COMMUNICATIONS

General Equilibrium Analysis of Excise Taxes: Comment*

In a recent paper, Paul Wells has presented an attempt at a general-equilibrium analysis of excise tax incidence, for a two-person two-factor two-commodity world, employing an ingenious geometrical technique similar in some respects to those commonly employed in international trade theory.[1] Such attempts to broaden the theoretical approach to tax incidence are of course to be welcomed, and Wells has opened a promising new line of attack: but unfortunately, in constructing his geometrical model Wells has incorporated a fundamental confusion between income and output which invalidates his analysis, or at least makes it depend for its validity on a peculiarly unrealistic assumption about the way in which income is distributed in his model economy. Revision of the model in the direction of orthodoxy is, however, relatively simple: it does not alter the qualitative conclusions reached as to the factors which govern the distribution of the burdens and benefits from excise taxes between individuals, though it does lead to the conclusion that the conditions under which an individual's benefit from an excise tax can outweigh its burden on him are more restrictive than Wells' analysis implies.

The fundamental confusion referred to is embodied in Wells' notion of a "division-of-output function" (KL in Figure 1, p. 347), which "states the ratio in which output is divided between A and B for all possible output combinations" (p. 346) in the community transformation curve. This cannot be correct. What the function must show is the division of *income*, or command over output, when factors are rewarded at their marginal productivities for the levels of production of the two goods shown by the corresponding point on the transformation curve. Alternately, and as Wells uses it in his subsequent analysis, the function shows the amounts of the two commodities each individual would receive if each were given a share in the total *output of each commodity* equal to the share he receives of aggregate income.

In his analysis of market equilibrium in the absence and presence of excise taxes, Wells takes the point on the division-of-output function corresponding to the production point on the transformation curve as representing the initial stocks of output held by the two individuals in the economy, and proceeds to show how they would modify these stocks through exchange in the market along familiar contract-box lines. This procedure implicitly assumes that factor owners are remunerated in pro rata shares of aggregate physical output; and it gives the model several peculiar features, one of which is that exchange takes place, and taxable capacity exists, only to the extent that individuals

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[1] Paul Wells, "A General Equilibrium Analysis of Excise Taxes," *Am. Econ. Rev.*, June 1955, XLV, 345-59. All page references in what follows are to this article.

wish to consume commodities in different proportions from the community average at the same price ratio.[2]

Obviously, the relevance of the model depends on the appropriateness of the assumption that factor owners are rewarded by the transfer to them of pro rata shares of physical output. This assumption seems highly unrealistic for the type of market economy with which Wells is concerned, in which production and exchange are separated;[3] it would be more reasonable to assume that factor owners are remunerated in generalized purchasing power, which they are free to spend as they like on commodities purchased from the production sector, and that excise taxes are levied on all such purchases of the taxed good.

In terms of Wells' geometrical construction, the division of income between the two individuals would be represented, not by a point on the division-of-output function, but by a line through that point having the same slope as the marginal rate of transformation, which would indicate the combinations of goods the individuals could purchase (at factor cost) with their incomes from factor ownership. Equilibrium in the absence of taxes would be determined, as in Wells' construction, by the tangency of indifference curves of the two individuals with this line at the same point on it. But the introduction of a tax on a particular commodity would be represented, not by a different line from the division-of-output point, but by two lines, one for each individual, starting from the intercepts of the income-division-line with the two sides of the contract box representing the untaxed commodity; and equilibrium individual consumption with excise taxation would be represented by tangencies of individual indifference curves with these lines.

The construction is illustrated in Figure 1, which represents a position of general equilibrium with excise taxation on clothing. In the diagram, PP' is the social transformation curve between food and clothing, and P_t is the

[2] In Wells' example, the farmer is assumed to want relatively more clothing and relatively less food than the worker; clearly the reverse might be true, making the farmer a seller of clothing and buyer of food (on the argument of the model) and requiring a modification of the analysis. Further, the introduction of a tax might lead to the elimination of exchange or a reversal of trading roles, possibilities not considered by Wells. Another peculiar feature of the model, emerging from the assumptions mentioned above, is that the Paretian optimum conditions are violated for one individual only (p. 353; Wells actually says "for at least one individual" but his analysis provides no basis for the insertion of the phrase "at least").

