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Numerical Specification of Applied General Equilibrium Models: Estimation, Calibration, and Data

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NUMERICAL SPECIFICATION OF APPLIED
GENERAL EQUILIBRIUM MODELS:
ESTIMATION, CALIBRATION, AND DATA

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This paper contains preliminary findings from research work still in progress and should not be quoted without prior approval of the author.

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UNIVERSITY OF WESTERN ONTARIO

Numerical Specification of Applied
General Equilibrium Models:
Estimation, Calibration, and Data

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October 1981

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I. INTRODUCTION

Most recent work on applied general equilibrium models has focused on solution of models and interpretation of findings in terms of policy implications. Only limited attention has been given to the issue of deciding upon an appropriate numerical specification before computation proceeds.

In this paper we discuss alternative approaches to parameter specification in applied general equilibrium models. Although the models currently in use span a number of applied fields,¹ a 'standard' procedure has evolved among modellers of 'calibration' of the whole model to a benchmark observation coupled with use of 'literature' estimates for certain key parameters (particularly elasticities). A sequence of data adjustments is frequently used to 'force' equilibrium conditions on observed data before calibration begins. Most of these models involve dimensionalities which are quite outside those which econometricians are used to, and estimation of all model parameters using a stochastic specification and time series data is usually ruled out as infeasible.

We explore some of the issues raised by these procedures. We consider whether for small-scale general equilibrium models stochastic estimation procedures can be used. We discuss both system estimation and estimation of subsystems of models, and in the process review some of the econometric

¹ In tax policy, the models of the U.S. by Fullerton, Shoven, and Whalley [1978, 1980] and the U.K. by Piggott and Whalley [1976, forthcoming] have been used in a range of tax change evaluations. In trade policy, models by Miller and Spencer [1977], Broadway and Treddenick [1978], Carrin, Gunning and Waelbroeck [1980], Deardorff and Stern [1979], and Brown and Whalley [1980] have been applied to evaluation of changes in protectionist policies. In development, alternative policy strategies for South Korea, Turkey, Columbia, and other countries have been studied by Adelman and Robinson [1977], Derivs and Robinson [1978], de Melo [1978], and others. A closely related modelling project in Australia (Dixon, Parmenter, Ryland, Sutton [1978]) has developed a multipurpose modelling capability for analysis of a wide range of policy options.

literature which seems to us relevant. For models of the scale currently being used, stochastic estimation of complete models appears to us to be infeasible. For smaller scale variants time series econometric estimation appears possible and we report some estimates for a two-sector model of the U.S. Because the calibration procedures in use in existing 'large' models are somewhat sparsely documented we also describe these methods, summarizing some of the characteristics of the data sets used.

In the first part of our paper we discuss econometric approaches to model estimation in small, simple general equilibrium systems. We discuss estimation of a classical pure exchange economy and extend the analysis to incorporate production. We discuss both system and subsystem estimation, reviewing some of the literature on demand and production functions. We also comment on estimation of linear demand-supply systems.

In the second part of our paper we outline the broad characteristics of some of the applied general equilibrium models currently in use and describe the calibration procedures used. We present a numerical example of a 'benchmark equilibrium' and outline how data of this type are used to generate parameter values. In practice, construction of a benchmark data set for a particular model involves a substantial reorganization of conventional national income and other accounts and we discuss this, suggesting the label 'general equilibrium accounting' to differentiate 'benchmark' accounts from traditional accounts. We comment on the absence of any statistical test of the specifications chosen through such a procedure.

In a third section we report on comparisons of some preliminary results due to Mansur [1981] of both stochastic estimation and deterministic calibration for a small dimensional general equilibrium tax model of the U.S. economy. In the final section of our paper, we synthesize the discussion and evaluate alternative approaches to parameter selection. We also emphasize the limits which data availability and reliability place on the modelling exercise.

II. STOCHASTIC ESTIMATION

The natural reaction of anyone faced with the problem of specifying parameter values for a general equilibrium model to be used for counterfactual policy or other analyses is to think in terms of stochastic estimation of the model (through either system or single equation methods). This, however, is not the procedure generally adopted for existing applied models where 'calibration' of the model to an equilibrium data set is followed, combined with a literature search or 'best guess' procedure for key parameters (usually elasticities). No test of model to data is employed and sensitivity analysis is widely used for parameters whose values are uncertain and/or pivotal to results.

In this section we outline some approaches to stochastic estimation of general equilibrium models highlighting some of the problems which have motivated the widespread use of calibration. Before discussing possible estimation procedures, we first outline stochastic formulations of two widely used simple general equilibrium models to put the material which follows in better focus.

A. Simple General Equilibrium Models in Stochastic Form

(a) Classical pure exchange economy.

The simplest general equilibrium structure is the traditional two person pure exchange economy represented in an Edgeworth box diagram. Each of two consumers have an endowment of each of two goods. The endowment point usually does not coincide with the contract curve and trade between the individuals moves the consumption point from the initial endowment point to a point on the contract curve (which is also Pareto optimal). Diagrammatically this can be represented by the intersection of the offer curves of the two individuals, the intersection point characterizing a competitive equilibrium.

This model can be formulated algebraically in terms of the excess demand functions, $f_i^j(\pi)$, for the two individuals, $j = A, B$, for each of the two goods, $i = X, Y$. Each individual's set of excess demand functions will separately satisfy budget balance; $\sum_{i=X, Y} \pi_i f_i^j(\pi) = 0$, $j=A, B$ for any pair of non-negative prices π_i . Because relative prices are all that are relevant in such models, a common normalization used is that the sum of prices equal unity, i.e., $\pi_X + \pi_Y = 1$.

A convenient functional form which can be used to represent individual excess demand functions is the constant elasticity offer curve (see Johnson [1964] Gorman [1957]). In the two good case, the restriction of budget balance implies that the elasticities of the two excess demand functions are not independent but are related to the elasticity of the offer surface. Thus, in constant elasticity form, the deterministic two good model can be written as

$$\begin{aligned}
 f_X^A &= X^A \pi^A \\
 f_Y^A &= Y^A \pi^A \\
 \sigma_Y^A + \sigma_X^A &= -1
 \end{aligned}
 \quad
 \begin{aligned}
 f_X^B &= X^B \pi^B \\
 f_Y^B &= Y^B \pi^B \\
 \sigma_X^B + \sigma_Y^B &= -1
 \end{aligned}$$

where π is the relative price of good X in terms of Y , the σ 's are the elasticities of excess demand functions for each good by each agent and X^A, Y^A, X^B, Y^B are unit parameters. In stochastic form, assuming multiplicative disturbances, this model becomes

$$\begin{aligned}
 f_X^A &= X^A \pi^A V_X^A & f_X^B &= X^B \pi^B V_X^B \\
 f_Y^A &= Y^A \pi^A V_Y^A & f_Y^B &= Y^B \pi^B V_Y^B \\
 \sigma_X^A + \sigma_Y^A &= -1 & \sigma_X^B + \sigma_Y^B &= -1 \\
 E[V_X^A] = E[V_Y^A] &= 1 & E[V_X^B] = E[V_Y^B] &= 1
 \end{aligned}$$

This model as specified is not identified, though if it is estimated in the presence of changing policies it can become identified. A natural area of application for such a model is to the simultaneous estimation of import demand and export supply elasticities in a two region model, and the incorporation of changing tariffs in both regions can serve to identify the model. Another approach to use of the model is to specify a functional form for agent demand functions for each commodity rather than excess demand functions (such as Cobb-Douglas or CES). In this case exogenous shifts in the endowment point for both consumers serve to identify the model.

Although one would perhaps have thought that the two-person pure exchange economy as the most basic model in general equilibrium analysis (and some would say, all of economics) would be widely used in econometric work. We find it remarkable that (to our knowledge) no stochastic analogue of this model has ever been estimated.

(b) One person, two good, general equilibrium with production.

A second widely used general equilibrium model is the two good, two factor general equilibrium model incorporating one consumer (or multiple consumers with identical homothetic preferences). Equilibrium in this economy involves demand supply equality conditions for both goods and factors, along with zero profit conditions.

This model in deterministic form can be specified as follows¹

Demand side	Demands X^D, Y^D determined from $\max U(X, Y)$ s.t. $P_X X + P_Y Y = P_L \bar{L} + P_K \bar{K}$
Production side	$Y^S = F_Y(K_Y, L_Y)$ $X^S = F_X(K_X, L_X)$
Zero profit conditions	$P_Y Y^S = P_K K_Y + P_L L_Y$ $P_X X^S = P_K K_X + P_L L_X$
Demand-supply equalities	$Y^D = Y^S$ $X^D = X^S$ $\bar{K} = K_X + K_Y$ $\bar{L} = L_X + L_Y$

This system is represented diagrammatically by the familiar production possibilities frontier diagram. In order to work with such a system "convenient" functional forms (such as Cobb-Douglas or CES) are frequently used for demands and production. In stochastic form, additive disturbances usually appear in demand functions while multiplicative disturbances enter production functions.

In contrast to conventional estimation of sets of demand functions which satisfy budget balance, estimation of the above model involves simultaneously estimating demand functions (subject to budget balance) and a production system subject to fixed factor endowments. By using data on fixed factor endowments in each period and assuming the functional form for production functions to be unchanged through time, the model is identified and estimation can proceed. As

¹The notation used is self-evident and not defined to conserve space.

with the pure exchange economy, to our knowledge there have been no attempts to estimate the standard two-good production model.

B. System Estimation of Simple GE Models

We now suppose that an attempt is to be made to estimate an entire general equilibrium model through system FIML methods. We consider a simple two factor, two good model with fixed endowments of labour (\bar{L}_t) and capital (\bar{K}_t) at time period t . The model will also incorporate utility maximization by consumers, cost minimization by producers, along with the relevant policy structure (if any). Let us suppose, for now, that we can specify a complete stochastic model in general form as:

$$F_t[Y_t, X_t, B] = e_t \quad (\text{II-1})$$

where Y_t is a $(1 \times M)$ dimensional row vector of endogenous variables, and X_t is a $(1 \times K)$ vector of exogenous variables. $F_t[\cdot]$ is a $(1 \times M)$ vector function $[f_{1t}, f_{2t}, \dots, f_{Mt}]$ which, in the neighborhood of the true parameter values B , is assumed to be uniformly bounded and twice differentiable with uniformly bounded derivatives. The $(1 \times M)$ vector of errors, e_t , is assumed to be multivariate normal with expected value zero and variance-covariance matrix Σ . The identifiability of the parameters of this system has to be ensured but for now is assumed to be satisfied.¹

Equation (II-1) can be rewritten

$$f_{it}[y_{1t}, \dots, y_{Mt}, x_{1t}, \dots, x_{Kt}, B_i] = e_{it}$$

¹General equilibrium systems usually contain a large number of endogenous variables along with relatively few exogenous ones, which can cause identification problems. Restrictions on parameters enter through constant returns to scale, budget balance, homogeneity, and symmetry. We assume, for now, that these independent restrictions along with exclusion restrictions provides enough information to identify the complete model.

where, $i=1, \dots, M$ and $t=1, \dots, T$.

$\{x_{k_t}\}$, $k=1, \dots, K$ are predetermined variables

$\{y_{m_t}\}$, $m=1, \dots, M$ are endogenous variables

and $\{B_i\}$ is a vector of parameter values

Under the assumption that the Jacobian of the transformation from the disturbances to the observed random variables y_t is non-vanishing and the errors follow a multi-normal distribution, a likelihood function can be derived.

The concentrated likelihood function can be expressed as:

$$L^*(B) = \text{constant} + \sum_t \log |\det J_t| - \frac{1}{2} \log |\det \hat{\Sigma}|$$

Here it is assumed that $E(e_{it}) = 0$, for all $i=1, \dots, M$; $t=1, \dots, T$; $E(e_t e'_t) = \hat{\Sigma}$ is of full rank; $E(e_t e_{t'}) = 0$ if $t=t'$, and $\hat{\Sigma}_{mi} = \hat{\Sigma}_{im} = \frac{1}{T} \sum_t f_{it} f_{mt}$ is derived from the first order conditions of likelihood function

$$J_{i,m,t} = \left(\frac{\delta f_i}{\delta y_m} \right)_t$$

J_t is the matrix of such derivatives.

A number of algorithms are available to maximize such a likelihood function such as Eisenpress and Greenstadt [1966], or Chow [1973] who use a Newton method or Brendt et al [1974] who use a gradient maximization method. The later algorithm by Brendt et al has the advantage that convergence to a local maximum of the likelihood function is guaranteed, and unlike the Newton method employed by Eisenpress and Greenstadt [1966] and Chow [1973], does not involve the evaluation of third derivatives. This method requires the evaluation of the model up to second derivatives while third derivatives are eliminated by taking advantage of the fundamental statistical relation that the asymptotic variance-covariance matrix of a maximum likelihood

estimator is equal to the variance-covariance matrix of the gradient of the likelihood function.

Although most satisfactory from a statistical point of view, such a system approach is of limited applicability for most general equilibrium models. A number of reasons are involved.

(i) For a wide class of GE models the likelihood function is not well defined. For example, a complete general equilibrium model normally has market clearing conditions requiring that labor and capital employed in all sectors together add up to the exogenously given factor endowments. Errors in the input demand functions are thus not independent of each other and we cannot define a likelihood function for the model since we do not have independently distributed error terms.

(ii) For most applied general equilibrium models the number of parameters to be estimated increases very rapidly with each increase in the number of sectors or consumers. With a moderate sample size, for most applied general equilibrium models the number of independent parameters to be estimated will exceed the number of available data points. Some of the larger models to be discussed in the next section have numbers of parameters in the thousands. The degrees of freedom problem is especially acute if the translog framework is used in modelling both the production and consumer behavior. For example, in the 36 sector model of Jorgenson and Fraumeni [1980] there are 38 relative share parameters for each sector (these are relative shares of inputs in the value of the output of each sector) and also relative shares of commodity groups in the value of total consumer expenditure. Besides these share parameters, there are "second order" parameters that correspond to the measures of

substitutability in production and consumption. Jorgenson-Fraumeni estimate these parameters subject to separability restrictions, leading to a hierarchy of sub-models. In their empirical implementation about 50 such parameters are estimated for each sector. An additional 50 or so parameters are needed to specify the consumer behavior; the total number of second-order (or elasticity) parameters being approximately 2,000 for the 36 sector model considered by Jorgenson and Fraumeni. Thus for models of this scale, any full information method for complete model estimation has limited implementation possibilities.

An obvious procedure in light of these problems is to apply full information methods to sub-systems of the complete general equilibrium model; dividing the model in such a way that structural equations whose coefficients or error variances are related to each other are part of the same subsystem. This hopefully enables cross equation constraints for each of the subsystems to be imposed. The most natural choice of subsystems are demand systems and production structures that are embedded into the complete GE formulation. We now turn to methods for separate estimation of subsystems and return later to review literature on demand and production subsystem estimates.

C. Estimation of GE Subsystems (Method I)

The specification of subsystems in any general equilibrium model will vary from case to case. Subsystems may be linear or nonlinear in variables, nonlinear in parameters or nonlinear in both variables and parameters.

