

Luna's Magic Reference

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Contents

1	Data Structure	2
1.1	KD tree	2
1.2	Splay	2
1.3	Link-cut tree	2
2	Formula	2
2.1	Zellers congruence	2
2.2	Lattice points below segment	2
2.3	Adaptive Simpson's method	2
3	Number theory	3
3.1	Fast power module	3
3.2	Euclidean algorithm	3
3.3	Discrete Fourier transform	3
3.4	Number theoretic transform	3
3.5	Chinese remainder theorem	3
3.6	Linear Recurrence	3
3.7	Baby step giant step algorithm	3
3.8	Miller Rabin primality test	3
3.9	Pollard's Rho algorithm	4
4	Geometry	4
4.1	Point	4
4.2	Line	4
4.3	Circle	4
4.4	Centers of a triangle	5
4.5	Fermat point	5
4.6	Convex hull	5
4.7	Half plane intersection	5
4.8	Minimum circle	5
4.9	Intersection of a polygon and a circle	5
4.10	Union of circles	5
5	Graph	6
5.1	Hopcroft-Karp algorithm	6
5.2	Kuhn-Munkres algorithm	6
5.3	Maximum flow	6
5.4	Minimum cost flow	6

1 Data Structure

1.1 KD tree

```

1 /* kd_tree : finds the k-th closest point in  $O(kn^{1-\frac{1}{k}})$ .
2 Usage : Stores the data in p[]. Call function init (n,
3 k). Call min_kth (d, k). (or max_kth) (k is 1-
4 based)
5 Note : Switch to the commented code for Manhattan
6 distance.
7 Status : SPOJ-FAILURE Accepted.*/
8 template <int MAXN = 200000, int MAXK = 2>
9 struct kd_tree {
10     int k, size;
11     struct point { int data[MAXK], id; } p[MAXN];
12     struct kd_node {
13         int l, r; point p, dmin, dmax;
14         kd_node() {}
15         kd_node(const point &rhs) : l (-1), r (-1), p (rhs) {
16             dmin (rhs), dmax (rhs) {}
17         }
18     };
19     void merge (const kd_node &rhs, int k) {
20         for (register int i = 0; i < k; ++i) {
21             dmin.data[i] = std::min (dmin.data[i], rhs.dmin.
22                 data[i]);
23             dmax.data[i] = std::max (dmax.data[i], rhs.dmax.
24                 data[i]);
25         }
26         long long min_dist (const point &rhs, int k) const {
27             register long long ret = 0;
28             for (register int i = 0; i < k; ++i) {
29                 if (dmin.data[i] <= rhs.data[i] && rhs.data[i] <=
30                     dmax.data[i]) continue;
31                 ret += std::min (1ll * (dmin.data[i] - rhs.data[i]
32                     ) * (dmin.data[i] - rhs.data[i]),
33                     1ll * (dmax.data[i] - rhs.data[i]) * (dmax.
34                         data[i] - rhs.data[i]));
35             }
36             // ret += std::max (0, rhs.data[i] - dmax.data[i])
37             // + std::max (0, dmin.data[i] - rhs.data[i]);
38             return ret;
39         }
40         long long max_dist (const point &rhs, int k) {
41             long long ret = 0;
42             for (int i = 0; i < k; ++i) {
43                 int tmp = std::max (std::abs (dmin.data[i] - rhs.
44                     data[i]), std::abs (dmax.data[i] - rhs.data[i]
45                     ));
46                 ret += 1ll * tmp * tmp;
47             }
48             // ret += std::max (std::abs (rhs.data[i] - dmax.
49                 data[i]) + std::abs (rhs.data[i] - dmin.data[i]));
50             return ret;
51         }
52     };
53     struct result {
54         long long dist; point d; result() {}
55         result (const long long &dist, const point &d) :
56             dist (dist), d (d) {}
57         bool operator > (const result &rhs) const { return
58             dist > rhs.dist || (dist == rhs.dist && d.id >
59                 rhs.d.id); }
60         bool operator < (const result &rhs) const { return
61             dist < rhs.dist || (dist == rhs.dist && d.id <
62                 rhs.d.id); }
63     };
64     long long sqrdist (const point &a, const point &b) {
65         long long ret = 0;
66         for (int i = 0; i < k; ++i) ret += 1ll * (a.data[i]
67             - b.data[i]) * (a.data[i] - b.data[i]);
68         // for (int i = 0; i < k; ++i) ret += std::abs (a.
69             data[i] - b.data[i]);
70         return ret;
71     };
72     int alloc() { tree[size].l = tree[size].r = -1;
73         return size++; }
74     void build (const int &depth, int &rt, const int &l,
75         const int &r) {
76         if (l > r) return;
77         register int middle = (l + r) >> 1;
78         std::nth_element (p + l, p + middle, p + r + 1, [=]
79             (const point &a, const point &b) { return a.
80                 data[depth] < b.data[depth]; });
81         tree[rt] = alloc(); kd_node (p[middle]);
82         if (l == r) return;
83         build ((depth + 1) % k, tree[rt].l, l, middle - 1);
84         build ((depth + 1) % k, tree[rt].r, middle + 1, r);
85         if (!tree[rt].l) tree[rt].merge (tree[tree[rt].l], k);
86         if (!tree[rt].r) tree[rt].merge (tree[tree[rt].r], k);
87     };
88     std::priority_queue<result>, std::vector<result>, std
89         ::less<result>> heap_l;
90     std::priority_queue<result>, std::vector<result>, std
91         ::greater<result>> heap_r;
92     void min_kth (const int &depth, const int &rt, const
93         int &m, const point &d) {
94         result tmp = result (sqrdist (tree[rt].p, d), tree[
95             rt].p);
96         if ((int)heap_l.size() < m) heap_l.push (tmp);
97         else if (tmp < heap_l.top()) {
98             heap_l.pop();
99             heap_l.push (tmp);
100         }
101         int x = tree[rt].l, y = tree[rt].r;
102         if (~x && ~y && sqrdist (d, tree[x].p) > sqrdist (d,
103             tree[y].p)) std::swap (x, y);
104         if (~x && ((int)heap_l.size() < m || tree[x].
105             min_dist (d, k) < heap_l.top().dist))
106             min_kth ((depth + 1) % k, x, m, d);
107         if (~y && ((int)heap_l.size() < m || tree[y].
108             min_dist (d, k) < heap_l.top().dist))
109             min_kth ((depth + 1) % k, y, m, d);
110     };
111     void max_kth (const int &depth, const int &rt, const
112         int &m, const point &d) {
113         result tmp = result (sqrdist (tree[rt].p, d), tree[
114             rt].p);
115         if ((int)heap_r.size() < m) heap_r.push (tmp);
116         else if (tmp > heap_r.top()) {
117             heap_r.pop();
118             heap_r.push (tmp);
119         }
120         int x = tree[rt].l, y = tree[rt].r;

