Lmxyy

For Manual/Intelligence

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Code Library for Lmxyy[1]

Version 2.0

Lmxyy
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[1]https://github.com/lmxyy/Code-Library-for-Lmxyy

目录

Chapter 1

Algorithms

1.1 1D1D Dynamic Programming

```
1  // noi2009 诗人小 G
2  #include<cstring>
3  #include<cstdio>
4  #include<cstdlib>
5  using namespace std;
6
7  #define limit (1e18)
8  #define maxn 100010
9  #define maxm 40
10 int N,L,P,pre[maxn],top;
11 char s[maxm];
12 long double f[maxn];
13 struct node { int l,r,key; }stack[maxn];
14
15 inline long double qsm(int a,int b)
16 {
17     long double  ret = 1;
18     while (b--) ret *= 1.0*a;
19     return ret;
20 }
21
22 inline long double calc(int a,int b)
23 {
24     return f[b]+qsm(abs(pre[a]-pre[b]-L),P);
25 }
26
27 inline int find(int a)
28 {
29     int l = 1,r = top,mid;
30     while (l <= r)
31     {
32         mid = (l + r) >> 1;
33         if (stack[mid].l<=a&&stack[mid].r>=a) return stack[mid].key;
```

```
34          if (a < stack[mid].l) r = mid - 1;
35          else l = mid + 1;
36      }
37  }
38
39  inline void updata(int now)
40  {
41      int l = 1,r;
42      while (top)
43      {
44          if (calc(stack[top].l,stack[top].key) >= calc(stack[top].l,now))
45              --top;
46          else
47          {
48              l = stack[top].l,r = stack[top].r;
49              while (l <= r)
50              {
51                  int mid = (l + r) >> 1;
52                  if (calc(mid,stack[top].key) >= calc(mid,now)) r = mid - 1;
53                  else l = mid + 1;
54              }
55              stack[top].r = r;
56              break;
57          }
58      }
59      if (l <= N) stack[++top] = (node){l,N,now};
60  }
61
62  inline void dp()
63  {
64      f[0] = 0;
65      stack[top = 1] = (node) {1,N,0};
66      for (int i = 1;i <= N;++i)
67      {
68          int key = find(i);
69          f[i] = calc(i,key);
70          updata(i);
71      }
72  }
73
74  int main()
75  {
76      freopen("1563.in","r",stdin);
77      freopen("1563.out","w",stdout);
78      int T; scanf("%d",&T);
79      while (T--)
80      {
81          scanf("%d %d %d\n",&N,&L,&P);
82          L++;
```

```
        for (int i = 1;i <= N;++i)
        {
            scanf("%s",s);
            pre[i] = strlen(s)+1+pre[i-1];
        }
        dp();
        if (f[N] > limit) printf("Too hard to arrange\n");
        else printf("%.0Lf\n",f[N]);
        printf("--------------------\n");
    }
    fclose(stdin); fclose(stdout);
    return 0;
}
```

1.2 Dynamic Minimal Spanning Tree

```
// 每次修改一条边，每次修改一条边权值，求最小生成树
#include<algorithm>
#include<cstring>
#include<vector>
#include<iostream>
#include<cstdio>
#include<cstdlib>
using namespace std;

typedef long long ll;
const int maxn = 100010; const ll inf = 1LL<<40;
int N,M,Q,father[maxn],cnt[maxn],reid[maxn]; ll ans[maxn];

inline int find(int a) { if (father[a] != a) return father[a] = find(father[a]); return
↪  father[a]; }

inline int gi()
{
    char ch; int ret = 0,f = 1;
    do ch = getchar(); while (!(ch >= '0'&&ch <= '9')&&ch != '-');
    if (ch == '-') f = -1,ch = getchar();
    do ret = ret*10+ch-'0',ch = getchar(); while (ch >= '0'&&ch <= '9');
    return ret*f;
}

struct Edge
{
    int a,b,id; ll c;
    inline Edge() = default;
    inline Edge(int _a,int _b,int _id,ll _c):a(_a),b(_b),id(_id),c(_c) {}
    inline void read(int i) { a = gi(),b = gi(),c = gi(); id = i; }
    friend inline bool operator <(const Edge &x,const Edge &y) { return x.c < y.c; }
}edge[22][maxn],tmp[maxn],bac[maxn];
```

```
33
34  struct Operation
35  {
36      int x; ll y;
37      inline Operation() = default;
38      inline Operation(int _x,ll _y):x(_x),y(_y) {}
39      inline void read() { x = gi(),y = gi(); }
40  }opt[maxn];
41
42  inline void construct(int &tot,ll &sum)
43  {
44      sort(tmp+1,tmp+tot+1);
45      for (int i = 1;i <= tot;++i)
46          father[tmp[i].a] = tmp[i].a,father[tmp[i].b] = tmp[i].b;
47      vector <Edge> vec;
48      for (int i = 1;i <= tot;++i)
49      {
50          int u = find(tmp[i].a),v = find(tmp[i].b);
51          if (u != v) father[u] = v,vec.push_back(tmp[i]);
52      }
53      for (int i = 0;i < (int)vec.size();++i)
54          father[vec[i].a] = vec[i].a,father[vec[i].b] = vec[i].b;
55      for (int i = 0;i < (int)vec.size();++i)
56      {
57          Edge e = vec[i];
58          if (e.c != -inf) father[find(e.a)] = find(e.b),sum += e.c;
59      }
60      vec.clear();
61      for (int i = 1;i <= tot;++i)
62      {
63          int u = find(tmp[i].a),v = find(tmp[i].b);
64          if (u != v)
65          {
66              tmp[i].a = u,tmp[i].b = v;
67              vec.push_back(tmp[i]);
68          }
69      }
70      for (int i = 0;i < (int)vec.size();++i) tmp[i+1] = vec[i];
71      for (int i = 1;i <= tot;++i) reid[tmp[i].id] = i;
72      tot = (int)vec.size();
73  }
74
75  inline void destruct(int &tot)
76  {
77      sort(tmp+1,tmp+tot+1);
78      for (int i = 1;i <= tot;++i)
79          father[tmp[i].a] = tmp[i].a,father[tmp[i].b] = tmp[i].b;
80      vector <Edge> vec;
81      for (int i = 1;i <= tot;++i)
```

```
82      {
83          int u = find(tmp[i].a),v = find(tmp[i].b);
84          if (u != v) father[u] = v,vec.push_back(tmp[i]);
85          else if (tmp[i].c == inf) vec.push_back(tmp[i]);
86      }
87      for (int i = 0;i < (int)vec.size();++i) tmp[i+1] = vec[i];
88      tot = (int)vec.size();
89  }
90
91  inline void work(int l,int r,int dep,ll sum)
92  {
93      int tot = cnt[dep];
94      for (int i = 1;i <= tot;++i) tmp[i] = edge[dep][i];
95      if (l == r)
96      {
97          bac[opt[l].x].c = opt[l].y;
98          for (int i = 1;i <= tot;++i)
99          {
100             tmp[i].c = bac[tmp[i].id].c;
101             father[tmp[i].a] = tmp[i].a;
102             father[tmp[i].b] = tmp[i].b;
103         }
104         sort(tmp+1,tmp+tot+1);
105         for (int i = 1;i <= tot;++i)
106         {
107             int u = find(tmp[i].a),v = find(tmp[i].b);
108             if (u != v) sum += tmp[i].c,father[u] = v;
109         }
110         ans[l] = sum; return;
111     }
112     for (int i = 1;i <= tot;++i)
113         tmp[i].c = bac[tmp[i].id].c,reid[tmp[i].id] = i;
114     for (int i = l;i <= r;++i) tmp[reid[opt[i].x]].c = -inf;
115     construct(tot,sum);
116     for (int i = l;i <= r;++i)
117         tmp[reid[opt[i].x]].c = inf;
118     destruct(tot);
119     for (int i = 1;i <= tot;++i) edge[dep+1][i] = tmp[i];
120     int mid = (l+r)>>1; cnt[dep+1] = tot;
121     work(l,mid,dep+1,sum); work(mid+1,r,dep+1,sum);
122 }
123
124 int main()
125 {
126     // freopen("B.in","r",stdin);
127     N = gi(),M = gi(),Q = gi();
128     for (int i = 1;i <= M;++i) bac[i].read(i),edge[0][i] = bac[i];
129     for (int i = 1;i <= Q;++i) opt[i].read();
130     for (int i = 1;i <= N;++i) father[i] = i;
```

```
    cnt[0] = M; work(1,Q,0,0);
    for (int i = 1;i <= Q;++i) printf("%lld\n",ans[i]);
    return 0;
}
```

1.3 Plug-like Dynamic Programming

```
// ural 1519
#include<cstdio>
#include<cstdlib>
#include<cstring>
#include<iostream>
#include<algorithm>
using namespace std;

typedef long long ll;
const int maxn = 14,maxs = 300010;
int mp[maxn][maxn],N,M,cur,last,total[2];
int size,ex,ey,head[maxs],nxt[maxs],Hash[maxs];
ll f[2][maxs],state[2][maxs];

inline void init()
{
    memset(mp,0,sizeof mp); ex = ey = 0;
    size = cur = 0; last = 1;
    total[cur] = 1;
    state[cur][1] = 0;
    f[cur][1] = 1;
}

inline void calc(ll s,ll inc)
{
    int pos = s%maxs;
    for (int i = head[pos];i;i = nxt[i])
        if (state[cur][Hash[i]] == s)
        {
            f[cur][Hash[i]] += inc;
            return;
        }
    ++total[cur];
    state[cur][total[cur]] = s;
    f[cur][total[cur]] = inc;
    nxt[++size] = head[pos];
    head[pos] = size;
    Hash[size] = total[cur];
}

inline ll work()
{
```

```
43      ll ret = 0;
44      for (int i = 1;i <= N;++i)
45      {
46          for (int k = 1;k <= total[cur];++k) state[cur][k] <<= 2;
47          for (int j = 1;j <= M;++j)
48          {
49              memset(head,0,sizeof head);
50              size = 0; cur ^= 1,last ^= 1;
51              total[cur] = 0;
52              for (int k = 1;k <= total[last];++k)
53              {
54                  ll s = state[last][k],num = f[last][k];
55                  int p = (s>>((j-1)<<1))%4,q = (s>>(j<<1))%4;
56                  if (!mp[i][j]) { if (!p&&!q) calc(s,num); }
57                  else if (!p&&!q)
58                  {
59                      if (mp[i+1][j]&&mp[i][j+1])
60                          calc(s+(1<<((j-1)<<1))+2*(1<<(j<<1)),num);
61                  }
62                  else if (!p&&q)
63                  {
64                      if (mp[i][j+1]) calc(s,num);
65                      if (mp[i+1][j]) calc(s-q*(1<<(j<<1))+q*(1<<((j-1)<<1)),num);
66                  }
67                  else if (p&&!q)
68                  {
69                      if (mp[i+1][j]) calc(s,num);
70                      if (mp[i][j+1]) calc(s-p*(1<<((j-1)<<1))+p*(1<<(j<<1)),num);
71                  }
72                  else if (p == 1&&q == 1)
73                  {
74                      int b = 1;
75                      for (int t = j+1;t <= M;++t)
76                      {
77                          int v = (s>>(t<<1))%4;
78                          if (v == 1) ++b; else if (v == 2) --b;
79                          if (b == 0)    { s -= 1*(1<<(t<<1)); break; }
80                      }
81                      calc(s-(1<<((j-1)<<1))-(1<<(j<<1)),num);
82                  }
83                  else if (p == 2&&q == 2)
84                  {
85                      int b = 1;
86                      for (int t = j-2;t >= 0;--t)
87                      {
88                          int v = (s>>(t<<1))%4;
89                          if (v == 2) ++b; else if (v == 1) --b;
90                          if (b == 0)    { s += 1*(1<<(t<<1)); break; }
91                      }
```

```
92                      calc(s-2*(1<<((j-1)<<1))-2*(1<<(j<<1)),num);
93                  }
94                  else if (p == 1&&q == 2) { if (i == ex&&j == ey) ret += num; }
95                  else if (p == 2&&q == 1)
96                      calc(s-2*(1<<((j-1)<<1))-(1<<(j<<1)),num);
97              }
98          }
99      }
100     return ret;
101 }
102
103 int main()
104 {
105     freopen("1519.in","r",stdin);
106     while (scanf("%d%d",&N,&M) != EOF)
107     {
108         init();
109         for (int i = 1;i <= N;++i)
110             for (int j = 1;j <= M;++j)
111             {
112                 char ch; do ch = getchar(); while (ch != '.'&&ch != '*');
113                 if (ch == '.') ex = i,ey = j,mp[i][j] = 1;
114             }
115         cout << work() << endl;
116     }
117     return 0;
118 }
```