[3] The assumption might be made that factors are remunerated in kind and allocated to industries so that the proportions of A-owned to B-owned factors are the same in both industries—an allocation which could be justified by chance considerations. This would ensure each owner a pro rata share of physical output to hold or exchange; but the introduction of a tax would then give factor owners the incentive to evade taxes by reallocating factors to industries in accord with their demands for final output. A possible alternative assumption, that the productive sector has two parts, one of which uses A's factors and turns over their output to him, and the other of which uses B's factors and surrenders its output to him, would not lead to pro rata shares in physical output, since the two sectors would have different comparative advantages and be led to specialize partially or completely in food and clothing respectively. This assumption would also give rise to tax evasion through the productive sector, since taxation would give an incentive to factor owners to charge different prices for the same factor to the different productive sectors.

equilibrium social production point. D is Wells' division-of-output point, and MM' (with slope equal to the marginal rate of transformation at P_t) is the income-division line. With social production P_t, individual A receives an income equal in value, measured at factor cost, to any combination of goods along MM' (referred to the origin O). With this income he could purchase OM of food, the untaxed good; but his purchases of clothing are subject to the excise tax, and so the consumption possibilities open to him are represented by the consumption-possibility line MN rather than by MM', the difference in slope between the two lines corresponding to the difference between the market price and factor cost of clothing as determined by the rate of the tax. His equilibrium consumption combination, shown by the consumption point P_a, is determined by the tangency of one of his indifference curves (I_a) with his consumption-possibility line MN. Similarly, B's income is shown by MM' (referred to the origin P_t) but his consumption possibilities are shown by the consumption possibility line $M'N'$ and his equilibrium consumption point P_b is determined by the tangency of one of his indifference curves (I_b) with his consumption-possibility line. The gap between the two individuals' consumption points, representing P_aQ of food and QP_b of clothing, corresponds to the amounts of the two goods consumed by the taxing authority.[4] The tax has two apparent effects on welfare: besides the extraction of the tax proceeds from personal consumption, it violates the Paretian optimum condition of equality between the marginal rates of transformation in production and substitution in consumption.[5]

A third possible effect of the tax is a shift in the distribution of income brought about by a shift of demand and production from one good to the other. This shift, and the resulting "benefit" to one of the factor owners, may go either way, depending on whether or not the reduction in consumer demand resulting from the imposition of the tax is or is not offset by the increased governmental demand for the taxed commodity.[6] The point may be illustrated by reference to Figure 1. If the tax on clothing were removed, consumers' demands for goods would be shown by tangency of their indifference curves with the income-division line. If the tangency point for A lay to the left of that for B, there would be an excess demand for food at current prices, and food prices and production would have to increase to restore equilibrium; this would imply an increase in the relative income share of individual A, who is assumed to own relatively more of the factor (land)

[4] The construction can easily be extended to deal with the case of excise taxes levied on both goods, or of any kind of income tax. It should perhaps be emphasized that the application of this technique to tax problems requires the assumption that factor supplies are completely inelastic.

[5] In contrast to the situation in Wells' model, Paretian optimum conditions remain fulfilled as between the two consumers.