```

```

74     if (~x && ~y && sqrdist (d, tree[x].p) < sqrdist (d
75         , tree[y].p)) std::swap (x, y);
76     if (~x && ((int)heap_r.size() < m || tree[x].
77         max_dist (d, k) >= heap_r.top().dist))
78         max_kth ((depth + 1) % k, x, m, d);
79     if (~y && ((int)heap_r.size() < m || tree[y].
80         max_dist (d, k) >= heap_r.top().dist))
81         max_kth ((depth + 1) % k, y, m, d);
82     void init (int n, int k) { this -> k = k; size = 0;
83         int rt = 0; build (0, rt, 0, n - 1); }
84     result min_kth (const point &d, const int &m) {
85         heap_l = decltype (heap_l) (); min_kth (0, 0, m,
86             d); return heap_l.top (); }
87     result max_kth (const point &d, const int &m) {
88         heap_r = decltype (heap_r) (); max_kth (0, 0, m,
89             d); return heap_r.top (); }

```

1.2 Splay

```

1 void push_down (int x) {
2     if (~n[x].c[0]) push (n[x].c[0], n[x].t);
3     if (~n[x].c[1]) push (n[x].c[1], n[x].t);
4     n[x].t = tag (); }
5 void update (int x) {
6     n[x].m = gen (x);
7     if (~n[x].c[0]) n[x].m = merge (n[n[x].c[0]].m, n[x].
8         m);
9     if (~n[x].c[1]) n[x].m = merge (n[x].m, n[n[x].c[1]].
10         m);
11 void rotate (int x, int k) {
12     push_down (x); push_down (n[x].c[k]);
13     int y = n[x].c[k]; n[x].c[k] = n[y].c[k ^ 1]; n[y].c[
14         k ^ 1] = x;
15     if (n[x].f != -1) n[n[x].f].c[n[n[x].f].c[1] == x] =
16         y;
17     n[y].f = n[x].f; n[x].f = y; if (~n[x].c[k]) n[n[x].c
18         [k]].f = x;
19     update (x); update (y); }
20 void splay (int x, int s = -1) {
21     push_down (x);
22     while (n[x].f != s) {
23         if (n[n[x].f].f != s) rotate (n[n[x].f].f, n[n[x].
24             f].c[1] == n[x].f);
25         rotate (n[x].f, n[n[x].f].c[1] == x);
26         update (x);
27         if (s == -1) root = x; }

```

1.3 Link-cut tree

```

1 void access (int x) {
2     int u = x, v = -1;
3     while (u != -1) {
4         splay (u); push_down (u);
5         if (~n[u].c[1]) n[n[u].c[1]].f = -1, n[n[u].c[1]].p
6             = u;
7         n[u].c[1] = v;
8         if (~v) n[v].f = u, n[v].p = -1;
9         update (u); u = n[v] = u.p; }
10     splay (x); }

```

2 Formula

2.1 Zellers congruence

```

1 /* Zeller's congruence : converts between a calendar
2 date and its Gregorian calendar day. (y >= 1) (0 =
3 Monday, 1 = Tuesday, ..., 6 = Sunday) */
4 int get_id (int y, int m, int d) {
5     if (m < 3) { --y; m += 12; }
6     return 365 * y + y / 4 - y / 100 + y / 400 + (153 * (
7         m - 3) + 2) / 5 + d - 307; }
8 std::tuple<int, int, int> date (int id) {
9     int x = id + 1789995, n, i, j, y, m, d;
10     n = 4 * x / 146097; x -= (146097 * n + 3) / 4;
11     i = (4000 * (x + 1)) / 1461001; x -= 1461 * i / 4 -
12         31;
13     j = 80 * x / 2447; d = x - 2447 * j / 80;
14     x = j / 11;
15     m = j + 2 - 12 * x; y = 100 * (n - 49) + i + x;
16     return std::make_tuple (y, m, d); }

```

2.2 Lattice points below segment

```

1 /* Euclidean-like algorithm : computes the sum of
2  $\sum_{i=0}^{n-1} \lfloor \frac{a+bi}{m} \rfloor$ . */
3 long long solve (long long n, long long a, long long b,
4     long long m) {
5     if (b == 0) return n * (a / m);
6     if (a >= m) return n * (a / m) + solve (n, a % m, b,
7         m);
8     if (b >= m) return (n - 1) * n / 2 * (b / m) + solve
9         (n, a, b % m, m);
10     return solve ((a + b * n) / m, (a + b * n) % m, m, b)
11         ; }

```

2.3 Adaptive Simpson's method

```

1 /* Adaptive Simpson's method : integrates f in [l, r].
2 */
3 struct simpson {
4     double area (double (*f) (double), double l, double r
5         ) {
6         double m = 1 + (r - l) / 2;
7         return (f (l) + 4 * f (m) + f (r)) * (r - l) / 6; }
8     double solve (double (*f) (double), double l, double
9         r, double eps, double a) {
10         double m = 1 + (r - l) / 2;

```

```

8 double left = area (f, l, m), right = area (f, m, r)
9 if (fabs (left + right - a) <= 15 * eps) return left
10 + right + (left + right - a) / 15.0;
11 return solve (f, l, m, eps / 2, left) + solve (f, m,
12 r, eps / 2, right); }
13 double solve (double (*f) (double), double l, double
14 r, double eps) {
15 return solve (f, l, r, eps, area (f, l, r)); } }

```

3 Number theory

3.1 Fast power module

```

1 /* Fast power module :  $x^n$  */
2 int fpm (int x, int n, int mod) {
3 int ans = 1, mul = x; while (n) {
4 if (n & 1) ans = int (1LL * ans * mul % mod);
5 mul = int (1LL * mul * mul % mod); n >>= 1; }
6 return ans; }

```

3.2 Euclidean algorithm

```

1 /* Euclidean algorithm : solves for  $ax + by = \gcd(a, b)$ . */
2 void euclid (const long long &a, const long long &b,
3 long long &x, long long &y) {
4 if (b == 0) x = 1, y = 0;
5 else euclid (b, a % b, y, x), y -= a / b * x; }
6 long long inverse (long long x, long long m) {
7 long long a, b; euclid (x, m, a, b); return (a % m +
8 m) % m; }

```

3.3 Discrete Fourier transform

```

1 /* Discrete Fourier transform : the naffarious you-know
2 -what thing.
3 Usage : call init for the suggested array size, and
4 solve for the transform. (use f!=0 for the inverse)
5 */
6 template <int MAXN = 1000000>
7 struct dft {
8 typedef std::complex <double> complex;
9 complex e[2][MAXN];
10 int init (int n) {
11 int len = 1;
12 for (; len <= 2 * n; len <= 1);
13 for (int i = 0; i < len; ++i) {
14 e[0][i] = complex (cos (2 * PI * i / len), sin (2
15 * PI * i / len));
16 e[1][i] = complex (cos (2 * PI * i / len), -sin (2
17 * PI * i / len)); }
18 return len; }
19 void solve (complex *a, int n, int f) {
20 for (int i = 0; i < n; ++i) {
21 if (i > j) std::swap (a[i], a[j]);
22 for (int t = n >> 1; (j ^= t) < t; t >>= 1); }
23 for (int i = 2; i <= n; i <= 1)
24 for (int j = 0; j < n; j += i) {
25 for (int k = 0; k < i; ++k) {
26 complex A = a[j + k];
27 complex B = e[f][n / i * k] * a[j + k + (i >> 1)];
28 a[j + k] = A + B;
29 a[j + k + (i >> 1)] = A - B; }
30 if (f == 1) {
31 for (int i = 0; i < n; ++i) a[i] = complex (a[i].
32 real () / n, a[i].imag ()); } } } }

