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THE UNITED KINGDOM TAX SYSTEM 1968–1970: SOME FIXED POINT INDICATIONS OF ITS ECONOMIC IMPACT

BY JOHN WHALLEY[1]

Proposals for the computation of competitive equilibria in the presence of taxation contained in recent joint work by the author are applied to a model of the United Kingdom economy and tax system for the period 1968–1970. Difficulties of model specification and parameterization are also discussed. Results provide indications of efficiency, distributional, and welfare impacts for a number of alternative tax changes.

1. INTRODUCTION

GENERAL EQUILIBRIUM ANALYSIS is now widely accepted as the theoretical basis for discussion of incidence and efficiency effects of taxation policies.[2] While misgivings may exist concerning the assumptions required, the overall consistency and elegance of the approach stand in sharp contrast to the sparseness of alternative methods of analyzing the impact of a whole tax structure on the micro-economic behavior of the economy. However, in spite of extensive discussion of the approach, quantitative estimates of general equilibrium impacts of taxation are sparse and when found usually concern themselves with the effects of a small change in a single tax.

A constraint on the ability to simultaneously consider the general equilibrium impacts of nonmarginal changes in multiple taxation instruments has been the lack of adequate methods for determining the resulting changes in equilibrium prices and quantities. This situation has been substantially changed by recent developments in computational methods following from Scarf's [**7**] work.

Extensions of that approach due to Shoven and Whalley [**10** and **11**], which explicitly incorporate taxation are used in this paper to analyze some general equilibrium impacts of the United Kingdom tax system for the period 1968–1970. Results of the overall system impacts of taxation arrangements in the United Kingdom are reported and serve to demonstrate the practical application of these techniques.

It should be emphasized that the ability to compute equilibrium prices and quantities by no means overcomes all obstacles to the micro-economic analysis of

[1] This work examines some implications of a general equilibrium model constructed for use in a Ph.D. thesis on the effects of tax changes introduced in 1973 in the United Kingdom. I am grateful to Professors H. E. Scarf and W. C. Brainard, to John Shoven, and to an anonymous referee for their help and comments at various stages in this work. I am also grateful to a number of seminar groups to whom these results have been presented and who have provided helpful comments. It should also be mentioned that the SSRC have supported the extension of this work to a more complex and detailed modelling. Some preliminary findings from these elaborations are contained in a progress report by Piggott and Whalley [**6**]; a more detailed monograph will appear in due course.

[2] In the introductory chapter of his public finance text Shoup [**8**, p. 15], for instance, comments "The habit of partial equilibrium analysis, strongly ... entrenched in most of public finance analysis after J. S. Mill and before Keynes ... has been carried over all too easily but invalidly to describe and analyse (changes) that requires a state of mind, if not the actual engine, of general equilibrium analysis."

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taxation systems; it merely lifts an important binding constraint. Many other difficulties remain, and these must not be underestimated. A complete model must be parameterized in numerical form prior to solution, and economic statistics are not organized in such a way that this is a simple and well-defined task. Difficulties of core storage, execution time, and data manipulation can transform what may appear to an outside observer as a modest undertaking into a major effort. Even when a numerical solution is finally obtained, the index number difficulties of comparing one equilibrium situation with another can be severe. In addition, any model, even if labelled "general equilibrium," must inevitably exclude features of reality on grounds of tractability alone. Nevertheless, the broad approach of attempting to provide numerical estimates of effects previously discussed qualitatively is one that has merit and the results given here represent a movement in that general direction which will, no doubt, be expanded upon as numerically soluble models become richer in detail. The model reported on here has already been the subject of some elaboration and extension, and a progress report on this activity has already been presented by Piggott and Whalley [**6**]. A more detailed monograph will appear in due course.

The modelling used here focuses on the combined static effects of distortions of commodity and factor prices resulting from all the major taxation instruments which existed in the United Kingdom during the period 1968–1970. The efficiency and distributional impacts of this combination of taxes are evaluated by replacing this existing tax structure by a single-rate broadly based sales tax yielding an unchanged level of aggregate tax receipts. Fifteen commodity groups chosen to capture the major distortionary effects of tax policies, and nine consumer groups each with separate preferences and endowments are specified within the model.

In Section 2 of the paper, the structure of general equilibrium models incorporating taxation is discussed and reference is made to the computation of equilibria in such circumstances. The computational methods used here are explained more fully in an appendix to the paper. Section 3 outlines the structure of the specific model of the United Kingdom economy and taxation system for the period 1968–1970 and discusses the numerical specification of the parameters of the model. Section 4 then presents a sequence of results, a summary of which is given in some concluding remarks.

Quite aside from the effects of taxation policies excluded because of the level of aggregation used, by no means all impacts of the United Kingdom taxation system are captured in the model presented here as the development of computable, large scale general equilibrium models is an ongoing but slow process of relaxing several constraints. The static effects of commodity distortions are analyzed but no inter-temporal effects via distortion of savings decisions of consumers are considered. No consideration is given to the effects of the taxation system on the supply of factors via distortion of the labor-leisure choice and an intermediate production technology is not incorporated into the model.[3] For these reasons, although the

[3] One of the extensions which has been considered by Piggott and Whalley [**6**] is the incorporation of intermediate production.

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major elements of a taxation system are modelled within a general equilibrium framework, estimates of impacts on the economic efficiency of the economy are in all probability lower bound estimates of "true" effects.

2. GENERAL EQUILIBRIA WITH TAXATION

The general equilibrium framework used here to assess the impacts of the United Kingdom tax system on the economy during the period 1968–1970 has been elaborated upon by the present author in a series of joint articles [**10**, **11**, and **12**]. A brief summary of the structure of static general equilibrium models which include taxation might be helpful at this point; the computational matters referred to in this literature are discussed in an appendix to this paper.

The static Walrasian general equilibrium model without taxation has a long history in economic analysis. In an economy with n commodities, a separate market for each commodity is distinguished where trading in that commodity (buying and selling) takes place at a single nonnegative market price Π_i ($i = 1, \ldots, n$). Traders entering any market on the demand side of the economy are thought of as determining the quantities of each commodity they wish to purchase from an examination of all market prices which they observe. They finance their purchases by sales of endowments of commodities. These endowments are treated as privately owned by consumers who, taken in aggregate, initially hold nonnegative quantities ω_i of each commodity. Sales of ω_i at the prices Π_i generate consumer incomes which, in turn, finance intended purchases.

The quantities of each commodity demanded by all the consumers in the economy are represented by the market demand functions $\xi_i\ (\Pi)\ (i = 1, \ldots, n)$, where the $\xi_i(\Pi)$ are assumed to be nonnegative continuous functions of $\Pi (\Pi = (\Pi_1, \ldots, \Pi_n))$, which are also homogeneous of degree zero in Π. The homogeneity assumption allows consideration of price vectors all normalized to a constant sum, such as unity. Market demand functions are also assumed to satisfy Walras law, that

$$\sum_{i=1}^{n} \Pi_i \xi_i(\Pi) = \sum_{i=1}^{n} \Pi_i \omega_i \quad \text{at any vector } \Pi.$$

On the production side of the economy m productive activities are assumed,[4] each of which may be operated by enterprise managers at any nonnegative level of intensity $X_j \geqslant 0$ ($j = 1, \ldots, m$). The term a_{ij} will be used to denote[5] the utilization of the ith good in the jth activity if that activity is operated at unit intensity. To rule out unlimited production possibilities, the set $X (X = (X_1, \ldots, X_m))$ such that $\sum_{j=1}^{m} a_{ij} X_j + \omega_i \geqslant 0$ for all $i = 1, \ldots, n$ must be bounded.