1.4 Slop Optimization

```
1  #include<algorithm>
2  #include<cstring>
3  #include<iostream>
4  #include<cstdio>
5  #include<cstdlib>
6  using namespace std;
7
8  typedef long long ll;
9  const int maxn = 500010; const ll inf = 1LL<<60;
10 int N,K,A[maxn]; ll pre[maxn],f[maxn];
11
12 struct Point
13 {
14     ll x,y;
15     inline Point() = default;
16     inline Point(ll _x,ll _y):x(_x),y(_y) {}
17     friend inline Point operator -(const Point &a,const Point &b) { return
   ↪  Point(a.x-b.x,a.y-b.y); }
18     friend inline ll operator /(const Point &a,const Point &b) { return a.x*b.y-a.y*b.x; }
```

```
};

inline ll calc(const Point &a,int b) { return -a.x*b+a.y; }

struct Queue
{
    Point array[maxn]; int h,t;
    inline Queue() = default;
    inline void init() { h = t = 0; }
    inline void pop_front(int i) { while (t-h >= 2&&calc(array[h+1],i) > calc(array[h+2],i))
    ↪  ++h; }
    inline void push(const Point &a,int i) { while (t-h >=
    ↪  2&&(a-array[t-1])/(array[t]-array[t-1]) >= 0) --t; array[++t] = a; }
    inline Point front() const { return array[h+1]; }
}team;

inline int gi()
{
    char ch; int ret = 0,f = 1;
    do ch = getchar(); while (!(ch >= '0'&&ch <= '9')&&ch != '-');
    if (ch == '-') f = -1,ch = getchar();
    do ret = ret*10+ch-'0',ch = getchar(); while (ch >= '0'&&ch <= '9');
    return ret*f;
}

int main()
{
    // freopen("E.in","r",stdin);
    for (int T = gi();T--;)
    {
        N = gi(),K = gi(); team.init();
        for (int i = 1;i <= N;++i) A[i] = gi();
        for (int i = 1;i <= N;++i) pre[i] = pre[i-1]+A[i];
        for (int i = 1;i <= N;++i)
        {
            if (i >= K)
            {
                if (f[i-K] != inf)
                    team.push(Point(A[i-K+1],f[i-K]-pre[i-K]+(ll)(i-K)*A[i-K+1]),i);
                team.pop_front(i);
                f[i] = calc(team.front(),i)+pre[i];
            }
            else f[i] = inf;
        }
        cout << f[N] << endl;
    }
    return 0;
}
```

1.5 Three-dimension Partial Order

```
//三维偏序，CDQ 分治
#define lowbit(a) (a&-a)
int M,N,A,B,tree[maxn];

inline void ins(int a,int b) { for (;a < maxn;a += lowbit(a)) tree[a] = max(tree[a],b); }
inline void clear(int a) { for (;a < maxn;a += lowbit(a)) tree[a] = 0; }
inline int calc(int a) { int ret = 0; for (;a;a -= lowbit(a)) ret = max(tree[a],ret); return
↪  ret; }

struct Node
{
    int x,y,z,res;
    inline Node(int _x = 0,int _y = 0,int _z = 0,int _res = 0):x(_x),y(_y),z(_z),res(_res) {}
    inline void update() { ++x,++y,++z; }
}E[maxn];

inline bool cmpx(const Node &a,const Node &b)
{
    if (a.x != b.x) return a.x < b.x;
    else if (a.y != b.y) return a.y > b.y;
    else return a.z > b.z;
}
inline bool cmpy(const Node &a,const Node &b) { return a.y < b.y; }

inline void work(int l,int r)
{
    if (l == r) { E[l].res = max(E[l].res,1); return; }
    int mid = (l+r) >> 1,p = l;
    work(l,mid);
    sort(E+l,E+mid+1,cmpy);
    sort(E+mid+1,E+r+1,cmpy);
    for (int i = mid+1;i <= r;++i)
    {
        for (;p <= mid&&E[p].y < E[i].y;++p) ins(E[p].z,E[p].res);
        E[i].res = max(E[i].res,calc(E[i].z-1)+1);
    }
    while (p > l) clear(E[--p].z);
    sort(E+mid+1,E+r+1,cmpx);
    work(mid+1,r);
}

inline int run()
{
    for (int i = 1;i <= N+M;++i) E[i].update();
    sort(E+1,E+N+M+1,cmpx); work(1,N+M);
    int ret = 0;
    for (int i = 1;i <= N+M;++i) ret = max(ret,E[i].res);
    return ret;
```

```
48 }
```

Chapter 2

Computational Geometry

2.1 Circle Intersection

```cpp
//modified
const double eps = 1e-7,pi = acos(-1.0);
int N,M; double area[maxn]; // area[k] -> area of intersections >= k.

inline int dcmp(double a)
{
    if (-eps <= a&&a <= eps) return 0;
    else if (a > 0) return 1; else return -1;
}

struct Point
{
    double x,y;
    inline Point() = default;
    inline Point(double _x,double _y):x(_x),y(_y) {}
    inline void read() { x = gi(),y = gi(); }
    inline double norm() const { return sqrt(x*x+y*y); }
    inline double angle() const { return atan2(y,x); }
    inline Point unit() const { double len = norm(); return Point(x/len,y/len); }
    friend inline Point operator-(const Point &a,const Point &b) { return
    ↪  Point(a.x-b.x,a.y-b.y); }
    friend inline Point operator+(const Point &a,const Point &b) { return
    ↪  Point(a.x+b.x,a.y+b.y); }
    friend inline Point operator*(const Point &a,double b) { return Point(a.x*b,a.y*b); }
    friend inline Point operator*(double b,const Point &a) { return Point(a.x*b,a.y*b); }
    friend inline Point operator/(const Point &a,double b) { return Point(a.x/b,a.y/b); }
    friend inline double operator/(const Point &a,const Point &b) { return a.x*b.y-a.y*b.x; }
};
struct Circle
{
    Point C; double r; int sgn;
    inline Circle() = default;
```

```
31      inline Circle(const Point &_C,double _r,int _sgn):C(_C),r(_r),sgn(_sgn) {}
    ↪  // sgn 代表该圆的权值，默认 1
32      friend inline bool operator==(const Circle &a,const Circle &b)
33      {
34          if (dcmp(a.r-b.r)) return false;
35          if (dcmp(a.C.x-b.C.x)) return false;
36          if (dcmp(a.C.y-b.C.y)) return false;
37          if (a.sgn != b.sgn) return false;
38          return true;
39      }
40      friend inline bool operator!=(const Circle &a,const Circle &b) { return !(a == b); }
41  }cir[maxn];
42
43  inline Point rotate(const Point &p,double cost,double sint)
44  {
45      double x = p.x,y = p.y;
46      return Point(x*cost-y*sint,x*sint+y*cost);
47  }
48  inline pair <Point,Point> crosspoint(const Point &ap,double ar,const Point &bp,double br)
49  {
50      double d = (ap-bp).norm(),cost = (ar*ar+d*d-br*br)/(2*ar*d),sint = sqrt(1-cost*cost);
51      Point v = ((bp-ap).unit())*ar;
52      return make_pair(ap+rotate(v,cost,-sint),ap+rotate(v,cost,sint));
53  }
54  inline pair <Point,Point> crosspoint(const Circle &a,const Circle &b) { return
    ↪  crosspoint(a.C,a.r,b.C,b.r); }
55
56  inline bool overlap(const Circle &a,const Circle &b) { return dcmp(a.r-b.r-(a.C-b.C).norm()) >=
    ↪  0; } // b 是不是在 a 里面
57  inline bool intersect(const Circle &a,const Circle &b)
58  {
59      if (overlap(a,b)) return false;
60      if (overlap(b,a)) return false;
61      return dcmp((a.C-b.C).norm()-a.r-b.r) < 0;
62  }
63
64  struct Event
65  {
66      Point p; double a; int d;
67      inline Event() = default;
68      inline Event(const Point &_p,double _a,double _d):p(_p),a(_a),d(_d) {}
69      friend inline bool operator <(const Event &a,const Event &b) { return a.a < b.a; }
70  };
71
72  inline void solve()
73  {
74      for (int i = 1;i <= M;++i) area[i] = 0;
75      for (int i = 1;i <= M;++i)
76      {
```

```
77          int cnt = cir[i].sgn; if (cnt<0) cnt = 0; vector <Event> event;
78          for (int j = 1;j < i;++j) if (cir[i] == cir[j]) cnt += cir[j].sgn;
79          for (int j = 1;j <= M;++j)
80              if (j != i&&cir[i] != cir[j]&&overlap(cir[j],cir[i])) cnt += cir[j].sgn;
81          for (int j = 1;j <= M;++j)
82              if (j != i&&intersect(cir[i],cir[j]))
83              {
84                  pair <Point,Point> res = crosspoint(cir[i],cir[j]); swap(res.first,res.second);
85                  double alpha1 = (res.first-cir[i].C).angle(),alpha2 =
    ↪  (res.second-cir[i].C).angle();
86                  event.push_back(Event(res.second,alpha2,cir[j].sgn));
87                  event.push_back(Event(res.first,alpha1,-cir[j].sgn));
88                  cnt += (alpha2 > alpha1)*cir[j].sgn;
89              }
90          if (!event.size()) area[cnt] += pi*cir[i].r*cir[i].r*cir[i].sgn;
91          else
92          {
93              sort(event.begin(),event.end());
94              event.push_back(event.front());
95              for (int j = 0;j+1 < (int)event.size();++j)
96              {
97                  cnt += event[j].d;
98                  area[cnt] += event[j].p/event[j+1].p/2*cir[i].sgn;
99                  double alpha = event[j+1].a-event[j].a;
100                 if (alpha < 0) alpha += 2*pi;
101                 if (!dcmp(alpha)) continue;
102                 area[cnt] += alpha*cir[i].r*cir[i].r/2*cir[i].sgn;
103                 area[cnt] += -sin(alpha)*cir[i].r*cir[i].r/2*cir[i].sgn;
104             }
105         }
106     }
107 }
108
109 // origin
110 struct Event {
111     Point p;
112     double ang;
113     int delta;
114     Event (Point p = Point(0, 0), double ang = 0, double delta = 0) : p(p), ang(ang),
    ↪  delta(delta) {}
115 };
116 bool operator < (const Event &a, const Event &b) {
117     return a.ang < b.ang;
118 }
119 void addEvent(const Circle &a, const Circle &b, vector<Event> &evt, int &cnt) {
120     double d2 = (a.o - b.o).len2(),
121            dRatio = ((a.r - b.r) * (a.r + b.r) / d2 + 1) / 2,
122            pRatio = sqrt(-(d2 - sqr(a.r - b.r)) * (d2 - sqr(a.r + b.r)) / (d2 * d2 * 4));
123     Point d = b.o - a.o, p = d.rotate(PI / 2),
```

```
124             q0 = a.o + d * dRatio + p * pRatio,
125             q1 = a.o + d * dRatio - p * pRatio;
126     double ang0 = (q0 - a.o).ang(),
127            ang1 = (q1 - a.o).ang();
128     evt.push_back(Event(q1, ang1, 1));
129     evt.push_back(Event(q0, ang0, -1));
130     cnt += ang1 > ang0;
131 }
132 bool issame(const Circle &a, const Circle &b) { return sign((a.o - b.o).len()) == 0 && sign(a.r
    ↪  - b.r) == 0; }
133 bool overlap(const Circle &a, const Circle &b) { return sign(a.r - b.r - (a.o - b.o).len()) >=
    ↪  0; }
134 bool intersect(const Circle &a, const Circle &b) { return sign((a.o - b.o).len() - a.r - b.r) <
    ↪  0; }
135 Circle c[N];
136 double area[N];  // area[k] -> area of intersections >= k.
137 Point centroid[N];  //k 次圆的质心
138 bool keep[N];
139 void add(int cnt, DB a, Point c) {
140     area[cnt] += a;
141     centroid[cnt] = centroid[cnt] + c * a;
142 }
143 void solve(int C) {
144     for (int i = 1; i <= C; ++ i) {
145         area[i] = 0;
146         centroid[i] = Point(0, 0);
147     }
148     for (int i = 0; i < C; ++i) {
149         int cnt = 1;
150         vector<Event> evt;
151         for (int j = 0; j < i; ++j) if (issame(c[i], c[j])) ++cnt;
152         for (int j = 0; j < C; ++j) {
153             if (j != i && !issame(c[i], c[j]) && overlap(c[j], c[i])) {
154                 ++cnt;
155             }
156         }
157         for (int j = 0; j < C; ++j) {
158             if (j != i && !overlap(c[j], c[i]) && !overlap(c[i], c[j]) && intersect(c[i], c[j]))
    ↪  {
159                 addEvent(c[i], c[j], evt, cnt);
160             }
161         }
162         if (evt.size() == 0u) {
163             add(cnt, PI * c[i].r * c[i].r, c[i].o);
164         } else {
165             sort(evt.begin(), evt.end());
166             evt.push_back(evt.front());
167             for (int j = 0; j + 1 < (int)evt.size(); ++j) {
168                 cnt += evt[j].delta;
```

```
                add(cnt, det(evt[j].p, evt[j + 1].p) / 2, (evt[j].p + evt[j + 1].p) / 3);
                double ang = evt[j + 1].ang - evt[j].ang;
                if (ang < 0) {
                    ang += PI * 2;
                }
                if (sign(ang) == 0) continue;
                double ang0 = evt[j].a,ang1 = evt[j+1].a;
                add(cnt, ang * c[i].r * c[i].r / 2, c[i].o +
                    Point(sin(ang1) - sin(ang0), -cos(ang1) + cos(ang0)) * (2 / (3 * ang) *
    ↪  c[i].r));
                add(cnt, -sin(ang) * c[i].r * c[i].r / 2, (c[i].o + evt[j].p + evt[j + 1].p) /
    ↪  3);
            }
        }
    }
    for (int i = 1; i <= C; ++ i)
        if (sign(area[i])) {
            centroid[i] = centroid[i] / area[i];
        }
}
```