[6] It is not clear whether Wells recognizes the possibility that production may actually shift toward the taxed commodity; his geometrical analysis indicates an unambiguous shift of production towards the untaxed commodity, due to his assumption that taxes are spent only on the untaxed good, but his verbal argument (p. 356) allows the possibility of a different expenditure of tax proceeds and explicitly mentions the possibility of no production change, while he emphasizes that his investigation of government demand is incomplete.

used relatively more in food production. Conversely, if the tangency point for A lay to the right of that for B, there would be an excess demand for clothing which would have to be remedied by a production shift towards clothing and a corresponding increase in B's income share. More concretely, it can be shown that the imposition of a tax will require a production shift away from or towards the taxed commodity, according to whether the elasticity of consumer demand for that commodity is greater or less than the proportion of tax proceeds spent by the government on that commodity (government expenditure being valued at factor cost.)[7] On the assumption that normally demand elasticities exceed unity and that the untaxed good is not "inferior"

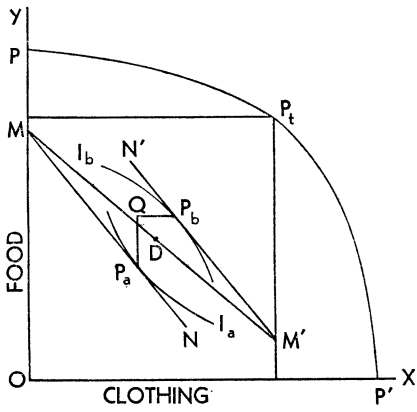

FIGURE 1

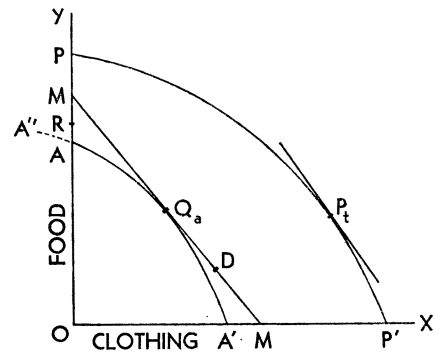

FIGURE 2

in government consumption, the tax must however produce a production shift towards the untaxed commodity. The same conclusion would follow from the assumption that the government's preference for the taxed good is no stronger than that of the taxpayers.[8]

[7] For low tax rates, the reduction in quantity demanded by consumers is $e.t.q$, where e is the elasticity of demand, t the tax rate (as a proportion of factor cost) and q the quantity initially purchased; tax proceeds are approximately $t.q.p$, where p is the untaxed price, equal to factor cost; and the change in quantity demanded is $(cp\text{-}e)tq$, where c is the physical quantity of the taxed good purchased by the government when its tax income increases by one unit, and cp is the money value of this quantity, or the proportion of an increment of tax proceeds which is spent on the taxed good, value and expenditure being measured at factor cost. The quantity demanded increases or decreases according to whether cp is greater or less than e; the economic meaning of this is stated in the text. A more accurate formula for the proceeds of the tax, which takes account of the consequential change in consumption, is $(1\text{-}\ et)\ tpq$.

The term cp may be interpreted as a governmental marginal propensity to spend on the taxed good. Since the consumer demand elasticity contains both a marginal propensity to spend and a positive substitution term, it follows that a necessary condition for demand for the taxed commodity to increase is that the government have a stronger preference for (higher marginal propensity to spend on) the taxed good than does the tax-paying community.

[8] The problem dealt with in this paragraph, and the analysis of it, exactly parallel a familiar problem in international trade theory, namely the conditions under which the imposition of a tariff turns a country's terms of trade against it.

Under the "normal" conditions defined in the preceding paragraph, the individual who owns relatively more of the factor relatively more required in the production of the untaxed good will derive an income "benefit" from the tax, in the form of an increase in his relative income share. The possibility arises that this benefit may outweigh the burden of the tax on the individual, leaving him better off than he would be in the absence of taxation. One way of expressing this proposition is to state that the increase in income may more than offset the increase in the price he pays for the taxed commodity. But this statement is misleading: it can be shown that, for a certain range of production shift, the benefit can only outweigh the burden if the effect of the tax is to reduce the market price of the taxed good (including the tax) below what it would be in the absence of the tax.

