ENSC488: Introduction to Robotics
Simon Fraser University, Spring 2020

Demo 2

Group #4
Member 1: Muhannad Fahmy #301323517
Member 2: David Munoz-Paniagua #301447689
Member3: Chi Hong Yip #301326384

Lab Project Demo 2 Instructions

Materials to be submitted:

- This page with the name and student no. of your group members written at the top.
- A succinct description of your trajectory planner specifying if it is a joint or Cartesian space planner, interpolation scheme used, type of continuity it ensures at the start, goal, and via points. Final equations with the polynomial coefficients as unknowns and the expressions for the polynomial coefficients should be clearly stated. This should not take more than two pages.
- Filled in teammate evaluation sheet.

Demonstrations:

Your program should demonstrate the followings:

- **Move** to a given joint configuration. This will act as the start configuration for the trajectory to be planned (this was part of your Demo 1).
- **Planning:** Plan a trajectory for the robotic arm from current Tool frame to a given goal Tool frame via three intermediate Tool frames provided by the user. The total time duration (in seconds) for the trajectory will also be specified by the user. Please note that the tool frame will be given as the position and orientation parameter values, of Tool frame with respect to Station frame i.e., a four vector - (x, y, z, φ].
- **Plotting:** Plot and show the planned and the executed trajectories (in joint space) sampled at a reasonable time resolution. It would also be useful to have a top view of the Cartesian space trajectory, i.e., an x-y plot of the path followed by the Tool with time as implicit parameter. For all plotting, you may use MATLAB or Excel. Please note that sampling resolution for plotting should not be too high (or you can potentially run into memory issues). It would be nice to have it interactive, i.e., send joint values to MATLAB as you move the robot, so that the plot is seen as the robot moves. But it is also ok, to simply do it offline, i.e. dump the values to a file and then use MATLAB to plot it after the robot motion is finished. Do make sure that it does not take you more than a few tens of seconds to massage/edit the data to plot. We have limited time for demos.
- **Grasp/Ungrasp:** Have the ability to execute a Grasp/Ungrasp command at the Goal Frame. We will use this to show a pick&place operation.

We have coded a joint space planner. Initially we thought to attempt a cartesian planner which is why the calculation of the Appendix exists. This also has helped us know that small values of θ_2 should be avoided if we can (the robot has a singularityt when this hits zero). However, we ran out of time to implement a weighting function in the configuration selection section so this was not implemented.

Our planning routine asks the user for four points (three via points, and a goal) plus the total time in which this must be accomplished. The distance between positions is calculated and the fraction it represents is used to portion the total time accordingly ensuring that more time is given to cover greater distances.

Our previously developed inverse kinematic calculation is used to determine options for each transition and the best option for each is stored.; the changes for each joint are gathered together.

We decided to use a cubic spline interpolation between these five points with continuity in first and second derivatives (velocity and acceleration).

We chose to avail ourselves of the **`alglib`**[†] library since it provides an easy way to implement the interpolation and allows the retrieval of zero, first and second order derivative coefficients at a given point within. We defined a sampling rate of 25 frames per second, evaluate the coefficents at each of these points and store the values.

We now instruct the robot to move through each one of the frames in turn using the `MoveWithConfVelAcc` function. In order for the move to be as smooth as possible we issue the new move instruction a few milliseconds ahead of the time when the movement is expected to finish by means of a `sleep` command.

Equations

The generic expression for a cubic function with its first and second order derivatives is:

$$\theta_i(t)=a_i+b_i t+c_i t^2+d_i t^3$$
$$\dot{\theta}_i(t)=b_i+c_i t+d_i t^2$$
$$\ddot{\theta}_i(t)=c_i+d_i t$$

We require that the begining and end of each segment be at the same location and have the same velocity, additionally, the velocity at the start and end of travel should be zero so:

$$\theta_1(0)=a_1 \qquad \dot{\theta}_1(0)=b_1=0$$
$$\theta_1(f)=a_1+b_1 t+c_1 t^2+d_1 t^3=\theta_2(0)=a_2(0) \qquad \dot{\theta}_1(f)=b_1+c_1 t+d_1 t^2=\dot{\theta}_2(0)=b_2(0)$$
$$\theta_2(f)=a_2+b_2 t+c_2 t^2+d_2 t^3=\theta_3(0)=a_3(0) \qquad \dot{\theta}_2(f)=b_2+c_2 t+d_2 t^2=\dot{\theta}_3(0)=b_3(0)$$
$$\theta_3(f)=a_3+b_3 t+c_3 t^2+d_3 t^3=\theta_4(0)=a_4(0) \qquad \dot{\theta}_3(f)=b_3+c_3 t+d_3 t^2=\dot{\theta}_4(0)=b_4(0)$$
$$\theta_4(f)=a_4+b_4 t+c_4 t^2+d_4 t^3=\theta_5(0)=a_5(0) \qquad \dot{\theta}_4(f)=b_4+c_4 t+d_4 t^2=\dot{\theta}_5(0)=b_5=0$$