```

3.4 Number theoretic transform

```

1 /* Number theoretic transform : NTT for any module.
2 Usage : Perform NTT on 3 modules and call crt () to
3 merge the result. */
4 template <int MAXN = 1000000>
5 struct ntt {
6 int MOD[3] = {1045430273, 1051721729, 1053818881},
7 PRT[3] = {3, 6, 7};
8 void solve (int *a, int n, int f, int mod, int prt) {
9 for (int i = 0; i < n; ++i) {
10 if (i > j) std::swap (a[i], a[j]);
11 for (int t = n >> 1; (j ^= t) < t; t >>= 1); }
12 for (int i = 2; i <= n; i <= 1) {
13 static int exp[MAXN]; exp[0] = 1;
14 exp[1] = fpm (prt, (mod - 1) / i, mod);
15 if (f == 1) exp[1] = fpm (exp[1], mod - 2, mod);
16 for (int k = 2; k < (i >> 1); ++k) {
17 exp[k] = int (1LL * exp[k - 1] * exp[1] % mod); }
18 for (int j = 0; j < n; j += i) {
19 for (int k = 0; k < i; ++k) {
20 int &A = a[j + k], &B = a[j + k + (i >> 1)];
21 int A = pA, B = int (1LL * pB * exp[k] % mod);
22 pA = (A + B) % mod;
23 pB = (A - B + mod) % mod; } } }
24 if (f == 1) {
25 int rev = fpm (n, mod - 2, mod);
26 for (int i = 0; i < n; ++i) a[i] = int (1LL * a[i]
27 * rev % mod); } }
28 int crt (int *a, int mod) {
29 static int inv[3][3];
30 for (int i = 0; i < 3; ++i) for (int j = 0; j < 3;
31 ++j)
32 inv[i][j] = (int) inverse (MOD[i], MOD[j]);
33 static int x[3];
34 for (int i = 0; i < 3; ++i) { x[i] = a[i];
35 for (int j = 0; j < i; ++j) {
36 int t = (x[i] - x[j] + MOD[i]) % MOD[i];
37 if (t < 0) t += MOD[i];

```

```

34 x[i] = int (1LL * t * inv[j][i] % MOD[i]); } }
35 int sum = 1, ret = x[0] % mod;
36 for (int i = 1; i < 3; ++i) {
37 sum = int (1LL * sum * MOD[i - 1] % mod);
38 ret += int (1LL * x[i] * sum % mod);
39 if (ret >= mod) ret -= mod; }
40 return ret; } }

```

3.5 Chinese remainder theorem

```

1 /* Chinese remainder theorem : finds positive integers
2 x = out.first + k * out.second that satisfies x %
3 in[i].second = in[i].first. */
4 struct crt {
5 long long fix (const long long &a, const long long &b
6 ) { return (a % b + b) % b; }
7 bool solve (const std::vector <std::pair <long long,
8 long long>> &in, std::pair <long long, long long>
9 &out) {
10 out = std::make_pair (1LL, 1LL);
11 for (int i = 0; i < (int) in.size (); ++i) {
12 long long n, u;
13 euclid (out.second, in[i].second, n, u);
14 long long divisor = gcd (out.second, in[i].second);
15 if ((in[i].first - out.first) % divisor) return
16 false;
17 n *= (in[i].first - out.first) / divisor;
18 n = fix (n, in[i].second);
19 out.first += out.second * n;
20 out.second *= in[i].second / divisor;
21 out.first = fix (out.first, out.second); }
22 return true; } }

```

3.6 Linear Recurrence

```

1 /* Linear recurrence : finds the n-th element of a
2 linear recurrence.
3 Usage : vector <int> - first n terms, vector <int> -
4 transition function, calc (k) : the kth term mod
5 MOD.
6 Example : In : {2, 1}, {2, 1} :
7 a1 = 2, a2 = 1, an = 2an-1 + an-2, Out : calc (3) = 5,
8 calc (10007) = 959155122 (MOD 1E9+7) */
9 struct linear_rec {
10 const int LOG = 30, MOD = 1E9 + 7; int n;
11 std::vector <int> first, trans;
12 std::vector <std::vector <int>> bin;
13 std::vector <int> add (std::vector <int> &a, std:::
14 vector <int> &b) {
15 std::vector <int> result (n * 2 + 1, 0);
16 for (int i = 0; i <= n; ++i) for (int j = 0; j <= n;
17 ++j)
18 if ((result[i + j] += 1LL * a[i] * b[j] % MOD) >=
19 MOD) result[i + j] -= MOD;
20 for (int i = 2 * n; i > n; --i) {
21 for (int j = 0; j < n; ++j)
22 if ((result[i - 1 - j] += 1LL * result[i] * trans[
23 j] % MOD) >= MOD) result[i - 1 - j] -= MOD;
24 result[i] = 0; }
25 result.erase (result.begin() + n + 1, result.end());
26 return result; }
27 linear_rec (const std::vector <int> &first, const std
28 :vector <int> &trans) : first (first), trans (
29 trans) {
30 n = first.size(); std::vector <int> a (n + 1, 0); a
31 [1] = 1; bin.push_back (a);
32 for (int i = 1; i < LOG; ++i) bin.push_back (add (bin
33 [i - 1], bin[i - 1])); }
34 int solve (int k) {
35 std::vector <int> a (n + 1, 0); a[0] = 1;
36 for (int i = 0; i < LOG; ++i) if (k >> i & 1) a =
37 add (a, bin[i]);
38 int ret = 0;
39 for (int i = 0; i < n; ++i) if ((ret += (long long)
40 a[i + 1] * first[i] % MOD) >= MOD) ret -= MOD;
41 return ret; } }

```

3.7 Baby step giant step algorithm

```

1 /* Baby step giant step algorithm : Solves  $a^x = b \pmod c$ 
2 in  $O(\sqrt{c})$ . */
3 struct bsgs {
4 int solve (int a, int b, int c) {
5 std::unordered_map <int, int> bs;
6 int m = (int) sqrt ((double) c) + 1, res = 1;
7 for (int i = 0; i < m; ++i) {
8 if (bs.find (res) != bs.end ()) bs[res] = i;
9 res = int (1LL * res * a % c); }
10 int mul = 1, inv = (int) inverse (a, c);
11 for (int i = 0; i < m; ++i) mul = int (1LL * mul *
12 inv % c);
13 res = b % c;
14 for (int i = 0; i < m; ++i) {
15 if (bs.find (res) != bs.end ()) return i * m + bs[
16 res];
17 res = int (1LL * res * mul % c); }
18 return -1; } }

```

3.8 Miller Rabin primality test

```

1 /* Miller Rabin : tests whether a certain integer is
2 prime. */
3 struct miller_rabin {
4 int BASE[12] = {2, 3, 5, 7, 11, 13, 17, 19, 23, 29,
5 31, 37};
6 bool check (const long long &prime, const long long &
7 base) {
8 long long number = prime - 1;
9 for (; ~number & 1; number >>= 1);
10 long long result = llfpm (base, number, prime);

