The government earns investment income from shares held in domestic firms, since crown corporations are included in the firm sector. Additional revenue comes from borrowing.

Government expenses each period include predetermined and endogenous budget items. The first category includes interest payments on the (historically accumulated) debt, transfers to households, transfers to non-residents and government labour demand, which is set according to an exogenous time path, since the government's factor demands are assumed not to be derived from an optimizing model. The second category includes transfers to firms and spending on goods and services. Transfers to firms are assumed to be based on sector-specific output price subsidies, and are thus determined by current economic activity. The budget for goods and services is the residual after total revenue has been dispersed elsewhere. Shares in real expenditures are mostly fixed, but shares of natural gas, fuel and electricity are allowed to vary according to relative prices by an estimated CES aggregator with an elasticity of -0.05.

Canada is assumed to be a small open economy, so demands for exports and supplies of imports are assumed to be perfectly elastic at prevailing world prices. The world interest rate is assumed to be the US prime rate, and foreign savings are assumed to be available elastically. This means that the foreign sector automatically clears any surplus or deficit in the domestic savings market each period, so the domestic interest rate is effectively exogenous.

The dividend rate paid on shares in foreign firms is the US prime rate plus an exogenous equity premium of 3%. The current account is defined as the nominal value of exports minus imports, minus net dividend and interest payments to foreigners, net of withholding taxes.

The capital account is defined as net foreign savings. The current account and capital account balances must sum to zero each period. Since the exchange rate is the numéraire, this constraint is satisfied as a consequence of Walras' law.

Gross substitutability between all pairs of goods is not imposed in either CES or NQ. This means that sufficient conditions for existence of a unique equilibrium are not met. Existence was not a problem, although uniqueness could not be proven, apart from the evidence provided by the fact that, in each experiment, numerous different starting values always generated the same equilibrium.

4. The model with CES functional forms

4.1. The industry models

The technology of each industry f is summarized by a short-run CES profit function with constant returns to capital:

$$\pi^f(\mathbf{p}_t,\tau_t,\tau_t^{\pi},K_t^f) = K_t^f\Big(\alpha_1\big[p_t^1\big(1+\tau_t^{f^1}\big)\big]^{\rho}+\dots \\ +\alpha_n\big[p_t^n\big(1+\tau_t^{f^n}\big)\big]^{\rho}\Big)^{1/\rho}\big(1-\tau_t^{f^{\pi}}\big) \tag{4.1}$$

The notation is as follows: $\mathbf{p}_t = (p_t^1,\dots,p_t^n,\, p_t^k)^{\top}$ is a vector of $n+1$ prices, one for each of n variable netputs v_t^i at time t, and one representing the price at which the capital stock is valued at time t. $\tau_t = (\tau_t^{f^1},\dots,\tau_t^{f^n})^{\top}$ is an n-length vector of industry-specific indirect tax rates associated with the variable netputs at time t. The industry-specificity arises because different firms have different partitions of inputs and outputs, and outputs are not taxed the same as inputs. $\tau_t^{f^{\pi}}$ is the direct tax rate on profits paid by industry f at time t. K_t^f is firm f's capital stock at time t, which is assumed to be fixed over the period. While (4.1) is the after-tax level of profits on variable inputs and outputs, chosen optimally given current taxes and prices, there will be other components of industry profits, namely depreciation and capital consumption allowances, bond interest, and so forth. Since these are fixed, they are omitted from (4.1), but in the CGE model they are included in the calculation of net earnings.

The use of a profit function rather than a cost function is somewhat non-standard. Formally, the two representations of technology are equivalent, but the profit function has the advantage of generating net supply and demand functions and the *ex post* rental rate directly, and can easily accommodate multiple outputs.

Since (4.1) is linearly homogeneous in prices, we can multiply each term in the square brackets by $(1-\tau_t^{f^{\pi}})$. For ease of notation, the tax-adjusted prices will henceforth be denoted by a superscript $\sim$, i.e.:

$$\tilde{p}_t^i = p_t^i\big(1+\tau_t^{f^i}\big)\big(1-\tau_t^{f^{\pi}}\big), \quad i=1,\dots,n \tag{4.2}$$

For further ease of notation we will drop the industry superscripts henceforth, except where needed for clarity.

We assume (4.1) is non-negative. To be a valid neoclassical profit function it must be positive, continuous, twice differentiable, monotonic (increasing in output prices and decreasing in input prices), homogeneous of degree one and globally convex. Positivity, continuity, differentiability and homogeneity are clearly satisfied. The monotonicity conditions are satisfied if the associated α_i parameters for net outputs are positive and for net inputs are negative. Finally, to ensure convexity, we will require:

$$0 < \rho < 1 \tag{4.3}$$

The proof of the convexity of (4.1) given (4.3) is in Appendix A of McKitrick (1996).

Using (4.2) in (4.1) and applying Hotelling's Lemma[8] generates an expression for the optimal variable netput v^* as a function of current prices, taxes, and capital:

$$v^*(\mathbf{p}_t,\tau_t,\tau_t^{\pi},K_t) = K_t\,\alpha_i\,\tilde{p}_t^{i(\rho-1)}\left(\alpha_1\tilde{p}_t^{1\rho} + \ldots + \alpha_n\tilde{p}_t^{n\rho}\right)^{(1/\rho-1)}, \quad i = 1,\ldots,n \tag{4.4}$$

Since (4.1) is linearly homogenous, Euler's theorem implies that

$$\sum_{i=1}^{n} v^*(\mathbf{p}_t,\tau_t,\tau_t^{\pi},K_t)\cdot\tilde{p}_t^i = \pi(\mathbf{p}_t,\tau_t,\tau_t^{\pi},K_t) \tag{4.5}$$

To form an econometric model, Eq. (4.1) is dropped, and both sides of (4.4) are divided by K_t (to reduce heteroskedasticity), obtaining:

$$\frac{v^*(\mathbf{p}_t,\tau_t,\tau_t^{\pi},K_t)}{K_t} = \alpha_i\,\tilde{p}_t^{i(\rho-1)}\left(\alpha_1\tilde{p}_t^{1\rho} + \ldots + \alpha_n\tilde{p}_t^{n\rho}\right)^{(1/\rho-1)}, \quad i = 1,\ldots,n \tag{4.6}$$

The equations in (4.6) constitute a very parsimonious system, with only $n + 1$ coefficients in n equations. The assumption that patterns of substitution among all netputs (plus exports and imports) can be summarized by a single parameter is unduly restrictive, and in practice would yield very inefficient estimates, especially if some pairs are complements. To improve the flexibility of this system it will be necessary to use multi-stage nesting functions. This approach supposes that firms combine basic inputs into intermediate aggregates, which are then combined into higher-level aggregates, until all non-labour inputs are grouped into a single energy-materials aggregate, which will be denoted EM_t, and all produced commodities into a single output aggregate, which will be denoted Y_t. Labour (denoted HL_t) is a basic factor and is not grouped into an intermediate nest.[9] Note that these are industry-specific aggregates, but we leave off the f superscript for convenience.