[4] In the model of the United Kingdom economy described in Section 3, this finite listing of activities is replaced by continuous production functions. This approach and notation is used in this section for consistency with the literature referenced earlier.

[5] Inputs into the production process are denoted by negative values of a_{ij} and outputs from the production process by positive values. The possibility of costlessly disposing of each of the n commodities may be explicitly incorporated by requiring that among the m activities appear a subset of n activities which have all zero entries except for a -1 corresponding to the index of the commodity.

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For this model equilibria can be shown to exist which are both Pareto optimal and able to be interpreted as the outcome of competitive processes where rival producers try to undercut each other so long as they do not make losses on any productive activity, and market prices change if there is a discrepancy between market demands and supplies. This notion of equilibrium is formalized in a definition of competitive equilibrium given by two sets of conditions which must prevail at an equilibrium vector of prices (Π^*) and activity levels (X^*).

(i) Demands equal supplies (including disposals if they occur):

$$\xi_i(\Pi^*) = \omega_i + \sum_{j=1}^{m} a_{ij} X_j^* \qquad (i = 1, \ldots, n).$$

(ii) No activity yields any producer the possibility of positive profit with those activities in use just breaking even:

$$\sum_{i=1}^{n} \Pi_i^* a_{ij} \leq 0 \qquad (=0 \text{ if } X_j^* > 0) \qquad (j = 1, \ldots, m).$$

A tax system may be incorporated into this model[6] by assigning to consumers and producers vectors of ad valorem tax rates which they must pay on commodity purchases, commodity sales, or income receipts. The tax payments accumulated by the central agency (the government) are treated as being income either of the central agency itself (in whole or in part), or of consumers who receive them by way of transfer payments. Thus, all tax payments collected through the various components of the tax system reappear as some trader's income (either personal or government income), and in equilibrium the government budget must be balanced as any government income remaining after transfers to persons must be spent on commodity purchases. The introduction of taxation into the economy will mean that the economy need not operate at a Pareto-optimal or even at an efficient allocation.

On the demand side of the economy, each of Q consumers is assigned a vector of nonnegative ad valorem tax rates $e^q = (e_1^q \ldots e_n^q)$ which must be paid by that consumer on expenditures on each of the n commodities. As transfer payments financed by tax receipts also enter the determination of consumer demands via incomes, the total tax revenue R is treated as being distributed among consumers according to a proportional distribution scheme $R^q = r^q R$; $r^q \geq 0$; $\Sigma_{q=1}^{Q} r^q = 1$.[7] The effect of this is that the market demand functions are dependent not only upon market prices but also upon the tax revenue R, and this is reflected in the notation $\xi_i(\Pi, R)$; where (Π, R) denotes the vector $(\Pi_1, \ldots, \Pi_n, R)$. The market demand functions $\xi_i(\Pi, R)$ are (as before) assumed to be nonnegative continuous functions but are now assumed to be homogeneous of degree zero in the vector (Π, R); the motivation being that a doubling of all prices and tax revenue doubles both incomes and consumer prices so that physical quantities demanded are

[6] In broad terms, the remainder of this section follows the structure suggested by Shoven and Whalley [11] in their analysis of economies with taxation.

[7] In this distribution scheme the government may be treated as one of the "consumers"; i.e., the share of the tax proceeds retained by the government is captured by one of the parameters r^q.

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unchanged. In contrast to a model in which taxation is absent, the focus is on relative market prices and total tax receipts, and a normalization $\Sigma_{i=1}^{n} \Pi_i + R = 1$, $R \geq 0$, can thus be used in place of the earlier normalization $\Sigma_{i=1}^{n} \Pi_i = 1$. In addition to vectors of expenditure tax rates, each of the Q consumers may be assigned a nonnegative income tax rate k^q which is charged on the value of taxable income.[8]

Walras law is still assumed to be satisfied by the demand functions $\xi_i(\Pi, R)$, but now in the sense that total net of income tax consumer incomes equal total gross of expenditure tax purchases at any vector (Π, R), i.e.,

$$\sum_{q=1}^{Q} \sum_{i=1}^{n} \Pi_i \xi_i^q(\Pi, R)(1+e_i^q) = \sum_{q=1}^{Q} \left(\sum_{i=1}^{n} \Pi_i \omega_i^q + R^q \right)(1-k^q)$$

where $\xi_i^q(\Pi, R)$ refer to the individual rather than the market demand functions.

On the production side of the economy, vectors of ad valorem tax rates T^j can be assigned to each of the productive activities where $T^j = (t_{1j}, \ldots, t_{nj})$, $j = 1, \ldots, m$. Any activity j operated a unit intensity where the producer faces a vector of prices Π will incur a tax liability $\Sigma_{i=1}^{n} a_{ij} t_{ij} \Pi_i$. The restriction that $a_{ij} t_{ij} \geq 0$ for any pair (i, j) ensures the nonnegativity of producer tax receipts from operation of any activity.[9]

As before, competitive equilibria can be examined but now these equilibria are defined in the presence of taxation. The prices which are thought of as ruling as market prices when taxation is introduced into the economy have to be defined with a little care. The prices included in the vector (Π, R) are those prices which are equalized across all the traders on any market. Thus prices Π_i are sellers' prices for inputs (prices net of producer input taxes) and wholesale prices for outputs (prices net of consumer output taxes but gross of any producer output taxes). An equilibrium in the presence of taxation is thus a vector $((\Pi^*, R^*), (X^*))$ such that:

(i) At least one commodity price is strictly positive:

$$\sum_{i=1}^{n} \Pi_i^* + R^* = 1; \quad \Pi_i^*, R^* \geq 0; \quad \Pi_i^* > 0 \quad \text{for at least one } i.$$

(ii) Demands equal supplies for all commodities (including disposals if they occur):

$$\xi_i(\Pi^*, R^*) = \omega_i + \sum_{j=1}^{m} a_{ij} X_j^* \qquad (i = 1, \ldots, n).$$

[8] Characteristics of income taxation systems such as an annually exempt amount of income, or expenditures on particular commodities can be incorporated into the model, although these are ignored in the remainder of this section in order to simplify the notation. Such features do appear in the model of the United Kingdom economy and tax system considered in subsequent sections.

[9] In the paper by Shoven and Whalley [**11**] subsidies were excluded in this manner as a general proof of the existence of a competitive equilibrium or a computational procedure guaranteed to find an equilibrium solution was not otherwise possible. For some public policy issues, general properties of these models may be of limited relevance and computation of equilibria in the presence of particular subsidies is usually possible. Subsidies are explicitly incorporated in the extensions considered by Piggott and Whalley [**6**].

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(iii) No activity yields any producer the possibility of any positive profit after payment of producer taxes, with those activities in use just breaking even:

$$\sum_{i=1}^{n} a_{ij}(1-t_{ij})\Pi_i^* \leq 0 \qquad (=0 \text{ if } X_j^* > 0) \qquad (j=1,\ldots,m).$$

In equilibrium the budget will be balanced, and so

$$R^* = \sum_{i=1}^{n}\sum_{j=1}^{m} a_{ij}t_{ij}\Pi_i^* X_j^* + \sum_{i=1}^{n}\sum_{q=1}^{Q} e_i^q \Pi_i^* \xi_i^q(\Pi^*, R^*) + \sum_{q=1}^{Q} k^q\left(\sum_{i=1}^{n} \Pi_i^* \omega_i^q + R^{q*}\right).$$

This framework has been followed in the model for which results are reported in this paper by parameterizing a general equilibrium model of the United Kingdom economy and United Kingdom tax system for the period 1968–1970. Components of the United Kingdom tax system are replaced by a single-rate broadly based sales tax which does not distort the economy away from an efficient or Pareto optimal allocation as under the observed tax system. Competitive equilibria are computed under the replacement tax scheme and comparisons between equilibria are made in order to assess the impact on the United Kingdom economy of the tax system during the period in question. The next section reports on the specific structure and parameterization of the model, a further section follows in which some results are reported.