Let us consider a subsystem of the general model (II-1) which consists of $L(< M)$ equations which in normalized form can be expressed as

$$y_{it} = f(G_{it}, B) + e_{it} \quad (\text{II-2})$$

where y_{it} is the t^{th} observation on the dependent variable in the i^{th} equation and e_{it} follows a normal distribution as described earlier. G_{it}

is a g_i^i -component vector of observations at time t on endogenous functions appearing in the i^{th} equation. Each of these functions contains endogenous variables (i.e., random variables correlated with e_{it}^i) and exogenous variables or combinations of the two. $G_{it}^i = [g_{1,t}^i, \dots, g_{g_i,t}^i]$, where in general $g_{jt}^i = g_j^i[y_t, x_t]$ and $y_t = (y_{1,t}, \dots, y_{M,t})$ is the set of endogenous variables of the system (II-1). x_t is the set of all predetermined variables.

Implicit in the construction of the subsystems is the assumption that some of the y 's are determined outside the subsystem but remain endogenous to the complete model. In a demand or expenditure system, for instance, income is not endogenously determined, but is endogenously in the larger complete GE model. In general terms if we treat y_1, \dots, y_L as endogenous variables, then the subsystem predetermined variables $y_{L+1,t}, \dots, y_{M,t}$ need careful treatment. One might hope that it is possible to treat the variables $y_{L+1,t}, \dots, y_{M,t}$ as exogenous or predetermined for this subsystem and proceed with NLFIML estimation.¹ However, strictly speaking, this would give us consistent estimators only if the variables $y_{L+1,t}, \dots, y_{M,t}$ are uncorrelated with e_{it}^i 's (of the relevant subsystem), a condition that would not generally be satisfied for GE models. In general, these variables are correlated in these models.

In this case we can use a nonlinear three-stage least squares estimator--a form of instrumental variable estimator often referred to as minimum distance estimator. Here we first transform the structural form of any given subsystem and then minimize the sum of squares of errors through an iterative procedure. We begin with the structural form of the subsystem (II-1) represented as

¹ It is worth noting that this would give us the conventional estimates of demand and production systems (as described later).

$$y - f(G, B) = e \quad (11-2')$$

Premultiplying each equation by a matrix Z' ($K \times T$) such that

$$E(Z' \cdot e) = 0, \quad (11-3)$$

the transformed system is

$$(I_L \otimes Z') [y - f(G, B)] = [I_L \otimes Z'] e$$

Let

$$E[e e'] = \Omega \otimes I_T; \text{ where } \Omega = \begin{bmatrix} \omega_{11} & \cdots & \omega_{1L} \\ \vdots & \ddots & \vdots \\ \omega_{L1} & & \omega_{LL} \end{bmatrix} \quad (L \times L)$$

$$\begin{aligned} E(e_{it} \cdot e_{jt'}) &= 0 \text{ for } i \neq j \text{ and/or } t = t' \\ &= \omega_{ij} \text{ for } i = j \text{ and } t = t' \end{aligned}$$

For the transformed system the variance covariance matrix can be represented as

$$\begin{aligned} E[I_L \otimes Z' \cdot e e' \cdot (I_L \otimes Z)] \\ &= [I_L \otimes Z'] [\Omega \otimes I_T] [I_L \otimes Z] \\ &= \Omega \otimes (Z' Z) \end{aligned}$$

The minimum distance estimator can be obtained by minimizing

$$\begin{aligned} J(B) &= [y - f(G, B)]' [I_L \otimes Z] [\Omega \otimes Z' Z]^{-1} \\ &\quad [I_L \otimes Z'] [y - f(G, B)] \\ &= (y - f)' S (y - f) \end{aligned} \quad (II-4)$$

where $f = f(G, B)$ and

$$S = [I_L \otimes Z] [\Omega \otimes Z' Z]^{-1} [I_L \otimes Z']$$

Under certain regularity conditions [following Amemiya (1977, 1974), Jorgenson and Laffont (1974)] it can be shown that the estimated value \hat{B} converges in probability to the corresponding true value B^* and $\sqrt{T} (\hat{B} - B^*)$ converges in distribution to

$$N[0, \{H' (\Omega \otimes M)^{-1} H\}^{-1}]^1$$

where, $M = \lim_{T \rightarrow \infty} \frac{1}{T} Z' Z$, which is assumed to exist and be nonsingular, and

$$H_i = \text{plim}_{T \rightarrow \infty} \frac{1}{T} Z' \frac{\delta f_i}{\delta B} \text{ uniformly in } B.$$

This implies

$$\text{plim}_{T \rightarrow \infty} \frac{1}{T} Z' \frac{\delta f}{\delta B} = \begin{bmatrix} H_1 \\ \vdots \\ H_L \end{bmatrix} = H \text{ is of rank } R \text{ uniformly in } B,$$

where

$$\left[\frac{\delta f_i}{\delta B} \right] = \frac{\delta f'}{\delta B} = \begin{bmatrix} \frac{\delta f_i}{\delta B'} (z_{i1}, B), \dots, \frac{\delta f_i}{\delta B'} (z_{iT}, B) \\ \vdots \\ \frac{\delta f_i}{\delta B} (z_{i1}, B), \dots, \frac{\delta f_i}{\delta B} (z_{iT}, B) \end{bmatrix}$$

Minimization of equation (II-4) is usually performed by an iterative procedure as the equation system is highly nonlinear in both parameters and variables. A Gauss-Newton method, or a variant, is normally used. The iterations take the form

$$\hat{B}_{(n)} = \hat{B}_{(n-1)} + \left[\frac{\delta f'}{\delta B} \cdot S \cdot \frac{\delta f}{\delta B'} \right] \cdot \frac{\delta f'}{\delta B} \cdot S \cdot (y - f) \quad (\text{II-5})$$

where $f(*)$ and $\frac{\delta f}{\delta B}$ are evaluated at the estimated parameter values of the $(n-1)^{\text{th}}$ iteration; i.e., at $\hat{B}_{(n-1)}$. Further discussion of the properties of this estimator occurs in Amemiya (1974, 1977), Brendt et al (1974) and Jorgenson and Laffont (1974).

¹ For the existence of such an estimator with this property further regularity assumptions are needed [see Amemiya (1974, 1977), Jorgenson and Laffont (1974)]. These are that e_t 's are identically independently distributed

random vectors and $\frac{1}{T} \left(\frac{\delta F}{\delta B_i \delta B'_i} \right) \cdot Z$ converges in probability to a constant matrix uniformly in B for $i = 1, 2, \dots, R$ where B_i is the i^{th} element of B .

D. The Selection of Instruments in Subsystem Estimation and Method II

In the discussion in the previous section we ignored the issue of selecting the components of the matrix Z , the matrix of instruments. Any set of variables that are uncorrelated with the errors (so as to satisfy condition II-3), but at the same time are highly correlated with the endogenous functions G_{jt} 's, should qualify as instruments. We suggest two methods of selecting the instruments. The first method is somewhat analogous to that suggested by Goldfeld and Quandt (1968), Kelejian (1971) and Amemiya (1974), is to regress each G_{jt} on the elements of a polynomial of degree r in X_t . The underlying regression model can be expressed as

$$G_{jt} = Z_{jt} + \eta_{jt}$$

where,

$$\begin{aligned} Z_{jt} = & \theta_0, j + \theta_{1,1} X_{1,t} + \dots + \theta_{1,K_1} X_{K_1,t} + \dots + \theta_{r,1} X_{1,t}^r \\ & + \dots + \theta_{r,K_1} X_{K_1,t}^r \end{aligned}$$

$$j=1, \dots, g_1; t=1, \dots, T$$

Here the X 's are the elements of X_t , and η_{jt} are residuals which are not correlated with the elements Z_{jt} . Defining

$$Q_t = [1, X_{1,t}, \dots, X_{K_1,t}, \dots, X_{K_1,t}^r]$$

and θ_j , the associated vector of parameters, the instruments we are considering are

$$\hat{z}_{jt} = Q_t \hat{\theta}_j$$

where, $\hat{\theta}_j = (Q'Q)^{-1} Q' G_j$; Q being the matrix formed with the vectors Q_t , and G_j is the vector with corresponding elements G_{jt} for $t=1, \dots, T$. The matrix Z would then consist of low order polynomials of all the exogenous variables of the complete system.

In the method above, the instruments are obtained by regressing the endogenous functions (g_{jt} 's) on the lower order polynomials of the exogenous variables of the complete system. The procedure accommodates the simultaneity, and thus the estimators are consistent. This method does not, however, directly incorporate the restrictions of the general equilibrium model, such as full employment of factors.

An alternative method is to impose general equilibrium restrictions directly on the instruments. Using the method described earlier a set of consistent estimators can be obtained for the subsystems of the model. Adopting the parameters from one set of subsystem estimates we can impose the conditions specified in the others that would ensure the restrictions of the general equilibrium model. General equilibrium solution algorithms can be employed at this stage to produce appropriate sets of instruments. Using these equilibrium magnitudes (e.g., prices, income, etc.) as instruments, we can re-estimate earlier subsystems repeating the methods outlined above. In applications, solution algorithms apply to each year's data separately as the endowments of factors change from year to year. The process is repeated for each year's data with changed values of exogenous variables, yielding a time series for the instruments to be used in subsystem re-estimation in the next round.

In terms of our generalized notation for the complete model, $F_t(Y_t, X_t, B) = U_t$, we first estimate \hat{B} by some suitable method (e.g., the method outlined above) to produce consistent estimators. We then produce forecasts

$$\hat{Y}_t = g(X_t, \hat{B}) .$$

The estimates of the endogenous variables from this solved reduced form can be used as instruments (or in constructing appropriate instruments) for each of the submodels in re-estimation.

This alternative method is in essence an extension and partial modification of the methods described earlier. Reduced form estimates of endogenous functions (g_{jt} 's) are used in the formation of the matrix Z and once again minimum distance estimation is used on this newly constructed matrix. Properties of the estimators remained essentially the same as described for the earlier method.

E. Literature on Demand and Production Systems

While the previous section has highlighted some of the conceptual issues in estimation of general equilibrium systems or subsystems, perhaps the simplest way to obtain parameter values for use in a simple equilibrium model with production is to separately estimate the commodity demand system, and the production functions (the production system). The literature on applied consumption and production analysis is clearly relevant in this context, and in this section we briefly refer to some of this literature.

A complete general equilibrium system can be disaggregated into two broad subsystems interrelated through the equilibrium conditions. We consider a commodity demand system derived from the underlying (direct or indirect) utility function as characterizing consumer behavior. The production system characterizing producer behavior, given the technology underlying the production function, forms the second subsystem. The market equilibrium conditions then coordinate the separate subsystems.

The model decomposition used by Mansur [1981] is outlined in Figure I. Within Part A, A-1 describes the consumer's optimization process and A-2 the optimizing behavior of producers. The two subsystems of Part A can be expressed in general notation as

General Equilibrium Subsystem Decomposition from Mansur [1981]: Consumer's and Producer's Optimization Separated from Market Equilibrium Conditions¹

Part A: Consumer's Optimization Process

Consumer Demand System:

$$F_1(g_1, z_1, b_1) = e_1$$

Obtainable through consumers' utility maximization process.

g_1 : a vector of endogenous variables

z_1 : a vector of variables exogenous to the consumer decisions including commodity prices and income

b_1 : parameter vector which includes all relevant parameters of the specified demand system

e_1 : a vector of random error terms

Part B

Market General Equilibrium Conditions

$$Q_{it} = \sum_i a_{ij} Q_{jt} + g_{it}(z_1, b_1)$$

$$\sum_i L_{it} = \bar{L}_t$$

$$\sum_i K_{it} = \bar{K}_t$$

Conditions (1) and (2) together ensure that demand does not exceed supply for each of the products and inputs.

$$P_i = \sum_j a_{ij} P_j + w \cdot l_i + r \cdot k_i$$

or in vector notation,

$$P = [I - A]^{-1} (wI + rk)$$

This ensures zero profit conditions prevail in equilibrium for each activity operated at a positive level of intensity.

Part B: Producer's Optimization Process

The equation system from producer cost minimization behavior is represented in a matrix function:

$$F_2(g_2, z_2, b_2) = e_2$$

g_2 : vector of endogenous choice variable

z_2 : vector of variables exogenous to the producer's optimization activities

b_2 : a vector containing all the relevant parameters of the specified system

e_2 : a vector of random error terms.

¹This table is from Mansur [1981]. The decomposition given in this represents a class of general equilibrium models with two primary factors of production and fixed coefficient intermediate inputs.

$$F_1(g_1, z_1, b_1) = e_1 \quad (A-1)$$

$$F_2(g_2, z_2, b_2) = e_2 \quad (A-2)$$

Here g_s ($s = 1, 2$) are vectors of endogenous variables; prices and income are subsumed under the vector z along with other predetermined variables. By construction, different subsystems in part A do not have common parameters, and errors in one can be assumed to be independent of those of others. These two subsystems are linked to each other indirectly through the market equilibrium conditions in B.

Independent estimation of commodity demand and production systems would provide estimates of all the parameters that are needed to solve a complete general equilibrium model. If equilibrium conditions are ignored, we could restrict ourselves to the separate estimation of demand and production systems. The issue facing users of subsystem estimates is thus to determine exactly what the degree of unreliability is if the simultaneity between subsystems is ignored.

Demand Systems

In empirical analyses of demand systems, income (Y) and prices (P , in vector form) are taken to be exogenously given. The budget balance condition along with other restrictions taken from the theory of consumer demand act as constraints on the system, which in compact notation can be represented as

$$\underline{x} = f(\underline{Y}, \underline{P}) + \underline{u}$$

$$\underline{Y} = \underline{P}' \underline{x}$$

where \underline{x} is a vector of quantities of n-goods and services demanded.

There are a large number of possible functional forms for the demand system, and different approaches can be adopted in specifying the system. Two classes of functional form for utility functions are direct and indirect utility functions with demand functions generated from utility maximization.¹ A third approach starts with directly specified demand equations and imposes theoretical restrictions from utility maximization in the process of estimation (see, Nasse [1973], Barten [1964], etc.).

The theoretical restrictions from utility maximization (which depend on the form of the utility function), along with the budget constraint imply cross equation restrictions. Joint estimation is thus necessary and full information methods such as FIML or variants of GLS methods (e.g., Zellner [1962]) can be employed. One equation is conventionally dropped from the system to avoid the problem of singularity of the variance-covariance matrix (due to the budget constraint). These issues are widely discussed in applied consumption analysis, an overall survey of which

¹ Stone (1954), Pollak and Wales (1969), Deaton (1972) deal with demand systems originated from direct utility functions. Johansen (1969) formulates a general additive utility function which implies that the Direct Translog of Christensen et al (1975), Stone-Geary, and Cobb-Douglas type utility functions are special cases. Houthakker (1960) introduced the indirect utility function and empirically estimated the derived demand system. Various forms of Leontief reciprocal indirect utility functions are suggested and estimated by Diewert (1969, 1974), Gussman (1972), Darrough (1975); for translog utility functions, see Christensen et al (1975), Jorgenson and Lau (1975), Christensen and Manser (1975).

also reveals that variations in income and prices generally can explain only some of the variation in observed demand (see Barten [1977], Desai [1976] and Bridge [1971]). The role of prices determining demand behavior would seem, on the basis of econometric studies on demand, to be less than theory might suggest although prices become somewhat more important if a finely divided classification of commodities is used. For broad aggregates price indices over time tend to move in the same direction following similar patterns, making substitution possibilities hard to detect from the data. For highly aggregate models, econometric studies suggest that it may be reasonable to use simple demand specifications with limited or restricted (price) substitution effects.