Associated with EM_t, and Y_t, are tax- and subsidy-adjusted, weighted average prices $\tilde{p}_t^Y$, and $\tilde{p}_t^{EM}$. These prices will differ among sectors, as the input mix will differ, as will net tax rates.

The parameters for each industry were estimated using the non-linear simultaneous equations regression routine in SHAZAM (White, 1978). Correction for first-order autocorrelation was applied, with a separate rho parameter identified for each equation. Parameter estimates are shown in Table 3.

We now turn to the structure of the lower-level nesting functions. To form an

[8] Here we are differentiating with respect to the tax-inclusive prices $\tilde{p}_t^i$, not the original prices.

[9] The exception, which will be discussed below, is the refining sector, for which labour is not as significant an input as crude oil. Hence labour is nested at an intermediate stage and other mineral products (XO), which includes crude, enters the profit function directly.

Table 3
CES profit function parameter estimates for six industries

Variable	Industry					
	Agriculture	Mining	Refining	Utilities	Mfg	Services
α_Y	0.459	0.695	0.941	0.492	2.338	2.779
	(0.11)	(0.05)	(0.22)	(0.15)	(0.10)	(0.23)
α_L	−0.058	−0.092	−0.615[a]	−0.032	−1.063	−0.965
	(0.01)	(0.02)	(0.19)	(0.14)	(0.13)	(0.08)
α_{EM}	−0.172	−0.327	−0.327	−0.086	−0.682	−0.995
	(0.04)	(0.04)	(0.05)	(0.03)	(0.08)	(0.09)
ρ	0.751	0.722	1.000	0.483	0.409	0.202
	(0.07)	(0.04)	(*)	(0.18)	(0.09)	(0.08)
R^2: Y	0.854	0.929	0.981[b]	0.841	0.857	0.802
HL	0.986	0.990	0.944[b]	0.903	0.988	0.848
EM	0.953	0.902	0.986[b]	0.843	0.950	0.902
Restrictions:	$\alpha_Y > 0$	$\alpha_Y > 0$	$\rho = 1$	$\alpha_L < 0$		
	$\rho > 0$	$\rho > 0$				

Numbers in parentheses are asymptotic standard errors on the untransformed coefficients.
Sign restrictions implemented by squaring. Restriction $\rho < 1$ was binding on refining sector so parameter value was forced.
[a] Commodity aggregates differ from other sectors: labour nested in EM, α_L is actually α_{XO}, the coefficient for the other minerals input, representing purchases of crude oil.
[b] All observations weighted by $(1/t)^2$ to remove non-stationary error trend.

intermediate composite input or output from h elements, the firm chooses that combination which minimizes costs, subject to a productivity constraint defined by a CES frontier. This problem is written as:

$$\min \tilde{\mathbf{q}}_t^\top \mathbf{x}_t \text{ w.r.t } \mathbf{x}_t \text{ s.t. } \left(\gamma_1 x_t^{1^\mu} + \ldots + \gamma_h x_t^{h^\mu}\right)^{1/\mu} = \Theta_t \tag{4.7}$$

where $\tilde{\mathbf{q}}_t$, is a vector of h tax-inclusive prices associated with the h netputs in the vector $\mathbf{x}_t$, Θ_t, defines a production possibility frontier, and μ and the γ_is are parameters. Solving (4.7) yields the ratios

$$\frac{x_t^i}{x_t^h} = \left(\frac{\gamma_i}{\gamma_h}\right)^{-\sigma}\left(\frac{\tilde{q}_t^i}{\tilde{q}_t^h}\right)^{\sigma}, \qquad i = 1, \ldots, h-1 \tag{4.8}$$

where $\sigma = 1/(\mu - 1)$. Since we are only estimating the γs in ratio form, γ_h will be normalized to unity, and the other γs henceforth interpreted subject to this normalization. Defining $g_i \equiv \gamma_i^{-\sigma}$, we can convert (4.8) into a set of $h - 1$ real share functions $w_t^i = x_t^i/(x_t^1 + \ldots + x_t^h)$, which are written:

$$w_t^i = \frac{g_i\left(\dfrac{\tilde{q}_t^i}{\tilde{q}_t^h}\right)^{\sigma}}{\left[g_1\left(\dfrac{\tilde{q}_t^1}{\tilde{q}_t^h}\right)^{\sigma} + \ldots + g_{h-1}\left(\dfrac{\tilde{q}_t^{h-1}}{\tilde{q}_t^h}\right)^{\sigma} + 1\right]}, i = 1, \ldots, h-1 \tag{4.9}$$

The parameters g_i and σ are estimated directly, since we do not need to recover γ_i or μ. Since the shares must sum to one, only the $h-1$ equations in (4.9) need be estimated.

In general, h had to be set very low, as inclusion of more than three elements tended to make the elasticity parameter very unstable. The CES system requires the interactions of all the prices to be characterized by a single parameter, a restriction which the data only support when two or three inputs are grouped at a time. This required extensive nesting to be done to avoid each model degenerating into a Leontief system. The nesting patterns were chosen to best fit the data, and consequently they differed among industries.

Altogether, 17 systems of nesting equations for industry outputs, and 34 systems of nesting equations for industry inputs, were estimated. Further details of the estimation, including parameter estimates and standard errors, are available in a technical appendix from the author. The nesting patterns were chosen to best fit the data, and consequently they differed among industries.

Negative substitution elasticities were generally observed among inputs, and positive substitution elasticities among outputs, as would be expected. Refining inputs tended to be complements or in fixed proportions, reflecting the limited scope for varying production processes in that industry. The import-domestic demand bundles in the manufacturing sector tended to have weak estimates of the elasticity parameters, reflecting in part the fact that the assignment of most imports to that sector was arbitrary. The elasticities of the higher-level nests tended to have much lower variances.

This completes the derivation of the CES model of firm sector production. The other aspect of firm behaviour to be modeled is the demand for investment goods. Each sector maintains a stock of immobile capital which is used for current production. Additions to the stock of capital are made by purchasing durables (MD) and services (SV), which are combined in a CES aggregator to form a capital good. Deductions from the capital stock occur through depreciation and, in the case of rapidly declining sectors, the selling-off of some of current capital. In the latter case, the capital good is decomposed through the same CES aggregator into durables and services, which are then sold on the open market.

Since the present study is focused on short-run effects, the investment model will be kept simple. The durables-services aggregator is derived using the same CES optimizing problem as in (4.7), yielding share functions of the form in (4.9). The estimation was done using the non-linear regression routine in SHAZAM (White, 1978), and the parameters are reported in Table B.7 of McKitrick (1996).