3. AN OUTLINE[10] OF THE STRUCTURE AND METHODS OF PARAMETERIZATION OF A GENERAL EQUILIBRIUM MODEL OF THE UNITED KINGDOM ECONOMY AND TAX SYSTEM FOR THE PERIOD 1968–1970[11]

A. *Structure*

To utilize the framework outlined in Section 2 for an assessment of the impact of the tax system in the United Kingdom during the period 1968–1970, a general

[10] Specific values for the tax rates and other parameters of the model are not presented in this section due to the number of parameters involved. These are given in [**15**], and to a lesser extent in [**16** and **17**], to which the interested reader is referred. This lack of reporting should not disguise the importance of the parametric specification and structure for the set of results given later. It would seem an inevitable feature of general equilibrium analysis of this form which seeks "realism" that a large number of parameter values will be involved and that a readable summary of the model must leave much of the detail of the numerical specification unreported.

[11] A number of comments on this choice of time period are in order. The effort involved in parameterizing a model of this form is substantial and the process severely constrained by data availability. At the time the model was originally formulated, the period 1968–1970 represented the most recently available complete set of data. Three years were chosen (and an averaging procedure followed) to balance out particular years' observations. 1968–1970 seemed reasonably distant from tax changes introduced in 1965 that major adjustments would have taken place, and sufficiently far from tax changes introduced in 1973 (but announced earlier) that anticipatory effects could be ruled out. The extensions considered by Piggott and Whalley [**6**] yield a data requirement which has thus far made it impossible to use a similar averaging procedure due to the volume of work involved.

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equilibrium model must be specified detailing both its structure and parameter values. Clearly, the overall design of such a model must aim to capture the major[12] discriminatory elements of the taxation system in question, and the approach used here reflects an attempt to meet this objective in the light of data constraints and restrictions imposed by computational methods. This model has been used by the present author in an earlier examination of the impact of specific tax changes introduced in the United Kingdom in 1973.

The general equilibrium model of the United Kingdom economy and the tax system is a purely static construct which has nine[13] domestic industrial groupings and seven classes of personal sector consumers who are classified schematically[14] from rich to poor. The industrial groupings chosen not only capture major discriminatory features of the United Kingdom tax system but also reflect some of the difficulties of assembling data for a disaggregated general equilibrium model incorporating taxation. On the consumer demand side of the economy the groupings of individuals are determined by their (net of tax deduction) taxable income as reported in tax return survey data compiled by the Inland Revenue.

On the production side of the model, the finite listing of activities introduced in Section 2 is replaced by a sequence of continuous production functions. This procedure has several advantages; it allows fine measurement of factor substitution effects in response to tax changes, it enables national accounts data to be more easily used, and it reduces the computational problems of dimensionality involved in pre-specifying a long list of activities. Constant elasticity of substitution functions are used for each industry in which two factor inputs (capital and labor

[12] The discriminatory features of a tax system can, of course, only be captured in their entirety for a finely disaggregated model for which data are not available (to say nothing of the computational techniques for the dimensions involved). The nine domestic industry model considered here captures major commodity distortions but, as mentioned earlier, there are some notable exclusions. The distortion of the labor-leisure choice through the income tax system is not included due to the difficulty of adequately parameterizing labor supply responses to tax differentials. Although this was attempted by the present author in joint work [**10**] on assessments of impacts of United States capital income tax, the results are highly stylized and can obscure the more easily definable commodity tax distortions. In addition, no explicit treatment appears here for regional or subsidy policy, which in the United Kingdom are both important. The model considered by Piggott and Whalley [**6**] is considerably more detailed and covers some 33 domestic commodities and 100 domestic household groupings.

[13] These are Food and Agriculture, Petroleum and Chemicals, Metals and Mechanical Engineering, Light Engineering and Textiles, Construction, Transport and Communications, Retail and Wholesale Trade, Housing Services and Real Estate, Professional and Other Services. The financial sector is excluded from the model due to problems with national accounts data. See Whalley [**15**].

[14] Some elaboration on the word "schematically" is in order. An aggregation over income ranges used in a 1969–1970 survey of personal income is adopted. This has some undesirable features which cannot be avoided and are responsible for the use of the word in question. The concept of income in the survey differs from that used in the model, chiefly through the exclusion of the imputed income from owner-occupied dwellings. Income is also reported in the survey by range of income net of deductions and allowances on a "taxable unit" basis. Thus a single individual with a mortgage may appear as a poor taxable unit while a married couple both working and renting a flat may appear as a rich taxable unit. Moreover, as this paper is concerned with the general equilibrium impacts of tax replacements in a model where individual incomes are determined by sale of privately owned assets plus transfer payments received from the government, movements in relative prices may move some individuals between the income ranges defined in the Survey of Personal Incomes. There is no obvious way in which these data problems can be overcome; some further comment is provided in Whalley [**15**].

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services)[15] enter, i.e.,

$$Y_i = \gamma_i\{\delta_i K_i^{-\rho_i} + (1-\delta_i)L_i^{-\rho_i}\}^{-1/\rho_i} \qquad (i = 1, \ldots, 9).$$

Y_i refers to the output of the ith industry, and K_i and L_i are the usage of capital and labor services. γ_i is a constant defining units of measurement for outputs, δ_i is a "distribution" parameter assigning weights between factor inputs, and ρ_i is a parameter expressing the ease (or difficulty) of substituting capital for labor services. This form of production structure excludes intermediate production, a feature of some importance which must be borne in mind in evaluating the results in Section 4.

On the demand side of the economy seven groups of individuals within the United Kingdom personal sector are considered; for each, demand functions are derived by maximization of a utility function subject to a budget constraint. These groups have different utility functions which, in turn, reflect differences in preferences across income classes. Constant elasticity of substitution utility functions are used of the form

$$U^q = \left[\sum_{i=1}^{N} (b_i^q)^{z^q} \xi_i^{q(z^q-1)/z^q}\right]^{(z^q-1)/z^q} \qquad (q = 1, \ldots, 7),$$

where U^q is the utility index for the qth group, ξ_i^q is the consumption of the ith commodity by the qth group, b_i^q is a "weighting" parameter on the ith commodity by the qth individual reflecting the intensity of preferences for various commodities, and Z^q is a parameter which determines the ease with which substitutions can be made between commodities in the utility generating process. The utility functions are defined over N commodities rather than nine (the number of domestic industrial groupings) as a foreign trade sector with heterogeneous foreign products[16] is included in the model along with a limited form of public sector production. These additional outputs enter the utility functions along with the domestically-produced outputs. The methods by which parametric values are chosen for the utility and production functions are explained in more detail below and also in Whalley [**15** and **17**]. In the case of the utility functions, these methods appeal to family expenditure data as a guide to preference variations across income ranges. Preference parameters are also chosen so that they satisfy an overall consistency condition with respect to the basic data for the period 1968 to 1970. This is explained more fully below.