To pursue the analysis of this problem, it is necessary to investigate the behavior of the income-division line as production shifts towards food and away from clothing. This can be done rather simply, by drawing on the factor-price-equalization literature, and specifically on the proposition that in a two-country two-factor two-commodity free-trade world, with linear homogeneous production functions the same for each commodity in the two countries and characterized by the property that a commodity which makes relatively intensive use of a factor at one factor-price ratio does so at all factor-price ratios, absolute factor prices will be equalized as long as both countries continue to produce both commodities, even though factors are immobile between countries.[9] Reversing the argument for the present problem, the income obtained by a factor owner, and his share in total income, under different patterns of aggregate production can be determined from the transformation curve for the factors he himself owns, because his income would be no different if his factors could be combined only with each other, and their products sold, than if the factors themselves were sold freely and combined with factors owned by the other member of the community. The equivalence holds strictly only for patterns of aggregate production and commodity prices making it profitable to employ this set of factors in producing both goods; but the principle can be extended to take care of cases falling outside this restriction.

The procedure is illustrated in Figure 2, where PP' represents, as before, the social transformation curve, and AA' is the transformation curve for A's factors only. For the aggregate production point P_t, A's income would be the same as if his factors were employed in producing at the production point Q_a (the point at which A's marginal rate of transformation between products would be the same as B's and the community's); the income-division line would be MM' and Wells' division-of-output point would be at D.

For shifts of aggregate production towards food, up to the point at which the marginal social rate of transformation became equal to the slope of AA' at A, the income-division line would rotate around AA', and its intercept with the Y-axis would move towards the point A. From this it follows that individ-

[9] See P. A. Samuelson, "International Trade and the Equalisation of Factor Prices," *Econ. Jour.*, June 1948, LVIII, 163-84 and "International Factor Price Equalisation Once Again," *ibid.*, June 1949, LIX, 181-92; also I. F. Pearce, "The Factor Price Equalisation Myth," *Rev. Econ. Stud.*, 1951-52, XIX(2), 111-23.

ual A could only be made better off by a tax-induced shift of production towards food if the shift, and the accompanying reduction in the relative marginal cost of clothing production, went far enough to make the new tax-inclusive price of clothing lower than the old tax-free price. For suppose the tax shifts production so that the income-division line intercepts the Y-axis at R instead of M; A's consumption possibilities will now be shown by a consumption-possibility line through R, reflecting market prices including the tax, and it would only be possible for A to reach a higher indifference curve along this line than he did along the old income-division line if the new line intersected the old—which would require that the new market price of clothing, including the tax, be lower than the former tax-free price.[10]

The point A corresponds to complete specialization of A's resources on food production; alternatively, it is defined by the condition that factor prices are such that maximum-profit food production employs factors in the same ratio as A happens to own them. With a further shift of production towards food, the consequential variation in relative factor prices would make it profitable to hire additional labor from B to cooperate with A's land in producing food. This could be represented by a continuation of the AA' curve to the left of the *Y-axis* (AA″ in Figure 2), the cost of the hired labor appearing as negative clothing production by A. The income-division line would be determined by a tangency as before, and A's net income measured in food would be given by the intercept of the income-division line with the Y-axis. But the tangency point would now lie to the left of the Y-axis, and shifts of production towards food would now increase A's income measured in food; hence it would now be possible for the tax to make A better off on balance, even though the market price of clothing had been increased by the tax.

To summarize: for tax-induced shifts in production which leave the profitable land-to-labor ratio in food production higher than the ratio of land-to-labor possessed by A, the tax can only benefit A on balance if its effect is to reduce the market price (including tax) of clothing: for shifts which make the profitable land-to-labor ratio in food production lower than the ratio of land-to-labor possessed by A, the tax may benefit A on balance even though it raises the market price of clothing. These results underline the crucial importance of differences in asset-holdings as between individuals to the theory of tax incidence. They also suggest the need for an investigation of the conditions which would permit a tax to reduce the market price of the taxed good, an investigation which cannot be pursued here.

HARRY G. JOHNSON*

[10] This is of course only a necessary, not a sufficient, condition for A to benefit on balance from the tax.

* The author, formerly a fellow of King's College, Cambridge, is now professor of economic theory at the University of Manchester.