Question 2 [2 x 5Marks]

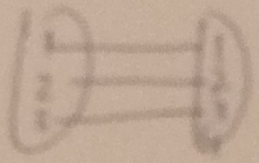
a. Find a function defined on the set of nonnegative integers that can be used to define the sequences whose first six terms are given below

1, $-\frac{1}{3}, \frac{1}{5}, -\frac{1}{7}, \frac{1}{9}, -\frac{1}{11}$

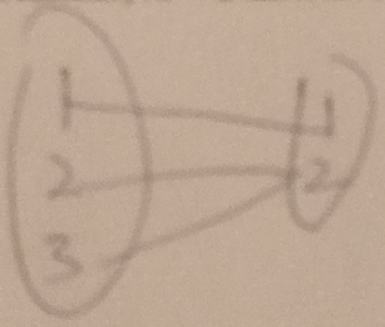
$f(n) = (-1)^{n+1} / 2(n-1)+1$

Let $X = \{1, 2, 3\}$, $Y = \{1, 2, 3, 4\}$ and $Z = \{1, 2\}$

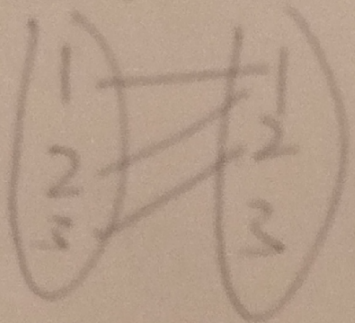
b. Define a function $f: X \to Y$ that is 1-1 but not onto



c. Define a function $f: X \to Z$ that is not 1-1 but onto



d. Define a function $f: X \to X$ that is neither 1-1 nor onto



e. Define a function $f: X \to Y$ that is both 1-1 and onto

NA possible

**Question 3 [5+5Marks]**

For what values of $n$ does the complete graph $K_n$ with $n$ vertices have

(a) an Euler circuit?

Euler circuit requires Even degree for all node

All Kn with ~~n~~ odd n will have all nodes with even degree

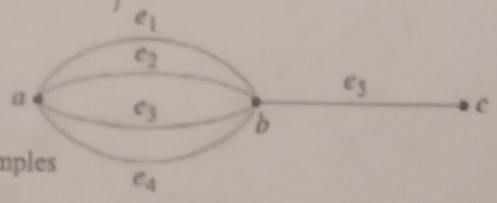
n = 3, 5, 7, 9 ...

(b) a Hamiltonian circuit? Justify your answers.

All Kn will have Hamiltonian because we can find a path that starts from any node and ends at same node without repeating.

**Question 4 [2x3 Marks]**

Consider the following graph.




a. How many paths are there from a to c? give examples

~~4~~ 1 a, b c

b. How many trails are there from a to c? give examples

4

c. How many walks are there from a to c? give examples (Matrix multiplication not expected)

Infinitely many

**Question 5 [10 Marks]**

Prove or disprove the following statement, In any simple graph, there is an even number of vertices with odd degree

Let E be the sum of all degrees of all vertices even degree

$E = deg(v_1) + deg(v_2) + \dots + d(v_n)$

Let O be the sum of all degrees of all vertices with odd degree

$O = deg(v_1) + deg(v_2) + \dots + deg(v_n)$

Adding Both Sides

$T = E + O$

$O = T - E$

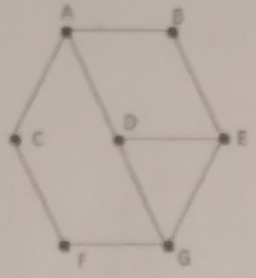
T is even quantity (by handshake Theorem)

E is even as it is sum of Even

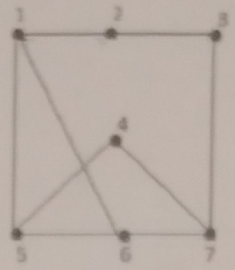
So O is Even

**Question 6 [10+ 5 Marks]**

a) Determine whether the following two graphs are isomorphic. If they are, give functions g: V(G) → V(G') and h: E(G) → E(G') that define the isomorphism. If they are not, give an invariant for graph isomorphism that they do not share.

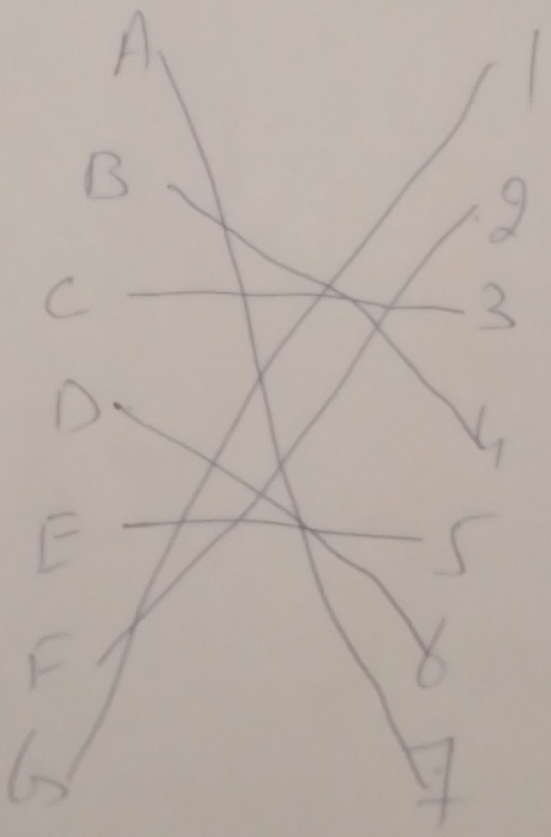



G




G'

f(A) = 7
f(B) = 4
f(C) = 3
f(D) = 6
f(E) = 5
f(F) = 2
f(G) = 1



b) Prove that if $x^3 + 3x^2 + 5x \geq 2$ or $x^4 + x^2 = 8$, then $x \geq 0$