[15] The description of factor inputs (and consumer endowments) as "services" should be noted. It is assumed that there is an endowment of capital services all of which must be depleted in a single time period. This treatment puts on one side difficulties of the empirical treatment of "old" and "new" capital with stocks of assets being carried across time periods. It also allows profit data to be used to give a measure of capital services avoiding difficulties with unreliable capital stock data in non-manufacturing industries. There are, however, disadvantages to such an approach and a fuller discussion is given in Whalley [**15**]. Like many of the difficulties encountered in the empirical specification of this model these are problems little discussed in the literature as general equilibrium models are not usually directly confronted with the difficulties of empirical work.

[16] Six such commodities are considered reflecting the traded goods from the list of nine domestic commodities reported in Footnote 13. Construction, Retail and Wholesale Trade, and Housing Services and Real Estate are treated as nontraded commodities.

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The public sector enters the model both through the tax system itself and through public expenditures financed by tax receipts. The tax system which provides the government's finance for its expenditures comprises a collection of producer and consumer taxes of ad valorem form levied on commodities and an income tax where tax rates may vary between agents as outlined in Section 2. In the general equilibrium structure used here financial assets are excluded but the government budget is nonetheless brought into balance by an adjustment to government expenditures.[17] The model treatment of particular legal taxes which operated in the United Kingdom during the period may be briefly summarized as follows:

United Kingdom Tax System[18] by Legal Form and Treatment in Model

Legal Tax	Treatment in Model
1. Corporation tax	1. Tax on use of capital in various industries
2. Rates (form of property tax)	2. Tax on use of capital in various industries
3. National insurance and related contributions (form of social security tax)	3. Tax on use of labor in various industries
4. Selective employment tax (abolished in 1973)	4. Tax on use of labor in various industries
5. Purchase tax (abolished in 1973)	5. Consumer expenditure tax on classes of commodities
6. Specific excises and tariffs	6. Consumer expenditure tax on classes of commodities
7. Personal income tax	7. Separate personal income tax system identified with tax rates varying across income groupings

The ways in which these separate legal taxation arrangements introduce distortions into the economy is explained in Whalley [**15**] where the combined effects of these taxes are also discussed.

It is important to emphasize that the tax system viewed in legal form and effective form is considerably different and the data exercise involved in obtaining a reasonable numerical description of the effective tax rates is lengthy, tedious, and dependent in places on both data and assumptions of unknown reliability. The basic data on payments of legal taxes compiled according to the industry classification and individual groupings used here necessitate cross-classification proce-

[17] This treatment involves a relatively small adjustment for data reflecting the position in the late 1960's, but would involve a substantial adjustment in the 1970's. In the extensions considered by Piggott and Whalley [**6**], a treatment of financial transactions of both the public and external sectors in terms of underlying "real" counterparts is considered.

[18] Neither death duties nor capital gains tax are included in the model.

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dures[19] and changes from legal tax bases to the tax bases assumed in the model,[20] all of which impose further limitations on the reliability of the specification.

In addition to problems of data reliability, there is an assumption implicit in these specifications of the equality of average and marginal tax rates. Exception can (and has been) taken to this feature of the model, but as with other aspects of the approach it is a price of numerical tractability. Stiglitz [**14**], for instance, has recently presented an argument for viewing corporate taxes as lump-sum taxes which if accepted is not consistent with the treatment used here. The intricacies of corporate taxation and company financial policies are excluded in this model, which does not imply that these matters are unimportant, but rather not amenable to the present approach. The strength of the approach, in that simultaneous interactions between multiple tax instruments can be considered, inevitably means that the richness of the specification of each must suffer to some extent. Details of all taxation parameters used are available in [**15**], where there is also a discussion of further interpretative difficulties in viewing all legal taxes in equivalent ad valorem form.

On the expenditure side of the public sector account government tax receipts are used to finance both the provision of public authority goods and services and transfers to the personal sector of the economy. Transfers to groups of individuals reflect the size distribution of transfer payments in the period 1968–1970. Public authority expenditures on goods and services roughly match those recorded in the national income accounts[21] for the same period.

Three final complications affecting the structure of the model should be mentioned. Firstly, for the United Kingdom economy overseas trade accounts for a substantial portion of economic activity and so a foreign trade sector is incorporated into the model. One other country (the rest of the world (R.O.W.)) is assumed to exist with which the United Kingdom trades in heterogeneous products (all United Kingdom and R.O.W. products are heterogeneous). This heterogeneity is adopted in response to the empirical observation of the United Kingdom both importing and exporting similar commodities which, if homogeneous, would be inconsistent with competitive assumptions. The rest of the world is taken to be "big" relative to the United Kingdom so that R.O.W. prices change very little as the United Kingdom changes its tax system.

Secondly, United Kingdom nationalized industries cause some complications on the production side of the model. While in practice these industries tend to

[19] This is particularly the case with rates and selective employment tax.

[20] As with income tax where income as defined here includes the imputed income from owner-occupied houses.

[21] The valuation of public sector output is a problem which has long occupied social statisticians and is not "resolved" here. The national accounts approach is to value output by valuing inputs, the procedure also followed here. The public sector is thought of as providing a public good for which there is no direct articulation of demands; the government is treated as having a utility function defined over the factors of production used in the provision of the public good. This utility function is maximized subject to a government budget constraint. It is worth noting that as tax changes are introduced in the model, the capital intensity of public sector factor purchases can thus change as relative factor prices change.

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have autonomy in output decisions, their freedom in pricing and factor hiring decisions is more limited so that profit maximization is not as appropriate a decision rule as for other industries. This problem is resolved here by including public corporations in an "extended" public sector.[22] There is thus some public sector output (gas, electricity, railway services, etc.) which is sold at market prices to individuals and which enters their utility functions. A portion of public sector factor hirings is treated as being used up during the process of providing public sector output and is assumed to equal in value terms the sales value of this output.

Lastly, a model in which real public sector expenditures enter means that the "size" of the public sector is of some importance in the assessment of impacts of changes in the tax system. Factor services which move from employment in the public sector to the private sector surrender a value marginal product in the public sector equalized to the net of tax private sector return for a gross of tax value marginal product when employed in the private sector. While this may, in practice, be an accurate description of the operation of public sector processes it can give a substantial upward bias to estimates of the efficiency gain from the removal of the distortionary features of a tax. If no tax were to be introduced in place of the distortionary tax abolished, it would be unclear what component of any estimates of impacts on the economy would represent the operation of the tax alone rather than changes in the "size" of the public sector. For this reason a replacement tax is considered in all cases where there is an abolition of taxation instruments. Where a discriminatory tax is removed, a single-rate broadly based replacement sales tax preserves the "real" yield of the tax system.[23] This introduces some further computational complications which are outlined in the Appendix along with the fuller discussion of computational methods.

B. *Parameterization*

In addition to outlining the structure of the model, it is necessary to explain how individual parameter values have been specified for all demand and production functions, and tax rates selected. This involves a substantial exercise in data classification in addition to choosing a general approach.

When faced with the task of providing a complete parametric specification of a general equilibrium model, an understandable initial reaction is to examine the empirical work of other researchers in an effort to combine various extraneous estimates of demand and production function parameters into a set of parameters for the model. There are, in the author's opinion, a number of difficulties with this procedure which makes it of limited use.

[22] A different treatment is adopted in Piggott and Whalley [**6**] where nationalized industries are treated as part of the "industrial sector," and in this way "subsidy" policy towards nationalized industries is introduced.