The investment demand is derived as follows. The main determinants of investment are assumed to be deviations of current profits and interest rates around long-run values. For the present purposes the long-run value will be assumed to be the average from 1961 to 1989. We must take care to ensure that the investment equation is homogeneous of degree zero in all prices to ensure that it is a valid demand function and that existence conditions for the general equilibrium are not violated. Consider the following quadratic investment function:

$$\frac{i_t}{K_t} = \beta_0 + \beta_1 \Delta\pi_t + \beta_{11}\Delta\pi_t^2 + \beta_2 \Delta r_t + \beta_{22}\Delta r_t^2 + \beta_{22}\Delta\pi_t \Delta r_t \tag{4.10}$$

where $\Delta\pi_t$, and Δr_t, represent, respectively, deviations of profits per dollar of capital stock and real interest rates around their long-run average values. Since $\Delta\pi_t$, is homogeneous of degree zero in prices, (4.10) is also. When all variables attain their long-run values,

$$i_t = \beta_0 K_t$$

therefore $\hat{\beta}_0$ provides an estimate of the sector-specific depreciation rate.

Eq. (4.10) was estimated for each sector using the linear regression routine in SHAZAM (White, 1978). An economy-wide estimate of the long run real interest rate was used (0.026, the 1961–1989 mean rate on 90 day prime corporate paper adjusted for changes in the gdp deflator), and sector-specific values of long run return rates were computed. Allowing the long-run nominal rate of return to capital to differ between industries recognizes that different risk, regulatory and opportunity cost characteristics may keep rental rates from converging, even over an extended horizon.

The results from the investment regressions are in Table B.7 of McKitrick (1996). Investments in the refineries and utilities sectors were treated differently, as (4.10) appeared to have very little explanatory power. It was instead supposed that these sectors plan for a smoothly trending capacity increase, so that investment is a function of time. Defining a variable $t_5 \equiv t/(t+5)$, where t is the number of years since 1960, the regression model was written:

$$\frac{i_t}{K_t} = \theta_0 + \theta_1 \cdot t_5 + \theta_2 \cdot t_5^2 \tag{4.11}$$

subject to the restriction $\theta_2 = -\theta_1$. This ensures that, in the long run, as $t_5 \to 1$, $(i_t/K_t) \to \theta_0$, so the intercept provides an estimate of the depreciation rate. These parameter estimates are reported in Table B.8 of McKitrick (1996).

4.2. The consumer model

The household has an endowment of financial wealth at time t, which is denoted W_t^h. It consists of domestic bonds (B_t^h), equity or shares in domestic firms (E_t^h) and foreign investments (F_t^h). The portfolio shares are taken as given. The overall net rate of return on W_t^h is denoted $\tilde{r}_t^W$. The 'price' of savings is the cost of securing one dollar for spending next period. It is denoted $\tilde{p}_t^s$ and is defined as

$$\tilde{p}_t^s = 1/(1 + \tilde{r}_t^W) \tag{4.12}$$

assuming expectations about inflation are static. Savings consists of the entire amount set aside for future consumption (not just additions to financial wealth). That is, financial wealth at the end of the period (W_{t+1}^h) represents the dollar value of what could be consumed next period in excess of income. 'Real savings' S_t^h will be defined as end-of-period financial wealth deflated by the price of savings. The change in financial wealth each period, denoted ΔW_t^h, is therefore

$$\Delta W_t^h = \tilde{p}_t^s S_t^h - W_t^h \tag{4.13}$$

The household has an endowment of time each period, which is denoted H_t. This is to be allocated between labour HL_t and leisure HR_t. The after-tax cost of a unit of leisure, $\tilde{p}_t^{HR}$, is the foregone net wage rate:

$$\tilde{p}_t^{HR} = p_t^{HL}(1 - \tau_t^H) \tag{4.14}$$

where τ_t^H is the combined income and payroll tax rate on household earnings. Note that $\tilde{p}_t^{HR}$ is the tax-inclusive price of leisure, while $\tilde{p}_t^{HL}$ is the price of labour faced by firms, inclusive of payroll and profits taxes.

Let C_t^h denote one unit of the household's total bundle of consumption commodities purchased at time t. Its tax-inclusive price is denoted $\tilde{p}_t^C$, which is a weighted average of the tax inclusive prices of all household consumables, the weights being the real shares of each commodity in the total bundle.

Finally, denote by F_t^h the aggregate purchase of consumption goods at time t plus the contract for future consumption, i.e. $F_t^h = C_t^h + S_t^h$. The price of this aggregate is the nominal value of non-leisure purchases divided by their respective deflated quantities:

$$\tilde{p}_t^F = \frac{\tilde{p}_t^C C_t^h + \tilde{p}_t^S S_t^h}{C_t^h + S_t^h} \tag{4.15}$$

The first optimizing problem faced by the consumer is to allocate the total current endowment of wealth between purchases of leisure HR_t and the full-consumption aggregate F_t^h. The budget constraint is written:

$$\tilde{p}_t^F F_t^h + \tilde{p}_t^{HR} HR_t \leq \tilde{p}_t^{HR} H_t + W_t^h + XI_t^h \tag{4.16}$$

where XI_t^h denotes all predetermined and exogenous sources of household income at time t, including investment income and government transfers. By setting up the budget constraint in this way, much of the intertemporal structure of the household's allocation problem is suppressed, while still making it possible to derive a simple savings demand function. In particular, households are not allowed here to borrow against future earnings, and no attempt is made to distinguish between durables and non-durables.

The CES utility function is written:

$$U(F_t^h, HR_t) = \left(\alpha_F F_t^{h^\rho} + \alpha_{HR} HR_t^\rho\right)^{1/\rho} \tag{4.17}$$

Maximizing (4.17) subject to (4.16) yields the share spent on leisure:

$$w_t^{HR} = \frac{1}{1 + \left(\dfrac{\alpha_{HR}}{\alpha_F}\right)^{1/(\rho-1)} \left(\dfrac{\tilde{p}_t^F}{\tilde{p}_t^{HR}}\right)^{\rho/(\rho-1)}} \tag{4.18}$$

This equation is suitable for econometric estimation. Since α_{HR} and α_F enter the share function as a ratio, α_{HR} was normalized to unity and the relative value of α_F computed.

A measure of total labour supplied, HL_t, was obtained by taking total before-tax household work earnings (i.e. the total wage bills across all firms and the government) and dividing by a before-tax wage rate index taken from the Statistics Canada publication *Aggregate Productivity Measures* (Catalogue 15-204). Estimated average hours worked per week were obtained from the same publication. Assuming individuals have a weekly time endowment of 100 h each, the proportions devoted to work and leisure were calculated (denote these as Ω_t^{HL} and Ω_t^{HR} respectively). The total time endowment H_t was calculated by dividing HL_t, by Ω_t^{HL}. Leisure was then computed as $H_t - HL_t$. The marginal tax rate on income was set equal to the average tax rate, which was calculated by dividing the total direct taxes on persons received by all governments, by the total household income from all sources. $\tilde{p}_t^C$ was calculated using the nominal and real household consumption estimates in the input-output tables, including appropriate indirect tax margins. The price of S_t^h was calculated using (4.12), and S_t^h was calculated using (4.13). The shares of leisure purchases out of the total endowment at time t was used in (4.18) for estimating the parameters of the utility function. The regression was done using the non-linear regression routine in SHAZAM (White, 1978). A correction for first-order autocorrelation was applied. The results of this regression are shown in Table 4.