```

```

8 for (; number != prime - 1 && result != 1 && result
9   != prime - 1; number <= 1)
10   result = mul_mod(result, result, prime);
11 return result == prime - 1 || (number & 1) == 1; }
12 bool solve (const long long &number) {
13   if (number < 2) return false;
14   if (number < 4) return true;
15   if (~number & 1) return false;
16   for (int i = 0; i < 12 && BASE[i] < number; ++i) if
    (!check (number, BASE[i])) return false;
    return true; }

```

3.9 Pollard's Rho algorithm

```

1 /* Pollard Rho : factorizes an integer. */
2 struct pollard_rho {
3   miller_rabin is_prime;
4   const long long threshold = 13E9;
5   long long factorize (const long long &number, const
6     long long &seed) {
7     long long x = rand () % (number - 1) + 1, y = x;
8     for (int head = 1, tail = 2; ; ) {
9       x = mul_mod (x, x, number);
10      x = (x + seed) % number;
11      if (x == y) return number;
12      long long answer = gcd (abs (x - y), number);
13      if (answer > 1 && answer < number) return answer;
14      if (++head == tail) { y = x; tail <= 1; } } }
15 void search (const long long &number, std::vector <
16   long long > &divisor) {
17   if (number > 1) {
18     if (is_prime.solve (number)) divisor.push_back (
19       number);
20     else {
21       long long factor = number;
22       for (; factor >= number; factor = factorize (
23         number, rand () % (number - 1) + 1));
24       search (number / factor, divisor); search (factor,
25         divisor); } } }
26 std::vector <long long> solve (const long long &
27   number) {
28   std::vector <long long> ans;
29   if (number > threshold) search (number, ans);
30   else {
31     long long rem = number;
32     for (long long i = 2; i * i <= rem; ++i)
33       while (!(rem % i)) { ans.push_back (i); rem /= i;
34         }
35     if (rem > 1) ans.push_back (rem); }
36   return ans; }

```

4 Geometry

```

1 #define cd const double &
2 const double EPS = 1E-8, PI = acos (-1);
3 int sgn (cd x) { return x < -EPS ? -1 : x > EPS; }
4 int cmp (cd x, cd y) { return sgn (x - y); }
5 double sqr (cd x) { return x * x; }

```

4.1 Point

```

1 #define cp const point &
2 struct point {
3   double x, y;
4   explicit point (cd x = 0, cd y = 0) : x (x), y (y) {}
5   int dim () const { return sgn (y) == 0 ? sgn (x) < 0
6     : sgn (y) < 0; }
7   point unit () const { double l = sqrt (x * x + y * y);
8     return point (x / l, y / l); }
9   //counter-clockwise
10  point rot90 () const { return point (-y, x); }
11  //clockwise
12  point _rot90 () const { return point (y, -x); }
13  point rot (cd t) const {
14    double c = cos (t), s = sin (t);
15    return point (x * c - y * s, x * s + y * c); }
16  bool operator == (cp a, cp b) { return cmp (a.x, b.x)
17    == 0 && cmp (a.y, b.y) == 0; }
18  bool operator != (cp a, cp b) { return cmp (a.x, b.x)
19    != 0 || cmp (a.y, b.y) != 0; }
20  bool operator < (cp a, cp b) { return (cmp (a.x, b.x)
21    == 0) ? cmp (a.y, b.y) < 0 : cmp (a.x, b.x) < 0; }
22  point operator + (cp a) { return point (-a.x, -a.y); }
23  point operator + (cp a, cp b) { return point (a.x + b.
24    x, a.y + b.y); }
25  point operator - (cp a, cp b) { return point (a.x - b.
26    x, a.y - b.y); }
27  point operator * (cp a, cd b) { return point (a.x * b,
28    a.y * b); }
29  point operator / (cp a, cd b) { return point (a.x / b,
30    a.y / b); }
31  double dot (cp a, cp b) { return a.x * b.x + a.y * b.y
32    ; }
33  double det (cp a, cp b) { return a.x * b.y - a.y * b.x
34    ; }
35  double dis2 (cp a, cp b = point ()) { return sqr (a.x
36    - b.x) + sqr (a.y - b.y); }
37  double dis (cp a, cp b = point ()) { return sqrt (dis2
38    (a, b)); }

```

4.2 Line

```

1 #define cl const line &
2 struct line {
3   point s, t;
4   explicit line (cp s = point (), cp t = point ()) : s
5     (s), t (t) {}
6   bool point_on_segment (cp a, cl b) { return sgn (det (
7     a - b.s, b.t - b.s)) == 0 && sgn (dot (b.s - a, b.
8     t - a)) <= 0; }

```

```

6 bool two_side (cp a, cp b, cl c) { return sgn (det (a
7   - c.s, c.t - c.s)) * sgn (det (b - c.s, c.t - c.s)
8   ) < 0; }
9 bool intersect_judgment (cl a, cl b) {
10   if (point_on_segment (b.s, a) || point_on_segment (b.
11     t, a)) return true;
12   if (point_on_segment (a.s, b) || point_on_segment (a.
13     t, b)) return true;
14   return two_side (a.s, a.t, b) && two_side (b.s, b.t,
15     a); }
16 point line_intersect (cl a, cl b) {
17   double s1 = det (a.t - a.s, b.s - a.s), s2 = det (a.t
18     - a.s, b.t - a.s);
19   return (b.s * s2 - b.t * s1) / (s2 - s1); }
20 double point_to_line (cp a, cl b) { return fabs (det (
21   b.t - b.s, a - b.s)) / dis (b.s, b.t); }
22 point project_to_line (cp a, cl b) { return b.s + (b.t
23   - b.s) * (dot (a - b.s, b.t - b.s) / dis2 (b.t, b
24     .s)); }
25 double point_to_segment (cp a, cl b) {
26   if (sgn (dot (b.s - a, b.t - b.s)) * dot (b.t - a, b.t
27     - b.s)) <= 0 return fabs (det (b.t - b.s, a - b
28     .s)) / dis (b.s, b.t);
29   return std::min (dis (a, b.s), dis (a, b.t)); }
30 bool in_polygon (cp p, const std::vector <point> &po)
31 {
32   int n = (int) po.size (), counter = 0;
33   for (int i = 0; i < n; ++i) {
34     point a = po[i], b = po[(i + 1) % n];
35     //Modify the next line if necessary.
36     if (point_on_segment (p, line (a, b))) return true;
37     int x = sgn (det (p - a, b - a)), y = sgn (a.y - p.y)
38     , z = sgn (b.y - p.y);
39     if (x > 0 && y <= 0 && z > 0) counter++;
40     if (x < 0 && z <= 0 && y > 0) counter--; }
41   return counter != 0; }
42 double polygon_area (const std::vector <point> &a) {
43   double ans = 0.0;
44   for (int i = 0; i < (int) a.size (); ++i) ans += det
45     (a[i], a[(i + 1) % a.size ()]) / 2.0;
46   return ans; }