1

1 https://www.alglib.net/interpolation/spline3.php

Appendix - Velocities and Jacobian

$$
{}^0_1T=\begin{bmatrix} c_1 & -s_1 & 0 & 0 \\ s_1 & c_1 & 0 & 0 \\ 0 & 0 & 1 & h_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}\quad
{}^1_2T=\begin{bmatrix} c_2 & -s_2 & 0 & l_2 \\ s_2 & c_2 & 0 & 0 \\ 0 & 0 & 1 & h_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}\quad
{}^2_3T=\begin{bmatrix} -1 & 0 & 0 & l_3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & -h_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}\quad
{}^3_4T=\begin{bmatrix} c_4 & -s_4 & 0 & 0 \\ s_4 & c_4 & 0 & 0 \\ 0 & 0 & 1 & h_4 \\ 0 & 0 & 0 & 1 \end{bmatrix}\quad
{}^4_5T=\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & h_5 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^{i+1}\omega_{i+1}={}^{i+1}_{i}R\,{}^{i}\omega_i+\dot{\theta}_{i+1}\,{}^{i+1}\hat{Z}_{i+1}
\qquad
{}^{i+1}v_{i+1}={}^{i+1}_{i}R\left({}^{i}v_i+{}^{i}\omega_i\times{}^{i}P_{i+1}\right)
$$

$$
{}^1\omega_1=\begin{bmatrix} c_1 & -s_1 & 0 \\ s_1 & c_1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}+\begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_1 \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_1 \end{bmatrix}
\qquad
{}^1v_1=\begin{bmatrix} c_1 & -s_1 & 0 \\ s_1 & c_1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\left(\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}+\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}\times\begin{bmatrix} 0 \\ 0 \\ h_1 \end{bmatrix}\right)=\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}
$$

$$
{}^2\omega_2=\begin{bmatrix} c_2 & -s_2 & 0 \\ s_2 & c_2 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_1 \end{bmatrix}+\begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_2 \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_1+\dot{\theta}_2 \end{bmatrix}
\qquad
{}^2v_2=\begin{bmatrix} c_2 & -s_2 & 0 \\ s_2 & c_2 & 0 \\ 0 & 0 & 1 \end{bmatrix}\left(\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}+\begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_1 \end{bmatrix}\times\begin{bmatrix} l_2 \\ 0 \\ h_2 \end{bmatrix}\right)=\begin{bmatrix} -s_2 l_2\dot{\theta}_1 \\ c_2 l_2\dot{\theta}_1 \\ 0 \end{bmatrix}
$$

$$
{}^3\omega_3=\begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}\begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_1+\dot{\theta}_2 \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \\ -(\dot{\theta}_1+\dot{\theta}_2) \end{bmatrix}
\qquad
{}^3v_3=\begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}\left(\begin{bmatrix} -s_2 l_2\dot{\theta}_1 \\ c_2 l_2\dot{\theta}_1 \\ 0 \end{bmatrix}+\begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_1+\dot{\theta}_2 \end{bmatrix}\times\begin{bmatrix} l_3 \\ 0 \\ -h_3 \end{bmatrix}\right)
$$

$$
+\begin{bmatrix} 0 \\ 0 \\ \dot{h}_3 \end{bmatrix}=\begin{bmatrix} s_2 l_2\dot{\theta}_1 \\ c_2 l_2\dot{\theta}_1 \\ +l_3(\dot{\theta}_1+\dot{\theta}_2) \\ \dot{h}_3 \end{bmatrix}
$$

$$
{}^4\omega_4=\begin{bmatrix} c_4 & -s_4 & 0 \\ s_4 & c_4 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 0 \\ 0 \\ -(\dot{\theta}_1+\dot{\theta}_2) \end{bmatrix}+\begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_4 \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_4 \\ -(\dot{\theta}_1+\dot{\theta}_2) \end{bmatrix}
\qquad
{}^4v_4=\begin{bmatrix} c_4 & -s_4 & 0 \\ s_4 & c_4 & 0 \\ 0 & 0 & 1 \end{bmatrix}\left(\begin{bmatrix} s_2 l_2\dot{\theta}_1 \\ c_2 l_2\dot{\theta}_1 \\ +l_3(\dot{\theta}_1+\dot{\theta}_2) \\ \dot{h}_3 \end{bmatrix}+\begin{bmatrix} 0 \\ 0 \\ -(\dot{\theta}_1+\dot{\theta}_2) \end{bmatrix}\times\begin{bmatrix} 0 \\ 0 \\ h_4 \end{bmatrix}\right)
$$