[23] This replacement corresponds to what has been suggested by Musgrave through the term "differential incidence." There are obvious index number difficulties in the definition of "real" tax yield and a Laspeyres price index is used here. The computational difficulties that this replacement scheme introduces are further explored in Shoven and Whalley [**13**].

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A practical difficulty is that estimates are often surprisingly sparse and where available usually refer to a quite different classification of commodities (or industries, or households) from that used in the general equilibrium model. A further difficulty is that, quite aside from the classification problems, extraneous estimates of parameter values which are not unit-free are of limited applicability where differing units are used and conversion between units is not possible. For this reason use of extraneous estimates is restricted here to parameters which are unit-free (such as elasticities of substitution). Finally, there is the point that a sequence of extraneous parameter values will produce a specification of the model for which the performance need in no way resemble "overall behavior" for the chosen period. Thus, a model which is meant to exhibit some correspondence to an (assumed) equilibrium state in the United Kingdom before any tax changes are introduced, requires some check on the ability of the model to reproduce this state as an equilibrium solution.

As a broad approach to the problem of parameterization, a body of data has been assembled (production expenditure, income distribution, and tax data) which is assumed to represent an equilibrium state in the United Kingdom in the presence of an existing tax structure: A set of values for the parameters are then chosen which are capable of replicating this state as an equilibrium solution to the model; i.e., the equilibrium conditions of the model along with the data are used to determine parameter values. This use of data also leads directly to the definition of units in that these are chosen such that for all outputs and factor services the equilibrium prices are unity[24] at the observed distortionary equilibrium.

There are clearly a very large number of combinations of parameters that are consistent with a single "observed equilibrium" and the choice of parameters must be restricted in some way. The procedure which is used is to take extraneous estimates of unit-free parameters (such as elasticities) and calculate all other parameters by a reverse solution of the model. An observed use of factors by each industry at the (gross of tax) equilibrium prices implies weighting parameters in production functions once production elasticities are chosen. Equally, expenditure patterns by households imply utility function parameters once elasticities are chosen for utility functions. A procedure of solution of equilibrium conditions is thus followed for all but unit-free parameters of the model and in this way the necessary sequence of parameter values is fully specified once the extraneous values of unit-free parameters are given. As the empirical evidence on the extraneous parameters is in places sparse, the result of this process is still indeterminate if only extraneous values are used which have a firm empirical basis. Thus, a combination of "best guess" values is used and some sensitivity

[24] This units definition is a more extensive use of a procedure originally used by Harberger [**3**]. For the factor services the assumption is that there is a homogeneous physical unit (of unknown physical dimension) which is capable of generating £1 of net of tax income in all alternative uses. The mobility assumed in the model of factors between industries, and of goods between consumers, is thus inherent in the units definitions. This procedure has the twin features of an inability to articulate the underlying physical measures, while enabling national accounts data to be used directly once organized on a consistent basis for the detail involved. These issues are more fully discussed in Whalley [**15**].

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experiments are used to assess the robustness of results to the chosen values. A catalog of both extraneously chosen and calculated parameter values is given in Whalley [**15**].

Once the structure has been specified and parameter values chosen, the model can be used for tax replacement experiments. Combinations of taxes can be changed or abolished and replacement taxes which preserve the yield of the taxation system introduced and the resulting equilibria computed. In the next section a set of results from experiments to assess the overall impact of the whole of the tax system is reported.

4. RESULTS OF FIXED POINT CALCULATIONS OF THE EQUILIBRIUM BEHAVIOR OF THE UNITED KINGDOM ECONOMY UNDER A REPLACEMENT TAX SYSTEM

The methods described in the Appendix applied to the model outlined above yield competitive equilibria for the general equilibrium model of the United Kingdom economy under a number of alternative tax regimes. In this section results from the abolition of all components of existing taxation arrangements and their replacement by a yield-preserving broadly based tax are reported.

Assessing the impact of the United Kingdom tax system involves examination of changes between equilibria. The computational methods used here enable a full description of alternative equilibria to be calculated and so variations in all individual utility levels, consumer purchases of each commodity, factor employments by industry, industry production levels, incomes both before and after tax, and relative prices can be obtained. Clearly, a complete presentation of all these data is not possible and so a number of summary statistics have been calculated for each tax replacement considered. The behavior and properties of these summary statistics have been explored at a theoretical level (see, for example, [**1**]) only for much simpler models than that used here and so a precise correspondence to proven results cannot be claimed. A "welfare" interpretation of these statistics is not stressed as it has been in simpler models (see, for instance, [**3** and **4**]) as welfare "results" are given directly by utility level variations.

To give some idea of the impact of the tax system in terms of economy wide productive efficiency losses, the change in the value of production (GNP) between two equilibria (the 1968–1970 equilibrium and the tax replacement equilibrium) has been calculated. The index number difficulty of two possible sets of prices being available arises from the calculation of both the distortionary equilibrium prices, and the nondistortionary equilibrium prices. Each of the resulting indices (the Laspeyres and Paasche indices) is presented in the cases reported below and it will be seen that the discrepancies are in some instances substantial; indeed, sometimes a difference of sign enters. These two estimates are interpreted as providing the bounds on the efficiency impacts of tax replacements; in some of the cases these bounds are widely separated. A mean of the two indices (such as a geometric mean, the Fisher ideal index) could be used to give a single number but such is not reported here; equally, no Divisia indices are reported.[25]

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To give some assessment of the impact of tax replacements on personal and functional distributions of income two further figures are given. The share of income is given (gross of receipts of transfers from the government but net of income tax) which accrues to the top group of income recipients identified by their position in the 1968–1970 equilibrium. Movements in shares of other groups are available but are not reported here. Due to the use of a different income concept in this model relative to that reported in the underlying data it is not possible to identify the members of this group as a particular percentage of the total number of all households or taxable units. They are identified as that group which received 4.48 per cent of income[26] (gross of transfers, but net of personal income tax) in the 1968–1970 equilibrium. Equally, due to the treatment of the income tax system as a tax on a broader income base than that defined for tax purposes, there is a partial correction in the model for some of the features of the "erosion" of the income tax base referred to by Pechman [**5**] and others. Thus the removal of income taxes does not have as strongly a redistributive effect as one might have thought from an examination of the legal income tax rates. In fact, as will be seen below, the removal of the income tax system does not give as big a gain for this top group as removal of the system of taxation of income from capital (primarily due to corporate taxes affecting the size of potential dividend payments). A further insight into the distributional impact is provided by estimates of functional shares (the ratio of total receipts of capital income to total net of tax factor rewards). It is to be emphasized that these functional distribution figures for the 1968–1970 period are known to differ somewhat from those for more recent years,[27] but some indication of the potential size of functional impacts is of a continuing interest in the public finance literature, especially in the context of corporate taxes.

To give a more direct assessment of welfare effects in terms of the distributional impact of tax replacement, the percentage change in utility levels for the poorest and richest groups (again identified by their relative positions in the 1968–1970 equilibrium) is given. Again, variations in these levels for all groups are available but are not reported. These numbers have to be viewed with some scepticism as regards absolute values; any monotonic transformation of a group's utility function could be chosen which would leave the demand functions for that group unchanged but considerably alter the percentage utility change between equilibria. Nevertheless, it is important to emphasize that distributional studies, which

[25] The calculation of a Divisia index of quantity variations would need characteristics of an adjustment path between equilibria. The computational methods used here are purely calculation devices which yield no interpretable adjustment path; and furthermore only a limited number of points in the calculation process are identifiable and these have no meaningful interpretation. For this reason, calculation of Divisia indices is not a practical proposition here. It would be of interest in further calculations to decompose a "large" change (e.g., abolition of a tax) into several smaller changes, calculate each of the equilibria involved and make comparisons between "chained" and "unchained" indices. This, however, has not as yet been done either for this model or the extended variant considered by Piggott and Whalley [**6**].