```

4.3 Circle

```

1 #define cc const circle &
2 struct circle {
3   point c; double r;
4   explicit circle (point c = point (), double r = 0) :
5     c (c), r (r) {}
6   bool operator == (cc a, cc b) { return a.c == b.c &&
7     cmp (a.r, b.r) == 0; }
8   bool operator != (cc a, cc b) { return !(a == b); }
9   bool in_circle (cp a, cc b) { return cmp (dis (a, b.c)
10     , b.r) <= 0; }
11 circle make_circle (cp a, cp b) { return circle ((a +
12   b) / 2, dis (a, b) / 2); }
13 circle make_circle (cp a, cp b, cp c) { point p =
14   circumcenter (a, b, c); return circle (p, dis (p,
15     a)); }
16 //In the order of the line vector.
17 std::vector <point> line_circle_intersect (cl a, cc b)
18 {
19   if (cmp (point_to_line (b.c, a), b.r) > 0) return std
20     ::vector <point> ();
21   double x = sqrt (sqr (b.r) - sqr (point_to_line (b.c,
22     a)));
23   return std::vector <point> ({project_to_line (b.c, a)
24     + (a.s - a.t).unit () * x, project_to_line (b.c,
25     a) - (a.s - a.t).unit () * x}); }
26 double circle_intersect_area (cc a, cc b) {
27   double d = dis (a.c, b.c);
28   if (sgn (d - (a.r + b.r)) >= 0) return 0;
29   if (sgn (d - abs(a.r - b.r)) <= 0) {
30     double r = std::min (a.r, b.r); return r * r * PI; }
31   double x = (d * d + a.r * a.r - b.r * b.r) / (2 * d),
32     t1 = acos (min (1., max (-1., x / a.r))), t2 =
33     acos (min (1., max (-1., (d - x) / b.r)));
34   return a.r * a.r * t1 + b.r * b.r * t2 - d * a.r *
35     sin (t1); }
36 //Counter-clockwise with respect of vector OaOb.
37 std::vector <point> circle_intersect (cc a, cc b) {
38   if (a.c == b.c || cmp (dis (a.c, b.c), a.r + b.r) > 0
39     || cmp (dis (a.c, b.c), std::abs (a.r - b.r)) <
40     0) return std::vector <point> ();
41   point r = (b.c - a.c).unit ();
42   double d = dis (a.c, b.c);
43   double x = ((sqr (a.r) - sqr (b.r)) / d + d) / 2;
44   double h = sqrt (sqr (a.r) - sqr (x));
45   if (sgn (h) == 0) return std::vector <point> ({a.c +
46     r * x});
47   return std::vector <point> ({a.c + r * x - r.rot90 ()
48     * h, a.c + r * x + r.rot90 () * h}); }
49 //Counter-clockwise with respect of point a.
50 std::vector <point> tangent (cp a, cc b) { circle p =
51   make_circle (a, b.c); return circle_intersect (p,
52     b); }
53 //Counter-clockwise with respect of point Oa.
54 std::vector <line> extangent (cc a, cc b) {
55   std::vector <line> ret;
56   if (cmp (dis (a.c, b.c), std::abs (a.r - b.r)) <= 0)
57     return ret;
58   if (sgn (a.r - b.r) == 0) {
59     point dir = b.c - a.c; dir = (dir * a.r / dis (dir))
60     .rot90 ();
61     ret.push_back (line (a.c - dir, b.c - dir));
62     ret.push_back (line (a.c + dir, b.c + dir)); }
63   else {
64     point p = (b.c * a.r - a.c * b.r) / (a.r - b.r);
65     std::vector pp = tangent (p, a), qq = tangent (p, b)
66     ;
67     if (pp.size () == 2 && qq.size () == 2) {
68       if (cmp (a.r, b.r) < 0) std::swap (pp[0], pp[1]),
69         std::swap (qq[0], qq[1]);

```



```

46: ret.push_back (line (pp[0], qq[0]));
47: ret.push_back (line (pp[1], qq[1])); } }
48: return ret; }
49: //Counter-clockwise with respect of point Oa.
50: std::vector<line> intangtent (cc c1, cc c2) {
51: point p = (b.c * a.r + a.c * b.r) / (a.r + b.r);
52: std::vector pp = tangent (p, a), qq = tangent (p, b);
53: if (pp.size () == 2 && qq.size () == 2) {
54: ret.push_back (line (pp[0], qq[0]));
55: ret.push_back (line (pp[1], qq[1])); }
56: return ret; }

```

4.4 Centers of a triangle

```

1: point incenter (cp a, cp b, cp c) {
2: double p = dis (a, b) + dis (b, c) + dis (c, a);
3: return (a * dis (b, c) + b * dis (c, a) + c * dis (a,
4: b)) / p; }
5: point circumcenter (cp a, cp b, cp c) {
6: point p = b - a, q = c - a, s (dot (p, p) / 2, dot (q,
7: q) / 2);
8: return a + point (det (s, point (p.y, q.y)), det (
9: point (p.x, q.x), s)) / det (p, q); }
10: point orthocenter (cp a, cp b, cp c) { return a + b +
11: c - circumcenter (a, b, c) * 2; }

```

4.5 Fermat point

```

1: /* Fermat point : finds a point P that minimizes
2: |PA|+|PB|+|PC|. */
3: point fermat_point (cp a, cp b, cp c) {
4: if (a == b) return a; if (b == c) return b; if (c ==
5: a) return c;
6: double ab = dis (a, b), bc = dis (b, c), ca = dis (c,
7: a);
8: double cosa = dot (b - a, c - a) / ab / ca;
9: double cosb = dot (a - b, c - b) / ab / bc;
10: double cosc = dot (b - c, a - c) / ca / bc;
11: double sq3 = PI / 3.0; point mid;
12: if (sgn (cosa + 0.5) < 0) mid = a;
13: else if (sgn (cosb + 0.5) < 0) mid = b;
14: else if (sgn (cosc + 0.5) < 0) mid = c;
15: else if (sgn (det (b - a, c - a) < 0) mid =
16: line_intersect (line (a, b + (c - b).rot (sq3)),
17: line (b, c + (a - c).rot (sq3)));
18: else mid = line_intersect (line (a, c + (b - c).rot (
19: sq3)), line (c, b + (a - b).rot (sq3)));
20: return mid; }

```

4.6 Convex hull

```

1: //Counter-clockwise, with minimum number of points.
2: bool turn_left (cp a, cp b, cp c) { return sgn (det (b
3: - a, c - a)) >= 0; }
4: std::vector<point> convex_hull (std::vector<point> a
5: ) {
6: int cnt = 0; std::sort (a.begin (), a.end ());
7: std::vector<point> ret (a.size (), point ());
8: for (int i = 0; i < (int) a.size (); ++i) {
9: while (cnt > 1 && turn_left (ret[cnt - 2], a[i], ret
10: [cnt - 1])) --cnt;
11: ret[cnt++] = a[i]; }
12: int fixed = cnt;
13: for (int i = (int) a.size () - 1; i >= 0; --i) {
14: while (cnt > fixed && turn_left (ret[cnt - 2], a[i],
15: ret[cnt - 1])) --cnt;
16: ret[cnt++] = a[i]; }
17: return std::vector (ret.begin (), ret.begin () + cnt
18: - 1); }