It is now widely believed (Barten [1977]) that since consumer demand models (in the forms in which these are currently estimated) concentrate only on one side of the market, additional strength (or explanatory power) might come from the simultaneous considerations of demand and supply. Although no systematic investigation has been made in this direction by econometricians, work in this area can be said to be moving in the GE direction. Currently, most applied consumption literature report independent estimation exercises, with little attention being given to this as a part or subsystem of an overall GE model. Nevertheless, the theoretical understanding and consensus developed over the last three decades on estimation of sets of demand equations does provide a valuable starting point to applied general equilibrium model builders in specifying the consumption side of models.

Production Relations

On the production side, there are a number of well-known papers on the estimation of production functions. Most of these emphasize the

Marschak-Andrews [1944] approach which utilizes the optimizing behavior of the producer in estimation.¹ The estimation of the parameters of any production system should be based on the derived input demand system since firms cost minimize. The input demand functions employed in this empirical literature are either obtained directly from the production function using cost minimization, or indirectly from cost functions using Shephard's lemma. If we simultaneously estimate production functions for several sectors, and impose the condition that factors are fully employed, the endowment constraint (if relevant) would be binding and impose further cross equation restrictions across the errors of input demand equations. In existing econometric literature the focus is only on the input demand functions for particular sectors.

For both production and cost function approaches a variety of functional forms have been used. Using the CES specification of Arrow, Chenery, Minhas and Solow [1961] (under constant returns to scale) and cost minimization, the production system in intensive form (i.e., per unit of labor) can be expressed as

¹ Marschak and Andrews [1944] showed that under perfect competition and profit maximizing conditions OLS does not yield consistent estimates of the parameters of Cobb-Douglas production functions. For a competitive firm maximizing profit (or minimizing cost) the choice of inputs reflects cost minimization, along with the technological relation characterizing the production function. Thus independent estimation of the technological relation alone does not provide consistent estimates. Hoch [1958], Kmenta [1964] and Mundlak [1963] provided consistent estimators for Cobb-Douglas production function parameters assuming that the 'technical' disturbance (associated with the production function) and the 'economic' disturbance (in the profit maximizing equations) are uncorrelated. In another classic paper, Zellner, Kmenta and Drèze [1966] show that under the assumption of expected profit maximization and perfect competition, OLS does provide consistent and unbiased estimates of the parameters. Most of this literature, although cast in terms of Cobb-Douglas production functions, can be applied to other specifications such as CES.

$$q_i = \frac{Q_i}{L_i} = A_i [\alpha_i k_i^{-\rho_i} + (1 - \alpha_i)]^{-1/\rho_i}$$

$$k_i = \frac{K_i}{L_i} = \left[\frac{1 - \alpha_i}{\alpha_i} \right]^{\frac{1}{1+\rho_i}} \cdot \left(\frac{P_L}{P_K} \right)^{\frac{1}{1+\rho_i}}$$

where,

Q_i : value added in the i^{th} sector

K_i : amount of capital used in i^{th} sector

L_i : amount of labor used in i^{th} sector

α_i : the share parameter

ρ_i : the elasticity parameter

A_i : the scale parameters and

P_K and P_L : are the prices of capital and labor inputs.

The CES production system can be estimated either in the direct form or in its intensive form (with or without logarithmic transformations).

Berndt and Wood (1975), Christensen et al (1971, 1973) use translog cost functions, while other forms such as generalized Cobb-Douglas cost functions are discussed by Diewert (1971 and 1973). In its general form the cost function can be expressed as

$$C = C(Q, P_1, \dots, P_m)$$

where C is total cost, P_i 's ($i = 1, \dots, m$) are the input prices and Q , the flow of gross output of j^{th} product. Using a translog cost function with symmetry and constant returns to scale imposed, the cost function can be expressed as

$$\ln C = \ln \alpha_0 + \ln Q + \sum_i \alpha_i \ln P_i + \frac{1}{2} \sum_{i,j} \gamma_{ij} \ln P_i \ln P_j$$

Linear homogeneity in prices imposes the restrictions

$$\sum_i \alpha_i = 1$$

and $\sum_j \gamma_{ij} = 0$ for all i

Using Shephard's Lemma, input demand equations are obtained which can be expressed in cost share form as

$$S_i = \alpha_i + \sum_j \gamma_{ij} \ln P_j; i, j = 1, \dots, n$$

and $\sum_i S_i = 1$

where the S_i are the shares of inputs in the total cost of producing Q .

In a recent piece Jorgenson and Fraumeni [1980] employ translog price functions, impose homogeneity (of degree one) and derive sectoral value shares of factors. Since share elasticities with respect to prices are symmetric, further cross equation restrictions are imposed. As the cost or value shares of the factors of production for each sector always sum to unity, in both Berndt and Wood [1975] and Jorgenson-Fraumeni [1980], the sum of the disturbances across the input cost shares or value share equations is zero at each observation. To avoid singularity of the covariance matrix they drop one equation arbitrarily (for each sector) and assume the disturbances in the remaining equations to be independently and identically normally distributed with zero mean and variance-covariance matrix Ω .

This literature on input demand systems emphasizes producer behavior and cost minimization, but ignores economy-wide restrictions which the general equilibrium approach suggests. This is perfectly consistent with the behavior of a single competitive price taking firm, but limits the potential use of these methods for general equilibrium models which incorporate analysis of broad aggregate sectors with economy-wide restrictions. At an aggregate industry level input prices are unlikely to be exogenous, and exogenously given factor endowments introduce additional cross-equation restrictions (among different sectors), particularly across the errors.¹ Thus in estimating production systems for use in multi-sector

¹ In terms of the production system specified earlier

$$\sum_i L_{it} = \bar{L}_t$$

and $\sum_i K_{it} = \bar{K}_t$

imply that all the errors to be associated with L_{it} 's or K_{it} 's (for all $i = 1, \dots, n$)

general equilibrium analyses it may be inappropriate to assume that prices are exogenous and that regressors in the input demand system are uncorrelated with the disturbances. If we are interested in obtaining consistent estimates of the parameters of the production subsystem of a general equilibrium model this simultaneity should be taken into account.¹

F. System Estimation of Linear Demand-Supply Systems:
Allingham's Approach

Most econometric work on commodity and production (or input demand) systems proceeds separately and is not developed in the context of general equilibrium modelling where classifications of the two systems must match and restrictions from the general equilibrium model need to be incorporated. Using an alternative approach, Allingham [1973] attempts to capture the general equilibrium market clearing process while giving somewhat less attention to the detailed specification of production and demand systems.

Allingham estimates a form of general equilibrium model where he uses a variant of a 'Keynesian type' ad hoc consumption function not based on utility maximization. Income, or expected income, determines aggregate consumption; its division among different categories then depends on relative prices (represented by the price for the category relative to the general consumer price level). Moreover, to avoid nonlinearities in both parameters and variables he uses a linear production function (linear in inputs). This has the two features that the marginal product of any factor is constant and the elasticity of substitution (between the factors) is undefined. Because of these simplifications and the limited treatment of

are not independent of each other. In the intensive form $\sum_i \theta_{it} k_{it} = \tilde{k}_t$ where $\sum_i \theta_i = 1$, which implies that $\sum_i \theta_{it} u_{it} = 0$ for all t.

¹ Berndt and Wood [1975] use instrumental variable method to deal with the simultaneity. The values of instruments are formed by regressing each of the regressors on a set of variables considered to be exogenous to the aggregate sector.

commodity and input demand systems (along with underlying cross equation restrictions), this model reduces to a system of linear simultaneous equations. This system does not, however, satisfy the usual properties of production and consumer demand systems, such as homogeneity, Walras' Law, and other restrictions which are key elements in much of the recent general equilibrium literature.

All these simplifications enable Allingham to use conventional limited information (single equation) methods, Two Stage Least Squares (2SLS) or Limited Information Maximum Likelihood (LIML). The lack of incorporation of prior restrictions coming from economic theory necessarily reduces the attractiveness of his model; however, this is (to our knowledge) the first attempt to incorporate both demand and production side features of general equilibrium modelling into a complete systems approach and estimate the model parameters using time series data.

Allingham evaluates the performance of his systems approach by comparing predictive performance against traditional macro models. He estimates his model with the U.K. data over the period 1956 through 1966, distinguishing ten producing agents (or industries) based on the Standard Industrial Classification (SIC) for 1958. After estimating the unknown parameters by using standard (linear) limited information methods, the estimated model is solved and quantitative static and dynamic properties of the system are analyzed.

His conclusion is that the complexities associated with general equilibrium modelling are justified in predicting equilibrium values of broad

aggregates and individual components, and also for investigating the process whereby equilibrium is attained. Allingham's approach, however, is quite different from the applied models currently in use for policy appraisal. As demand functions do not come from utility maximization, no welfare analysis of policy alternatives is possible; the absence of Walras' Law as a restriction on demand functions raises issues as to the internal feasibility of model solutions, and the absence of clearly defined production functions suggests that economy-wide production possibility sets may not be well defined. Nonetheless, Allingham's work represents a bold first step towards a desirable systems approach.

III. DETERMINISTIC 'CALIBRATION' PROCEDURES¹

From the discussion in the preceding section, system or subsystem estimation might seem the natural way to proceed in specifying a numerical general equilibrium model to be used for policy or other analysis. The 'facts of life' of applied general equilibrium modelling, however, are that less sophisticated procedures are usually employed. The common procedure is to 'calibrate' the model to a base year observation; 'calibration' meaning the ability of the model to reproduce base year data as a model solution. Calibration is augmented by literature search (and on occasion econometric estimation) for key model parameters, whose value is required before calibration can proceed. In practice, due to the widespread use of CES functions in applied models 'key' parameters are more or less synonymous with elasticities, and in some cases literature search provides limited, contradictory, or (at times) no information. Resort is usually made to 'sensitivity' analysis, in which alternative values of key variables are tried and model findings crudely evaluated for their robustness. In this section we outline these procedures and in the process briefly describe the structure of some of the applied models currently in use.

A. Summary of Existing Applied General Equilibrium Models in which Calibration is Used

To provide background for the later discussion in this section, we begin by outlining the main characteristics of some of the recent applied general equilibrium models. In Table 1 we briefly summarize the main characteristics of a number of models constructed for use in counterfactual policy analysis. The main areas of use are international trade, public finance, and development, but

¹ Similar discussion of 'calibration' in applied models appears in St. Hilaire and Whalley [1980], and Chapters 4 and 6 of Piggott and Whalley (forthcoming). We also use three tables drawn from Piggott and Whalley (forthcoming) by way of illustration in this section.

Table 1

Summary of Model Characteristics and Uses

Model	Demand Side		Production Side		Policy Interventions Interpreted
	Demand Functions	Disparation Structure	Functions for Production	No. of Industrial Commodities	
Adelman/Robinson [1978]	CES with elasticities adjusted to satisfy budget and other constraints	15 socioeconomic categories of income recipients	A two-level CES using a Cobb-Douglas aggregation function for labor -fixed coefficients for intermediate production involving 'composite' goods	29 producing sectors 4 firm size 7 labor skill categories	-tax and transfer experiments -agricultural experiments -trade and industrialization experiments -technology and manpower experiments -land reform -urban policy -development strategies
Dervis/Robinson [1978]	Cobb-Douglas demands for 'composite' goods, CES aggregation functions over comparable, domestic and imported products	2 groups of consumers (rural/urban)	Two level CES functions: fixed coefficient intermediate production involving 'composite' goods	19 sectors 3 labor groups	-higher exchange rate -manufactured exports and investment oriented policy package
Carr/Gunnits/Watbrook [1980]	CES aggregation over comparable domestic and imported products	2 sets of CES coefficients for household demands for composites	Cobb-Douglas value added functions plus fixed coefficient intermediate production involving 'composites'	5 sectors (2 rural 3 urban)	simulation experiments considering the impact of an increase in oil prices under price rigidity and flexibility
Deardorff/Stern [1979]	Cobb-Douglas demands for 'composite' goods, and rest of the world CES aggregation functions over 'comparable', domestic and imported products.	18 OECD countries	CES value added functions, fixed coefficient intermediate use for CES composites of home imports	22 tradeable 7 nontradeable industries and commodities	Tokyo round tariff changes under fixed exchange rate Non-Tariff Barriers -agricultural concessions -procurement liberalization
Walley [1980]	Nested CES	41 households (in largest variant)	CES value added functions, fixed coefficient intermediate use for CES composites of home imports	33 commodities and industries in each of 4 regions	changes in tariffs and non-tariff barriers and taxes in EEC, U.S. and Japan
Fullerton/Shawers/Walley [1980]	Nested CES/Cobb-Douglas	12 households	CES value added functions 16 commodities 17 industries	variations in U.S. tax policies (represented by 1973 data)	
Piggott/Walley [1980]	Nested CES	100 households (income, occupation and family characteristics) 3 regions	CES value added functions fixed coefficient intermediate production	33 commodities and industries	-3 assumptions on energy supply -1 assumption on energy demand -3 variations in trade elasticities
Menne [1980]	Derived demands from energy CES production functions	1 household	Nested CES production functions energy and non energy	109 industries and commodities 9 labor groups	-change exchange rate -tariff increase
IMPACT [1980]	Nested CES	3 level functions (between inputs of composite products)	1- Isotopic assumption 2- CES - between imported and domestic products 3- between primary factors		
Serra [1979]	Cobb-Douglas	8 consumer groups 4 income class rural/urban	3- CES between labor inputs CES value added functions with fixed coefficient specification for intermediate production	8 industries and commodities plus imports plus investment	-substitute value added tax for sales tax -substitute sales tax for income tax -lump-sum rebate for poor -change in income tax progressivity

increasingly other areas of economic activity including energy and financial market activity are being examined. Examples of issues to which these models have been applied are analyses of the incidence of taxes in domestic economies, and the effects of changes in tariffs on the international economy.

In these applications modifications to existing policies are expected to change relative prices in the (domestic and international) economy. The use of the model allows comparison between a historical equilibrium generated by existing policies which is assumed to be observable, and a hypothetical or counterfactual equilibrium which the model produces under a changed policy regime. Comparison of equilibria leads directly to applied welfare analysis producing estimates of welfare gains and losses for the groups identified in the model. In an alternative use the model may simply be interpreted as providing indications as to how the structure of the economy could be affected by the policy change.

The central characteristic of these models is the price endogenous equilibrium framework which typically involves separate specification of equation systems representing the demand and production side of the economy. In equilibrium all behavior is consistent with the equilibrium prices in that consumers maximize utility, producers maximize profits, and market demands equal market supplies. Policy evaluation proceeds by comparing a 'benchmark' equilibrium under existing policies to a new equilibrium under new policies. As the new policy alternatives considered are usually hypothetical such an equilibrium is termed 'counterfactual'. The benchmark equilibrium is assumed to be reflected in observable behavior and a micro consistent equilibrium data set is constructed using national accounts and other data sources both for purpose of making the comparison possible and providing a data base for model calibration.