From the estimation of (4.17) we can deduce that the absolute elasticity of substitution between leisure HL_t, and full consumption F_t^h is ~ 0.89, which is high compared to the early consumption-leisure elasticities cited in Shoven and Whalley (1992), but is close to the recent estimate for New Zealand (0.82) by Diewert and Lawrence (1994) using a flexible functional form method.

Having determined F_t^h, the consumer allocates it between current consumption C_t^h and future consumption S_t^h. Using a CES aggregation programme of the form (4.7), yields the standard share form (4.9) for estimation. C_t^h is then allocated among commodity categories using a series of nested CES aggregators. The nesting function elasticity estimates are shown in Table 5.

Table 4
Parameter estimates from the CES consumption-leisure utility model

Variable	Estimate	t-statistic[a]
α_F	1.91	10.89
ρ	-0.12	1.44
R^2	0.99	

[a]Asymptotic.

Table 5
Parameter estimates for the CES household consumption model

Inputs to nest	Estimate	t-statistic	Nest code
XG^h, XC^h, RF^h, UE^h	-0.22[a]	2.60	E_h
XO^h, RO^h, UO^h	-0.76[a]	8.20	O_h
MD^h, MN^h	-1.77	1.25	M_h
E_h, O_h, M_h, SV^h	-0.14[a]	4.08	D_h
D_h, WI^h	0.0	([b])	C_t^h
C_t^h, S_t^h	-1.29[a]	7.89	F_t^h

Notes: t-statistics asymptotic.
[a]All observations weighted by $(1/t)^2$ to remove non-stationary error trend.
[b]Parameter value set a priori.

The first nest combines natural gas, coal, refined fuels and electricity, forming the energy aggregate E_h. Not surprisingly, the elasticity among these is fairly low (-0.22), indicating a limited ability to switch among energy types in the short-run. By comparison, elasticities are relatively higher among non-manufactured goods (-0.76), among manufactured goods (-1.77), and between current and future consumption (-1.29). The elasticity among services, energy and the goods aggregates is low (-0.14), as would be expected among the major divisions of the consumer budget. Non-competing imports were assumed to enter the consumption aggregate in fixed proportions, since the share is small (about 0.5%) and fairly constant over the sample years.

5. The model with normalized quadratic functional forms

5.1. The industry models

The second version of the model is much the same as the first except that less nesting had to be undertaken. Since the functional form imposes no *a priori* restrictions on own- and crossprice elasticities, it is in principle possible to include as many elements in the aggregator function as desired. In practice, the number of parameters gets very large beyond inclusion of five elements, and degrees-of-freedom limitations prevented estimation of a single all-encompassing profit function for each sector. Diewert and Wales (1988) present a semi-flexible version of the normalized quadratic model which reduces the required number of parameters, which they recommend for use in CGE modeling. This method was not used for two reasons. First, each sector has between 12 and 20 net inputs/outputs to aggregate, and to keep the number of parameters to a manageable level requires restricting the rank of the cross-price coefficient matrix to not much more than five. The effect on the flexibility of the system from this amount of rank-restriction is not well understood. Nesting also reduces the flexibility of the system, but uses restrictions with more obvious effects, and this transparency was considered an advantage. Second, the computer code for a semi-flexible system with over 12 inputs is awkward to write and debug, especially if curvature must be imposed by restriction and the degree of rank-restriction is to be determined by trial and error. With six production sectors and a household sector to be estimated, time constraints necessitated using a nested share-function approach, the code for which is more manageable and which helps ensure that an optimum of the likelihood function can be found in a reasonably efficient manner.

The profit function for each industry is written (ignoring the industry superscript):

$$\pi(\tilde{\mathbf{p}}_t, t_5, K_t) = \left(\mathbf{c} \cdot \tilde{\mathbf{p}}_t + \mathbf{d} \cdot \tilde{\mathbf{p}}_t t_5 + \tilde{\mathbf{p}}_t \cdot \mathbf{B}\tilde{\mathbf{p}}_t / (2\mathbf{a} \cdot \tilde{\mathbf{p}}_t)\right) K_t \tag{5.1}$$

where $\mathbf{c}$, $\mathbf{d}$ and $\mathbf{a}$ are n-vectors of parameters, $\mathbf{B}$ is a symmetric $n \times n$ matrix of parameters, t_5 is a tapering time trend defined above in the utilities sector investment model and $\cdot$ denotes a vector dot product. As before, the $\sim$ superscript on prices denotes inclusion of sales and profits taxes. The parameters in $\mathbf{a}$ can be chosen arbitrarily while the remaining parameters are to be estimated. The following restrictions are applied to (5.1):

$$\mathbf{B} \cdot \mathbf{p}^* = \mathbf{0} \tag{5.2}$$

and

$$\mathbf{a} \cdot \mathbf{p}^* = 1 \tag{5.3}$$

where $\mathbf{p}^*$ is a reference price vector (defined here as a vector of 1s, corresponding to the 1989 prices) and $\mathbf{0}$ is a vector of zeroes. Diewert and Wales (1987) establish that if $\mathbf{B}$ is positive semidefinite, then (5.1) is a globally convex flexible profit function. That is, (5.1) subject to (5.2, 5.3) provides a globally convex second-order approximation to any underlying technology, with the approximation centred around the reference point $\mathbf{p}^*$. Convexity can be imposed (without destroying the flexibility of 5.1) by setting $\mathbf{B} = \mathbf{A}\mathbf{A}^\top$, where $\mathbf{A}$ is a lower triangular matrix of coefficients. Eq. (5.1) also satisfies continuity, differentiability and homogeneity, but not necessarily monotonicity. The latter requirement cannot be imposed, a problem which will be discussed later.

If we set $\mathbf{a} = (0, \ldots, 0, 1)^\top$ then (5.1) is referred to as an asymmetric generalized

McFadden profit function, which has all the same properties of the normalized quadratic except that we are assuming one price can be singled out as that against which all others are normalized, noting that the coefficient estimates from (5.1) are conditional on this assumption. The chief advantage of using the asymmetric form is that the first $n-1$ netput equations are linear in the parameters, and the nth equation can be the one dropped from the econometric model, simplifying the estimation. Using the subscript $-n$ to indicate that the nth row is dropped, Hotelling's lemma yields:

$$\mathbf{v}_{-n}(\tilde{\mathbf{p}}_t, t_5)/K_t = \mathbf{c}_{-n} + \mathbf{d}_{-n}t_5 + \mathbf{B}_{-n}\tilde{\mathbf{p}}_t/\tilde{p}_n \tag{5.4}$$

These equations, along with (5.1), form a complete econometric model (with the latter divided by K_t as well). The profit function was assumed to identify aggregate output, and inputs of labour, energy and materials. Nesting functions are used to disaggregate output, energy and materials down to the basic commodity levels. The parameters of (5.1) were estimated for each sector using the non-linear system estimation routine in SHAZAM. Each system was estimated without curvature imposed, but in every case examination of the eigenvalues of $\mathbf{B}$ indicated that global convexity was not attained, so it was imposed and the system re-estimated. The results of the estimations are shown in Table 6. The equations fit quite well, especially considering the fact that each equation is in netput-capital ratio form, with many t-statistics showing asymptotic significance and the R^2 values consistently high.