2.2 Common Formulas

```
//计算几何常用公式
inline int dcmp(double a)
{
    if (fabs(a) <= eps) return 0;
    else if (a > 0) return 1;
    else return -1;
}
struct Point
{
    double x,y;
    inline Point() = default;
    inline Point(double _x,double _y):x(_x),y(_y) {}
    inline Point unit() const
    {
        double len = norm();
        if (!dcmp(len)) return Point(1,0);
        else return *this/len;
    }
    inline double norm() const { return sqrt(x*x+y*y); }
    inline Point reflect(const Point &p) const
    {
        Point v = *this-p; double len = v.norm();
        v = v/len; return p+v*(1/len);
    }
    inline void read() { scanf("%lf %lf",&x,&y); }
    inline Point vertical() const { return Point(y,-x); }
```

```
27      inline double angle() const
28      {
29          double ret = atan2(y,x);
30          if (ret < 0) ret += 2*pi;
31          return ret;
32      }
33      friend inline bool operator ==(const Point &a,const Point &b) { return
    ↪  !dcmp(a.x-b.x)&&!dcmp(a.y-b.y); }
34      friend inline Point operator -(const Point &a,const Point &b) { return
    ↪  Point(a.x-b.x,a.y-b.y); }
35      friend inline Point operator +(const Point &a,const Point &b) { return
    ↪  Point(a.x+b.x,a.y+b.y); }
36      friend inline Point operator /(const Point &a,double b) { return Point(a.x/b,a.y/b); }
37      friend inline Point operator *(const Point &a,double b) { return Point(a.x*b,a.y*b); }
38      friend inline Point operator *(double b,const Point &a) { return Point(a.x*b,a.y*b); }
39      friend inline double operator /(const Point &a,const Point &b) { return a.x*b.y-a.y*b.x; }
40  };
41  struct Line
42  {
43      Point p,v; double slop;
44      inline Line() = default;
45      inline Line(const Point &_p,const Point &_v):p(_p),v(_v) {}
46      inline void update() { slop = v.alpha(); }
47      friend inline bool operator <(const Line &l1,const Line &l2)
48      { return l1.slop < l2.slop; }
49      inline double dis(const Point &a) { fabs((a-p)/v)/(v.len()); } //点到直线距离
50  };
51
52  inline bool OnLine(const Line &l,const Point &p) { return !dcmp(l.v/(p-l.p)); } //点在直线上
53
54  inline Point CrossPoint(const Line &a,const Line &b)  //直线交点
55  {
56      Point u = a.p - b.p;
57      double t = (b.v/u)/(a.v/b.v);
58      return a.p+a.v*t;
59  }
60
61  inline bool parallel(const Line &a,const Line &b) { return !dcmp(a.v/b.v); } //直线平行
62
63  inline Point rotate(const Point &p,double cost,double sint)
64  {
65      double x = p.x,y = p.y;
66      return Point(x*cost-y*sint,x*sint+y*cost);
67  }
68
69  inline Point reflect(const Point &a,const Line &l)
70  {
71      Point p = l.p,v = l.v; v = v.unit();
72      return (2*v*(a-p))*v-(a-p)+p;
```

```
}

inline void TangentPoint(const Point &c1,double r2,const Point &c1,double r2)
    //两圆严格相离，求的是外切
{
    Point v = c1-c2; double len = v.norm(); v = v/len;
    double cost = (r2-r1)/len,sint = mysqrt(1-cost*cost);
    // 两对切点 {c1+r1*rotate(v,cost,sint),c2+r2*rotate(v,cost,sint)},{c1+r1*rotate(v,cost,-sint),c2+r2*rotate(
}
```

2.3 Convex Hull

```
struct Point
{
    inline Point() = default;
    inline Point(double _x,double _y):x(_x),y(_y) {}
    inline Point unit() const
    {
        double len = norm();
        if (!dcmp(len)) return Point(1,0);
        else return *this/len;
    }
    inline double norm() const { return sqrt(x*x+y*y); }
    inline Point reflect(const Point &p) const
    {
        Point v = *this-p; double len = v.norm();
        v = v/len; return p+v*(1/len);
    }
    inline void read() { scanf("%lf %lf",&x,&y); }
    inline Point vertical() const { return Point(y,-x); }
    inline double angle() const
    {
        double ret = atan2(y,x);
        if (ret < 0) ret += 2*pi;
        return ret;
    }
    friend inline bool operator ==(const Point &a,const Point &b) { return
     !dcmp(a.x-b.x)&&!dcmp(a.y-b.y); }
    friend inline Point operator -(const Point &a,const Point &b) { return
     Point(a.x-b.x,a.y-b.y); }
    friend inline Point operator +(const Point &a,const Point &b) { return
     Point(a.x+b.x,a.y+b.y); }
    friend inline Point operator /(const Point &a,double b) { return Point(a.x/b,a.y/b); }
    friend inline Point operator *(const Point &a,double b) { return Point(a.x*b,a.y*b); }
    friend inline Point operator *(double b,const Point &a) { return Point(a.x*b,a.y*b); }
    friend inline double operator /(const Point &a,const Point &b) { return a.x*b.y-a.y*b.x; }
    friend inline bool operator <(const Point &a,const Point &b)
    {
        if (a.x != b.x) return a.x < b.x;
```

```
        else return a.y < b.y;
    }
}P[maxn],convex[maxn];

inline void ConvexHull()
{
    sort(P+1,P+N+1);  //x 第一关键字，y 第二关键字从小到大排序
    for (int i = 1;i <= N;++i)
    {
        while (m > 1&&(convex[m]-convex[m-1])/(P[i]-convex[m-1]) <= 0) --m;
        convex[++m] = P[i];
    }
    int k = m;
    for (int i = N-1;i;--i)
    {
        while (m > k&&(convex[m]-convex[m-1])/(P[i]-convex[m - 1]) <= 0) --m;
        convex[++m] = P[i];
    }
    if (N > 1) m--;
}
```

2.4 Cross Points of Circles

```
//圆圆求交，需先判定两圆有交
inline Point rotate(const Point &p,double cost,double sint)
{
    double x = p.x,y = p.y;
    return Point(x*cost-y*sint,x*sint+y*cost);
}

inline pair <Point,Point> CrossPoint(const Point &ap,double ar,const Point &bp,double br)
{
    double d = (ap-bp).norm();
    double cost = (ar*ar+d*d-br*br)/(2*ar*d),sint = sqrt(1-cost*cost);
    Point v = ((bp-ap)/(bp-ap).norm())*ar;
    return make_pair(ap+rotate(v,cost,-sint),ap+rotate(v,cost,sint));
}
```