The most common specification in this type of modelling involves demand and production functions drawn from the family of so-called 'convenient' functional forms. Cobb-Douglas are the simplest of these but this group also includes CES, LES, and CRESH systems, and may incorporate 'nesting' involving hierarchical functions. A major difference in these models from more conventional input-output analysis is that they explicitly incorporate extensive substitutability on both the demand and production side of the model.

An important ingredient in these models is the amount of detail which is incorporated. While some seek to provide a general purpose capability so that many different policy alternatives can be analyzed, others are oriented more to specific issues. With multipurpose models, incorporation of a large number of commodity and household groupings is usually viewed as important because of the complex commodity differentiation contained in the policy instruments to be analyzed. The IMPACT model of Australia, for instance, separately identifies 109 industries in an attempt to accommodate the detail of Australian tariff and other policies. In their model of the UK tax system, Piggott and Whalley identify 100 household types stratified by income, occupation, and family size in order to perform detailed distributional analysis of the effects of tax changes. In the multipurpose models the level of detail identified on the demand and production sides frequently reflects the maximum available detail in basic data.

For the selection of parameter values these models all employ procedures which we loosely describe as 'calibration'. For the particular functions assumed for demand and production, parameter values are chosen so that the model will exactly reproduce an assembled equilibrium data set as a solution to the model assuming there are no changes in policies from those

in operation in the base year. This 'observed' equilibrium is frequently termed a benchmark equilibrium data set, and we next turn to an elaboration of this concept.

B. The Concept of a Benchmark Equilibrium Data Set

In counterfactual equilibrium analysis, the numerical analogue of traditional comparative statics, the assumption of an 'observable' equilibrium leads directly to the construction of a data set which fulfills the equilibrium conditions for some form of general equilibrium model. A natural accounting framework consistent with general equilibrium models is to record transactions occurring in the separate markets which comprise the economy. A benchmark equilibrium data set is a collection of data in which equilibrium conditions of an assumed underlying equilibrium model are satisfied.

General equilibrium analysis is perhaps the most widely used theoretical framework for economy-wide microeconomic analysis, but is only explicitly recognized in the construction of current national income accounts in the aggregate income-expenditure identity, not in any of the sub-aggregate detail in the accounts. The orientation of conventional national accounts can reasonably be described as the determination of macroeconomic aggregates. The detailed information presented in most national accounts, while clearly of enormous value to economists, nonetheless is largely a by-product of the process of assembly of macro aggregates and typically does not aim at consistency in the various areas of detail which general equilibrium analysis requires.

If equilibrium is reflected in an assembled set of accounts, demands equal market supplies for all commodities, and supplies and demands can be separately disaggregated by agent. Each agent, in turn, has incomes and expenditures consistent with their budget constraint.

Four sets of equilibrium conditions satisfied by most of the constructed benchmark equilibrium data sets are

- i) Demands equal supplies for all commodities.
- ii) Non-positive profits are made in all industries.¹
- iii) All domestic agents (including the government) have demands which satisfy their budget constraints.
- iv) The economy is in zero external sector balance.

These conditions are not all satisfied in intermediate transactions accounts (input/output data) and other data published by agencies which produce national accounts data. In input-output data, sector budget conditions are not explicit, nor is an external sector balance condition satisfied. Demand supply equalities by commodity do not appear in national accounts data. Household expenditure data are usually inconsistent with production side data; classifications differ and totals do not agree.

In constructing benchmark data sets various adjustments are therefore necessary to the blocks of data involved that are available separately but are not arranged on any synchronized basis. The nature of these adjustments varies from case to case as alternate sets of benchmark accounts are constructed to fit differing models.

¹This typically involves treating the residual profit return to equity as a contractual cost as is implicit in most input-output transactions tables.

Differences in measurement concepts from national accounts practice frequently arise for particular items. One example is the measurement of input use by industry since unadjusted national accounts measures of the use of capital by industry are inappropriate for use in general equilibrium models. Another example arises with the imputation of retained earnings through to households as savings, which is necessary to examine production and exchange in terms of underlying real counterparts.

Further difficulties arise with differences in classification between various inconsistent data sets. An example is the incompatibility between categories of consumers' expenditures in family expenditure data and the classification of products of industries in GDP accounts on which final consumer expenditures by product are recorded in input-output data. A further difficulty is that producer output classifications refer to measures of the value of output on a net of retail and wholesale margin and net of transportation costs basis while consumer expenditure classifications are on a gross basis. Classification difficulties also arise with taxation data which in a number of instances are collected on an administrative rather than statistical basis.

Another form of adjustment arises with the need to guarantee mutual consistency between inconsistent data sets. Most benchmark data sets rely heavily on the 'RAS' adjustment method¹ for these modifications. Examples of areas where this technique is applied are where household demands for individual products do not equal the supplies of firms where costs of industries are not equal to

¹The term 'RAS' derives from the Row and Column Sum method discussed in detail in Bacharach [1970]. This is a method of using an initial guess of a matrix where given row and column constraints must be met and moving to a consistent matrix. This procedure is typically followed in updating input-output tables from previous years' tables to take account of new national accounts data.

sales (after modifications to published intermediate transactions accounts), and where household incomes do not equal expenditures.

In Table 2 we provide an example of interlocking benchmark accounts for an artificial economy with four industries, four goods, and three consumer groups. In the consistent set of production side accounts in this economy the value of GNP is 29 and the total value of production is 44. Zero profit conditions are satisfied for each industry as is a zero external sector balance condition. On the left of the table we highlight the equilibrium consistency conditions satisfied by the data.

The data sets used in the applied models typically contain much more detail and complexity than in the example discussed above. The data usually refer to a single year, although some averaging across years is done in constructing portions of those data sets where substantial volatility occurs.

In Table 3 we list the types of data used in the models outlined in Table 1. The documentation of data sources and adjustments used in these models is in varying states of completeness since complex, detailed modifications are often involved. The paper by St. Hilaire and Whalley [1980], however, is worth mentioning at this point since it is solely concerned with describing data modifications and procedures used in the construction of a benchmark data set for tax policy analysis for Canada. The benchmark approach is also outlined in this paper.

While benchmark data sets are usually constructed with one particular model in mind, many different model specifications are consistent with the same benchmark data set. In the following sections we discuss choice of model and specifications of parameter values consistent with benchmark data.

A Simple Example of a Benchmark Equilibrium Data Set from Piggott and Whalley (1981)

Demand Side			Production Side			Public Sector			Expenditures			Consistency Conditions Satisfied by Data		
1) Purchases of products by consumer groups at consumer prices			1) Inter Industry Transactions			1) Final Consumer Receipts			1) Demands equal supplies for all products.			1) Demands equal supplies for all products.		
Consumer Groups			Industries			Demand ^a			2) Each industries total costs equal total sales.			2) Each industries total costs equal total sales.		
Value of Purchases at Consumer Prices			Output for Intermediate Use			6			3) Each consumers groups purchases equals disposable income for the group.			3) Each consumers groups purchases equals disposable income for the group.		
A	B	C	Industries			6	4	4	4) The endowments of consumers match factor usage.			4) The endowments of consumers match factor usage.		
1 6 2	7		1 2 1	3	4	6	4	7	5) The Government's budget is balanced.			5) The Government's budget is balanced.		
2 2 3	6		1 2 3	1	2	10	7	19	6) The value of final demands equals the sum of value added.			6) The value of final demands equals the sum of value added.		
Products	3 4	1 3	Products	2 4	3 1	10	7							
4 1	3	4	3 2	1 1	1	5								
Disposable Income	11	9	4 2	1 1	0	4								
2) Consumer Disposable Income			Intermediate Goods			10	6	3	27					
Consumer Groups	A	E	Total			4	3	2	22					
Income From Capital	5	1	7			6	4	3						
Labour Income	3	3	4			10	7	2						
Transfers Received	6	7	6			19	10	4						
less Income Tax Paid	3	2	2			7	1	3						
Disposable Income	11	9	17				1	2						
3) Consumer Taxes Paid			Total											
Value of Purchases at Consumer Prices			Capital Service Usage											
Consumer Taxes			Labour Service Usage											
at Producer Prices			Indirect Business Taxes											
			Total											
1	7	1	6											
2	6	2	4											
Products	3	8	1											
4	6	3	7											
Total	29	7	22											

^a A number of features not included in this example must be incorporated into the data set used in the model such as 1) Real Government Expenditures; 2) Foreign Trade; 3) Consumer savings; 4) Investments by business, consumers and government; 5) Depreciation; 6) Inventory Accumulation; 7) Financial transactions between the personnel business, public and external sectors; 8) More complex taxation and subsidy arrangements (e.g., a value-added tax).

Table 3
Data Used In Applied Models Listed In Table 1

<u>Model</u>	<u>Country(ies)</u>	<u>Year</u>	<u>Base Year Data</u>	<u>Extraneous Use of Elasticities</u>	<u>Production Data</u>	<u>Demand Data</u>	<u>Policy Data</u>
Adelman/ Robinson [1977]	Korea	1968	Data reconciled to 1968 Input Output data and selected National accounts estimates	Expenditure elasticities, price elasticities of demand, Labour supply elasticities	Input-Output, National accounts, Capital stock, Employment Survey, Financial Survey, Wage Survey	Urban Household Survey Farm Household Survey Input-Output accounts	Policy scenarios, no data required
Dertis/ Robinson [1978]	Turkey	1973	Approximate calibration to base year (1973)	Production side and trade elasticities	Input-Output, Capital Stock, Wage and Employment, Governmen- tal expenditure and Income	Input Output final demand	Policy scenarios, no data required
Carrin/ Ganning/ Waebroek [1980]	9 groups of developing countries and the rest of the world	1978	Data used for projec- tions in World Bank Development Report for 1978	Income and price elasticities; trade elasticities	World Bank base year data	World Bank data	Simulation experiments, no data required
Deardorff/ Stern [1979]	18 OECD countries and rest of the world	1976	Comparative static analysis rather than calibration to base year elasticities data set	Import demands, production function elasticities	U.S. Input-Output, national acc. (GDP), Trade data, Tariffs, Employment	Trade data for each industry -level of aggregate expenditure by country	-Post Kennedy round tariffs -IMF offer rate tariffs (ad valorem) -Non tariff barriers data
Whalley [1980]	EEC, U.S., Japan, rest of the world	1973	Constructed benchmark data set for interna- tional and domestic economics for 1973	Production function elasticities, demand elasticities, trade elasticities	Input-Output Plus value added data	Household expenditure data disaggregated using foreign trade data	Tariffs, ad valorem equivalent of non-tariff barriers, and domestic taxes
Pullerton/ Shoven/ Whalley [1980]	U.S.	1973	Constructed benchmark data set for 1973	Production and demand elasticities	Input-Output plus value added data	Household expenditure data	Production, expenditure, and income taxes
Piggott/ Whalley [1980]	U.K.	1973	Constructed benchmark data set for 1973	Production and demand elasticities	Input-Output plus value added data	Household expenditure data	U.K. taxes and subsidies
Mamuc [1980]	3 regions: Industrialized -LDC oil exp. -LDC oil imp.	1976	Approximate calibration to 1976 data in World Development Report	Trade elasticities	GNP; energy production and consumption; exports and imports; trade deficit (World Development Report)	-energy consumption -imports	Various quantity scenarios
IMPACT [1977]	Australia	1968/69	1968/69 Input-Output data base	extensive elasticities file (estimation and literature search)	Input-Output accounts	Input-Output accounts	1% devaluation 1% increase in all ad valorem tariff rates
Serra [1979]	Mexico	1968	Approximate consistency of model to 1968 data from Mexico	Production side only Value added	Input-Output and Value added	Household survey adjusted to National account	Ad Valorem taxes

C. Choice of Functional Form and Selection of Parameter Values

The choice of functional forms for demand and production functions in applied general equilibrium models is guided in practice by the restrictions which ease of solution impose. The major considerations in choice of functional form are that they be consistent with the basic model assumptions and the maximizing responses of agents must be able to be repeatedly solved for in the sequences of calculations involved in equilibrium computations. Tractable functional forms must clearly be used to describe behavior patterns of both producers and consumers. Household utility maximization problems must be readily soluble, as must industry cost minimization problems.¹

Inevitably, the well-known family of convenient functional forms provides the candidate specifications for general equilibrium policy models of the form discussed here. Demand and cost functions derived from Cobb-Douglas, Stone-Geary, and CES (either single stage or nested) utility and production functions tend to be used. More complex variants (such as Generalized Leontief functions) may also be considered although such functions substantially increase execution times required for equilibrium calculations. The widespread choice of CES functions reflects a tradeoff which model builders face between complexity and tractability. The use of CES functions (nested or unnested) allows corresponding Cobb-Douglas functions to be separately

¹ In fact to determine equilibria computationally it is simpler to work directly with the consumer demand functions for commodities, and the per unit output factor demand functions of producers.

considered as special cases. The CES function is a directly additive function which implies certain restrictions on the corresponding demand functions.¹ Nested CES utility functions are widely used (additive functions of additive functions) to derive demand functions, which modify the nature of these restrictions somewhat but they still apply within nests. The derived CES demand functions have unitary income elasticities which for some policy issues may be inappropriate. While this feature can be relaxed through the use of a Stone-Geary variant of CES, these functions are not widely used. These models can therefore be thought of as drawing heavily on CES functions for demand and production and the main issue in practice faced in model specification is the choice of parameter values in these functions.

The procedure of calibration of model to a benchmark data set follows from the structural characteristic that utility and production functions yield household demand and industry cost functions which depend directly or indirectly upon all prices. Even if we are able to conceive of a reduced form for one of these models being represented by tractable interdependent excess demand functions, estimation of the model using conventional methods rapidly becomes difficult if not impossible for some of the reasons mentioned

¹An important property of the demand functions derived from a single stage CES utility function is what Deaton [1974] refers to as 'Pigou's Law'. This is an approximate constant proportionality between income and own price elasticities for any demand functions derived from directly additive preferences (of which a single stage CES function is an example). Other restrictions are also imposed by additivity most notably the absence of inferiority and the unambiguous sign of compensated cross price elasticities. If preference functions are nested rather than single stage CES functions, these restrictions apply only to demands for the implied composite goods rather than for the individual goods. With this structure the restrictions implied by direct additivity will no longer hold exactly although they still apply within nests.

in Section II. For the parameters of any particular structural equation to be identified, a large number of excluded exogenous variables or other identifying restrictions are required and even if identification is possible time series are required of a length infeasible for estimation of even modest models. While partitioning of the model prior to estimation partially overcomes some of these problems, a major objection to partitioning is that in estimation production and demand functions the exogeneity of variables not central to the equation(s) being considered is assumed; and the estimates so determined are then used in a model which explicitly recognizes their joint endogeneity.

Estimation also produces difficulties with the compatibility of units used in the general equilibrium model and separate estimation exercises. Units for capital and labour services are defined in some of these models as those amounts capable of generating a return of one dollar in any possible use net of taxes, and gross of subsidies. These units are defined for a particular year for which the data are assumed to represent an equilibrium for the economy. While physical units are implied by such an assumption they cannot be specified in a form that makes a conversion to other well-defined physical units (such as tons) possible. Single equation estimation which produces unit dependent estimates are of limited value in the model unless the units involved are capable of being converted from one to another.