The nesting functions are derived from a cost minimization program which is identical to that in (4.7) without assuming a functional form for production. If the production function is linearly homogeneous, (4.7) has a dual constant-returns-to-scale cost function defined over h input prices, which we will assume has the normalized quadratic form:

$$c(\tilde{\mathbf{p}}_t, t_5, y_t) = \left(\mathbf{s}\cdot\tilde{\mathbf{p}}_t + \mathbf{g}\cdot\tilde{\mathbf{p}}_t t_5 + \tilde{\mathbf{p}}_t\cdot\mathbf{W}\tilde{\mathbf{p}}_t/(2\mathbf{m}\cdot\tilde{\mathbf{p}}_t)\right)y_t \tag{5.5}$$

where $\mathbf{s}$, $\mathbf{g}$ and $\mathbf{m}$ are parameter vectors of length h and $\mathbf{W}$ is a symmetric $h\times h$ parameter matrix. Restrictions $\mathbf{s}\cdot\mathbf{p}^* = 1$, $\mathbf{g}\cdot\mathbf{p}^* = 0$, $\mathbf{W}\cdot\mathbf{p}^* = \mathbf{0}$ and $\mathbf{m}\cdot\mathbf{p}^* = 1$ are imposed. If $\mathbf{W}$ is negative (positive) semidefinite then Diewert and Wales (1987, theorem 10) show that (5.5) is globally concave (convex), which corresponds with a nesting system for minimizing input costs (maximizing output revenues). Furthermore, Diewert and Wales (1988) show that (5.5) has the property of being 'technical-progress flexible' at the point $\mathbf{p}^*$, which is not destroyed even if concavity must be imposed (by setting $\mathbf{W} = -\mathbf{Z}\mathbf{Z}^\top$ where $\mathbf{Z}$ is a lower triangular matrix of coefficients).

Choosing $\mathbf{m} = (0,\ldots,0,1)^\top$ allows us to exploit the asymmetry of the first $h-1$ conditional demand functions. This yields $h-1$ share equations of the form:

$$\begin{aligned}&\mathbf{w}_{-n}(\tilde{\mathbf{p}}_t, t_5)/K_t\\ &\quad = \mathbf{I}_{p_{-n}}(\mathbf{s}_{-n} + \mathbf{g}_{-n}t_5 + \mathbf{W}_{-n}\tilde{\mathbf{p}}_t/\tilde{p}_n)/\left(\mathbf{s}\cdot\tilde{\mathbf{p}}_t + \mathbf{g}\cdot\tilde{\mathbf{p}}_t t_5 + \tilde{\mathbf{p}}_t\cdot\mathbf{W}\tilde{\mathbf{p}}_t/\tilde{p}_n\right)\end{aligned} \tag{5.6}$$

where $\mathbf{I}_{p_{-n}}$ is an $(h-1)\times(h-1)$ identity matrix with the first $h-1$ prices down

Table 6
Profit function coefficients for normalized quadratic model

Coefficient	Agricul -ture	Mining	Refining	Utilities	Manufac- turing	Services
c1	0.932	0.741	1.076	0.145	−0.057	0.271
	(15.06)	(9.55)	(6.99)	(5.34)	(0.19)	(2.04)
c2	0.045	0.031	−0.850	−0.037	−0.228	−0.132
	(1.15)	(0.42)	(3.50)	(3.88)	(0.97)	(2.32)
c3	−0.078	0.006	−0.021	−0.008	−0.082	−0.016
	(6.41)	(0.49)	(5.40)	(1.15)	(2.71)	(1.85)
c4	−0.335	−0.315	−0.301	−0.031	−0.580	0.040
	(6.41)	(4.18)	(0.91)	(1.07)	(3.29)	(0.05)
d1	−0.768	−0.623	0.065	0.103	1.022	1.129
	(10.21)	(5.08)	(0.28)	(2.93)	(2.96)	(8.35)
d2	−0.096	−0.096	−1.174	−0.019	−0.113	−0.342
	(2.11)	(1.14)	(3.30)	(1.17)	(0.43)	(6.09)
d3	0.060	0.046	−0.002	−0.013	0.055	−0.050
	(4.11)	(0.73)	(0.33)	(1.47)	(1.69)	(5.25)
d4	0.276	0.232	0.985	0.006	0.322	−0.493
	(4.11)	(2.15)	(2.17)	(0.15)	(1.61)	(6.14)
b11[a]	0.024	0.005	0.001	0.023	0.951	3.138
	(0.80)	(2.26)	(0.07)	(2.94)	(8.34)	(7.77)
b12	0.017	−0.035	−0.005	−0.020	−0.962	−2.007
	(0.52)	(3.41)	(0.24)	(1.96)	(7.89)	(8.66)
b13	−0.003	−0.004	0.000	−0.009	−0.045	−0.048
	(0.26)	(1.21)	(0.33)	(2.38)	(2.79)	(1.44)
b22	0.145	0.278	0.025	0.027	1.110	1.511
	(4.90)	(0.31)	(0.00)	(1.70)	(4.46)	(7.87)
b23	0.001	0.041	0.001	0.003	0.017	0.051
	(0.23)	(2.14)	(0.00)	(1.55)	(4.36)	(2.61)
b33	0.018	0.009	0.00	0.006	0.008	0.004
	(8.52)	(0.00)	(0.00)	(0.00)	(0.00)	(0.77)

the diagonal. The restrictions cited above ensure that we need only estimate the $h - 1$ equations in (5.6) to obtain all the parameters in (5.5).

Twenty nesting systems of the form (5.6) were estimated for the six production

Table 6 (*Continued*)

Coefficient	Agricul -ture	Mining	Refining	Utilities	Manufac-turing	Services
R^2: Y	0.993	0.995	0.990	0.989	0.997	0.997
L	0.999	0.999	0.924[b]	0.997	0.999	0.999
E	0.998	0.994	0.962	0.898	0.998	0.996
π	0.945	0.917	0.778	0.977	0.906	0.982

Absolute asymptotic t-statistics in parentheses.
[a] Coefficient is i,j-th element of transformed (**B**) matrix but t-statistic is that of corresponding element of untransformed (**A**) matrix.
[b] Inputs of XO not L.
R^2, square of correlation between observed and predicted values.

sectors using SHAZAM. The number of elements ranged from 2 to 5. Correction for autocorrelation was applied in each case, as were time-trend weights. In most cases curvature had to be imposed: convexity for output nests and concavity for input nests. Despite the fact that the models are in share form the equations fit quite well, frequently yielding R^2 values above 0.95. Of the 215 parameters estimated, 117 were significant at 95%, 17 at 90%, and 81 were not significant. In two cases the **W** matrix was restricted to have rank $h - 2$ (instead of the maximum possible $h - 1$) using the semi-flexible method outlined in Diewert and Wales (1988) to aid in finding a convergent solution. Because of the great number of parameter estimates they are not reported, but they are available on request from the author.