```

4.7 Half plane intersection

```

1: /* Online half plane intersection : complexity O(n)
2: each operation. */
3: std::vector<point> cut (const std::vector<point> &c,
4: line p) {
5: std::vector<point> ret;
6: if (c.empty ()) return ret;
7: for (int i = 0; i < (int) c.size (); ++i) {
8: int j = (i + 1) % (int) c.size ();
9: if (turn_left (p.s, p.t, c[i])) ret.push_back (c[i])
10: }
11: if (two_side (c[i], c[j], p)) ret.push_back (
12: line_intersect (p, line (c[i], c[j]))); }
13: return ret; }
14: // Offline half plane intersection : complexity
15: O(n log n).
16: bool turn_left (cl l, cp p) { return turn_left (l.s, l
17: .t, p); }
18: int cmp (cp a, cp b) { return a.dim () != b.dim () ? (
19: a.dim () < b.dim () ? -1 : 1) : -sgn (det (a, b)); }
20: std::vector<point> half_plane_intersect (std::vector<vector
21: <line> h) {
22: typedef std::pair<point, line> polar;
23: std::vector<polar> g; g.resize (h.size ());
24: for (int i = 0; i < (int) h.size (); ++i) g[i] = std
25: ::make_pair (h[i].t - h[i].s, h[i]);
26: sort (g.begin (), g.end (), [&] (const polar &a,
27: const polar &b) {
28: if (cmp (a.first, b.first) == 0) return sgn (det (a.
29: second.t - a.second.s, b.second.t - a.second.s))
30: < 0;
31: else return cmp (a.first, b.first) < 0; });
32: h.resize (std::unique (g.begin (), g.end ()), [] (
33: const polar &a, const polar &b) { return cmp (a.
34: first, b.first) == 0; } - g.begin ());
35: for (int i = 0; i < (int) h.size (); ++i) h[i] = g[i
36: ].second;

```

```

22: int fore = 0, rear = -1; std::vector<line> ret (h.
23: size (), line ());
24: for (int i = 0; i < (int) h.size (); ++i) {
25: while (fore < rear && !turn_left (h[i],
26: line_intersect (ret[rear - 1], ret[rear]))) --
27: rear;
28: while (fore < rear && !turn_left (h[i],
29: line_intersect (ret[fore], ret[fore + 1]))) ++
30: fore;
31: ret.push_back[++rear] = h[i]; }
32: while (rear - fore > 1 && !turn_left (ret[fore],
33: line_intersect (ret[rear - 1], ret[rear]))) --
34: rear;
35: while (rear - fore > 1 && !turn_left (ret[rear],
36: line_intersect (ret[fore], ret[fore + 1]))) ++
37: fore;
38: if (rear - fore < 2) return std::vector<point> ();
39: std::vector<point> ans; ans.resize (rear + 1);
40: for (int i = 0; i < rear + 1; ++i) ans[i] =
41: line_intersect (ret[i], ret[(i + 1) % (rear + 1)
42: ]);
43: return ans; }

```

4.8 Minimum circle

```

1: circle minimum_circle (std::vector<point> p) {
2: circle ret; std::random_shuffle (p.begin (), p.end ())
3: );
4: for (int i = 0; i < (int) p.size (); ++i) if (!
5: in_circle (p[i], ret)) {
6: ret = circle (p[i], 0); for (int j = 0; j < i; ++j)
7: if (!in_circle (p[j], ret)) {
8: ret = make_circle (p[j], p[i]); for (int k = 0; k <
9: j; ++k)
10: if (!in_circle (p[k], ret)) ret = make_circle (p[i
11: ], p[j], p[k]); } }
12: return ret; }

```

4.9 Intersection of a polygon and a circle

```

1: struct polygon_circle_intersect {
2: double sector_area (cp a, cp b, const double &r) {
3: double c = (2.0 * r * r - dis2 (a, b)) / (2.0 * r *
4: r);
5: return r * r * acos (c) / 2.0; }
6: double area (cp a, cp b, const double &r) {
7: double dA = dot (a, a), dB = dot (b, b), dC =
8: point_to_segment (point (), line (a, b));
9: if (sgn (dA - r * r) <= 0 && sgn (dB - r * r) <= 0)
10: return det (a, b) / 2.0;
11: point tA = a.unit () * r, tB = b.unit () * r;
12: if (sgn (dC - r) > 0) return sector_area (tA, tB, r)
13: ;
14: std::vector<point> ret = line_circle_intersect (
15: line (a, b), circle (point (), r));
16: if (sgn (dA - r * r) > 0 && sgn (dB - r * r) > 0)
17: return sector_area (tA, ret[0], r) + det (ret[0],
18: ret[1]) / 2.0 + sector_area (ret[1], tB, r);
19: if (sgn (dA - r * r) > 0) return det (ret[0], b) /
20: 2.0 + sector_area (tA, ret[0], r);
21: else return det (a, ret[1]) / 2.0 + sector_area (ret
22: [1], tB, r); }
23: double solve (const std::vector<point> &p, cc c) {
24: double ret = 0.0;
25: for (int i = 0; i < (int) p.size (); ++i) {
26: int s = sgn (det (p[i] - c.c, p[(i + 1) % p.size ()
27: ] - c.c));
28: if (s > 0) ret += area (p[i] - c.c, p[(i + 1) % p.
29: size ()] - c.c, c.r);
30: else ret -= area (p[(i + 1) % p.size ()] - c.c, p[i
31: ] - c.c, c.r); }
32: return std::abs (ret); } }

```

4.10 Union of circles

```

1: template<int MAXN = 500> struct union_circle {
2: int C; circle c[MAXN]; double area[MAXN];
3: struct event {
4: point p; double ang; int delta;
5: event (cp p = point (), double ang = 0, int delta =
6: 0) : p(p), ang(ang), delta(delta) {}
7: bool operator < (const event &a) { return ang < a.
8: ang; }
9: void addevent (cc a, cc b, std::vector<event> &evt,
10: int &cnt) {
11: double d2 = dis2 (a.c, b.c), d_ratio = ((a.r - b.r)
12: * (a.r + b.r) / d2 + 1) / 2;
13: p_ratio = sqrt (std::max (0., -(d2 - sqr(a.r - b.r)
14: ) * (d2 - sqr(a.r + b.r)) / (d2 * d2 * 4)));
15: point d = b.c - a.c, p = d.rot (PI / 2), q0 = a.c + d
16: * d_ratio + p * p_ratio, q1 = a.c + d * d_ratio
17: - p * p_ratio;
18: double ang0 = atan2 ((q0 - a.c).y, (q0 - a.c).x),
19: ang1 = atan2 ((q1 - a.c).y, (q1 - a.c).x);
20: evt.emplace_back (q1, ang1, 1); evt.emplace_back (q0,
21: ang0, -1); cnt += ang1 > ang0; }
22: bool same (cc a, cc b) { return sgn (dis (a.c, b.c))
23: == 0 && sgn (a.r - b.r) == 0; }
24: bool overlap (cc a, cc b) { return sgn (a.r - b.r -
25: dis (a.c, b.c)) >= 0; }
26: bool intersect (cc a, cc b) { return sgn (dis (a.c, b.
27: c) - a.r - b.r) < 0; }
28: void solve () {
29: std::fill (area, area + C + 2, 0);
30: for (int i = 0; i < C; ++i) {
31: int cnt = 1; std::vector<event> evt;
32: for (int j = 0; j < i; ++j) if (same (c[i], c[j]))
33: ++cnt;
34: for (int j = 0; j < C; ++j) if (j != i && !same (c[
35: i], c[j]) && overlap (c[j], c[i])) ++cnt;