For all these reasons, the equilibrium solution concept of the model is used as the dominant restriction in the process of parameter selection by 'calibrating' the model to the benchmark equilibrium data set. The fundamental assumption made is that the economy is in equilibrium in a particular year. By modifying the National accounts and other blocks of data for that year, a benchmark data set is generated in which all equilibrium conditions inherent in the model are satisfied. The requirement that the

set of parameter values used in the model be capable of replicating this 'observed equilibrium' as an equilibrium solution to the model is then imposed as a restriction on parameter values selected. Parameter values are determined in a non-stochastic manner by solving the equations which represent the equilibrium conditions of the model using the data on prices and quantities which characterize the benchmark equilibrium.

One potential difficulty with this procedure is that the models used may exhibit multiplicity of equilibria, so that even if the test of replication is not satisfied there is no way to rule out the admissibility of a given specification. This is one of the several difficulties which the possibility of multiplicity of equilibria causes in the use of these models. A number of ad hoc procedures designed to investigate the possibility of non-uniqueness in these models have been used (starting computational procedures at different places, approaching solutions at different speeds and along different paths, displacing solutions once found and testing the successful relocation). In no case known to the authors has non-uniqueness been established in these models although this does not 'prove' that it is absent.¹

¹ Kehoe [1980] has shown that for general equilibrium models with production an index can be associated with any equilibrium which is either +1 or -1 with the property that the sum of the indices will be +1. There is a suggestion that -1 equilibria are unstable. In a simple numerical example involving four commodities and four households with Cobb-Douglas demands and activities, Kehoe has also shown a case of non-uniqueness which does not seem to be in any way an extreme or implausible specification. In this example, the equilibrium prices are widely separated between the equilibria suggesting that non-uniqueness may possibly not be as likely an occurrence as the numerical ad hoc tests seem to indicate.

D. Determination of Parameter Values through Calibration to a Benchmark Equilibrium Data Set

Whether the observed equilibrium alone is sufficient to uniquely determine the parameter values depend upon the functional forms used. The benchmark equilibrium data set, which contains "equilibrium" share observations, can only be generated by one set of Cobb-Douglas functions. For CES production and demand functions, however, extraneous estimates of elasticities of substitution (which are unit free) need to be incorporated into the procedure to serve with the equilibrium replication requirement as identifying restrictions on the model. The choice of elasticity values critically affects results obtained with these models, and the values chosen are discussed later. Additional functional complexity thus implies calibration using an expanding set of previously specified extraneous parameter estimates.

The requirement placed on the set of estimated parameter values for production and demand functions in the process of calibration is that they be capable of reproducing the complete benchmark data set as an equilibrium solution to the model. The 'calibration' procedure thus uses the equilibrium conditions of the model and the benchmark equilibrium data set to solve for parameter estimates.

After selection of functional forms for the model, the first step in calibration involves the separation of the benchmark transactions data into separate price and quantity observations. The benchmark equilibrium data set obtained by adjusting diverse data sets into a mutually consistent form provides observations on equilibrium transactions in value terms. To obtain

information separately on equilibrium prices and quantities, a unit's convention must be adopted to separate observations on price quantity combinations into component parts.

Factors of production are treated in most of the applied models as perfectly mobile between alternative uses, and the allocation of factors by industry in equilibrium will equalize the returns received net of taxes and gross of subsidies in all industries. A convenient and widely used definition of physical units for all factors is that amount of a factor that can in equilibrium earn a reward of 1 currency unit (\$1) net of taxes and before receipt of subsidies in any of its alternative uses. Units for commodities are similarly defined as those amounts which in equilibrium sell for \$1 net of all consumer taxes and subsidies.

The assumption that marginal revenue products of factors are equalized in all uses in equilibrium permits factor payments data by industry to be used as observations on physical quantities of factors for use in the determination of parameters for the model.¹ In this way observed equilibrium transactions (products of prices and quantities) are separated out into price and quantity observations. An observed equilibrium is characterized by an equilibrium price vector of unity, and ownership of a unit of labour or capital services yields a net income of \$1.

Using these data it is then possible to calculate production function parameters from the benchmark equilibrium observations of capital and labour services in each industry. We consider the case of CES value added functions for

¹ While underlying physical units of measurement are implied by such a procedure, their physical dimensions remain undefined as there are no weight or volume measures one can appeal to. With labour services, for instance, different people will be of different productivities and provide different quantities of labour services; counting the number of workers in an industry is an inappropriate measure of labour used by industry.

each of j industries. These functions are given by

$$(1) \quad Y_j = \gamma_j [\delta_j K_j^{-\rho_j} + (1 - \delta_j) L_j^{-\rho_j}]^{\frac{1}{\rho_j}}$$

where γ_j is a constant defining units of measurement, δ_j is a weighting parameter, $\sigma_j (= \frac{1}{1+\rho_j})$ is the elasticity of substitution, K_j and L_j are capital and labour service inputs, and Y_j is the industry scale of operation.

From the benchmark equilibrium data set, values for K_j and L_j can be obtained and (in the case of a tax model) factor tax rates t_j^K and t_j^L calculated. As units are chosen for productive factors such that $P_K = P_L = 1$ (where P_K and P_L refer to the net of tax factor prices) at the benchmark equilibrium, prices associated with the equilibrium quantities are known.

Once a value of the elasticity parameter σ_j is selected for each industry,¹ the values of the share parameters δ_j are given by

$$\delta_j = \left[\frac{\frac{1}{\sigma_j} (1 + t_j^K)}{\frac{1}{\sigma_j} (1 + t_j^L)} \right] \left[1 + \left[\frac{\frac{1}{\sigma_j} (1 + t_j^K)}{\frac{1}{\sigma_j} (1 + t_j^L)} \right] \right]$$

Values for γ_j are then derived from the zero profit conditions for each industry given the units definition for outputs.

Parameters for household demand functions are estimated in a similar manner from the benchmark equilibrium data on purchases of commodities by households. The procedure is analogous to that for production functions, except that individual consumer demand functions rather than first-order conditions for cost minimization problems are used to estimate parameter values.

¹The selection of these values is discussed later.

Taking a two nested variant of CES consumer demand functions, the ratio of expenditures by household q on any two commodities i, j , within the same nest, ℓ , gives an equation involving the bottom level weighting parameters in each household utility function.

$$\frac{(p_i^q)^{\sigma_{\ell}} x_i^q}{(p_j^q)^{\sigma_{\ell}} x_j^q} = \frac{(b_i^q)^{\frac{1}{\sigma_{\ell}}}}{(b_j^q)^{\frac{1}{\sigma_{\ell}}}}$$

where the x 's represent benchmark demands, b 's are weighting parameters of the lower level CES utility function and σ_{ℓ} , is the value of the elasticity of substitution within the nest ℓ .

Using the elasticity value within the nest σ_{ℓ} , the ratios of the coefficients b_i^q, b_j^q can be calculated. A normalization of the coefficients within the nest to sum to unity their individual values is frequently used.

For top level utility function weighting parameters, the benchmark data on the sum of expenditures on components of the nest can be used. If $\bar{P}_{\ell} \bar{x}_{\ell}^q$ is the expenditure by household q on the nest ℓ , the ratio of expenditures on any two nests ℓ, ℓ' gives a similar equation involving top level weighting parameters

$$\frac{\bar{P}_{\ell} \bar{x}_{\ell}^q}{\bar{P}_{\ell'} \bar{x}_{\ell'}^q} = \frac{(b_{\ell}^q)^{\frac{1}{\sigma}}}{(b_{\ell'}^q)^{\frac{1}{\sigma}}} \cdot \frac{(p_{\ell}^q)^{\sigma}}{(p_{\ell'}^q)^{\sigma}}.$$

A value for σ , the elasticity of substitution across nests, is selected and the coefficients b_{ℓ}^q calculated as for lower level nests. This same procedure can be extended to three or higher level nested CES functions.

By using the complete benchmark equilibrium data set in this way to generate parameter values for production and demand functions,¹ the equilibrium computed by the model before any policy changes will replicate the benchmark equilibrium data set exactly. This is assured as the equilibrium conditions have been used directly in the non-stochastic determination of parameter values. A practical advantage of this procedure is that the equilibrium solution of the estimated model is known ex ante and its recalculation serves as a check on the correctness of programming and on error propagation difficulties.

A 'typical' procedure of parameter selection thus involves the construction of a benchmark equilibrium data set consistent with the model along with a set of extraneously chosen elasticities of substitution in demand and production functions. These two pieces of information are then combined to give a procedure for selecting non-elasticity parameter values such that the entire set of parameters chosen for the model equations produce an equilibrium solution identical in all respects to the benchmark equilibrium data.

E. Specifying Extraneous Elasticity Values

Because of the widespread use of CES functions in applied models, calibration procedures need elasticity values to be prespecified and heavy reliance is placed on literature searches (or extraneous estimation of elasticities of substitution) in production and demand functions. The set of elasticity values used are critical parameters in determining the general equilibrium impacts of policy changes generated by these models. Discussion of their values usually precedes presentation of model results and sensitivity analysis on key parameters is frequently employed. Here we review some of the values used in the hope this may provide useful information to other researchers.

¹ Ignoring the problem of non-uniqueness mentioned earlier.

(i) Production Function Elasticities

Most models incorporate CES value added functions for each industry. For each industry in the model it is therefore necessary to specify a separate value for the elasticity of substitution between capital and labour.

Since the introduction of the CES function in the early 1960s, there has been a continuing debate as to whether the elasticity of substitution for manufacturing industry is approximately unity. If unity is a correct value, the more complex CES form can be replaced by the simpler Cobb-Douglas form which has unitary elasticity of substitution. This debate has concentrated primarily on substitution elasticities for aggregate manufacturing rather than component industries as specified in these models. This debate is nonetheless important in assessing the choice of values.

Early estimation of the elasticity of substitution in manufacturing industry by Arrow, Chenery, Minhas, and Solow (ACMS) [1961], involved a pooled cross country data set of observations on output per man and wage rates for a number of countries. The same production function was assumed to apply in all countries, and the first order condition from the industry cost minimization problem equating the marginal product of labour to the wage rate was used to estimate the elasticity of substitution. Their results indicated that the elasticity of substitution was below one, but that the difference between the estimated coefficient and unity was not significant at a 90% level. This was used as support for the position that Cobb-Douglas production functions are a reasonable specification of aggregate production functions.

Following ACMS, a number of econometric studies have estimated substitution elasticities for manufacturing industry (primarily in the U.S.) by a variety of methods and produced results with substantial disagreement.¹ Cross section studies, many of which use statewide data, produce estimates which are close to unity, but time series studies produce lower estimates typically differing from cross section by a factor of around 2. Also, estimates of substitution elasticities appear to vary systematically with the choice of estimating equation. Using the marginal product of capital relationship produces lower estimates than using the marginal product of labour.

A number of explanations of this difference have been offered, such as lagged adjustment, technical change and problems in measurement of inputs, serial correlation in time series data, and cyclical variations in utilization rates. At present no single explanation is widely accepted. A recent attempt by Berndt [1976] to reconcile alternative elasticity estimates used six different functional forms, five alternate measures of capital prices, and two estimation methods. His main finding was that estimates of substitution elasticities "are extremely sensitive to differences in measurement and data construction," and concurred with an earlier remark of Nerlove [1967] that "even slight variations in the period or concepts tend to produce drastically different estimates of the elasticity." With this degree of uncertainty over estimates for manufacturing in aggregate, obtaining estimates for individual industries is therefore hazardous.

¹The discussion here draws heavily on the summary presented by Berndt [1976] in his attempt to reconcile alternative estimates of elasticities of substitution in aggregate production functions.

A common procedure is to construct 'central tendency' tables for elasticity estimates by industry drawn from the literature. A catalogue of industry estimates of substitution elasticities has been recently compiled by Caddy [1976] in connection with the IMPACT project and is currently one of the most widely used source in the applied models. In Table 4 we show the table constructed by Piggott and Whalley [1981] for use in their model of the U.K. economy and tax system. This table is compiled for all estimates in a given industry, and separately for cross section and time series estimates. In building the table a small number of estimates are rejected as being implausible (due to a wrong sign, for instance), and the remainder are classified according to the industries used in the model. An important point to note is that for some of the industries used in the applied models no estimates exist in the literature because of the problems of measurement of outputs (such as financial services, government, and other service industries).

A further point worth noting is that these elasticity values represent technological relationships and when alternate general equilibria are calculated for policy variations the adjustments between equilibria are assumed to be complete. An assumption of smooth substitutability between capital and labour services in any industry in the short run is clearly not appropriate. Much capital is industry specific and cannot be easily adapted for alternative use and complete substitution only takes place in the longer run as capital depreciates and is not replaced. Some adjustments between equilibria may be relatively small and thus are capable of being made quickly, others may require much longer periods of time.¹ The time scale for the adjustments in applied models

¹ Given depreciation rates on the housing stock of a little over 1% per year, removal of tax preferences to owner occupied housing, for instance, could result in adjustments which may take 30 years or more to complete.

Table 4

'Central Tendency' Estimates of Elasticities of Substitution by Industry used by Piggott and Whalley [1981]

<u>INDUSTRIES</u>	<u>CENTRAL TENDENCY VALUES¹</u>		
	<u>Overall</u>	<u>Cross Section</u>	<u>Time Series</u>
Agriculture and Fishing	.607 (29, .13)	.809 (17, .10)	.322 (12, .03)
Coal Mining	-	-	-
Other Mining and Quarrying	-	-	-
Food	.789 (58, .17)	.937 (14, .13)	.433 (17, .08)
Drink	.657 (30, .15)	.879 (17, .11)	.368 (13, .06)
Tobacco	.848 (12, .24)	1.309 (3, .24)	.694 (9, .13)
Mineral Oils	.827 (24, .17)	1.002 (13, .15)	.621 (11, .13)
Other Coal and Petroleum Products	-	-	-
Chemicals	.827 (42, .16)	1.009 (26, .11)	.531 (16, .12)
Metals	.806 (79, .16)	.967 (51, .09)	.511 (28, .14)
Mech. Engineering	.587 (35, .11)	.663 (21, .11)	.451 (14, .09)
Instr. Engineering	.893 (16, .14)	1.053 (10, .12)	.627 (6, .06)
Elec. Engineering	.750 (32, .13)	.811 (24, .11)	.568 (8, .17)
Ship Building	.808 (21, .35)	1.043 (14, .33)	.341 (7, .04)
Vehicles	.810 (25, .31)	1.040 (15, .33)	.471 (9, .08)
Textiles	.914 (67, .18)	1.093 (46, .11)	.520 (21, .11)
Clothing	1.106 (25, .17)	1.221 (20, .14)	.649 (5, .05)
Leather, Fur, etc.	.940 (50, .13)	1.058 (35, .09)	.664 (15, .08)
Timer, Furniture, etc.	.843 (76, .13)	.974 (56, .07)	.475 (20, .10)
Paper, Printing, and Publishing	.908 (65, .14)	1.057 (48, .08)	.489 (17, .08)
Manufacturing n.e.s.	.944 (76, .17)	1.067 (54, .11)	.641 (22, .196)
Construction	-	-	-
Gas, Electricity and Water	-	-	-
Transport	-	-	-
Communications	-	-	-
Retail Trade	-	-	-
Wholesale Trade	-	-	-
Banking and Insurance, etc.	-	-	-
Housing Services (Private)	-	-	-
Housing Services (Local Auth.)	-	-	-
Public Service	-	-	-
Professional Services	-	-	-
Other Services	-	-	-

¹Estimates are taken from Caddy [1976] and are classified according to the industries

is usually left ill-defined although it is assumed that sufficient time elapses for all adjustments between equilibria to be complete.