5.2. The consumer model

The flexible consumer model starts with an expenditure function which aggregates leisure, savings, energy and non-energy goods. Leisure and savings are defined in Section 4.2. The expenditure function is written:

$$e(\tilde{\mathbf{p}}_t, t_5, u_t) = \mathbf{a} \cdot \tilde{\mathbf{p}}_t + \left(\mathbf{b} \cdot \tilde{\mathbf{p}}_t + \mathbf{d} \cdot \tilde{\mathbf{p}}_t t_t + \tfrac{1}{2}\tilde{\mathbf{p}}_t \cdot \mathbf{C}\tilde{\mathbf{p}}_t / \tilde{p}_t^n\right) u_t \tag{5.7}$$

where **a**, **b**, and **d** are n-length parameter vectors, **C** is an $n \times n$ parameter matrix, $t_5 (= t/(t+5))$ is a tapering time trend and u_t is the consumer's utility level at time t. Money-metric scaling can be imposed by assuming $\mathbf{a} \cdot \mathbf{p}^* = \mathbf{d} \cdot \mathbf{p}^* = 0$, $\mathbf{b} \cdot \mathbf{p}^* = 1$ and $\mathbf{C} \cdot \mathbf{p}^* = \mathbf{0}$. Eq. (5.7) is of the Gorman polar form and thus is consistent with aggregation of individual quasi-homothetic preferences. Diewert and Wales (1993) show that (5.7) satisfies most of the appropriate regularity conditions for consumer demand equations as long as **C** is negative semidefinite. Monotonicity, however, cannot be imposed on (5.7). This is not a problem for within-sample inferences, however it can cause difficulties in forecasting and simulation. Recent advances in the development of flexible functional forms with global monotonicity (e.g. Fry et al., 1996) raise the encouraging prospect that a fully regular flexible functional form may soon be available for modeling applications.

The Marshallian demand equations corresponding to (5.7) are:

$$\mathbf{x}_{-n}(\tilde{\mathbf{p}}_t, t_5, y) = \mathbf{a}_{-n} + (\mathbf{b}_{-n} + \mathbf{d}_{-n} t_5 + \mathbf{C}_{-n} \tilde{\mathbf{p}}_{-n} / \tilde{p}^n) \times \left((y - \mathbf{a} \cdot \tilde{\mathbf{p}}) / \left(\mathbf{b} \cdot \tilde{\mathbf{p}} + \mathbf{d} \cdot \tilde{\mathbf{p}} t_5 + \tfrac{1}{2} \tilde{\mathbf{p}} \cdot \mathbf{C} \tilde{\mathbf{p}} / \tilde{p}^n \right) \right) \tag{5.8}$$

where the $-n$ subscript denotes the removal of the nth row and y is the consumer's total wealth at time t. The time subscripts have been suppressed. Pre-multiplying each $x^i(\cdot)$ by p^i/y converts (5.8) into a set of three share equations. These were estimated using SHAZAM, and the results are in Table 7.

The nesting functions for the consumer are of the form (5.6). Three sets of nests were used to break the goods aggregates down to basic commodity levels. Gas, coal, oil and electricity combine to form the energy aggregate. Durables, non-durables, services and non-competing imports combine to form the produced goods aggregate, while other minerals, other refineries products and other utilities combine to form the 'raw' goods aggregate. The produced goods and the 'raw' goods are then combined to form the non-energy goods aggregate. The parameters of these nests are also in Table 7.

6. The comparative policy simulations

The key question to be addressed here is whether the model constructed with normalized quadratic functional forms gives pretty much the same results as the model constructed with CES functional forms. If found, such results would provide evidence that the information contained in a CGE model is not primarily located in the functional structure. In combination with careful use of sensitivity analysis this would help justify the present lack of concern over the econometric critique of CGE modeling. However, as we will see, the switch from the CES to the NQ model yields substantial changes in simulation results, both at aggregate and industry-specific levels. Thus, choice of functional form appears to be influential in CGE model performance. For all their similarities, the CES and NQ models can be viewed as different descriptions of the general equilibrium structure of the Canadian economy.

Values of exogenous variables were specified for the year 2000. This includes industry capital stocks, population, world prices, tax and subsidy rates, the government deficit and labour demand, and start-of-period financial sector data. All such values were constructed by extrapolating on the most recent data available. The exogenous variables were fixed across all policy simulations. The general equilibrium for the year 2000 was computed by the model, providing a base case data set against which policy-induced changes could be computed. Firm sector investment demands and savings were then set to the base level in the policy simulation, as were household savings, the real value of government purchases, and the government deficit.

Table 7
Coefficients for normalized quadratic consumer models

Variable	Nested goods				Exp Fn	
	Energy	Produced	Raw	P + R	Coefficient	Value
c1	0.031	0.048	0.037	0.993	a1	−10 891.0
	(10.81)	(1.24)	(7.15)	(1057.70)		(0.03)
c2	0.040	0.239	0.186		a2	161 700.0
	(18.33)	(34.89)	(3.96)			(0.45)
c3	0.650	0.693			a3	10 238.0
	(152.76)	(20.63)				(0.54)
d1	−0.004	0.273	−0.021	−0.002	b1	0.151
	(0.24)	(1.76)	(3.00)	(1.33)		(0.68)
d2	−0.050	−0.255	−0.071		b2	0.262
	(15.52)	(15.12)	(1.16)			(0.09)
d3	0.003	−0.015			b3	−0.011
	(0.12)	(0.10)				(0.40)
b11	−0.005	−0.095	−0.008	−0.001	d1	−0.056
	(2.25)	(1.18)	(3.13)	(0.43)		(0.19)
b12	0.001	0.015	−0.002		d2	−0.148
	(0.81)	(0.76)	(0.83)			(0.06)
b13	−0.002	0.028			d3	0.017
	(0.32)	(0.41)				(0.61)
b22	−0.001	−0.002	−0.035		b11	−0.006
	(2.77)	(0.00)	(3.93)			(0.05)
b23	0.005	−0.005			b12	−0.025
	(2.53)	(0.00)				(0.42)
b33	−0.022	−0.008			b13	0.00
	(0.00)	(0.00)				(0.003)
Goods nested	*XG*	*MD*	*XO*	*P*	b22	−0.103
	XC	*MN*	*RO*	*R*		(0.05)
	RF	*SV*	*UO*	Nest: *M*		
	UE	*WI*	Nest: *R*		b23	0.004
	Nest: *E*	Nest: *P*				(0.10)
					b33	−0.001
						(0.00)

Absolute asymptotic *t*-statistics in parentheses.