2.5 Cross Points of Line and Circle

```
//a b 直线两点，o 圆心
//若 a b 为线段，则 0 <= t1,t2 <= 1
inline void CrossPoint(const Point &a,const Point &b,const Point &o,double r,Point *ret,int
    ↪  &num)
{
    double X0 = o.x,Y0 = o.y;
    double X1 = a.x,Y1 = a.y;
    double X2 = b.x,Y2 = b.y;
    double dx = X2-X1,dy = Y2-Y1;
```

```
    double A = dx*dx+dy*dy;
    double B = 2*dx*(X1-X0)+2*dy*(Y1-Y0);
    double C = (X1-X0)*(X1-X0)+(Y1-Y0)*(Y1-Y0)-r*r;
    double delta = B*B-4*A*C+eps;
    num = 0;
    if (delta >= 0)
    {
        double t1 = (-B-sqrt(delta))/(2*A);
        double t2 = (-B+sqrt(delta))/(2*A);
        ret[++num] = Point(X1+t1*dx,Y1+t1*dy);
        ret[++num] = Point(X1+t2*dx,Y1+t2*dy);
    }
}
```

2.6 Graham Scanning Algorithm

```
//凸包上最大四边形面积
#include<cmath>
#include<algorithm>
#include<cstring>
#include<iostream>
#include<cstdio>
#include<cstdlib>
using namespace std;

const int maxn = 2010;
int N,M; double ans;

struct Point
{
    double x,y;
    Point() = default;
    Point(double _x,double _y):x(_x),y(_y) {}
    inline void read() { scanf("%lf %lf",&x,&y); }
    friend inline Point operator -(const Point &a,const Point &b) { return
    ↪  Point(a.x-b.x,a.y-b.y); }
    friend inline double operator /(const Point &a,const Point &b) { return a.x*b.y-a.y*b.x; }
    friend inline double operator <(const Point &a,const Point &b)
    {
        if (a.x != b.x) return a.x < b.x;
        else return a.y < b.y;
    }
}P[maxn],convex[maxn];

inline int gi()
{
    char ch; int ret = 0,f = 1;
    do ch = getchar(); while (!(ch >= '0'&&ch <= '9')&&ch != '-');
    if (ch == '-') f = -1,ch = getchar();
```

```
33      do ret = ret*10+ch-'0',ch = getchar(); while (ch >= '0'&&ch <= '9');
34      return ret*f;
35  }
36
37  inline void ConvexHull()
38  {
39      int m = 0;
40      sort(P+1,P+N+1);  //x 第一关键字，y 第二关键字从小到大排序
41      for (int i = 1;i <= N;++i)
42      {
43          while (m > 1&&(convex[m]-convex[m-1])/(P[i]-convex[m-1]) <= 0) --m;
44          convex[++m] = P[i];
45      }
46      int k = m;
47      for (int i = N-1;i;--i)
48      {
49          while (m > k&&(convex[m]-convex[m-1])/(P[i]-convex[m - 1]) <= 0) --m;
50          convex[++m] = P[i];
51      }
52      if (N > 1) m--; M = m;
53  }
54
55  inline void Graham()
56  {
57      for (int i = 1;i <= M;++i) convex[i+M] = convex[i];
58      int p1,p2,p3,p4;
59      for (p1 = 1;p1 <= M;++p1)
60      {
61          p2 = p1+1;
62          p3 = p2+1;
63          p4 = p3+1;
64          for (;p3 < p1+M-1;++p3)
65          {
66              Point v = convex[p3]-convex[p1];
67              while (p2 < p3&&fabs((convex[p2]-convex[p1])/v) <
    ↪  fabs((convex[p2+1]-convex[p1])/v)) ++p2;
68              while (p4 < p1+M&&fabs((convex[p4]-convex[p1])/v) <
    ↪  fabs((convex[p4+1]-convex[p1])/v)) ++p4;
69              ans = max(ans,fabs((convex[p2]-convex[p1])/v)+fabs((convex[p4]-convex[p1])/v));
70          }
71      }
72      ans = ans/2;
73  }
74
75  int main()
76  {
77      N = gi();
78      for (int i = 1;i <= N;++i) P[i].read();
79      ConvexHull();
```

```
    Graham();
    printf("%.3f\n",ans);
    return 0;
}
////////////////////////////////////////////////////////////////////
inline void jam()  //凸包上最大四边形面积
{
    for (int i = 1;i <= m;++i) ch[i+m] = ch[i];  //凸包倍长
    for (int p1 = 1,p2,p3,p4;p1 <= m;++p1)
    {
        p2 = p1 + 1;
        p3 = p2 + 1;
        p4 = p3 + 1;
        for (;p3 < p1 + m - 1;++p3)
        {
            Line l = ((SEG) { ch[p1],ch[p3] }).extend();//枚举对角线，线段变成直线
            while (p2 < p3 && l.dis(ch[p2]) < l.dis(ch[p2 + 1])) ++p2;//点到直线距离
            while (p4 < p1 + m && l.dis(ch[p4]) < l.dis(ch[p4 + 1])) ++p4;
            ans = max(ans,(l.dis(ch[p2])+l.dis(ch[p4]))*(ch[p1] - ch[p3]).len()/2);//更新答案
        }
    }
}
```

2.7 Half Plane Intersection

```
//半平面交，直线左侧半平面，注意最后是 tail-head <= 0 还是 tail-head <= 1
inline int dcmp(double a)
{
    if (-eps <= a&&a <= eps) return 0;
    else if (a > 0) return 1; else return -1;
}

struct Point
{
    double x,y;
    inline Point() = default;
    inline Point(double _x,double _y):x(_x),y(_y) {}
    inline void read() { x = gi(),y = gi(); }
    inline Point vertical() const { return Point(-y,x); }
    inline Point unit() const
    {
        double len = norm();
        if (!dcmp(len)) return Point(1,0);
        else return *this/len;
    }
    inline double norm() const { return sqrt(x*x+y*y); }
    inline double angle() const { return atan2(y,x); }
    friend inline Point operator+(const Point &a,const Point &b) { return
    ↪  Point(a.x+b.x,a.y+b.y); }
```

```
24      friend inline Point operator-(const Point &a,const Point &b) { return
    ↪  Point(a.x-b.x,a.y-b.y); }
25      friend inline Point operator*(const Point &a,double b) { return Point(a.x*b,a.y*b); }
26      friend inline Point operator*(double b,const Point &a) { return Point(a.x*b,a.y*b); }
27      friend inline double operator/(const Point &a,const Point &b) { return a.x*b.y-a.y*b.x; }
28  }P[maxn],pp[maxn],pol[maxn];
29
30  struct Line
31  {
32      Point p,v;
33      inline Line(const Point _p = Point(),const Point _v = Point()):p(_p),v(_v) {}
34      inline double slop() const { return v.angle(); }
35      friend inline bool operator<(const Line &a,const Line &b) { return a.slop() < b.slop(); }
36  }line[maxn],qq[maxn];
37
38  inline bool onleft(const Line &L,const Point &p)
39  {
40      return dcmp(L.v/(p-L.p)) > 0;
41  }
42  inline bool parallel(const Line &a,const Line &b) { return !dcmp(a.v/b.v); }
43  inline Point crosspoint(const Line &a,const Line &b)
44  {
45      Point u = a.p-b.p;
46      double t = (b.v/u)/(a.v/b.v);
47      return a.p+(a.v*t);
48  }
49
50  inline int half_plane_intersection()
51  {
52      sort(lines+1,lines+tot+1);  //直线按斜率排序
53      int head,tail;
54      qq[head = tail = 1] = lines[1];
55      for (int i = 2;i <= tot;++i)
56      {
57          while (head < tail&&!onleft(lines[i],pp[tail-1])) --tail;
58          while (head < tail&&!onleft(lines[i],pp[head])) ++head;
59          qq[++tail] = lines[i];
60          if (parallel(qq[tail],qq[tail-1]))
61          {
62              tail--;
63              if (onleft(qq[tail],lines[i].p)) qq[tail] = lines[i];
64          }
65          if (head < tail) pp[tail-1] = crosspoint(qq[tail],qq[tail-1]);
66      }
67      while (head < tail && !onleft(qq[head],pp[tail-1])) --tail;
68      if (tail-head <= 0) return 0;
69      pp[tail] = crosspoint(qq[tail],qq[head]);
70      for (int i = head;i <= tail;++i) pol[++m] = pp[i];  //半平面交点
71      pol[0] = pol[m];
```

```
72      return m;
73  }
```

2.8 Intersecting Area of Circle and Polygon

```
1   const int maxn = 510;
2   const double eps = 1e-9;
3
4   inline int dcmp(double a)
5   {
6       if (a > eps) return 1;
7       else if (a < -eps) return -1;
8       else return 0;
9   }
10
11  struct Point
12  {
13      double x,y;
14      Point() = default;
15      Point(double _x,double _y):x(_x),y(_y) {}
16      inline double norm() const { return sqrt(x*x+y*y); }
17      inline Point unit() const { double len = norm(); return Point(x/len,y/len); }
18      friend Point operator +(const Point &a,const Point &b) { return Point(a.x+b.x,a.y+b.y); }
19      friend Point operator -(const Point &a,const Point &b) { return Point(a.x-b.x,a.y-b.y); }
20      friend Point operator *(const Point &a,double b) { return Point(a.x*b,a.y*b); }
21      friend Point operator *(double b,const Point &a) { return Point(a.x*b,a.y*b); }
22      friend Point operator /(const Point &a,double b) { return Point(a.x/b,a.y/b); }
23      friend double operator /(const Point &a,const Point &b) { return a.x*b.y-b.x*a.y; }
24      friend double operator *(const Point &a,const Point &b) { return a.x*b.x+a.y*b.y; }
25      inline void read() { scanf("%lf %lf",&x,&y); }
26  }P[maxn],A,B;
27  int N; double K;
28
29  inline double getSectorArea(const Point &a,const Point &b,double r)
30  {
31      double c = (2*r*r-((a-b)*(a-b)))/(2*r*r);
32      double alpha = acos(c);
33      return r*r*alpha/2.0;
34  }
35
36  inline pair <double,double> getSolution(double a,double b,double c)
37  {
38      double delta = b*b-4*a*c;
39      if (dcmp(delta) < 0) return make_pair(0,0);
40      else return make_pair((-b-sqrt(delta))/(2*a),(-b+sqrt(delta))/(2*a));
41  }
42
43  inline pair <Point,Point> getIntersection(const Point &a,const Point &b,double r)
44  {
```

```
    Point d = b-a;
    double A = d*d,B = 2*(d*a),C = (a*a)-r*r;
    pair <double,double> s = getSolution(A,B,C);
    return make_pair(a+(d*s.first),a+(d*s.second));
}

inline double getPointDist(const Point &a,const Point &b)
{
    Point d = b-a;
    int sA = dcmp(a*d),sB = dcmp(b*d);
    if (sA*sB <= 0) return (a/b)/((a-b).norm());
    else return min(a.norm(),b.norm());
}

double getArea(const Point &a,const Point &b,double r)
{
    double dA = a*a,dB = b*b,dC = getPointDist(a,b),ans = 0;
    if (dcmp(dA-r*r) <= 0&&dcmp(dB-r*r) <= 0) return (a/b)/2;
    Point tA = a.unit()*r,tB = b.unit()*r;
    if (dcmp(dC-r) > 0) return getSectorArea(tA,tB,r);
    pair <Point,Point> ret = getIntersection(a,b,r);
    if (dcmp(dA-r*r) > 0&&dcmp(dB-r*r) > 0)
    {
        ans += getSectorArea(tA,ret.first,r);
        ans += (ret.first/ret.second)/2;
        ans += getSectorArea(ret.second,tB,r);
        return ans;
    }
    if (dcmp(dA-r*r) > 0) return (ret.first/b)/2+getSectorArea(tA,ret.first,r);
    else return (a/ret.second)/2.0+getSectorArea(ret.second,tB,r);
}

double getArea(int n,Point *p,const Point &c,double r)
{
    double ret = 0;
    for (int i = 0;i < n;++i)
    {
        int sgn = dcmp((p[i]-c)/(p[(i+1)%n]-c));
        if (sgn > 0) ret += getArea(p[i]-c,p[(i+1)%n]-c,r);
        else ret -= getArea(p[(i+1)%n]-c,p[i]-c,r);
    }
    return fabs(ret);
}
```