[26] This could be taken as very loosely representing the top 1 per cent of taxable units. See "National Income and Expenditure 1969" (HMSO).

[27] See, for instance, "National Income and Expenditure 1963–1973" (HMSO), Table 26, p. 32.

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focus solely on income rather than also looking at welfare levels can be misleading if a direct welfare interpretation of movements in income distributions is claimed. Whalley [**17**], for instance, in examining particular tax changes in the United Kingdom for 1973 finds groups for which incomes rise and utility levels fall, and vice versa, due to the change in consumer prices. In the results reported here it is found that for certain tax replacements the total incomes for a particular group of consumers may rise more than for another tax replacement while utility levels rise less.

A number of results from tax replacements are reported on in Table I for the "best guess" set of parameters and Table II attempts to provide some indication of the robustness of results. The results appearing in each table differ by the type of tax in the United Kingdom tax system which is abolished; a separate tax change is reported in each column. In all cases a broadly based sales tax[28] paid by all

TABLE I

ASSESSMENT OF THE IMPACT OF THE UNITED KINGDOM TAX SYSTEM

(All figures in percentage terms)

"Best Guess" Parameterization[a]	Tax Replacement 1	2	3	4	5	6
Δ GNP (percentage terms)—nondistortionary prices	0.78	−0.13	−0.47	−0.21	0.40	−0.45
Δ GNP (percentage terms)—distortionary prices	2.73	0.12	−0.47	−0.22	2.96	3.05
Share of income (net of tax, gross of transfers) received by top group (recipients of 4.48 per cent of income in distortionary equilibrium)	4.92	4.36	4.48	5.66	4.85	6.46
Relative share of capital (23.12 per cent in distortionary equilibrium)	26.88	21.83	23.10	22.96	25.47	27.37
Δ Utility (percentage terms)—richest group	17.17	−0.59	−2.07	19.45	16.73	48.86
Δ Utility (percentage terms)—poorest group	2.41	0.49	−0.63	−10.77	2.87	−4.97

Replacement in all cases by single-rate broad based sales tax

Tax Replacement 1: Abolish distortionary capital taxation.
Tax Replacement 2: Abolish distortionary labor taxation.
Tax Replacement 3: Abolish distortionary output taxation.
Tax Replacement 4: Abolish income taxation.
Tax Replacement 5: Abolish distortionary capital and labor taxation.
Tax Replacement 6: Abolish all existing taxation.

[a] See text for details of methods of parameterization. Details of parameter values are given in Whalley [**15**].

[28] The type of tax which is introduced in place of the distortionary tax abolished may be varied and, in fact, does affect the results although not too markedly. The interested reader is referred to Whalley [**15**] where distortionary capital income taxation is replaced by a broadly based tax on capital income in all industries in addition to the replacement considered here.

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TABLE II

ASSESSMENT OF THE IMPACT OF THE UNITED KINGDOM TAX SYSTEM UNDER PARAMETRIC VARIATIONS IN THE MODEL

(All figures in percentage terms)

Parametric variation I All domestic production elasticities equal 0.5[a]	Tax Replacement 1	2	3	4	5	6
Δ GNP (percentage terms)—nondistortionary prices	0.67	−0.13	−0.47	−0.22	0.28	−0.52
Δ GNP (percentage terms)—distortionary prices	2.56	0.12	−0.47	−0.23	2.78	2.90
Share of income (net of tax, gross of transfers) received by top group (recipients of 4.48 per cent of income in distortionary equilibrium)	4.87	4.36	4.48	5.66	4.77	6.38
Relative share of capital (23.12 per cent in distortionary equilibrium)	25.23	21.85	23.10	22.94	24.85	26.89
Δ utility (percentage terms)—richest group	15.06	−0.58	−2.07	19.39	14.68	46.92
Δ utility (percentage terms)—poorest group	2.32	0.49	−0.63	−10.77	2.77	−5.01
Parametric variation II All domestic production elasticities equal 2.0						
Δ GNP (percentage terms)—nondistortionary prices	1.25	−0.27	−0.46	−0.19	0.92	−0.02
Δ GNP (percentage terms)—distortionary prices	4.03	−0.01	−0.46	−0.19	4.20	4.05
Share of income (net of tax, gross of transfers) received by top group (recipients of 4.48 per cent of income in distortionary equilibrium)	5.43	4.34	4.48	5.67	5.27	6.87
Relative share of capital (23.12 per cent in distortionary equilibrium)	30.53	21.65	23.10	23.00	28.73	29.95
Δ utility (percentage terms)—richest group	29.56	−1.16	−2.04	19.64	28.01	59.81
Δ utility (percentage terms)—poorest group	3.01	−0.40	−0.63	−10.75	3.47	−4.66

Replacement in all cases by single-rate broad based sales tax

Tax Replacement 1: Abolish distortionary capital taxation.
Tax Replacement 2: Abolish distortionary labor taxation.
Tax Replacement 3: Abolish distortionary output taxation.
Tax Replacement 4: Abolish income taxation.
Tax Replacement 5: Abolish distortionary capital and labor taxation.
Tax Replacement 6: Abolish all existing taxation.

[a] See Footnote a, Table I, and the text.

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consumers on purchases of all commodities[29] in the United Kingdom is introduced at a single rate which maintains the yield of the tax system in "real" terms.[30] Each table contains results from each of six tax replacements denoted as Cases 1 through 6 which are each described at the bottom of the table. For each of these cases the six summary statistics discussed above are presented in the columns which form the table. The summary statistics are described on the left-hand side. Table I considers what is termed the "best guess" parameterization of the model reported in detail by Whalley [**15**]. Table II treats the same cases as in Table I with the same replacement tax; the underlying parameters of the model are, however, altered. In Table II all the production function elasticities of substitution are set at first 0.5, and then at 2.0. These tables attempt to provide information on the sensitivity of estimates to parametric values, and may perhaps be viewed as loosely providing bounds on the estimates with respect to a range of values for one of the sets of parameters.[31]

Tables I and II may be taken as an indication that the overall static efficiency losses resulting from the distortions considered in this model[32] which operated in the United Kingdom tax system during the period 1968–1970 are bounded from below by −0.6 per cent GNP and from above by 4.1 per cent GNP. One might perhaps suggest that the effects would be more likely to be in the region of 1 to 3 per cent of GNP. The range 1 to 3 per cent of GNP is a particular construction placed on the figures appearing in Tables I and II by the author with which some might disagree. Column 6 in Tables I and II gives the results from the total replacement of the United Kingdom tax system by a broadly based sales tax at a single rate on all commodities and the above ranges come from these figures. Rows 1 and 2 in Tables I and II indicate that the major portion of the gains come from a removal of the distortionary features of the system of taxation of capital income, much more so than from any of the other components of the tax system. In fact, in results not reported on here the indication is that of the two legal taxes that make up the distortionary capital income tax system the rating system (property taxation) yields a slightly bigger gain if abolished than does corporation tax. Distortionary labor income taxation (with distortions operating primarily through selective employment tax) does not seem to result in a serious loss (Column 2 in Tables I and II). There is, in fact, a significant element of counterbalancing of distortions between the two components of the factor tax

[29] In those cases where the existing output tax system is not removed but some other component of the tax system is removed, the single rate tax acts as a surtax on the existing output tax structure.