(ii) Demand Function Elasticities

On the demand side household demand functions derived from staged CES utility functions are commonly used in the applied models. These functions specify constant substitution elasticities between sub-groups from the list of commodities. Where a multi-level 'nesting' structure is used for each household, the 'bottom' level nest may contain comparable domestically produced and imported goods between which a degree of substitutability is assumed consistent with import demand price elasticity estimates. In the 'intermediate' nest would be blocks of commodities between which various elasticities of substitution are assumed. A constant elasticity of substitution then prevails at the top level between commodity blocks. As many as four levels of hierachial nesting are currently in use in some models.

Few econometric estimates of substitution elasticities for CES demand functions of the staged variety exist, and few (if any) demand function systems are separately estimated by household type. Therefore, a set of indirect procedures has been used in the applied models to obtain elasticity values which have some claim to plausibility. These procedures involve collection of central tendency estimates from a literature survey of (both compensated and uncompensated) own price elasticities of demand, by product, for aggregate household sector demand functions. Substitution elasticities are then chosen for the various levels of nests to approximately calibrate to these as point estimates of the demand functions of the model at the benchmark equilibrium.

The procedure of calibrating point estimates of own price elasticities at the benchmark equilibrium to estimates of substitution elasticities can be illustrated most simply in the case of a single stage CES demand function. The N commodity demand functions derived from maximization of a single stage CES utility function subject to a household budget constraint are:

$$x_i = \frac{a_i I}{p_i^\sigma \cdot \sum_j a_j p_j^{1-\sigma}} \quad (i=1, \dots, N)$$

where I is household income, a_i are weighting parameters, σ is the elasticity of substitution, and p_i is the commodity price of good i .

Taking derivatives through the demand function

$$\frac{\partial x_i}{\partial p_i} = -\sigma a_i I p_i^{-\sigma-1} \cdot \left(\sum_j a_j p_j^{1-\sigma} \right)^{-1} - a_i I p_i^{-\sigma} \left(\sum_j a_j p_j^{1-\sigma} \right)^{-2} \cdot (1-\sigma) a_i p_i^{-\sigma}$$

gives the expression for the uncompensated own price elasticity

$$\frac{\partial x_i}{\partial p_i} \cdot \frac{p_i}{x_i} = -\sigma - \frac{a_i \cdot (1-\sigma)}{p_i^{(\sigma-1)} \cdot \sum_j a_j p_j^{(1-\sigma)}} \cdot$$

The (uncompensated) cross price elasticities can be shown to be

$$\frac{\partial x_i}{\partial p_k} \cdot \frac{p_k}{x_i} = - \frac{a_k (1-\sigma)}{p_k^{(\sigma-1)} \cdot \sum_j a_j p_j^{(1-\sigma)}}$$

At the benchmark equilibrium, producer prices are equal to unity because of the units definition adopted for outputs. Consumer prices may not exactly equal unity because of the consumer taxes; however, if they are assumed to be approximately equal to unity, and the a_i are chosen such that $\sum_j a_j = 1$,

$$\frac{\partial X_i}{\partial p_i} \cdot \frac{p_i}{X_i} \approx -\sigma - a_i (1-\sigma)$$

$$\frac{\partial X_i}{\partial p_k} \cdot \frac{p_k}{X_i} \approx - a_k (1-\sigma)$$

In those cases where weighting parameters are small (and σ is not too different from unity)

$$\frac{\partial X_i}{\partial p_i} \cdot \frac{p_i}{X_i} \approx -\sigma \quad \frac{\partial X_i}{\partial p_k} \cdot \frac{p_k}{X_i} \approx 0.$$

In the case of two level staged CES functions, an expression for the uncompensated own price elasticity at the benchmark equilibrium is obtained

$$\frac{\partial X_i}{\partial p_i} \cdot \frac{p_i}{X_i} \approx -\sigma_k - a_{jk} \cdot (1-\sigma_k) - a_{jk} \cdot (1-\sigma)[-\sigma-b_k(1-\sigma)]$$

where commodity i is the j^{th} commodity in the k^{th} nest; a_{jk} and b_k are the bottom and top level weighting parameters respectively; and σ_k and σ are bottom and top level elasticity values respectively. If each a_{jk} is reasonably small, and σ_k and σ not too far from 1, this gives

$$\frac{\partial X_i}{\partial p_i} \cdot \frac{p_i}{X_i} \approx -\sigma_k.$$

Under these approximations the own price elasticity of a commodity or composite of commodities is determined primarily by the elasticity of substitution in the lowest level of the nesting in which it appears. Similar but more complex expressions can be obtained from three level staged CES functions.

The procedure used for specifying substitution elasticities is to utilize these expressions to generate values for elasticities of substitution from own price elasticity estimates by product. Because all commodities within any nest have the same compensated own price elasticity, this procedure is somewhat limited in scope unless extensive nesting is used to provide sufficient

elasticity parameters to prespecify. Frequently the nesting is arranged so that elasticities for 'key' commodities, or commodity blocks can be set (e.g., labour supply, savings, energy demand).

Central tendency estimates are obtained from literature estimates in a manner similar to that adopted for production function elasticities. The values used on the demand side by Piggott and Whalley [1981] in their U.K. model are shown in Table 5. The majority of elasticity estimates reported in the literature are for demand function systems for which price elasticity estimates are only available as the point estimates at sample means. For this reason, the central tendency figures are differentiated by estimating equation. Moreover, these estimates relate to aggregate demand functions rather than household demand functions. Most estimates are based on time series data and the stratification between time series and cross-section estimates important for production function estimates does not appear in this table.

(iii) Other Elasticities

Depending upon the orientation of the particular applied model, other elasticities besides those discussed are also important elements of the parameter set and are treated in a similar way with literature search followed by some form of calibration as needed.

In the trade models import and export demand price elasticities are critical parameters and parameters of CES functions are calibrated to literature estimates. The recent compendium of estimates due to Stern, Francis, and Schumacher [1976] is widely used. In some of the trade modelling income elasticities of import demand functions are important, and LES variants of CES forms are used with calibration of intercept LES parameters to literature income elasticities.

In the tax models two important elasticities parameters are labour supply and savings elasticities. In analysis of labour supply taxation issues, the common procedure is to define utility functions over leisure and goods and choose a value for the leisure-goods substitution elasticity consistent

Table 5

Central Tendency Values for Own Price Elasticities of Household Demand Functions Used by Piggott and Whalley [1981]

(All Compensated Own Price Elasticity Estimates; Figures in Parentheses refer to the number of studies included and the Variance of Estimate)

<u>Commodities</u>	<u>Linear Expenditure System Estimates</u>	<u>Log Linear Demand Estimates</u>	<u>Other</u>	<u>Total</u>
Agriculture and Fishing	.334 (17, .03)	.420 (25, .05)	.562 (44, .08)	.468 (86, .07)
Coal	-	.321 (1, 0)	1.265 (2, .84)	.950 (3, .76)
Other Mining and Quarrying Products	.425 (1, 0)	.905 (3, .06)	.257 (2, .01)	.609 (6, .13)
Food	.353 (15, .03)	.580 (30, .19)	.476 (27, .08)	.494 (72, .13)
Drink	.617 (5, .07)	.780 (12, .25)	.464 (15, .06)	.607 (32, .16)
Tobacco	-	.611 (8, .15)	.431 (11, .04)	.507 (.9, .10)
Mineral Oils	.425 (1, 0)	.905 (3, .07)	.257 (2, .01)	.609 (6, .13)
Other Coal and Petroleum Products	1.283 (2, .01)	1.404 (3, .80)	1.978 (3, 1.41)	1.589 (8, .90)
Chemicals	.685 (1, 0)	.890 (1, 0)	.680 (3, .07)	.724 (5, .05)
Metals	-	1.522 (19, .42)	.989 (18, .40)	1.083 (51, .43)
Mech. Engineering	-	1.296 (16, .61)	1.068 (15, .43)	1.005 (45, .48)
Instr. Engineering	.606 (14, .15)	1.099 (17, .57)	1.240 (11, .54)	.972 (42, .49)
Elec. Engineering	-	1.388 (19, .377)	1.049 (17, .410)	1.060 (50, .44)
Ship Building	-	-	-	-
Vehicles	.606 (14, .15)	1.137 (19, .55)	1.099 (18, .40)	.985 (51, .44)
Clothing	.277 (16, .03)	.491 (26, .16)	.564 (19, .15)	.458 (61, .18)
Timber, Furniture, etc.	.570 (14, .09)	1.258 (19, .23)	.974 (20, .39)	.969 (53, .33)
Paper, Printing, and Publishing	.191 (1, 0)	.343 (5, .02)	.416 (5, .02)	.362 (11, .02)
Manufacturing n.e.s.	.578 (14, .02)	.527 (7, .11)	.626 (17, .12)	.592 (38, .09)
Construction	-	-	-	-
Gas, Electricity and Water	1.203 (1, 0)	.921 (9, .02)	.369 (10, .01)	.659 (20, .10)
Transport	.761 (4, .23)	1.027 (14, .26)	.994 (10, .16)	.977 (28, .23)
Communications	-	-	-	-
Retail Trade	-	-	-	-
Wholesale Trade	-	-	-	-
Banking and Insurance	-	.559 (3, .02)	.894 (1, 0)	.642 (4, .04)
Housing Services (Private)	.461 (15, .11)	.550 (29, .45)	.434 (9, .09)	.505 (53, .29)
Housing Services (Local Auth.)	-	-	-	-
Public Service	-	-	-	-
Professional Services	{ .488 (7, .08)	1.09 (27, .56)	.946 (16, .39)	.961 (50, .48)
Other Services				

with literature estimates of labour supply elasticities with respect to the net of tax wage. Literature estimates on this elasticity vary sharply by the group involved with prime age males having low if not negative elasticities, and secondary and older workers having higher elasticities (around .5). Fullerton, Shoven, and Whalley [1980] use Lewis' [1976] 'conjecture' of .15 for the whole workforce.

With intertemporal taxation issues the elasticity of savings with respect to the real net of tax rate of return becomes important. The approach in Fullerton, Shoven, and Whalley [1980] is to consider a sequence of equilibria through time with savings decisions depending on expected rates of return. Savings today augments the capital stock in all future periods. Fullerton, Shoven, and Whalley place substantial reliance on Boskin's [1978] recently estimated elasticity of 0.4 although the use of this elasticity in an alternative model by Summers [1980] has sharply different implications.

In other areas critical elasticities also arise. In energy modelling a major issue is the substitutability (or complementarity) between energy, capital, and labour. In development models regional features are sometimes incorporated and the elasticity of outward migration from rural areas becomes important.

In all these cases the procedure seems to be similar to the discussion above. Share parameters come from calibration, elasticity values come from a literature search or extraneous estimation.

IV. COMPARISON BETWEEN STOCHASTIC ESTIMATION AND
CALIBRATION FOR A ONE CONSUMER, TWO SECTOR
GE TAX MODEL USING U.S. DATA

In this section we present some results obtained by Mansur [1981] from time series estimation of a two sector general equilibrium tax model of the U.S. We are then able to compare his parameter estimates to estimates obtained by calibration to a benchmark data set for alternative years. While such a comparison is by no means conclusive, it does provide some initial indications as to how large the differences may be between the two procedures.

Mansur focuses on the well known one consumer, two sector general equilibrium model with differential taxation of income from capital due to Harberger [1962]. Although the literature on this model is primarily concerned with measuring the costs of distortions in capital use arising from non-neutralities of tax system, its use in the present context serves three purposes: (a) As an illustration of how estimation procedures suggested in Part II can be applied; (b) to allow comparison between parameter estimates obtained from 'calibration' to a benchmark equilibrium and those produced by estimation; (c) to allow comparison of measures of the cost of capital tax distortions for the alternative parameter estimates and further compare them with cost estimates due to Harberger [1966], Shoven and Whalley [1972], and Shoven [1976].

Mansur uses time series data for the U.S. economy (1948-65), most of the information coming from U.S. Department of Commerce publications (e.g., various issues of the Survey of Current Business). A similar commodity classification as used by Harberger [1962, 1968], Shoven and Whalley [1972] is used. The two sectors are the predominantly corporate (heavily taxed)

sector, and the lightly taxed sector (agriculture, housing or real estate, crude oil and gas). The corporate sector includes mining, manufacturing, transport, communication, contract construction, electric, gas and sanitary services, wholesale and retail trade, finance and insurance (except real estate) and services.¹

Mansur considers two simple specifications:²

- (i) demand and production systems derived from Cobb-Douglas utility and production functions;
- (ii) the demand system derived from a Cobb-Douglas utility function while the sector production functions are CES.

The associated input demand system is derived accordingly.

Table 6 reports calibrated benchmark equilibrium parameter estimates for the Cobb-Douglas case for alternative benchmark years. For each year reported data have to be adjusted to ensure the conditions of general equilibrium accounting (as described above). The 'RAS' method has been applied to

¹ Personal and business services and wholesale and retail trade are not overwhelmingly corporate in structure but treated as heavily taxed. Rosenberg [1969] has an approximate 30% tax rate on income from capital in these activities.

² On the demand side other variations were tried such as Linear Expenditure System, and CES demand system, but the estimation procedure either did not converge or produced estimates which were not of proper sign.

TABLE 6

Parameters of the Cobb-Douglas Production and Demand
System¹ Over Time From Benchmark Calibration

Year↓\Parameters	a ₁	a ₂	b ₁	b ₂
1948	0.516	0.330	0.160	0.840
1950	0.564	0.333	0.147	0.857
1955	0.600	0.276	0.126	0.874
1960	0.633	0.298	0.109	0.891
1962	0.636	0.271	0.115	0.885
1965	0.687	0.280	0.115	0.885

$$^1Q_i = A_i K_i^{a_i} L_i^{1-a_i} \quad : \text{sector production functions}$$

$$W = \prod_{i=1}^n X_i^{b_i} \quad : \text{household utility function}$$

Sector 1 is lightly and sector 2 heavily taxed.

yearly data obtained from national income accounts to ensure the equilibrium conditions and 'calibration' used to determine parameter values.

Table 7 presents parameter estimates obtained by employing a nonlinear instrumental variable method as described above (in Section II). Unlike the use of calibration methods there is no prior adjustment to data as involved in benchmarking. Estimates are also shown as obtained by applying FIML method to the subsystems of the complete model ignoring the simultaneity.