2.9 Intersection of Line and Convex Hull

```
//O(logN)
inline double getA(const Node &a)
{
```

```
    double ret = atan2(a.y,a.x);
    if (ret <= -pi/2) ret += 2*pi;
    return ret;
}

inline int find(double x)
{
    if (x <= w[1]||x >= w[m]) return 1;
    return upper_bound(w+1,w+m+1,x)-w;
}

inline bool intersect(const Node &a,const Node &b)
{
    int i = find(getA(b-a)),j = find(getA(a-b));
    if (dcmp((b-a)/(hull[i]-a))*dcmp((b-a)/(hull[j]-a)) > 0) return false;
    else return true;
}

inline void convex()
{
    for (int i = 1;i <= N;++i)
    {
        while (m > 1&&(hull[m]-hull[m-1])/(P[i]-hull[m-1]) <= 0) --m;
        hull[++m] = P[i];
    }
    int k = m;
    for (int i = N-1;i;--i)
    {
        while (m > k&&(hull[m]-hull[m-1])/(P[i]-hull[m-1]) <= 0) --m;
        hull[++m] = P[i];
    }
    if (N > 1) m--;
    for (int i = 1;i <= m;++i)
        w[i]= getA(hull[i+1]-hull[i]);
}
```

2.10 Minimal Product

```
// 最小乘积匹配
#include<algorithm>
#include<cstring>
#include<iostream>
#include<cstdio>
#include<cstdlib>
using namespace std;

const int maxn = 80,inf = 1<<29;
int N,ans,A[maxn][maxn],B[maxn][maxn];

```

```
12  inline int gi()
13  {
14      char ch; int ret = 0,f = 1;
15      do ch = getchar(); while (!(ch >= '0'&&ch <= '9')&&ch != '-');
16      if (ch == '-') f = -1,ch = getchar();
17      do ret = ret*10+ch-'0',ch = getchar(); while (ch >= '0'&&ch <= '9');
18      return ret*f;
19  }
20
21  struct KM
22  {
23      int w[maxn][maxn],lx[maxn],ly[maxn],match[maxn],way[maxn],slack[maxn]; bool used[maxn];
24
25      inline void init()
26      {
27          for (int i = 1;i <= N;++i)
28              match[i] = lx[i] = ly[i] = way[i] = 0;
29      }
30
31      inline void hungary(int x)
32      {
33          match[0] = x; int j0 = 0;
34          for (int j = 0;j <= N;++j)
35              slack[j] = -inf,used[j] = false;
36          do
37          {
38              used[j0] = true;
39              int i0 = match[j0],delta = -inf,j1 = 0;
40              for (int j = 1;j <= N;++j)
41                  if (!used[j])
42                  {
43                      int cur = -w[i0][j]-lx[i0]-ly[j];
44                      if (cur > slack[j]) slack[j] = cur,way[j] = j0;
45                      if (slack[j] > delta) delta = slack[j],j1 = j;
46                  }
47              for (int j = 0;j <= N;++j)
48              {
49                  if (used[j]) lx[match[j]] += delta,ly[j] -= delta;
50                  else slack[j] -= delta;
51              }
52              j0 = j1;
53          }
54          while (match[j0]);
55          do
56          {
57              int j1 = way[j0];
58              match[j0] = match[j1];
59              j0 = j1;
60          }
```

```
61          while (j0);
62      }
63
64      inline void work() { for (int i = 1;i <= N;++i) hungary(i); }
65
66      inline int get_ans()
67      {
68          int sum = 0;
69          for (int i = 1;i <= N;++i)
70          {
71              // if (w[match[i]][i] == inf) ; // 无解
72              if (match[i] > 0) sum += w[match[i]][i];
73          }
74          return sum;
75      }
76
77      inline void getp(int &x,int &y)
78      {
79          x = y = 0;
80          for (int i = 1;i <= N;++i)
81              x += A[match[i]][i],y += B[match[i]][i];
82      }
83  }km;
84
85  inline void work(int X1,int Y1,int X2,int Y2)
86  {
87      km.init();
88      for (int i = 1;i <= N;++i)
89          for (int j = 1;j <= N;++j)
90              km.w[i][j] = (X2-X1)*B[i][j]+(Y1-Y2)*A[i][j];
91      km.work();
92      if (km.get_ans() >= X2*Y1-X1*Y2) return;
93      int x,y; km.getp(x,y);
94      ans = min(ans,x*y);
95      work(X1,Y1,x,y); work(x,y,X2,Y2);
96  }
97
98  int main()
99  {
100     // freopen("B.in","r",stdin);
101     for (int T = gi();T--;)
102     {
103         N = gi(); ans = inf;
104         for (int i = 1;i <= N;++i) for (int j = 1;j <= N;++j) A[i][j] = gi();
105         for (int i = 1;i <= N;++i) for (int j = 1;j <= N;++j) B[i][j] = gi();
106         int X1,Y1,X2,Y2;
107         km.init();
108         for (int i = 1;i <= N;++i)
109             for (int j = 1;j <= N;++j)
```

```
                km.w[i][j] = A[i][j];
        km.work(); km.getp(X1,Y1);
        km.init();
        for (int i = 1;i <= N;++i)
            for (int j = 1;j <= N;++j)
                km.w[i][j] = B[i][j];
        km.work(); km.getp(X2,Y2);
        ans = min(X1*Y1,X2*Y2);
        work(X1,Y1,X2,Y2);
        cout << ans << endl;
    }
    fclose(stdin); fclose(stdout);
    return 0;
}
```

2.11 Planar Graph

```
// 包括平面图转对偶图
inline int dcmp(double a)
{
    if (fabs(a) <= eps) return 0;
    else if (a > 0) return 1;
    else return -1;
}
struct Point
{
    double x,y;
    inline Point(double _x = 0,double _y = 0):x(_x),y(_y) {}
    inline void read() { x = gi(),y = gi(); }
    friend inline Point operator-(const Point &a,const Point &b) { return Point(a.x-b.x,a.y-b.y); }
    friend inline double operator/(const Point &a,const Point &b) { return a.x*b.y-a.y*b.x; }
    inline double angle() { return atan2(y,x); }
}pp[maxn];
struct Segment
{
    int from,to,h,id,sur; // from 号点到 to 号点，h 为边权,suf 为这条有向边维出来的平面编号。
    inline Segment(int _from = 0,int _to = 0,int _h = 0,int _id = 0,int _sur = 0):from(_from),to(_to),h(_h),id(_id),sur(_sur) {}
    friend inline bool operator<(const Segment &a,const Segment &b) { return (pp[a.to]-pp[a.from]).angle() < (pp[b.to]-pp[b.from]).angle(); }
}edge[maxm*2];
vector <int> G[maxn];

inline void nadd(int u,int v,int h) { ++ncnt; G[u].push_back(ncnt); edge[ncnt] = Segment(u,v,h); }
inline void nins(int u,int v,int h) { nadd(u,v,h); nadd(v,u,h); }

inline bool cmp(int a,int b) { return edge[a] < edge[b]; }
```

```
29
30  inline void find_surface()
31  {
32      for (int i = 1;i <= N;++i) sort(G[i].begin(),G[i].end(),cmp);
33      for (int i = 1;i <= N;++i)
34      {
35          int nn = G[i].size();
36          for (int j = 0;j < nn;++j)
37              edge[G[i][j]].id = j;
38      }
39      for (int i = 2;i <= ncnt;++i)
40          if (!edge[i].sur)
41          {
42              ++tot; int j = i,p,nn; vector <Point> vec;
43              while (!edge[j].sur)
44              {
45                  edge[j].sur = tot; vec.push_back(pp[edge[j].from]);
46                  p = edge[j].to; nn = G[p].size();
47                  j ^= 1; j = G[p][(edge[j].id+1)%nn];
48              }
49              double res = 0; nn = vec.size();
50              for (j = 0;j < nn;++j)
51                  res += (vec[j]-vec[0])/(vec[(j+1)%nn]-vec[0]);
52              res /= 2; space[tot] = res;
    ↪  // 第 tot 个平面的有向面积，外面的大平面面积为正，其余为负，大平面可能有多个（平面图不连通）
53          }
54      // 开始建边，以 mst 为例
55      // for (int i = 2;i <= cnt;i += 2)
56      // {
57      //     if (space[edge[i].sur]<0&&space[edge[i^1].sur]<0)
58      //         arr[++all] = (ARR) { edge[i].sur,edge[i^1].sur,edge[i].h };
59      //     else arr[++all] = (ARR) { edge[i].sur,edge[i^1].sur,inf};
60      // }
61  }
62
63  // 点定位
64  struct Scan
65  {
66      double x,y; int bel,sign;
67      inline Scan(double _x = 0,double _y = 0,int _bel = 0,int _sign =
    ↪  0):x(_x),y(_y),bel(_bel),sign(_sign) {}
68      friend inline bool operator < (const Scan &a,const Scan &b)
69      {
70          if (a.x != b.x) return a.x < b.x;
71          else return a.sign > b.sign;
72      }
73  }bac[maxn*4];
74
75  struct Splay
```

```
{
    int num,root,ch[maxn][2],fa[maxn],key[maxn]; queue <int> team;

    inline int newnode()
    {
        int ret;
        if (team.empty()) ret = ++num;
        else ret = team.front(),team.pop();
        fa[ret] = ch[ret][0] = ch[ret][1] = 0;
        return ret;
    }

    inline void init() { num = 0; root = newnode(); key[root] = cnt; }

    inline void rotate(int x)
    {
        int y = fa[x],z = fa[y],l = ch[y][1] == x,r = l^1;
        if (z != 0) ch[z][ch[z][1] == y] = x;
        fa[x] = z; fa[y] = x; fa[ch[x][r]] = y;
        ch[y][l] = ch[x][r]; ch[x][r] = y;
    }

    inline void splay(int x)
    {
        while (fa[x] != 0)
        {
            int y = fa[x],z = fa[y];
            if (fa[y] != 0)
            {
                if ((ch[y][0] == x)^(ch[z][0] == y)) rotate(x);
                else rotate(y);
            }
            rotate(x);
        }
        root = x;
    }

    inline int lower_bound(const Point &p)
    {
        int now = root,ret = 0;
        while (now)
        {
            int k = key[now];
            if ((p-pp[edge[k].from])/(pp[edge[k].to]-pp[edge[k].from]) >= 0)
                ret = k,now = ch[now][0];
            else now = ch[now][1];
        }
        return ret;
    }
```

```
125
126     inline int find(int w)
127     {
128         int now = root;
129         double x = pp[edge[w].to].x,y = pp[edge[w].to].y;
130         double ang = (pp[edge[w].to] - pp[edge[w].from]).angle();
131         while (now)
132         {
133             int k = key[now];
134             if (k == w) return now;
135             NODE p = pp[edge[k].to] - pp[edge[k].from],q = pp[edge[k].from];
136             double xx = x - q.x,yy = q.y+xx/p.x*p.y;
137             if (equal(yy,y))
138             {
139                 double t = p.angle();
140                 now = ch[now][ang < t];
141             }
142             else now = ch[now][y > yy];
143         }
144     }
145
146     inline void erase(int w)
147     {
148         int p = find(w);
149         while (ch[p][0] || ch[p][1])
150         {
151             if (ch[p][0])
152             {
153                 rotate(ch[p][0]);
154                 if (p == root) root = fa[p];
155             }
156             else
157             {
158                 rotate(ch[p][1]);
159                 if (p == root) root = fa[p];
160             }
161         }
162         team.push(p);
163         ch[fa[p]][ch[fa[p]][1] == p] = 0;
164         fa[p] = 0;
165     }
166
167     inline void insert(int w)
168     {
169         int now = root,pre;
170         double x = pp[edge[w].from].x,y = pp[edge[w].from].y;
171         double ang = (pp[edge[w].to] - pp[edge[w].from]).angle();
172         double xx,yy;
173         while (true)
```

```
174         {
175             int k = key[now];
176             NODE p = pp[edge[k].to] - pp[edge[k].from],q = pp[edge[k].from];
177             xx = x - q.x,yy = q.y+xx/p.x*p.y;
178             if (equal(yy,y))
179             {
180                 double t = p.angle();
181                 pre = now,now = ch[now][ang > t];
182                 if (!now)
183                 {
184                     now = newnode();
185                     fa[now] = pre; ch[pre][ang > t] = now; key[now] = w;
186                     break;
187                 }
188             }
189             else
190             {
191                 pre = now,now = ch[now][y > yy];
192                 if (!now)
193                 {
194                     now = newnode();
195                     fa[now] = pre; ch[pre][y>yy] = now; key[now] = w;
196                     break;
197                 }
198             }
199         }
200         splay(now);
201     }
202 }S;
203
204 inline void locate()
205 {
206     int nn = 0;
207     for (int i = 2;i <= cnt;i += 2)
208     {
209         if (!dcmp(pp[edge[i].from].x-pp[edge[i].to].x)) continue;
210         bac[++nn] = Scan(pp[edge[i].from].x,pp[edge[i].from].y,i,2);
211         bac[++nn] = Scan(pp[edge[i].to].x,pp[edge[i].to].y,i,3);
212     }
213     scanf("%d",&T); double x,y;
214     // 查询 (x,y) 所在平面
215     for (int i = 1;i <= T;++i)
216     {
217         scanf("%lf %lf",&x,&y);
218         bac[++nn] = Scan(x,y,i,0);
219         scanf("%lf %lf",&x,&y);
220         bac[++nn] = Scan(x,y,i,1);
221     }
222     sort(bac+1,bac+nn+1);
```

```
223     pp[++n] = Point(-oo,-oo); pp[++n] = (oo,-oo);
224     edge[++cnt] = Edge(n-1,n);
225     S.init(); int p;
226     for (int i = 1;i <= nn;++i)
227     {
228         if (bac[i].sign == 2||bac[i].sign == 3)
229         {
230             if (bac[i].sign == 2) S.insert(bac[i].bel);
231             else S.erase(bac[i].bel);
232         }
233         else
234         {
235             p = S.lower_bound(Point(bac[i].x,bac[i].y));
236             query[bac[i].bel][bac[i].sign] = edge[p].sur;
237         }
238     }
239 }
```