In all the simulations below, for both the NQ and CES models, numerous starting prices were tried, and the same equilibrium was computed in each case. Thus non-uniqueness did not seem to be a problem. For the NQ model the range of starting values which would lead to an equilibrium (as opposed to the program crashing) was smaller than for the CES model. Because global monotonicity is not imposed on the NQ aggregator function, the derived demand and supply Eqs. (5.4),(5.8) can take inappropriate signs, leading for instance to negative firm demands for labour and negative consumer demands for goods, depending on the prices at which the excess demands are being evaluated. The output share of exported coal in the mining sector had to be truncated arbitrarily close to zero in order to prevent search problems in the NQ model. Also, the own-price output supply elasticity in mining and refining had to be constrained to values higher than the unrestricted estimated values. Otherwise, the equilibrium prices of the outputs in these sectors would be so high as to drive input demands negative in some sectors. These measures take away from the flexibility which motivated use of the NQ system in the first place. This could be remedied by using a functional form which allows imposition of global monotonicity at the estimation stage, while preserving curvature and flexibility. Development of such forms is ongoing: Barnett et al. (1996) and Terrel (1996) presents procedures for imposing within sample monotonicity, while Fry et al. (1996) present a procedure for ensuring estimated budget shares remain on the unit simplex. Meanwhile, in defence of the present model, in every case where a parameter was forced, the remaining parameters in that system were re-estimated conditional on the restriction, an informational gain which is impossible in the calibration approach.

For each variable, a simple indicator of similarity between the models is shown, with •• denoting the most resemblance, • denoting some and a blank denoting the least. The specific numerical criteria are shown at the bottom of Table 8. At the bottom of the column an 'Index of similarity' is shown, which is the ratio of the number of bullets in the column to the maximum possible.[10]

Three fiscal experiments, representing 'small', 'medium' and 'large' interventions, are presented in which both models are subjected to the same tax policy changes. In the first case, the average income tax rate is reduced from 22.0% to 20.0%. In the second case, a new 10.0% tax is added to all purchases of services. In the third case the GST is doubled to 14.0%. The size descriptors of these policies are in single quotation marks to indicate that they are tautological: a 'small' policy shock is defined as one on the order of a 2% reduction in average personal income taxes, and so forth.

The revenues raised by each tax are used to finance new government purchases of goods and services. The model is set to a 2000 base, meaning that the base case is computed assuming year 2000 values of all exogenous variables. The macroeconomic effects are listed in Table 8. For each policy shock the table shows

[10] It need hardly be stated that this index is only an approximate summary of the information in the tables, and it is not claimed that its cardinal magnitude has an exact interpretation. Its primary value is as a ranking device, and as a useful summary of the numerical results in the tables.

Table 8
Macroeconomic results for CES-NQ comparison (percent changes except as noted)

Variable	1: Average income tax cut from 22.0 to 20.0%			2: 10% new tax added to services purchases			3: GST doubled from 7.0 to 14.0%	
	CES	NQ	(=)	CES	NQ	(=)	CES	NQ
Real:								
GNP	0.0	0.0	●●	+0.1	+0.5	●●	−1.4	+0.3
Consumption	+2.3	+2.2	●●	−6.7	+2.2		−6.3	−5.1
Investment	+0.3	+0.1	●●	+0.2	−2.7		−0.6	−0.7
Government spending on goods and services	−27.7	−5.6		+61.4	+14.1		+36.9	+13.3
Exports	−0.1	−0.2	●●	+0.6	+0.3	●●	−1.2	−0.3
Imports	−1.3	0.0		−0.9	−1.0	●●	−3.4	−1.4
GNP deflator	−2.1	−0.4		+5.9	+3.4		+6.6	+6.8
Consumer price index	−1.7	0.0		+5.9	+4.1		+5.5	+5.7
Real wages	+2.1	+2.6	●●	−5.5	−5.6	●●	−5.8	−9.2
Return to capital[a]	+4.1	+8.0		−9.7	−21.7		−3.0	−12.0
Employment	0.0	0.0	●●	+0.3	+0.9	●	−1.4	+0.4
Utility	+0.6	+0.8	●●	−1.1	−3.3		−0.6	−3.6
Tax as a % of personal income	31.9	32.3	●●	39.2	39.1	●●	34.7	35.5

Table 8 (*Continued*)

Variable	1: Average income tax cut from 22.0 to 20.0%			2: 10% new tax added to services purchases			3: GST doubled from 7.0 to 14.0%	
	CES	NQ	(=)	CES	NQ	(=)	CES	NQ
Chg government revenue (1989 $bil):[b]								
Total	−16.5	−18.6	●●	+34.1	+45.3		+20.8	+53.0
CIT	+0.1	−0.1	●●	−0.7	−0.3	●●	+0.6	+3.7
PIT	−14.1	−18.4		−0.5	−1.4	●●	−1.0	0.0
HH indirect	−0.2	+0.3	●●	+29.4	+36.6		−0.3	+0.4
GST	−0.8	−0.2	●●	−0.1	−0.1	●●	+22.9	+48.5
Terms of trade index[c]	−0.1	0.0	●●	+0.2	+0.2	●●	+0.1	+0.3
Index of similarity			0.684			0.500		

[a] Return to capital includes capital gains and ex post rental payments.
[b] CIT, corporate income tax; PIT, personal income tax; HH, household.
[c] Terms of trade index = average price exports/average price imports.
(=) Indicator of similarity between CES and NQ model. The criteria are as follows: percent change variables, ● difference ≤ 1.0%, ●● differen
tax as a % of personal income, ● difference ≤ 1.0, ●● difference ≤ 0.5; change in government revenue, ● difference ≤ $8.0 bil, ●● difference
change in revenue components, ● difference ≤ $2.0 bil, ●● difference ≤ $1.0 bil.

the induced change in the CES model and the normalized quadratic (NQ) model. Looking first at GNP and its components, the small shock induced makes no change in either model, and the changes in consumption are comparable. The comparability of GNP and consumption breaks down through the medium and large shocks. The effects on government spending differ substantially in all cases.

Changes in price indices are, curiously, only comparable in the large shock. However, they always have the same sign. Real wage changes diverge in the large shock and returns to capital are different for all shock sizes. Employment changes diverge as the size of the shock increases.

The change in the value of the consumer's utility function is most comparable in the small policy shock. The predicted changes in the government's revenues span fairly large nominal intervals, although some comparability remains even in the third experiment. Terms of trade effects are very small in all simulations.

Table 8 establishes that the two models do not merely replicate one another's macroeconomic results. For a small shock there are appreciable similarities, but for larger shocks the two models are divergent, as indicated by the steady fall in the Similarity Index. Table 9 takes the comparison to the microeconomic level. The changes in total demand for refined fuels is the only one that remains comparable across scenarios. Otherwise by the third shock the models are conspicuously divergent in their predictions of total demand for major commodities. The discrepancies continue at the industry level. For Agriculture and Refining the models return similar results across all three experiments, but for the biggest sectors (Manufacturing and Services) the similarities do not persist, while for Utilities even the signs are opposite.