```

```

23 for (int j = 0; j < C; ++j) if (j != i && !overlap
    (c[j], c[i]) && !overlap (c[i], c[j]) &&
    intersect (c[i], c[j]))
24   addevent (c[i], c[j], evt, cnt);
25 if (evt.empty ()) area[cnt] += PI * c[i].r * c[i].r
    else {
26   std::sort (evt.begin (), evt.end ());
27   evt.push_back (evt.front ());
28   for (int j = 0; j + 1 < (int) evt.size (); ++j) {
29     cnt += evt[j].delta; area[cnt] += det (evt[j].p,
30       evt[j + 1].p) / 2;
31     double ang = evt[j + 1].ang - evt[j].ang; if (ang
32       < 0) ang += PI * 2;
    area[cnt] += ang * c[i].r * c[i].r / 2 - sin(ang)
      * c[i].r * c[i].r / 2; } } } } }

```

5 Graph

5.1 Hopcroft-Karp algorithm

```

1 /* Hopcroft-Karp algorithm : unweighted maximum
   matching for bipartition graphs with complexity
   O(m*sqrt(n)). */
2 template <int MAXN = 100000, int MAXM = 100000>
3 struct hopcroft_karp {
4   using edge_list = std::vector <int> [MAXN];
5   int mx[MAXN], my[MAXM], lv[MAXN];
6   bool dfs (edge_list <MAXN, MAXM> &e, int x) {
7     for (int y : e[x]) {
8       int w = my[y];
9       if (!w || (lv[x] + 1 == lv[w] && dfs (e, w))) {
10        mx[x] = y; my[y] = x; return true; } }
11     lv[x] = -1; return false; }
12 int solve (edge_list &e, int n, int m) {
13   std::fill (mx, mx + n, -1); std::fill (my, my + m,
14     -1);
15   for (int ans = 0; ; ) {
16     std::vector <int> q;
17     for (int i = 0; i < n; ++i)
18       if (mx[i] == -1) {
19         lv[i] = 0; q.push_back (i);
20       } else lv[i] = -1;
21     for (int head = 0; head < (int) q.size(); ++head) {
22       int x = q[head];
23       for (int y : e[x]) { int w = my[y]; if (~w && lv[w]
24         < 0) {
25         lv[w] = lv[x] + 1; q.push_back (w); } } }
26     int d = 0; for (int i = 0; i < n; ++i) if (!mx[i]
27       && dfs (e, i)) ++d;
28     if (d == 0) return ans; else ans += d; } } }