2.12 Polygon Class

```
1  inline bool PointOnSegment(const Point &t,const Point &a,const Point &b)
2  {
3      if (dcmp((t-a)/(b-a))) return false;
4      if (dcmp((t-a)*(t-b)) > 0) return false;
5      return true;
6  }
7
8  inline bool in(const Point &a,const Point &b,const Point &c)
9  {
10     double alpha = a.angle(),beta = b.angle(),gamma = c.angle(); // angle 返回 [0,2pi]
11     if (alpha <= beta) return dcmp(gamma-alpha) > 0&&dcmp(beta-gamma) > 0;
12     else return dcmp(gamma-alpha) > 0||dcmp(beta-gamma) > 0;
13 }
14
15 struct Polygon
16 {
17     int n; Point a[maxn];
18     inline Polygon() {}
19     inline void read()
20     {
21         n = gi();
22         for (int i = 0;i < n;++i) a[i].read();
23         a[n] = a[0];
24     }
25     // 点是否在多边形内部，内部为 1，外部为 0，边界为 2，不管顺逆时针
26     inline int Point_In(const Point &t) const
27     {
28         int num = 0;
29         for (int i = 0;i < n;++i)
```

```
30          {
31              if (PointOnSegment(t,a[i],a[i+1])) return 2;
32              int k = dcmp((a[i+1]-a[i])/(t-a[i]));
33              int d1 = dcmp(a[i].y-t.y),d2 = dcmp(a[i+1].y-t.y);
34              if (k > 0&&d1 <= 0&&d2 > 0) ++num;
35              if (k < 0&&d2 <= 0&&d1 > 0) --num;
36          }
37          return num != 0;
38      }
39      // 判断多边形的方向，true 为逆时针，false 为顺时针，用叉积判断哪个多
40      inline bool CalculateClockDirection()
41      {
42          int res = 0;
43          for (int i = 0;i < n;++i)
44          {
45              int p = i-1,s = i+1,sgn;
46              if (p < 0) p += n; if (s >= n) s -= n;
47              sgn = dcmp((a[i]-a[p])/(a[s]-a[i]));
48              if (sgn) { if (sgn > 0) ++res; else --res; }
49          }
50          return res > 0;
51      }
52      // 判断多边形方向，true 为逆时针，false 为顺时针，用 Green 公式
53      inline bool CalculateClockDirection()
54      {
55          double res = 0;
56          for (int i = 0;i < n;++i)
57              res -= 0.5*(a[i+1].y+a[i].y)*(a[i+1].x-a[i].x);
58          return res > 0;
59      }
60
61      // 线段 ab 是否有点严格在多边形内部，先判断线段是否与多边形边界有交，再判断 ab 是否与多边形有交，内部 false，外
62      inline bool can(int ia,int ib)
63      {
64          Point a = P[ia],b = P[ib],v = b-a;
65          if (in(P[ia+1]-a,P[ia-1]-a,b-a)||in(P[ib+1]-b,P[ib-1]-b,a-b)) return false;
66          for (register int i = 0;i < N;++i)
67          {
68              if (dcmp(v/(P[i]-a))*dcmp(v/(P[i+1]-a)) <
    ↪  0&&dcmp(vec[i]/(a-P[i]))*dcmp(vec[i]/(b-P[i])) < 0)
69                  return false;
70              if (PointOnSegment(a,P[i],P[i+1])||PointOnSegment(b,P[i],P[i+1])) return false;
71              if (PointOnSegment(P[i],a,b)||PointOnSegment(P[i+1],a,b)) return false;
72          }
73          return true;
74      }
75  }poly;
```

2.13 Union Area of Circles

```
//N 为开始圆的个数，M 为离散化后圆的个数，cnt 为去包含后圆的个数
int N,M,cnt;

struct Node
{
    double x,y;
    inline  Node(double _x = 0,double _y = 0):x(_x),y(_y) {}
    inline void read() { x = gi(),y = gi(); }
    inline double norm() const { return sqrt(x*x+y*y); }
    inline double angle() const { return atan2(y,x); }
    inline Node unit() const { double len = norm(); return Node(x/len,y/len); }
    friend inline Node operator-(const Node &a,const Node &b) { return Node(a.x-b.x,a.y-b.y); }
    friend inline Node operator+(const Node &a,const Node &b) { return Node(a.x+b.x,a.y+b.y); }
    friend inline Node operator*(const Node &a,double b) { return Node(a.x*b,a.y*b); }
    friend inline Node operator*(double b,const Node &a) { return Node(a.x*b,a.y*b); }
    friend inline double operator/(const Node &a,const Node &b) { return a.x*b.y-a.y*b.x; }
};
struct Circle
{
    Node C; double r;
    inline Circle(const Node &_C = Node(),double _r = 0):C(_C),r(_r) {}
    friend inline bool operator<(const Circle &a,const Circle &b)
    {
        if (dcmp(a.r-b.r)) return dcmp(a.r-b.r) < 0;
        else if (dcmp(a.C.x-b.C.x)) return dcmp(a.C.x-b.C.x) < 0;
        else return dcmp(a.C.y-b.C.y) < 0;
    }
    friend inline bool operator==(const Circle &a,const Circle &b)
    {
        if (dcmp(a.r-b.r)) return false;
        if (dcmp(a.C.x-b.C.x)) return false;
        if (dcmp(a.C.y-b.C.y)) return false;
        return true;
    }
}tc[maxn],cir[maxn];

inline Node rotate(const Node &p,double cost,double sint)
{
    double x = p.x,y = p.y;
    return Node(x*cost-y*sint,x*sint+y*cost);
}
inline pair <Node,Node> crosspoint(const Node &ap,double ar,const Node &bp,double br)
{
    double d = (ap-bp).norm(),cost = (ar*ar+d*d-br*br)/(2*ar*d),sint = sqrt(1-cost*cost);
    Node v = ((bp-ap).unit())*ar;
    return make_pair(ap+rotate(v,cost,-sint),ap+rotate(v,cost,sint));
}
```

```
48  inline pair <Node,Node> crosspoint(const Circle &a,const Circle &b) { return
    ↪  crosspoint(a.C,a.r,b.C,b.r); }
49
50  struct Event
51  {
52      Node p; double a; int d;
53      inline Event(const Node &_p = Node(),double _a = 0,double _d = 0):p(_p),a(_a),d(_d) {}
54      friend inline bool operator <(const Event &a,const Event &b) { return a.a < b.a; }
55  };
56
57  inline double work()
58  {
59      sort(tc+1,tc+M+1); M = unique(tc+1,tc+M+1)-tc-1;
60      for (int i = M;i;--i)
61      {
62          bool ok = true;
63          for (int j = i+1;j <= M;++j)
64          {
65              double d = (tc[i].C-tc[j].C).norm();
66              if (dcmp(d-fabs(tc[i].r-tc[j].r)) <= 0) { ok = false; break; }
67          }
68          if (ok) cir[++cnt] = tc[i];
69      }
70      // for (int i = M;i;--i) cir[++cnt] = tc[i];
71      double ret = 0;
72      for (int i = 1;i <= cnt;++i)
73      {
74          vector <Event> event;
75          Node boundary = cir[i].C+Node(cir[i].r,0);
76          event.push_back(Event(boundary,-pi,0));
77          event.push_back(Event(boundary,pi,0));
78          for (int j = 1;j <= cnt;++j)
79          {
80              if (i == j) continue;
81              double d = (cir[i].C-cir[j].C).norm();
82              if (dcmp(d-(cir[i].r+cir[j].r)) < 0)
83              {
84                  pair <Node,Node> res = crosspoint(cir[i],cir[j]);
85                  double x = (res.first-cir[i].C).angle(),y = (res.second-cir[i].C).angle();
86                  if (dcmp(x-y) > 0)
87                  {
88                      event.push_back(Event(res.first,x,1));
89                      event.push_back(Event(boundary,pi,-1));
90                      event.push_back(Event(boundary,-pi,1));
91                      event.push_back(Event(res.second,y,-1));
92                  }
93                  else
94                  {
95                      event.push_back(Event(res.first,x,1));
```

```
96                      event.push_back(Event(res.second,y,-1));
97                  }
98              }
99          }
100         sort(event.begin(),event.end());
101         int sum = event[0].d;
102         for (int j = 1;j < (int)event.size();++j)
103         {
104             if (!sum)
105             {
106                 ret += (event[j-1].p/event[j].p)/2;
107                 double x = event[j-1].a,y = event[j].a;
108                 double area = cir[i].r*cir[i].r*(y-x)/2;
109                 Node v1 = event[j-1].p-cir[i].C,v2 = event[j].p-cir[i].C;
110                 area -= (v1/v2)/2; ret += area;
111             }
112             sum += event[j].d;
113         }
114     }
115     return ret;
116 }
```