[30] See paper by Shoven and Whalley [**13**] where this notion of "real" is discussed in more detail.

[31] Whalley [**15**] also contains some results which report sensitivity of calculations to variations in elasticities of substitution in utility functions. For cases differing only slightly from those reported on here, "best guess" utility function elasticities are replaced by common values set first at 0.5, 1.0 (the Cobb-Douglas case), and then 2.0. The sensitivity of estimates of changes in the value of production are of a similar magnitude and direction to those from similar variations in the production function elasticities reported here.

[32] It should perhaps be emphasized, once again, that the distortion of the labor-leisure choice, the question of intertemporal effects, and the role of intermediate production are all left out of these calculations.

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system noted by Whalley [**16**] which is reflected in results in Column 5 of the two tables when both factor tax systems are abolished together.

The distributional impacts from these tax replacements appear to be significant. The share of the top group of income recipients varies between a range of 6.9 per cent and 4.3 per cent depending upon the best and worst possible tax replacements from the point of view of that group. This may not seem that large to those who have a view of the United Kingdom income tax system as steeply progressive, but two features must be borne in mind. Firstly, the income base considered here includes some "erosion" of the legal tax base, particularly through the inclusion in the income base of the imputed income from owner occupied houses, and secondly the steep progression in marginal tax rates in the United Kingdom tax system only captures a small number of individuals who will, in turn, form only a component of the top income group in this model. This second feature is compounded by an assumed treatment in the model (forced on grounds of tractability) of the top group as a single group facing the same income tax rate. It is striking, however, that in the bottom portion of Table II the top group of individuals would seem to gain as much from a removal of the capital income taxation system (through the concentration in ownership claims, particularly of equities) as from a removal of the income tax system. The figures for the impact on the functional distribution of income from the various tax replacements are also striking (Row 4 in Tables I and II). While Table I does not produce quite as large an effect as Table II the indication is that the relative share of capital can be affected by as much as nine percentage points through the choice of tax replacements. A total removal of the tax system (with a broadly based sales tax in its place) would affect the share of capital by up to seven percentage points.

The impacts of tax replacements on welfare levels are presented in Rows 5 and 6 of Tables I and II. It is to be re-emphasized that the exact figures given in these rows are of limited meaning as the numbers relate to the assumed preference functions for the groups of individuals and monotonic transformations of these functions which would preserve the demand functions for these groups would alter the percentage figures. Nevertheless, these figures are instructive on a number of counts. The only instances where both groups of individuals are better off in utility terms are in Cases 1 and 5 in each table where there is an efficiency gain from the removal of capital income taxation which, while going largely to the richer group, also benefits the poorer group whose receipts from transfers rise. The only change which is both pro-poor and anti-rich is the abolition of the labor income tax system and the changes are relatively small compared to the replacement of capital income tax. Abolishing income taxes alone is the most notably anti-poor of the possible measures (Case 4). The rich group seems to do considerably better from a total removal of the tax system than one would suppose from an addition of their gains from the piecewise replacement of the tax system. This is to be accounted for both by the nonlinearity of the preference functions and the direct compounding of the effects when considered as one package.

In Table I, Cases 2 and 3 yield examples where the greater loss in income terms

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for the richer group is accompanied by the smaller loss in utility terms due to the differential movements in output prices.

This emphasizes the point that income distribution calculations of the distributional impact of institutional changes such as tax changes have to be qualified in terms of their welfare interpretation. The change in the size distribution of "utilities" may not reflect the change in the size distribution of incomes particularly if substantial changes in relative consumer prices are anticipated. While for empirical purposes it is usually impractical to work directly with utility functions, a word of caution in the presentation of income variations would seem advisable where no account can be taken directly of changes in output prices.

5. SUMMARY

This paper reports results from some general equilibrium computations aimed at an assessment of the impact of the United Kingdom tax system on the United Kingdom economy during the period 1968–1970. Computational techniques for general equilibrium models with taxation developed in recent joint work by the author are applied to a model of the United Kingdom economy and tax system which is parameterized from national accounts data for the period. For reasons of both data reliability and tractability of the undertaking, these calculations exclude the distortion of the work-leisure choice in the income tax system, exclude intertemporal effects, and do not take joint production into account. The parameterization of the model is also inevitably schematic in certain respects and so no exact forecasting is either attempted or suggested but instead some guide is sought to ranges of likely effects.

Results indicate that for the model considered static efficiency losses from the industrial discrimination in the United Kingdom tax system lie in a wide range with an upper bound of around 4 per cent GNP. The major portion of any gain would appear to originate in the capital income tax system. The distributional impacts are obtained both in terms of the size distribution of incomes, the functional distribution of incomes, and the changes in welfare levels. For the top group of income earners (approximately 1 per cent of "taxable units") there is a difference of between 7 per cent and 4.3 percent in their shares of total income (net of tax, but gross of transfers) in the best and worst (from this group's point of view) of the possible tax replacements considered. Differences in the functional distribution of up to 9 per cent are obtained. Results also indicate that movements in the size distribution of incomes and movements in utility levels can be somewhat different due to variations in output prices as taxes are changed. Extensions of the approach which accommodate both more detail and alternative model specifications have been undertaken, and a progress report containing some preliminary findings is contained in Piggott and Whalley [**6**].

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APPENDIX ON COMPUTATIONAL PROCEDURES

Once the model outlined in Section 3 has been parameterized, a competitive equilibrium which is otherwise unobservable has to be computed and compared to an observed distortionary equilibrium (the United Kingdom during 1968–1970). It is for this form of problem that computational techniques developed by Scarf [**7**] for the approximation of a fixed point of a continuous mapping can be used. For the model analyzed here, however, routines designed to refine the approximation from a Scarf procedure can be used in their own right as computational devices (see [**10**, Appendix A]) and the way in which these devices have been adapted for use here is reported in this Appendix. It should be emphasized that termination routines (Newton methods) of the form used here have no argument available to guarantee finding a solution, as is true with a Scarf procedure and may work well locally but fail globally. The problem of nonconvergence has not been encountered here but it may, of course, occur in other problems. The important feature is that the routines used here have proved to be quicker in execution time for these kinds of problems than a convergence guaranteed simplicial subdivision procedure.[33]

The methods used here for computing equilibria given the new tax system and the parameters of the economy referred to in the text are here explained in more detail.[34] As defined in Section 2, a competitive equilibrium in the presence of taxation consists of three conditions: the positivity of at least one commodity price, demand supply equalities for all commodities, and zero profitability conditions (net of producer taxes) for all activities available in the economy. This is the equilibrium concept which is used here; the equal-yield replacement tax calculations discussed in the text require some small modifications to the correspondence and these are discussed in Shoven and Whalley [**13**].

The routines used here to calculate an equilibrium make use of the specific structure of the model of Section 3 in two crucial ways. Firstly, by working only in the space of factor prices and tax revenue it is possible to generate the corresponding output prices which guarantee zero profit in each industry for the cost-minimizing activity. This in turn ensures all other activities not in use (those represented by the industry production functions) make losses. Secondly, the cost-minimizing activities in each industry can be treated as operating at levels which meet consumer demands for industry outputs as determined at the output prices calculated from the factor prices. These two devices directly impose two portions of the equilibrium conditions at each stage of the search procedure for a true equilibrium and this considerably simplifies the computational requirements. Zero profitability in all industries is directly imposed, as are the demand supply equalities for industry outputs.[35] It is these two devices which make computation a manageable proposition for this model. The condition of the positivity of at least one price has never in practice been violated although not directly imposed by these methods.