Comparison of estimates in Tables 6 and 7 indicate that these different methods have somewhat similar parameter estimates. Benchmark equilibrium parameters show some degree of variation from year to year, but if we take the average of these yearly values, the mean is not very different from the values obtained by estimation. For the Cobb-Douglas case, the initial impression would seem to be that calibration to a benchmark is not too bad; although if one is to apply the benchmark method, it may be best to apply consistency adjustments to time averaged data. The treatment of technical change, however, may dictate further modifications to such a procedure.

In the CES case, benchmark calibration cannot be a complete substitute for econometric estimation because of the need to prespecify substitution elasticities. Benchmark calibration when applied to average data may give reasonably good estimates of the share and scale parameters but cannot estimate the elasticity parameters. The values selected for elasticity parameters influence calibration determined values of share and scale parameters. In the Cobb Douglas case this issue does not arise because the elasticity of substitution in both demand and production is unity.

Table 8 shows how share and scale parameters calculated through calibration are influenced by the variations in adopted elasticity values for the

Table 7PARAMETER VALUES FOR DIFFERENT ESTIMATION METHODS(Cobb-Douglas Production and Demand Systems)

Method Parameters → ↓	FIML ^{*1} (Ignoring Simultaneity)	NLIV ^{*1} (Method I)	NLIV ^{*1} (Method II)	Benchmark Equilibrium Parameters ^{*2}
a_1	0.606	0.627	0.647	0.516 - 0.678 (0.609)
a_2	0.369	0.307	0.325	0.332 - 0.272 (0.292)
b_1	0.122	0.124	0.124	0.160 - 0.109 (0.124)
b_2	0.878	0.876	0.876	0.890 - 0.840 (0.876)

*¹All coefficients are statistically significant at 1% level of significance.

*²The numbers shown side by side in each box represent the range of estimated values over different years as obtained under Benchmark Equilibrium method. The numbers within the parentheses represent the arithmetic mean of the whole range of estimated values.

Table 8

SENSITIVITY OF SHARE AND SCALE PARAMETERS DUE TO
VARIATIONS IN ELASTICITY VALUES UNDER BENCHMARK CALIBRATION*

Other Parameters Elasticity Values	a_1	a_2	A_1	A_2
0.333	0.547	0.048	4.0988	3.6383
0.500	0.531	0.136	4.1008	3.9338
1.000	0.516	0.329	4.1028	4.4412
2.000	0.508	0.464	4.1038	4.7885

* This is based on the benchmark equilibrium data set constructed for the year 1948. The elasticity values are arbitrarily chosen to assess the impact of variations on other parameters.

U.S. data used by Mansur. We note that estimated share parameter in sector 2 is surprisingly sensitive to changes in the adopted values of substitution elasticities. This point is important to keep in mind because of the limited consensus from literature searches on relevant elasticity values. Benchmark calibration, by using only elasticity values from literature searches can give the appearance of 'removing' the need for estimation. However, the fact remains that errors committed at the stage of elasticity selection then tend to be propagated onto other estimated parameters.

Table 9 reports estimated parameters for a model with CES production functions under different methods of estimation. For NLFIML method applied to model subsystems ignoring subsystem simultaneity, the estimators would generally be biased and estimates obtained appear to be biased downwards. For the corporate sector (sector 2), both the elasticity of substitution and share parameters are statistically insignificant in this case. For the NLIV methods both sets of estimated parameters are not very different from one another, although method I (NLIV) is computationally much simpler and involves significantly less work compared to NLIV method II. This table once again lends support to the observation that when averaged over time, the share parameters from calibration (with same elasticity value) tend to be not much different from those of NLIV methods, but the dependence on pre-selected elasticities is worth re-emphasis.

Table 10 reports welfare costs due to differential rates of taxation in the U.S. economy using a Cobb-Douglas production and utility function, and using estimated parameter values obtained under different methods.

TABLE 9

Stochastic Estimates of the Elasticity and Share Parameters
Under Various Methods of Estimation

Stochastic Methods	Parameter →	Elasticity (Production)		Share Parameters (Production)		Share Parameters (Demand)	
		σ_1	σ_2	α_1	α_2	b_1	b_2
NLFIML ¹		.2419	.2465 ²	.8648	.0015 ²	0.122	0.878
NLIV (Method I)		1.45	0.485	0.623	0.18	0.124	0.876
NLIV (Method II)		1.489	0.535	0.620	.16	0.124	0.876
Calibration ³ (with elasticities obtained by NLIV Method I)		1.45	0.485	0.574	0.15	0.124	0.876
Calibration ³ (with elasticities obtained by NLIV Method II)		1.489	0.535	0.574	0.12	0.124	0.976

¹ NLFIML applied to the subsystems ignoring the underlying simultaneity.

² The parameter is not statistically significant

³ The share parameters are the average figures and the elasticity estimates are taken from NLIV (method II).

Table 10
**COMPARISON OF THE ESTIMATES OF WELFARE LOSS DUE TO NON-NEUTRALITY OF TAXES ON INCOME
 FROM CAPITAL IN THE U.S. ECONOMY WITH COBB-DOUGLAS PRODUCTION AND DEMAND SYSTEMS***

Selected Years ↓	HARBERGER FORMULATION		SHOVEN - WHALLEY FORMULATION		% Variation Between Col. 5 and 6 (5)-(6) × 100 (5)	
	NLIV Estimators (Method II)	Harberger *** Parameters	% Variation Between Col. 3 and Col. 1 (3)-(1) × 100 (1)			
			NLIV Parameters	% Loss (FI)* (5)		
Loss (m)	% Loss (2)	Loss (m) (3)	(4)	% Loss (FI)* (6)	(7)	
1949	5451	2.29	5175	-5.06	0.558	
1952	6159	2.19	5874	-4.63	0.558	
1955	6501	2.15	6246	-3.92	0.569	
1958	6563	2.05	6328	-3.58	0.558	
1962	7841	2.09	7577	-3.37	0.598	
1965	9573	2.14	9292	-2.94	0.900	

* The estimates are based on the parameters reported in Table III-2.

** FI is the Fisher's index (the geometric mean of Laspyre's and Paasche's index) with no-tax situation as the base.

*** Share of capital (fK_2) in sector 2 is 0.2; $fK_1 = 0.54$; $T = 0.53$, the surcharge, share of labour in sector 2. $fL_2 = 0.8$, elasticity of substitution is unity for both sectors and price elasticity of demand for commodity 2 is unity.

This table shows that the measures of loss in Shoven-Whalley algorithmic approach tend to be lower with benchmark equilibrium parameters than those with NLIV estimators. The extent of underestimation increases rapidly in both absolute and relative terms over time. While for 1949 the benchmark parameters underestimate the loss by 25 percent, by 1965 this figure (for underestimation) increases to 162 percent. For Harberger's formula for measuring the costs of the tax distortion, we use parameter values close to those used by Harberger which are then compared to loss estimates from use of the NLIV estimators. There are represented in column 3 and column 1 respectively of Table 10. The difference between the two outcomes is not very significant and while the 'Harberger' parameters tend to underestimate the measured costs, the difference tends to be reduced over time (as indicated in column 4). The surprising degree of closeness in the measured costs is due to the fact that Harberger's method of assigning the parameter values for this Cobb-Douglas case yields parameters very close to the NLIV estimators.

For CES production functions the welfare costs are reported in Table 11 in a manner similar to those of Table 10. The figures in Table 11 are based on the parameters reported in Table 9. In the first ten columns we report Harberger's measures of costs under three different sets of parameter values. It appears that loss estimates are very sensitive to the price elasticity of demand for corporate sector products (see columns 6 and 8 of Table 11). NLIV-Method I parameters slightly but systematically underestimate the welfare loss compared to those from NLIV-Method II parameters. While the Harberger parameters with $E2 = 1/7$ (price elasticity of corporate sector products) underestimate the welfare loss compared to NLIV methods, the figures with $E2 = 1.0$ are significantly higher than those of NLIV counterparts.

Table 11
COMPARISON OF THE ESTIMATES OF WELFARE LOSSES DUE TO NON-NEUTRALITY OF
TAXES ON INCOME FROM CAPITAL IN THE U.S. ECONOMY, USING CES
PRODUCTION FUNCTIONS^{a1}

Years	Method ^b	HARBERGER FORMULATION						SLOVEN - WHALEY FORMULATION						NLIV Estimate (Method I)			NLIV Estimate (Method II)		
		NLIV Estimates (Method I)			NLIV Estimates (Method II)			Harberger Parameters ^{b2}			NLIV Estimate (Method I)			NLIV Estimate (Method II)			NLIV Estimate (Method I)		
		Loss (m)	% Loss (2)	Loss (m)	% Loss (3)	Col. 1 as % of Col. 3	Col. 1 as % of Col. 5	Loss (m)	E2 = $\frac{1}{7}$	Loss (m)	E2 = $\frac{1}{10}$	Loss (m)	E2 = $\frac{1}{10}$	Loss (m)	F1 ^{c3} (11)	Loss (m)	F1 ^{c3} (12)	Loss (m)	F1 ^{c3} (13)
1948	3611.44	1.48	3818.27	1.55	94.1	1962.09	.79	5126.27	2.11	.359	1522.60	.515	1397.8	.491	1392.8	.50	.004		
1949	3531.64	1.48	3711.11	1.56	95.2	1890.75	.80	5125.26	2.17	.390	1649.6	.567	1359.3	.49	1280.2	.47	.058		
1950	3967.91	1.52	4169.16	1.60	95.2	2094.46	.80	5804.53	2.22	.392	2354.0	.74	2247.5	.76	1331.5	.456	.408		
1951	3922.97	1.45	4124.11	1.52	95.1	2096.95	.77	5777.59	2.13	.401	1870.8	.53	2046.3	.61	1893.6	.49	.075		
1952	3937.96	1.40	4144.38	1.47	95.0	2102.93	.75	5873.89	2.09	.417	2101.6	.56	2162.0	.62	1907.4	.55	.118		
1953	3942.48	1.35	4155.58	1.43	94.9	2103.11	.72	5988.36	2.06	.441	2299.3	.57	1998.2	.54	1605.1	.45	.197		
1954	3836.56	1.35	4063.25	1.42	94.8	2062.12	.72	5850.98	2.05	.443	1844.7	.42	2254.5	.62	1592.8	.46	.294		
1955	4071.10	1.35	4296.92	1.42	96.8	2157.76	.72	6245.83	2.07	.454	2437.7	.58	2094.8	.55	1780.0	.49	.150		
1956	4353.95	1.35	4594.33	1.42	94.8	2283.38	.71	6593.53	2.07	.457	2680.8	.61	2244.1	.56	1676.1	.43	.253		
1957	4301.81	1.32	4542.51	1.39	94.7	2284.21	.70	6672.29	2.04	.469	2958.4	.72	4280.3	1.0	1733.7	.43	.593		
1958	4075.13	1.28	4302.97	1.35	94.7	2230.31	.70	6327.46	1.98	.470	2461.7	.56	2254.6	.54	1763.0	.45	.217		
1959	4051.15	1.41	5124.64	1.49	94.7	2530.25	.73	7486.32	2.17	.461	2498.1	.54	2409.4	.56	1700.4	.42	.294		
1960	4455.76	1.27	4708.80	1.36	94.6	2398.35	.68	6380.91	1.99	.483	2617.5	.56	2609.4	.52	1920.4	.44	.203		
1961	4336.70	1.27	4794.30	1.34	94.6	2664.10	.69	7109.92	1.99	.483	3405.5	.63	2525.4	.54	2012.1	.46	.201		
1962	4039.37	1.29	5113.96	1.36	94.6	2603.04	.69	7577.06	2.01	.482	3447.5	.58	2828.9	.56	2346.5	.47	.173		
1963	5025.96	1.29	5311.17	1.36	94.6	2692.84	.69	7867.23	2.01	.481	3517.2	.57	2842.3	.55	2138.4	.41	.248		
1964	5508.85	1.31	5829.70	1.39	94.5	3025.36	.72	8779.90	2.09	.506	3773.3	.60	5026.6	.59	2445.1	.48	.514		
1965	5846.41	1.30	6185.27	1.38	94.5	3229.20	.72	9291.49	2.07	.502	3393.8	.58	6289.1	1.36	2593.1	.45	.689		

^{a1}The parameters used are based on the estimates reported in Table 9. The same figures as shown in the footnote of Table 10 have been used as 'Harberger' parameters; the elasticities of substitution are also the same, i.e., unity.

^{a2}We consider two situations, the original Harberger assumption that demand elasticity for this corporate sector is only $-\frac{1}{7}$ and the other more reasonable assumption that demand elasticity is -1.0 for both sectors.

^{a3}F1 is the Fisher's Index (the geometric mean of Laspeyres and Paasche's Index), with no-tax situation as the base.

With the Shoven-Whalley algorithmic approach the loss estimates are somewhat similar to each other. Under all three sets of parameters the values of welfare loss tend to increase over time, following the pattern observed with the Harberger formulation. The absolute measures of loss are not significantly different from the others, though the benchmark method yields figures that are slightly downward biased compared to other methods. This lack of differences may once again be due to the fact that for the benchmark case we use elasticities from NLIV (method II) estimation procedure. As has been indicated above in the discussion of the Cobb-Douglas case, as long as the elasticities are the same, benchmark calibration tends to produce share parameters close to those from stochastic methods.

V. IMPLICATIONS FOR EXISTING APPLIED MODELS AND SPECIFICATION PROCEDURES

Any assessment of implications of our paper for current general equilibrium modelling efforts must inevitably be both inconclusive and highly subjective.

Our perspective on the issues raised can be summarized as follows:

(1) Literature on numerical application of general equilibrium analysis (primarily to policy issues) appears to us to have progressed from a concern with computation methods in the late 60s and early 70s to a recent spate of model building where the ability to solve and manipulate large models has taken primacy. While we see this as both a valuable and natural progression, the contribution this work can make to policy issues ultimately lies in the numerical perspectives offered on policy choices. The 'value' of these attempts at quantification depends crucially on the reasonableness or otherwise of the numerical specifications used, and it seems natural to us that more attention

be given in this area to the question of how one numerically specifies a general equilibrium model in a 'reasonable' manner, in addition to how one solves it.¹

(2) Modellers seem to be increasingly conscious of the key role played by elasticities in these models. The widespread use of CES and other tractable forms seems to us to be a natural development in these models,² but the paucity of econometric literature on many required parameters poses serious problems and dilemmas. 'Assumed' elasticities for 'illustrative' calculations do not provide convincing policy conclusions. On the other hand, it does not seem reasonable to suggest modellers suspend their work in order to devote themselves to prior estimation of elasticities. The accommodation might be to clearly display the absence of estimates where this occurs, and to limit modelling efforts where elasticities are the bottleneck (e.g., perhaps not work with detailed commodity or industrial groupings if no estimates exist for that detail, but use a more aggregated model).

(3) Calibration, as a method for selecting parameter values, appears to have the two weaknesses of requiring pre-selection of elasticities, and not providing any basis for a test of the specification since it fits perfectly the single data point used. Exclusive reliance on stochastic estimation also appears to have problems associated with it. For even 'mildly' elaborate models (commodity and industry groups each less than 10), prohibitively long time series are needed for system estimation, yet subsystem (or single equation) estimation will

¹ And we might add, a question almost totally ignored in the Hayek-Robbins-Lange debate in the 1930s on the feasibility on centralized calculation of solutions to "Paretian equations".