Chapter 3

Data Structure

3.1 Divide and Conquer on Tree

```
#include<cstring>
#include<iostream>
#include<cstdio>
#include<cstdlib>
using namespace std;

#define maxn (100010)
int best,cnt = 1,side[maxn],toit[maxn],next[maxn],large[maxn];
int sd[maxn],d[maxn],ns,nd,ans,N,K,size[maxn]; bool vis[maxn];

inline void add(int a,int b)
{ next[++cnt] = side[a]; side[a] = cnt; toit[cnt] = b; }
inline void ins(int a,int b)
{ add(a,b); add(b,a); }

inline void getroot(int now,int fa,int rest)
{
    size[now] = 1; large[now] = 0;
    for (int i = side[now];i;i = next[i])
    {
        if (toit[i] == fa||vis[toit[i]]) continue;
        getroot(toit[i],now,rest);
        size[now] += size[toit[i]];
        large[now] = max(large[now],size[toit[i]]);
    }
    large[now] = max(large[now],rest-size[now]);
    if (large[now] < large[best]) best = now;
}
inline int find_root(int now,int rest)
{ best = 0; getroot(now,0,rest); return best; }

inline void dfs(int now,int fa,int dep)
{
```

```
    size[now] = 1; nd = max(dep,nd); ++d[dep];
    for (int i = side[now];i;i = next[i])
        if (toit[i] != fa&&!vis[toit[i]])
            dfs(toit[i],now,dep+1),size[now] += size[toit[i]];
}

inline void subdivide(int now)
{
    vis[now] = true;
    for (int i = side[now];i;i = next[i])
    {
        if (vis[toit[i]]) continue;
        dfs(toit[i],now,1); ans += d[K];
        for (int j = 1;j < K;++j) ans += d[j]*sd[K-j];
        for (int j = 1;j <= nd;++j) sd[j] += d[j],d[j] = 0;
        ns = max(nd,ns); nd = 0;
    }
    memset(sd,0,4*(ns+1)); ns = 0;
    for (int i = side[now];i;i = next[i])
        if (!vis[toit[i]])
            subdivide(find_root(toit[i],size[toit[i]]));
}

int main()
{
    freopen("D.in","r",stdin);
    freopen("D.out","w",stdout);
    scanf("%d %d",&N,&K);
    for (int i = 1,a,b;i < N;++i) scanf("%d %d",&a,&b),ins(a,b);
    large[0] = 1<<30; subdivide(find_root(1,N));
    printf("%d",ans);
    fclose(stdin); fclose(stdout);
    return 0;
}
```

3.2 Dynamicly Divide and Conquer on Tree

```
// N 个点的树，每个点点权 0/1，询问两个 0 点之间最远距离，每次可以 flip 一个点的点权
#include<set>
#include<vector>
#include<algorithm>
#include<cstring>
#include<iostream>
#include<cstdio>
#include<cstdlib>
using namespace std;

const int maxn = 200010,inf = 1<<29,lhh = 4000037;
```

```
int N,cnt,nlight,tot,best,Root,side[maxn],toit[maxn],nxt[maxn],size[maxn],large[maxn],optimal[maxn];
int father[maxn],L[maxn],R[maxn],leaf[maxn],rechain[lhh],depth[lhh]; bool off[maxn],vis[maxn];
vector <int> son[maxn]; pair <int,int> Hash[lhh]; multiset <int> mx[maxn],S[maxn];

struct Value
{
    int a,b;
    inline Value() {}
    inline Value(int _a,int _b):a(_a),b(_b) {}
    friend inline Value operator +(const Value &x,const Value &y)
    {
        Value ret;
        if (x.a > y.a)
        {
            ret.a = x.a;
            if (x.b > y.a) ret.b = x.b;
            else ret.b = y.a;
        }
        else
        {
            ret.a = y.a;
            if (y.b > x.a) ret.b = y.b;
            else ret.b = x.a;
        }
        return ret;
    }
}tree[maxn*2];

inline void add(int a,int b) { nxt[++cnt] = side[a]; side[a] = cnt; toit[cnt] = b; }
inline void ins(int a,int b) { add(a,b); add(b,a); }

inline int gi()
{
    char ch; int ret = 0,f = 1;
    do ch = getchar(); while (!(ch >= '0'&&ch <= '9')&&ch != '-');
    if (ch == '-') f = -1,ch = getchar();
    do ret = ret*10+ch-'0',ch = getchar(); while (ch >= '0'&&ch <= '9');
    return ret*f;
}

inline int find(const pair <int,int> &key)
{
    int now = (2333*key.first+5003*key.second)%lhh;
    while (true)
    {
        if (Hash[now].first == 0||Hash[now] == key) return now;
        else ++now;
        if (now >= lhh) now -= lhh;
```

```
60      }
61  }
62
63  inline void getroot(int now,int rest,int fa)
64  {
65      size[now] = 1; large[now] = 0;
66      for (int i = side[now];i;i = nxt[i])
67      {
68          if (vis[toit[i]]||toit[i] == fa) continue;
69          getroot(toit[i],rest,now);
70          size[now] += size[toit[i]];
71          large[now] = max(large[now],size[toit[i]]);
72      }
73      large[now] = max(large[now],rest-size[now]);
74      if (large[now] < large[best]) best = now;
75  }
76  inline int find_root(int rest,int now) { best = 0; getroot(now,rest,0); return best; }
77
78  inline void dfs(int id,int root,int now,int fa,int dep)
79  {
80      S[id].insert(dep);
81      pair <int,int> key = make_pair(root,now); int pos = find(key);
82      rechain[pos] = id; depth[pos] = dep; Hash[pos] = key;
83      size[now] = 1;
84      for (int i = side[now];i;i = nxt[i])
85      {
86          if (vis[toit[i]]||toit[i] == fa) continue;
87          dfs(id,root,toit[i],now,dep+1);    size[now] += size[toit[i]];
88      }
89  }
90
91  inline void subdivide(int root)
92  {
93      optimal[root] = -inf; mx[root].insert(-inf);
94      L[root] = tot+1;
95      for (int i = side[root];i;i = nxt[i])
96      {
97          if (vis[toit[i]]) continue;
98          ++tot; dfs(tot,root,toit[i],root,1);
99          S[tot].insert(-inf);
100     }
101     R[root] = tot; vis[root] = true;
102     for (int i = side[root];i;i = nxt[i])
103     {
104         if (vis[toit[i]]) continue;
105         int tmp = find_root(size[toit[i]],toit[i]);
106         father[tmp] = root; son[root].push_back(tmp);
107         subdivide(tmp);
108     }
```

```
109 }
110
111 inline void build(int now,int l,int r)
112 {
113     if (l == r)
114     {
115         tree[now] = Value(*S[l].rbegin(),-inf);
116         leaf[l] = now; return;
117     }
118     int mid = (l+r)>>1;
119     build(now<<1,l,mid); build(now<<1|1,mid+1,r);
120     tree[now] = tree[now<<1]+tree[now<<1|1];
121 }
122
123 inline Value query(int now,int l,int r,int ql,int qr)
124 {
125     if (l == ql&&r == qr) return tree[now];
126     int mid = (l+r)>>1;
127     if (qr <= mid) return query(now<<1,l,mid,ql,qr);
128     else if (ql > mid) return query(now<<1|1,mid+1,r,ql,qr);
129     else return query(now<<1,l,mid,ql,mid)+query(now<<1|1,mid+1,r,mid+1,qr);
130 }
131
132 inline void upd(int &a,int b) { if (a < b) a = b; }
133
134 inline void modify(int pos,int dep,bool sign)
135 {
136     if (sign) S[pos].insert(dep); else S[pos].erase(S[pos].find(dep));
137     tree[leaf[pos]] = Value(*S[pos].rbegin(),-inf);
138     for (int now = leaf[pos]>>1;now;now >>= 1)
139         tree[now] = tree[now<<1]+tree[now<<1|1];
140 }
141 inline void modify(int pos)
142 {
143     int c = 0;
144     if (father[pos]) mx[father[pos]].erase(mx[father[pos]].find(optimal[pos]));
145     off[pos] ^= 1; if (off[pos]) nlight++; else nlight--;
146     if (L[pos] <= R[pos])
147     {
148         Value res = query(1,1,tot,L[pos],R[pos]);
149         optimal[pos] = max(res.a+res.b,*mx[pos].rbegin());
150         if (off[pos]) upd(optimal[pos],res.a);
151     }
152     if (father[pos]) mx[father[pos]].insert(optimal[pos]);
153     for (int now = father[pos];now;now = father[now])
154     {
155         int t = find(make_pair(now,pos));
156         int id = rechain[t],dep = depth[t];
157         modify(id,dep,off[pos]);
```

```
158         if (father[now]) mx[father[now]].erase(mx[father[now]].find(optimal[now]));
159         Value res = query(1,1,tot,L[now],R[now]);
160         optimal[now] = max(res.a+res.b,*mx[now].rbegin());
161         if (off[now]) upd(optimal[now],res.a);
162         if (father[now]) mx[father[now]].insert(optimal[now]);
163         ++c;
164     }
165 }
166
167 inline void redfs(int now)
168 {
169     for (int i = 0;i < (int)son[now].size();++i)
170         redfs(son[now][i]),mx[now].insert(optimal[son[now][i]]);
171     if (L[now] <= R[now])
172     {
173         Value res = query(1,1,tot,L[now],R[now]);
174         optimal[now] = max(res.a+res.b,*mx[now].rbegin());
175         if (off[now]) upd(optimal[now],res.a);
176     }
177 }
178
179 int main()
180 {
181     // freopen("A.in","r",stdin);
182     memset(off,true,sizeof off);
183     nlight = N = gi();
184     for (int i = 1;i < N;++i) ins(gi(),gi());
185     large[0] = inf;
186     subdivide(Root = find_root(N,1));
187     build(1,1,tot); redfs(Root);
188     for (int Q = gi();Q--;)
189     {
190         char opt; do opt = getchar(); while (opt != 'G'&&opt != 'C');
191         if (opt == 'G')
192         {
193             if (!nlight) puts("-1");
194             else if (nlight == 1) puts("0");
195             else printf("%d\n",optimal[Root]);
196         }
197         else modify(gi());
198     }
199     return 0;
200 }
```