```

5.2 Kuhn-Munkres algorithm

```

1 /* Kuhn-Munkres algorithm : weighted maximum ming
   algorithm for bipartition graphs with complexity
   O(N^3).
2 Note : The graph is 1-based. */
3 template <int MAXN = 500>
4 struct kuhn_munkres {
5   int n, w[MAXN][MAXN], lx[MAXN], ly[MAXN], m[MAXN],
6     way[MAXN], sl[MAXN];
7   bool u[MAXN];
8   void hungary(int x) {
9     m[0] = x; int j0 = 0;
10    std::fill (sl, sl + n + 1, INF); std::fill (u, u + n
11      + 1, false);
12    do {
13      u[j0] = true; int i0 = m[j0], d = INF, j1 = 0;
14      for (int j = 1; j <= n; ++j)
15        if (u[j] == false) {
16          int cur = -w[i0][j] - lx[i0] - ly[j];
17          if (cur < sl[j]) { sl[j] = cur; way[j] = j0; }
18          if (sl[j] < d) { d = sl[j]; j1 = j; } }
19      for (int j = 0; j <= n; ++j)
20        if (u[j]) { lx[m[j]] += d; ly[j] -= d; }
21      else sl[j] -= d;
22      j0 = j1; } while (m[j0] != 0);
23    do {
24      int j1 = way[j0]; m[j0] = m[j1]; j0 = j1;
25    } while (j0); }
26 int solve() {
27   for (int i = 1; i <= n; ++i) m[i] = lx[i] = ly[i] =
28     way[i] = 0;
29   for (int i = 1; i <= n; ++i) hungary (i);
30   int sum = 0; for (int i = 1; i <= n; ++i) sum += w[m
31     [i]][i];
32   return sum; } }

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5.3 Maximum flow

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1 /* Sparse graph maximum flow : isap.*/
2 template <int MAXN = 1000, int MAXM = 100000>
3 struct isap {
4   struct flow_edge_list {
5     int size, begin[MAXN], dest[MAXM], next[MAXM], flow[
6       MAXM];
7     void clear (int n) { size = 0; std::fill (begin,
8       begin + n, -1); }
9     flow_edge_list (int n = MAXN) { clear (n); }
10    void add_edge (int u, int v, int f) {
11      dest[size] = v; next[size] = begin[u]; flow[size] =
12        f; begin[u] = size++;
13      dest[size] = u; next[size] = begin[v]; flow[size] =
14        0; begin[v] = size++; } }
15   int pre[MAXN], d[MAXN], gap[MAXN], cur[MAXN];
16   int solve (flow_edge_list &e, int n, int s, int t) {
17     for (int i = 0; i < n; ++i) { pre[i] = d[i] = gap[i]
18       = 0; cur[i] = e.begin[i]; }
19     gap[0] = n; int u = pre[s] = s, v, maxflow = 0;

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15 while (d[s] < n) {
16   v = n; for (int i = cur[u]; ~i; i = e.next[i])
17     if (e.flow[i] && d[u] == d[e.dest[i]] + 1) {
18       v = e.dest[i]; cur[u] = i; break; }
19   if (v < n) {
20     pre[v] = u; u = v;
21     if (v == t) {
22       int dflow = INF, p = t; u = s;
23       while (p != s) { p = pre[p]; dflow = std::min (
24         dflow, e.flow[cur[p]]); }
25       maxflow += dflow; p = t;
26       while (p != s) { p = pre[p]; e.flow[cur[p]] -=
27         dflow; e.flow[cur[p] ^ 1] += dflow; } }
28   } else {
29     int mindist = n + 1;
30     for (int i = e.begin[u]; ~i; i = e.next[i])
31       if (e.flow[i] && mindist > d[e.dest[i]]) {
32         mindist = d[e.dest[i]]; cur[u] = i; }
33     if (!--gap[d[u]]) return maxflow;
34     gap[d[u] = mindist + 1]++; u = pre[u]; } }
35 return maxflow; } }
36 /* Dense graph maximum flow : dinic. */
37 template <int MAXN = 1000, int MAXM = 100000>
38 struct dinic {
39   struct flow_edge_list {
40     int size, begin[MAXN], dest[MAXM], next[MAXM], flow[
41       MAXM];
42     void clear (int n) { size = 0; std::fill (begin,
43       begin + n, -1); }
44     flow_edge_list (int n = MAXN) { clear (n); }
45     void add_edge (int u, int v, int f) {
46       dest[size] = v; next[size] = begin[u]; flow[size] =
47         f; begin[u] = size++;
48       dest[size] = u; next[size] = begin[v]; flow[size] =
49         0; begin[v] = size++; } }
50   int n, s, t, d[MAXN], w[MAXN], q[MAXN];
51   int bfs (flow_edge_list &e) {
52     std::fill (d, d + n, -1);
53     int l, r; q[l = r = 0] = s, d[s] = 0;
54     for (; l <= r; l++)
55       for (int k = e.begin[q[l]]; ~k; k = e.next[k])
56         if (!d[e.dest[k]] && e.flow[k] > 0) d[e.dest[k]]
57           = d[q[l]] + 1, q[++r] = e.dest[k];
58     return d[t] ? 1 : 0; }
59   int dfs (flow_edge_list &e, int u, int ext) {
60     if (u == t) return ext; int k = w[u], ret = 0;
61     for (; ~k; k = e.next[k], w[u] = k) {
62       if (ext == 0) break;
63       if (d[e.dest[k]] == d[u] + 1 && e.flow[k] > 0) {
64         int flow = dfs (e, e.dest[k], std::min (e.flow[k],
65           ext));
66         if (flow > 0) {
67           e.flow[k] -= flow, e.flow[k ^ 1] += flow;
68           ret += flow, ext -= flow; } } }
69     if (!k) d[u] = -1; return ret; }
70   int solve (flow_edge_list &e, int n_, int s_, int t_) {
71     int ans = 0; n = n_; s = s_; dinic::t = t_;
72     while (bfs (e)) {
73       for (int i = 0; i < n; ++i) w[i] = e.begin[i];
74       ans += dfs (e, s, INF); }
75     return ans; } }

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5.4 Minimum cost flow

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1 /* Sparse graph minimum cost flow : EK. */
2 template <int MAXN = 1000, int MAXM = 100000>
3 struct minimum_cost_flow {
4   struct cost_flow_edge_list {
5     int size, begin[MAXN], dest[MAXM], next[MAXM], cost[
6       MAXM], flow[MAXM];
7     void clear (int n) { size = 0; std::fill (begin,
8       begin + n, -1); }
9     cost_flow_edge_list (int n = MAXN) { clear (n); }
10    void add_edge (int u, int v, int c, int f) {
11      dest[size] = v; next[size] = begin[u]; cost[size] =
12        c; flow[size] = f; begin[u] = size++;
13      dest[size] = u; next[size] = begin[v]; cost[size] =
14        -c; flow[size] = 0; begin[v] = size++; } }
15   int n, s, t, prev[MAXN], dist[MAXN], occur[MAXN];
16   bool augment (cost_flow_edge_list &e) {
17     std::vector <int> queue;
18     std::fill (dist, dist + n, INF); std::fill (occur,
19       occur + n, 0);
20     dist[s] = 0; occur[s] = true; queue.push_back (s);
21     for (int head = 0; head < (int) queue.size(); ++head) {
22       int x = queue[head];
23       for (int i = e.begin[x]; ~i; i = e.next[i]) {
24         int y = e.dest[i];
25         if (e.flow[i] && dist[y] > dist[x] + e.cost[i]) {
26           dist[y] = dist[x] + e.cost[i]; prev[y] = i;
27           if (!occur[y]) {
28             occur[y] = true; queue.push_back (y); } } }
29     occur[x] = false;
30     return dist[t] < INF; }
31   std::pair <int, int> solve (cost_flow_edge_list &e,
32     int n_, int s_, int t_) {
33     n = n_; s = s_; t = t_; std::pair <int, int> ans =
34       std::make_pair (0, 0);
35     while (augment (e)) {
36       int num = INF;
37       for (int i = t; i != s; i = e.dest[prev[i] ^ 1]) {
38         num = std::min (num, e.flow[prev[i]]); }
39       ans.first += num;
40       for (int i = t; i != s; i = e.dest[prev[i] ^ 1]) {
41         e.flow[prev[i]] -= num; e.flow[prev[i] ^ 1] += num;
42         ans.second += num * e.cost[prev[i]]; } }
43     return ans; } }
44 /* Dense graph minimum cost flow : zkw. */
45 template <int MAXN = 1000, int MAXM = 100000>
46 struct zkw_flow {

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40 struct cost_flow_edge_list {
41     int size, begin[MAXN], dest[MAXM], next[MAXM], cost[
42         MAXM], flow[MAXM];
43     void clear (int n) { size = 0; std::fill (begin,
44         begin + n, -1); }
45     cost_flow_edge_list (int n = MAXN) { clear (n); }
46     void add_edge (int u, int v, int c, int f) {
47         dest[size] = v; next[size] = begin[u]; cost[size] =
48             c; flow[size] = f; begin[u] = size++;
49         dest[size] = u; next[size] = begin[v]; cost[size] =
50             -c; flow[size] = 0; begin[v] = size++; } };
51     int n, s, t, tf, tc, dis[MAXN], slack[MAXN], visit[
52         MAXN];
53     int modlable() {
54         int delta = INF;
55         for (int i = 0; i < n; i++) {
56             if (!visit[i] && slack[i] < delta) delta = slack[i];
57             slack[i] = INF; }
58         if (delta == INF) return 1;
59         for (int i = 0; i < n; i++) if (visit[i]) dis[i] +=
60             delta;
61         return 0; }
62     int dfs (cost_flow_edge_list &e, int x, int flow) {
63         if (x == t) { tf += flow; tc += flow * (dis[s] - dis
64             [t]); return flow; }
65         visit[x] = 1; int left = flow;
66         for (int i = e.begin[x]; ~i; i = e.next[i])
67             if (e.flow[i] > 0 && !visit[e.dest[i]]) {
68                 int y = e.dest[i];
69                 if (dis[y] + e.cost[i] == dis[x]) {
70                     int delta = dfs (e, y, std::min (left, e.flow[i])
71                         );
72                     e.flow[i] -= delta; e.flow[i ^ 1] += delta; left
73                         -= delta;
74                     if (!left) { visit[x] = false; return flow; }
75                 } else
76                     slack[y] = std::min (slack[y], dis[x] + e.cost[i]
77                         - dis[y]); }
78         return flow - left; }
79     std::pair <int, int> solve (cost_flow_edge_list &e,
80         int n_, int s_, int t_) {
81         n = n_; s = s_; t = t_; tf = tc = 0;
82         std::fill (dis + 1, dis + t + 1, 0);
83         do { do {
84             std::fill (visit + 1, visit + t + 1, 0);
85             } while (dfs (e, s, INF)); } while (!modlable ());
86         return std::make_pair (tf, tc);
87     } };

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