² Although we note the reservations expressed by some authors as to the restrictions these functions imply.

typically neglect some of the cross equation restrictions which are the essence of general equilibrium. Statistically well grounded estimation procedures may indeed only be implementable for models so simplified that they remove much (or even most) of the detail that interests the policymakers. This we see as a dilemma and offer no clear guidance.

(4) Our instinct is that numerical general equilibrium modelling for some time to come will remain the subjective elasticity-dependent 'art' into which it seems to be evolving. Despite that, it has a major contribution to make to policy evaluation much of which even today seems to proceed on hunches, prior belief, and data exercises with unclear (or missing) theoretical underpinnings. It is ridiculous to claim that numerical general equilibrium analysis can definitely 'solve' policy problems, especially as the assumptions (let alone the numerical applications) can be and are frequently questioned. However, taken with the appropriate grain of salt at the right time, we believe there is enormous potential for both extending and raising the level of debate on many social issues. If this potential is to be realized, existing procedures for choosing between alternative numerical specifications for their models would seem to need further refinement.

References

- Adelman, I. and S. Robinson, 1977, Income Distribution Policy in Developing Countries, Stanford, Stanford University Press.
- Allingham, M., 1973, Equilibrium and Disequilibrium; A Quantitative Analysis of Economic Interaction, Ballinger Publishing Co., Cambridge, Mass.
- Amemiya, T., 1974, "The Nonlinear Two Stage Least Squares Estimator," Journal of Econometrics.
- Amemiya, T., 1977, "The Maximum Likelihood and the Nonlinear Three Stage Least Squares Estimation in the General Nonlinear Simultaneous Equation Model," Econometrica, pp. 955-968.
- Armington, P., 1969, "A Theory of Demand for Products Distinguished by Place of Production," I.M.F. Staff Papers.
- Arrow, K. J., et al (1961), "Capital-Labor Substitution and Economic Efficiency," The Review of Economics and Statistics.
- Bacharach, M., 1970, Biproportional Matrices and Input-Output Change, Cambridge University Press.
- Barku, A. P., 1964, "Consumer Demand Functions Under Conditions of Almost Additive Preferences," Econometrica, 32, pp. 1-38.
- Barten, A. P., 1967, "Evidence on the Slutsky Conditions for Demand Equations," The Review of Economics and Statistics.
- Barten, A. P., 1969, "Maximum Likelihood Estimation of a Complete System of Demand Equations," European Economic Review.
- Barten, A. P., 1977, "The Systems of Consumer Demand Functions Approach: A Review," in, M. D. Intriligator (ed.), Frontiers of Quantitative Economics, North-Holland, Amsterdam, pp. 24-58.
- Berndt, E. R., 1976, "Reconciling Alternative Estimates of the Elasticity of Substitution," Review of Economics and Statistics 58, pp. 59-68.
- Berndt, E. R., B. Hall and J. A. Hausman, 1974, "Estimation and Inference in Non-Linear Structural Models," Annals of Economic and Social Measurement.

- Berndt, E. R. and D. O. Wood, 1975, "Technology, Prices and the Derived Demand For Energy," Review of Economics and Statistics, pp. 259-268.
- Boadway, R. and J. Treddenick, 1978, "A General Equilibrium Computation of the Effects of the Canadian Tariff Structure," Canadian Journal of Economics.
- Boskin, M. J., 1978, "Taxation, Savings and the Rate of Interest," Journal of Political Economy, pp. s3-s28.
- Bridge, J. L., 1971, Applied Econometrics, North-Holland, Amsterdam.
- Brown, A. and A. Deaton, 1972, "Surveys in Applied Economics: Models of Consumer Behavior," The Economic Journal.
- Brown, F. and J. Whalley, 1980, "General Equilibrium Evaluations of Tariff Cutting Proposals in the Tokyo Round and Comparisons with More Extensive Liberalization of World Trade," Economic Journal, Vol. 90, December, pp. 838-866.
- Caddy, V., 1976, "Empirical Estimation of the Elasticity of Substitution: A Review," Mimeograph, Industries Assistance Commission, Melbourne, Australia.
- Carrin, G., Gunning, J. W., and Waelbroek, J., 1980, "A General Equilibrium Model for the World Economy: Some Preliminary Results," Paper prepared for the 8th IIASA Global Modelling Conference, Laxenburg, Austria.
- Chow, G., 1973, "On the Computation of Full-Information Maximum Likelihood Estimates for Nonlinear Equation Systems," Review of Economics and Statistics.
- Christensen, L. R., D. W. Jorgenson and L. J. Lau (1971), "Conjugate Duality and Transcedental Logarithmic Production Function," Econometrica.
- Christensen, L. R., D. W. Jorgenson and L. J. Lau (1973), "Transcedental Logarithmic Production Frontiers," Review of Economics and Statistics.
- Christensen, L. R., D. W. Jorgenson and L. J. Lau, 1975, "Transcedental Logarithmic Utility Functions," American Economic Review.

Christensen, L. R., and M. E. Manser, 1975, "Estimating the U.S. Consumer Preference for Meat with Non-Additive Utility Function," Journal of Econometrics.

Darrough, M. N., 1975, "Intertemporal Allocation of Consumption, Savings and Leisure: The Japanese Case," Unpublished Ph.D. Dissertation, University of British Columbia.

Deardorff, A. V. and R. M. Stern, 1979, "An Economic Analysis of the Effects of the Tokyo Round of Multilateral Trade Negotiations on the United States and the Other Major Industrialized Countries," Report prepared for the Committee on Finance, United States Senate.

Deaton, A. S., 1972, "The Estimation and Testing of Systems of Demand Equations: A Note," European Economic Review.

Deaton, A., 1974, "A Reconsideration of the Empirical Implications of Additive Preferences," Economic Journal, Vol. 84, June, pp. 338-349.

De Melo, J., 1978, "Estimating the Cost of Protection: A General Equilibrium Approach," Quarterly Journal of Economics.

De Melo, J. and K. Dervis, 1977, "Modelling the Effects in a Dynamic Framework," Journal of Development Economics.

Dervis, K., 1975, "Substitution, Employment and Intertemporal Equilibrium in a Nonlinear Multi-Sectoral Planning Model for Turkey," European Economic Review.

Dervis, K. and S. Robinson, 1978, "The Foreign Exchange Gap, Growth and Industrial Strategy in Turkey: 1973-83," World Bank Staff Working Paper No. 306.

Desai, M., 1976, Applied Econometrics, Philip Allan Publisher Limited, Oxford. Dhrymes, P., 1962, "On Devising Unbiased Estimators for the Parameters of the Cobb-Douglas Production Function," Econometrica.

Diewert, W. E., 1969, "Functional Form in the Theory of Production and Consumer Demand," Unpublished Ph.D. Dissertation, University of California at Berkeley.

- Diewert, W. E., 1971, "An Application of the Shephard Duality Theorem: The Generalized Leontief Production Function," Journal of Political Economy.
- Diewert, W. E., 1973, "Separability and a Generalization of the Cobb-Douglas Cost, Production and Indirect Utility Functions," Technical Report No. 86, Institute of Mathematical Studies in Social Sciences, Stanford University, mimeo.
- Diewert, W. E., 1974, "Applications of Duality Theory," in, Frontiers of Quantitative Economics, Vol. II, M. D. Intriligator and D. A. Kendrick (eds.), North-Holland, Amsterdam.
- Dixon, P. B., B. R. Parmenter, G. J. Ryland and J. Sutton, 1977, Orani, A General Equilibrium Model of the Australian Economy: Current Specification and Illustrations of Use for Policy Analysis, Australia: Australian Government Publishing Service
- Eisenpress, H. and J. Greenstadt, 1966, "The Estimation of Nonlinear Econometric Systems," Econometrica.
- Fisher, F. M., 1966, The Identification Problem in Econometrics, McGraw-Hill Book Co.
- Fullerton, D., J. B. Shoven and J. Whalley, 1978, "General Equilibrium Analysis of U.S. Taxation Policy," in 1978 Compendium on Tax Research, Washington, D.C.: Office of Tax Analysis, U.S. Treasury.
- Fullerton, D., J. B. Shoven, and J. Whalley, 1980, "Dynamic General Equilibrium Impacts of Replacing the U.S. Income Tax with a Progressive Consumption Tax," N.B.E.R. Conference Paper No. 55, October.
- Goldfeld, S. and R. Quandt, 1968, "Nonlinear Simultaneous Equations: Estimation and Prediction," International Economic Review.
- Gorman, W. M., 1957, "Tariffs, Retaliation and Elasticity of Demand for Imports," Review of Economic Studies, XXV, pp. 137-162.

- Gussman, T. K., 1972, "The Demand for Durables, Nondurables, Services and the Supply of Labor in Canada: 1946-1969," Department of Economics, University of British Columbia, mimeo.
- Harberger, A. C., 1962, "The Incidence of Corporate Income Tax," Journal of Political Economy.
- Harberger, A. C., 1966, "Efficiency Effects of Taxes on Income from Capital," in, M. Kryzhaik (ed.), Effects of Corporation Income Tax.
- Hausman, J. A., 1975, "An Instrumental Variable Approach to Full-Information Estimators for Linear and Certain Nonlinear Econometric Models," Econometrica.
- Hoch, I., 1958, "Simultaneous Equation Bias in the Context of the Cobb-Douglas Production Function," Econometrica, 26, pp. 566-578.
- Houthakker, H. S., 1960, "Additive Preferences," Econometrica.
- Johansen, L., 1960, A Multi-Sectoral Study of Economic Growth, North-Holland Publishing Co., Amsterdam.
- Johansen, L., 1969, "On the Relationship Between Some Systems of Demand Functions," Liiketaloudellinen Aikakauskirga.
- Johnson, H. G., 1953, "Optimum Tariffs and Retaliation," Review of Economic Studies, XXI, pp. 142-153.
- Johnson, H. G., 1956, "General Equilibrium Analysis of Excise Taxes: Comment," American Economic Review, 46, pp. 151-156.
- Jorgenson, D. W. and Fraumeni, 1980, "Relative Prices and Technical Change," Discussion Paper Series, Harvard Institute of Economic Research, Harvard University, Cambridge, Massachusetts.
- Jorgenson, D. W. and Jean J. Laffont, 1974, "Efficient Estimation of Nonlinear Simultaneous Equations with Additive Disturbances," Annals of Economic and Social Measurement, pp. 615-652.
- Jorgenson, D. W. and L. J. Lau, 1975, "The Structure of Consumer Preferences," Annals of Economic and Social Measurement.

- Kehoe, T. J., 1980, "An Index Theorem for General Equilibrium Models with Production," Econometrica, July, pp. 1211-1233.
- Kelejian, H., 1971, "Two Stage Least Squares and Econometric Systems Linear in Parameters but Nonlinear in Endogenous Variables," Journal of American Statistical Association.
- Kendrick, J. W., 1973, "Postwar Productivity Trends in the United States, 1948-1969," National Bureau of Economic Research, General Series 98, New York.
- Kmenta, J., 1964, "Some Properties of Alternative Estimators of the Cobb-Douglas Production Function," Econometrica 32, pp. 183-188.
- Lau, L. J., 1976, "Complete Systems of Consumer Demand Functions Through Duality," Vol. III, in, M. D. Intriligator (ed.), Frontiers of Quantitative Economics, North-Holland, Amsterdam.
- Lewis, H. G., 1975, "Economics of Time and Labour Supply," The American Economic Review, LXV, No. 2.
- Manne, A. S., 1980, "A Three-Region Model of Energy, International Trade and Economic Growth," Department of Operations Research, Stanford University, California.
- Mansur, A., 1981, "On the Estimation of General Equilibrium Models," Department of Economics, University of Western Ontario, London, mimeo.
- Marschak, J., and W. Andrews, 1944, "Random Simultaneous Equations and the Theory of Production," Econometrica.
- Miller, M. H. and J. E. Spencer (1977), "The Static Economic Effects of the UK Joining the EEC: A General Equilibrium Approach," The Review of Economic Studies, February, pp. 71-94.
- Mundlak, Y., 1963, "Estimation of Production and Behavioral Functions from a Combination of Cross-Section and Time Series Data," Measurement in Economics, Stanford University Press, pp. 138-165.

Nasse, Ph., 1973, "Un Système Complet de Functions de Demande: Les Equations de Fourgeand et Nataf," Econometrica.

Nerlove, M., 1967, "A Survey of Recent Evidence on C.E.S. and Related Production Functions," in N.B.E.R., 31, Studies in Income and Wealth, The Theory and Empirical Analysis of Production.

Piggott, J. R., and J. Whalley, 1976, "General Equilibrium Investigations of U.K. Tax-Subsidy Policy: A Progress Report," in (eds.) M. J. Artis and A. R. Nobay, Studies in Modern Economic Analysis, Oxford, England; Blackwell.

Piggott, J. and J. Whalley, 1977, "The Numerical Specification of Large Scale Walrasian Policy Models," Unpublished mimeo, University of Western Ontario, London, Canada.

Piggott, John and J. Whalley, "Economic Effects of U.K. Tax-Subsidy Policies: A General Equilibrium Appraisal," To be published by Macmillan.

Pollak, R. A. and T. Wales, 1969, "Estimation of the Linear Expenditure System," Econometrica.

Rosenberg, L. G., 1969, "Taxation of Income from Capital by Industry Group," in, Arnold C. Harberger and M. J. Bailey (eds.), The Taxation of Income from Capital, The Brookings Institution, Washington, D.C.

St.-Hilaire, F. and J. Whalley, 1980, "A Microconsistent Equilibrium Data Set for Canada for Use in Tax Policy Analysis," Mimeo, University of Western Ontario.

Scarf, H., 1973, The Computation of Economic Equilibria, Cowles Foundation Research Monograph.

Serra-Puche, J. J., 1979, "A Computational General Equilibrium Model for the Mexican Economy: An Analysis of Fiscal Policies," Ph.D. dissertation, Yale University.

- Shoven, J. B., 1976, "The Incidence and Effects of Taxes on Income From Capital," Journal of Political Economy.
- Shoven, J. B. and J. Whalley, 1972, "A General Equilibrium Calculation of the Effects of Differential Taxation of Income from Capital in the U.S.," Journal of Public Economics.
- Sparks, R. W., 1969, "Systems of Demand Equations: An Empirical Comparison of Alternative Functional Forms," Econometrica.
- Stern, R. M., J. Francis and B. Schumacker, 1976, Price Elasticities in International Trade: An Annotated Bibliography, London: Macmillan for the Trade Policy Centre.
- Stone, R. D., 1954, "Linear Expenditure Systems and Demand Analysis: An Application to the Pattern of British Demand," Economic Journal.
- Summers, Lawrence, H., 1980, "Tax Policy in a Life-Cycle Mode," Paper presented at NBER Conference on Taxation of Capital, November 16-17, 1979, Cambridge, Massachusetts.
- Zellner, A., 1962, "An Efficient Method of Estimating Seemingly Unrelated Regressions and Tests for Aggregation Bias," Journal of American Statistical Association.
- Zellner, A., J. Kementa and J. Drèze, 1966, "Specification and Estimation of Cobb-Douglas Production Function Models," Econometrica.