These tables show that two CGE models which differ only in their functional structure[11] cannot be viewed as being only trivially different. By changing from nested CES functions to nested normalized quadratic functions, and consequently introducing into the model a wider range of price-interactions as well as different extrapolations of out-of-sample behaviour, the predicted responses to policy innovations are substantially altered.

7. Conclusions

The econometric critique of contemporary CGE modeling states (among other things) that the calibration approach leads to over-reliance on non-flexible functional forms. It has long been recognized by econometricians that functional forms in the CES class impose restrictions which are not theoretically justifiable and which are frequently rejected in formal tests. On these grounds the use of flexible functional forms are preferred for empirical modeling of firms' technology. Whether this matters for CGE modeling depends on whether CGE models are sensitive to the choice of functional form. A series of comparative simulations shows that the

[11] The CES and NQ models share a numerical structure in the sense that they are both estimated using a maximum likelihood criterion on the same database.

Table 9
Microeconomic results for CES-NQ comparison (percent changes)

Variable	1: Average income tax cut from 22.0 to 20.0%			2: 10% new tax added to services purchases			3: GST doubled from 7.0 to 14.0%	
	CES	NQ	(=)	CES	NQ	(=)	CES	NQ
Selected demands:[a]								
Natural gas	+0.6	−0.7		−1.2	+0.7		−4.1	−3.0
Refined fuels	−0.5	−0.4	●●	+0.6	+1.2	●	0.0	−0.5
Electricity	+0.2	−0.5	●	−0.4	+1.6		−2.8	+1.7
Durables	−1.0	−0.3	●	−0.1	+0.2	●●	−2.5	−0.3
Non-durables	−1.2	+0.2		+1.2	−1.3		−0.2	−0.9
Services	+0.2	+0.2	●●	−2.4	−1.1		−3.5	−1.7
Industry output:[b]								
Agriculture	+0.2	−0.1	●●	−1.7	−0.9	●	+0.4	−0.1
Mining	+0.9	−0.7		−1.4	+0.1		−3.9	−4.0
Refining	0.0	−0.2	●●	0.0	+0.2	●●	0.0	−0.9
Utilities	+1.5	0.0		−2.1	+0.4		−3.6	+0.3
Manufacturing	−0.7	−0.4	●●	+2.0	+1.4	●	0.0	+1.6
Services	−0.4	+0.1	●●	−0.8	−0.8	●●	−2.7	−1.6
Index of similarity			0.583			0.375		

(=) Denotes index of similarity between CES and NQ columns; see notes for Table 8.
[a] Includes imports.
[b] Includes exports.

functional structure appears to strongly influence the results from a policy simulation at both the industry-specific and macroeconomic levels, for large and small policy shocks. It therefore suggests that the use of first-order functional forms in a CGE model plays a non-trivial role in determining the results that will be obtained. Consequently, the preference for flexible functional forms in empirical microeconomics is a pertinent challenge for CGE modelers, which merits renewed attention.

Acknowledgements

Financial support was provided by the University of British Columbia, the Social Sciences and Humanities Research Council, the Tri-Council Eco-Research Secretariat and Environment Canada. This paper is a shorter version of the first chapter of my Ph.D. dissertation at UBC. Many helpful comments were received from John F. Helliwell, W. Erwin Diewert, John Kesselman, participants in seminars at UBC, the University of Guelph and the University of Waterloo, and an anonymous referee. Arlene Ells and Barb Zupancyk gave helpful research assistance.

References

Barnett, W.A., Kirova, M., Pasupathy, M., 1996. Technology modeling: curvature is not sufficient for regularity. Washington University, working paper no. 200.

Clements, K.W., 1980. A general equilibrium econometric model of an open economy. International Economic Review 21, 469–488.

Dean, A., Hoeller, P., 1992. Costs of reducing CO_2 emissions: evidence from six global models. OECD Economic Studies 19, 15–48.

Diewert, W.E., Wales, T.J., 1993. Linear and quadratic spline models for consumer demand functions. Canadian Journal of Economics XXVI, 77–106.

Diewert, W.E., Wales, T.J., 1988. A normalized quadratic semiflexible functional form. Journal of Econometrics 37, 327–342.

Diewert, W.E., Wales, T.J., 1987. Flexible functional forms and global curvature conditions. Econometrica 55, 43–48.

Diewert, W.E., Lawrence, D., 1994. The marginal costs of taxation in New Zealand. Report prepared for the New Zealand Business Round Table by Swan Consultants Pty Ltd., Canberra.

Fry, J., Fry, T., McLaren, K., 1996. The stochastic specification of demand share equations: restricting budget shares to the unit simplex. Journal of Econometrics 73, 377–385.

Hansen, L.P., Heckman, J., 1996. The empirical foundations of calibration. Journal of Economic Perspectives 10, 87–104.

Hazilla, M., Kopp, R.J., 1990. Social cost of environmental quality regulations: a general equilibrium analysis. Journal of Political Economy 98, 853–873.

Jorgensen, D.W., 1984. Econometric methods for applied general equilibrium analysis. In: Scarf, H., Shoven, J.B. (Eds.), pp. 139–203.

Jorgensen, D.W., Slesnick, D.T., Wilcoxen, P.J., 1992. Carbon taxes and economic welfare. Harvard Institute of Economic Research Discussion Paper 1589.

Lau, L., 1984. Comments on Mansur and Whalley's numerical specification of applied general equilibrium models. In: Scarf, H., Shoven, J.B. (Eds.), pp. 127–137.

Mansur, A., Whalley, J., 1984. Numerical specification of applied general equilibrium models: estimation, calibration and data. In: Scarf, H., Shoven, J.B. (Eds.), pp. 69–126.

McKitrick, R., 1996. The Econometric Critique of Applied General Equilibrium Modeling: A Comparative Assessment with Application to Carbon Taxes in Canada. Ph.D. Dissertation, the University of British Columbia, Vancouver.

Perroni, C., Rutherford, T.F., 1995. A comparison of the performance of flexible functional forms for use in applied general equilibrium analysis. University of Colorado working paper.

Pogany, P., 1996. The almost ideal demand system and its application in general equilibrium calculations. US International Trade Commission Office of Economics Working Paper 96-05-A.

Robinson, S., Soule, M.J., Weyerbrock, S., 1992. Import demand functions, trade volumes, and terms of trade effects in multi-country models. Unpublished paper, Department of Agricultural and Resource Economics, the University of California at Berkeley.

Shoven, J.B., Whalley, J., 1992. Applying General Equilibrium. The University Press, Cambridge.

Statistics Canada. Aggregate Productivity Measures Catalogue 15-204; Fixed Capital Flows and Stocks 13-211; National Balance Sheet Accounts 13-214; National Income and Expenditure Accounts 13-201.

Terrel, D., 1996. Incorporating monotonicity and concavity conditions in flexible functional forms. Journal of Applied Econometrics 11, 179–194.

Varian, H.R., 1984. Microeconomic Analysis, 2nd ed. Norton, New York.

White, K.J., 1978. A general purpose program for econometric methods — SHAZAM. Econometrica, January, 239–240.