Consider a sub-simplex of dimension Z where $Z<N$. All vectors $(\Pi_1, \ldots, \Pi_Z, R)$ on this sub-simplex contain the factor prices[36] and tax revenue and are normalized such that $\Sigma_{i=1}^{z} \Pi_i + R = F$, $0<F<1$. We consider a point on the sub-simplex $(\tilde{\Pi} = \tilde{\Pi}_1, \ldots, \tilde{\Pi}_Z, \tilde{R})$ which can be designated the "initial point." For convenience this can be taken as the observed equilibrium in the United Kingdom for the period 1968–1970. This point can be surrounded by a sequence of vectors $\tilde{\Pi}^1, \ldots, \tilde{\Pi}^z, \tilde{\Pi}^{z+1}$ each of which is perturbed from $\tilde{\Pi}$ sequentially in the coordinate corresponding to the index of its superscript. The vector $\tilde{\Pi}^1$ contains an element $\tilde{\Pi}_1^1$ decreased by x per cent from $\tilde{\Pi}_1$, $\tilde{\Pi}^2$ contains an element $\tilde{\Pi}_2^2$ decreased by x per cent from $\tilde{\Pi}_2$, and so on. For all these vectors, the normalization to sum to F is still applied.

[33] For the argument that a solution must be found, Scarf procedures rely on the finiteness of the number of possible "portions" of the simplex (primitive sets) which can be examined. In practice this has in the past involved taking a specific grid of points on the simplex with a mesh of constant size, although methods which allow some flexibility in this procedure have recently been developed. The ability to examine "pieces" of simplices of varying sizes in these current methods reduces execution times but at the cost of an *ex ante* convergence argument.

[34] See Shoven [**9**] who is responsible for the development of the class of routines outlined here, although some important simplifying devices used for this application have been adapted specifically to the model under consideration.

[35] And, hence, the excess demands for commodities are equal to zero. This "scaling" of cost minimizing activities will yield aggregate derived demands for productive factors and, hence, excess demands by subtracting economy-wide factor endowments.

[36] These will also contain the rest of the world factor prices in the model used here. In addition, the price charged for public sector output (nationalized industry outputs) is also included in this sub-simplex. This price is needed for the determination of demands for this class of commodities as no explicit production function is specified in the model.

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Corresponding to each of these vectors $\tilde{\Pi}^j$ there are Z derived excess demands for factors of production which can be determined, and one government budget imbalance (surplus or deficit). These can be represented by the $(Z+1)$ excess 'demands' $f_i(\tilde{\Pi}^j)$. If for any vector $\tilde{\Pi}^j$ all these $(Z+1)$ excess demands $f_i(\tilde{\Pi}^j)$ were equal to zero this would mean that the conditions for a competitive equilibrium would be met and a competitive solution would have been determined. This situation will typically not apply at the initial point and some systematic search procedure yielding an equilibrium price vector must be used.

For this purpose, all weighted combinations of the vectors $\tilde{\Pi}^1, \ldots, \tilde{\Pi}^{z+1}$ which also lie on the sub-simplex $\Sigma_{i=1}^{z} \tilde{\Pi}_i + \tilde{R} = F$ are considered. The weights on these vectors are denoted by α_j, where $\Sigma_{j=1}^{z+1} \alpha_j = 1$, $\alpha_j \geqslant -\theta$. The last of these conditions represents a negative bound on each weight α_j which prevents the weighted vector $\Sigma_{j=1}^{z+1} \alpha_j \tilde{\Pi}^j$ straying "too far" from $\tilde{\Pi}$.

Having constructed the weighted vector $\Sigma_{j=1}^{z+1} \alpha_j \tilde{\Pi}^j$, an assumption is used which it is known will be violated at each stage of the operations to be performed, but it is hoped will hold to a closer and closer approximation as the calculation procedure gets closer to an exact solution. The assumption is that the $(z+1)$ excess demands at $\Sigma_{j=1}^{z+1} \alpha_j \tilde{\Pi}^j$ are approximated by the weighted sums of excess demands at $\tilde{\Pi}^1, \ldots, \tilde{\Pi}^{z+1}$, or

$$f_i\left(\sum_{j=1}^{z+1} \alpha_j \tilde{\Pi}^j\right) \approx \sum_{j=1}^{z+1} \alpha_j f_i(\tilde{\Pi}^j) \qquad (i = 1, \ldots, z+1).$$

This allows a problem to be stated to

$$\min \varepsilon$$

subject to

$$\sum_{j=1}^{z+1} \alpha_j f_i(\tilde{\Pi}^j) \leqslant \varepsilon \qquad (i = 1, \ldots, z+1),$$

$$\sum_{j=1}^{z+1} \alpha_j = 1, \qquad \alpha_j \geqslant -\theta,$$

which can be solved through the solution of the transformed linear programming problem

$$\max \sum_{j=1}^{z+1} y_j$$

subject to

$$\sum_{j=1}^{z+1} y_j(f_i(\tilde{\Pi}^j) - C_i) \leqslant 1, \qquad y_j \geqslant 0,$$

where

$$y_j = \frac{\alpha_j + \theta}{\varepsilon + M}; \qquad \sum_{j=1}^{z+1} y_j = \frac{1 + (z+1)\theta}{\varepsilon + M}; \qquad M \gg 0; \qquad C_i = \frac{\left[\theta \sum_{j=1}^{z+1} f_i(\tilde{\Pi}^j) - M\right]}{1 + (z+1)\theta}.$$

From the second of these problems the solutions y_j are found from which $\hat{\alpha}_j$ are, in turn, calculated to yield a vector $\Sigma_{j=1}^{z+1} \hat{\alpha}_j \tilde{\Pi}^j$. Actual excess demands for the productive factors and the budget imbalance are evaluated at this vector and if not within a desired tolerance a new linear programming problem set up by a repetition of the process. The area of outward search from each new trial vector is sequentially reduced if the solution to the linear programming problem yields a vector interior to the search area ($\alpha_j > 0$ for all j). As the area of search diminishes, the assumption on excess demands at $\Sigma_{j=1}^{z+1} \alpha_j \tilde{\Pi}^j$ becomes a less severe approximation until a solution within a desired tolerance is found. For the model of the United Kingdom economy and tax system in Section 3 competitive equilibria within a tolerance of 0.000000002 on excess demands have been found within 2 seconds of execution time on a CDC 7600 where the largest demands exceed 50,000.0. Computational experience is thus encouraging for further applications of these methods.

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One problem which cannot be overcome by the choice of solution is the possibility of a non-unique competitive solution to the model. No proof of the uniqueness of a competitive equilibrium in a model such as that outlined in Section 3 is available.[37] In the absence of general proofs of uniqueness, experimentation to numerically test for non-uniqueness has been tried on the model and in no case has multiplicity of competitive equilibria been encountered. Equilibria have been approached at different speeds and from different directions, they have been displaced by different amounts once found and reapproached and the same equilibrium has been returned to in such tests.

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[37] It has in fact been suggested by some authors (see [2]) that distortions of the form considered here may explicitly introduce non-uniqueness of the competitive equilibria even if the equilibrium in the absence of taxation is unique.

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