$$
=\begin{bmatrix} c_4 s_2 l_2\dot{\theta}_1-s_4(c_2 l_2\dot{\theta}_1+l_3(\dot{\theta}_1+\dot{\theta}_2)) \\ s_4 s_2 l_2\dot{\theta}_1+c_4(c_2 l_2\dot{\theta}_1+l_3(\dot{\theta}_1+\dot{\theta}_2)) \\ \dot{h}_3 \end{bmatrix}
$$

$$
{}^5\omega_5=\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_4 \\ -(\dot{\theta}_1+\dot{\theta}_2) \end{bmatrix}+\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_4 \\ -(\dot{\theta}_1+\dot{\theta}_2) \end{bmatrix}
\qquad
{}^5v_5=\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\left(\begin{bmatrix} c_4 s_2 l_2\dot{\theta}_1-s_4(c_2 l_2\dot{\theta}_1+l_3(\dot{\theta}_1+\dot{\theta}_2)) \\ s_4 s_2 l_2\dot{\theta}_1+c_4(c_2 l_2\dot{\theta}_1+l_3(\dot{\theta}_1+\dot{\theta}_2)) \\ \dot{h}_3 \end{bmatrix}\right.
$$

$$
\left.+\begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_4 \\ -(\dot{\theta}_1+\dot{\theta}_2) \end{bmatrix}\times\begin{bmatrix} 0 \\ 0 \\ h_5 \end{bmatrix}\right)=\begin{bmatrix} c_4 s_2 l_2\dot{\theta}_1-s_4(c_2 l_2\dot{\theta}_1+l_3(\dot{\theta}_1+\dot{\theta}_2)) \\ s_4 s_2 l_2\dot{\theta}_1+c_4(c_2 l_2\dot{\theta}_1+l_3(\dot{\theta}_1+\dot{\theta}_2)) \\ \dot{h}_3 \end{bmatrix}
$$

$$
{}^5J=\begin{bmatrix} s_2 c_4 l_2-c_2 s_4 l_2-s_4 l_3 & -s_4 l_3 & 0 \\ s_2 s_4 l_2+c_2 c_4 l_2+c_4 l_3 & c_4 l_3 & 0 \\ 0 & 0 & 1 \end{bmatrix}
$$

$$ {}^{0}J = {}^{0}_{5}R\,{}^{5}J = \begin{bmatrix} -s_1(c_2 s_4 - c_4 s_2) - c_1(c_2 c_4 + s_2 s_4) & c_1(c_2 s_4 - c_4 s_2) - s_1(c_2 c_4 + s_2 s_4) & 0 \\ c_1(c_2 s_4 - c_4 s_2) - s_1(c_2 c_4 + s_2 s_4) & s_1(c_2 s_4 - c_4 s_2) + c_1(c_2 c_4 + s_2 s_4) & 0 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} s_2 c_4 l_2 - c_2 s_4 l_2 - s_4 l_3 & -s_4 l_3 & 0 \\ s_2 s_4 l_2 + c_2 c_4 l_2 + c_4 l_3 & c_4 l_3 & 0 \\ 0 & 0 & 1 \end{bmatrix} $$

For square matrices of equal size, $\det(AB) = \det(A)\det(B)$ so, $\det({}^{0}J) = \det({}^{0}_{5}R)\det({}^{5}J) = 1 \cdot [s_2 l_2 l_3]$ which is zero at $s_2 l_2 l_3 = 0 \quad \therefore \quad$ valudes of $\theta_2 \simeq 0$ are to be avoided.

$$ {}^{0}J = \begin{bmatrix} -l_3 s_{(\theta_1+\theta_2-2\theta_4)} - l_2 s_1 c_{(2\theta_2-2\theta_4)} - c_1 l_2 s_{(2\theta_2-2\theta_4)} & -l_3 s_{(\theta_1+\theta_2-2\theta_4)} & 0 \\ l_3 c_{(\theta_1+\theta_2-2\theta_4)} + l_2 c_1 c_{(2\theta_2-2\theta_4)} - s_1 l_2 s_{(2\theta_2-2\theta_4)} & l_3 c_{(\theta_1+\theta_2-2\theta_4)} & 0 \\ 0 & 0 & -1 \end{bmatrix} $$

$$ {}^{0}J^{-1} = \begin{bmatrix} \dfrac{-c_{(\theta_1+\theta_2-2\theta_4)}}{l_2 s_2} & \dfrac{-s_{(\theta_1+\theta_2-2\theta_4)}}{l_2 s_2} & 0 \\ \dfrac{l_3 c_{(\theta_1+\theta_2-2\theta_4)} + l_2 c_{(\theta_1+2\theta_2-2\theta_4)}}{l_2 l_3 s_2} & \dfrac{l_3 s_{(\theta_1+\theta_2-2\theta_4)} + l_2 s_{(\theta_1+2\theta_2-2\theta_4)}}{l_2 l_3 s_2} & 0 \\ 0 & 0 & -1 \end{bmatrix} $$