3.3 Heavy Path Decomposition

```
1 int side[maxn],toit[maxn<<1],nxt[maxn<<1],nxt[maxn<<1];
2 int timestamp,father[maxn],dfn[maxn],redfn[maxn],top[maxn],size[maxn];
3
```

```
4  void decompose(int now,int tp)
5  {
6      redfn[dfn[now] = ++timestamp] = now;
7      top[now] = tp; int heavy = 0;
8      for (int i = side[now];i;i = nxt[i])
9          if (toit[i] != father[now]&&size[toit[i]] > size[heavy]) heavy = toit[i];
10     if (!heavy) return; decompose(heavy,tp);
11     for (int i = side[now];i;i = nxt[i])
12         if (toit[i] != father[now]&&toit[i] != heavy) decompose(toit[i],toit[i]);
13 }
14
15 void dfs(int now)
16 {
17     size[now] = 1;
18     for (int i = side[now];i;i = nxt[i])
19         if (toit[i] != father[now])
20         {
21             father[toit[i]] = now,con[toit[i]] = i;
22             dep[toit[i]] = dep[now]+1,dfs(toit[i]);
23             size[now] += size[toit[i]] ;
24         }
25 }
26
27 // 对点操作
28 inline int query(int a,int b)
29 {
30     int ret = -inf;
31     while (top[a] != top[b])
32     {
33         if (dep[top[a]] < dep[top[b]]) swap(a,b);
34         ret = max(ret,ask(1,1,N,dfn[top[a]],dfn[a]));
35         a = father[top[a]];
36     }
37     if (dep[a] < dep[b]) swap(a,b);
38     ret = max(ret,query(1,1,N,dfn[b],dfn[a]));
39     return ret;
40 }
41
42 // 对边操作
43 inline int query(int a,int b)
44 {
45     int ret = -inf;
46     while (top[a] != top[b])
47     {
48         if (dep[top[a]] < dep[top[b]]) swap(a,b);
49         ret = max(ret,ask(1,1,N,dfn[top[a]],dfn[a]));
50         a = father[top[a]];
51     }
52     if (a == b) return ret;
```

```
    if (dep[a] < dep[b]) swap(a,b);
    ret = max(ret,ask(1,1,N,dfn[b]+1,dfn[a]));
    return ret;
}
```

3.4 K-Dimension Tree

```
struct Point
{
    double x,y; int id;
    inline Point() = default;
    inline Point(double _x,double _y,int _id):x(_x),y(_y),id(_id) {}
    inline void read(int i = 0) { scanf("%lf %lf",&x,&y); id = i; }
    inline double norm() { return sqrt(x*x+y*y); }
    friend inline Point operator+(const Point &a,const Point &b) { return
    ↪  Point(a.x+b.x,a.y+b.y); }
    friend inline Point operator-(const Point &a,const Point &b) { return
    ↪  Point(a.x-b.x,a.y-b.y); }
    friend inline double operator*(const Point &a,const Point &b) { return a.x*b.x+a.y*b.y; }
    friend inline double operator/(const Point &a,const Point &b) { return a.x*b.y-a.y*b.x; }
}P[maxn];

struct Rectangle
{
    double lx,rx,ly,ry;
    inline Rectangle() = default;
    inline Rectangle(double _lx,double _rx,double _ly,double
    ↪  _ry):lx(_lx),rx(_rx),ly(_ly),ry(_ry) {}
    inline void set(const Point &p) { lx = rx = p.x; ly = ry = p.y; }
    inline void merge(const Point &p)
    {
        lx = min(lx,p.x); rx = max(rx,p.x);
        ly = min(ly,p.y); ry = max(ry,p.y);
    }
    inline void merge(const Rectangle &r)
    {
        lx = min(lx,r.lx); rx = max(rx,r.rx);
        ly = min(ly,r.ly); ry = max(ry,r.ry);
    }
    // 最小距离，到 4 个角和 4 条边距离
    inline double dist(const Point &p)
    {
        if (p.x <= lx&&p.y <= ly) return (p-Point(lx,ly)).norm();
        else if (p.x <= rx&&p.y <= ly) return p.y-ly;
        else if (p.x >= rx&&p.y <= ly) return (p-Point(rx,ly)).norm();
        else if (p.x >= rx&&p.y <= ry) return p.x-rx;
        else if (p.x >= rx&&p.y >= ry) return (p-Point(rx,ry)).norm();
        else if (p.x >= lx&&p.y >= ry) return p.y-ry;
        else if (p.x <= lx&&p.y >= ry) return (p-Point(lx,ry)).norm();
```

```
40          else if (p.x <= lx&&p.y >= ly) return p.x-lx;
41          return 0;
42      }
43      // 最大距离，到 4 个角的距离
44      inline double dist(const Point &p)
45      {
46          double ret = 0;
47          ret += max((rx-p.x)*(rx-p.x),(lx-p.x)*(lx-p.x));
48          ret += max((ry-p.y)*(ry-p.y),(ly-p.y)*(ly-p.y));
49          return ret;
50      }
51  };
52
53  struct Node
54  {
55      int child[2]; Point p; Rectangle r;
56      inline Node() = default;
57      inline Node(const Point &_p,const Rectangle &_r):p(_p),r(_r) { r.set(p); memset(child,0,8);
    ↪  }
58      inline void set(const Point &_p) { p = _p; r.set(p); memset(child,0,8); }
59  }tree[maxn];
60
61  inline bool cmpx(const Point &a,const Point &b)
62  {
63      if (a.x != b.x) return a.x < b.x;
64      else return a.y < b.y;
65  }
66  inline bool cmpy(const Point &a,const Point &b)
67  {
68      if (a.y != b.y) return a.y < b.y;
69      else return a.x < b.x;
70  }
71
72  inline bool cmp(pair <double,int> a,pair <double,int> b)
73  {
74      int sgn = dcmp(a.first-b.first);
75      if (sgn) return sgn < 0;
76      else return a.second < b.second;
77  }
78
79  // 查询 k 大/小
80  inline void query(int now,const Point &p,int k,pair <double,int> ret[],bool dim = false)
81  {
82      if (dcmp(tree[now].r.dist(p)-ret[k].first) > 0) return;
83      pair <double,int> val = make_pair((p-tree[now].p).norm(),tree[now].p.id);
84      for (int i = 1;i <= k;++i)
85          if (cmp(val,ret[i]))
86          {
87              for (int j = k+1;j > i;--j) ret[j] = ret[j-1];
```

```
88              ret[i] = val; break;
89          }
90      if ((dim&&cmpx(p,tree[now].p))||(!dim&&cmpy(p,tree[now].p)))
91      {
92          if (tree[now].child[0]) query(tree[now].child[0],p,k,ret,dim^1);
93          if (tree[now].child[1]) query(tree[now].child[1],p,k,ret,dim^1);
94      }
95      else
96      {
97          if (tree[now].child[1]) query(tree[now].child[1],p,k,ret,dim^1);
98          if (tree[now].child[0]) query(tree[now].child[0],p,k,ret,dim^1);
99      }
100 }
101
102 // 查询最小/大
103 inline void query(int x,const Point &p,pair <double,int> ret,bool dim = false)
104 {
105     if (dcmp(tree[now].r.disp(p)-ret.first) > 0) return;
106     pair <double,int> val = make_pair((p-tree[now].p).norm(),tree[now].p.id);
107     if (cmp(val,ret)) ret = val;
108     if ((dim&&cmpx(p,tree[now].p))||(!dim&&cmpy(p,tree[now].p)))
109     {
110         if (tree[now].child[0]) query(tree[now].child[0],p,ret,dim^1);
111         if (tree[now].child[1]) query(tree[now].child[1],p,ret,dim^1);
112     }
113     else
114     {
115         if (tree[now].child[1]) query(tree[now].child[1],p,ret,dim^1);
116         if (tree[now].child[0]) query(tree[now].child[0],p,ret,dim^1);
117     }
118 }
119
120 inline int build(int l,int r,bool dim)
121 {
122     int now = ++size,mid = (l+r)>>1;
123     nth_element(vec.begin()+l-1,vec.begin()+mid-1,vec.begin()+r,dim?cmpx:cmpy);
124     tree[now].set(vec[mid-1]);
125     if (l < mid)
126     {
127         tree[now].child[0] = build(l,mid-1,dim^1);
128         tree[now].r.merge(tree[tree[now].child[0]].r);
129     }
130     if (r > mid)
131     {
132         tree[now].child[1] = build(mid+1,r,dim^1);
133         tree[now].r.merge(tree[tree[now].child[1]].r);
134     }
135     return now;
136 }
```

3.5 Leftlist Tree

```
// It's correct, but it needs be rewritten.
#include<iostream>
#include<cstdio>
#include<cstdlib>
using namespace std;

#define maxn (600010)
int N,M,root[maxn],size[maxn],v[maxn],dep[maxn],l[maxn],r[maxn],tot;

inline int Merge(int x,int y)
{
    if (!x||!y) return x+y;
    if (v[x]>v[y]) swap(x,y);
    r[x] = Merge(r[x],y);
    if (dep[l[x]] < dep[r[x]]) swap(l[x],r[x]);
    dep[x] = dep[r[x]]+1;
    return x;
}
inline int Init(int x) { v[++tot] = x; l[tot] = r[tot] = dep[tot] = 0; return tot;}
inline int Insert(int x,int y) { return Merge(x,Init(y)); }
inline int pop(int x) { return Merge(l[x],r[x]); }

inline int read()
{
    char ch; int f = 1,ret = 0;
    do ch = getchar(); while (!(ch >= '0'&&ch <= '9')&&ch != '-');
    if (ch == '-') f = -1,ch = getchar();
    do ret = ret*10+ch-'0',ch = getchar(); while (ch >= '0'&&ch <= '9');
    return ret*f;
}

int main()
{
    freopen("1050.in","r",stdin);
    freopen("1050.out","w",stdout);
    scanf("%d %d",&N,&M);
    for (int i = 1;i <= N;++i) root[i] = Init(read()),size[i] = 1;
    while (M--)
    {
        int opt = read();
        if (!opt)
        {
            int a = read()+1,b = read()+1;
            if (size[b]) root[a] = Merge(root[a],root[b]);
            size[a] += size[b]; size[b] = 0;
        }
        else if (opt == 1)
        {
```

```
            int a = read()+1;
            if (!size[a]) puts("-1");
            else printf("%d\n",v[root[a]]),root[a] = pop(root[a]),--size[a];
        }
        else
        {
            int a = read()+1; ++size[a];
            root[a] = Insert(root[a],read());
        }
    }
    fclose(stdin); fclose(stdout);
    return 0;
}
```

3.6 Link Cut Tree

```
inline bool isroot(int a) { return ch[fa[a]][0] != a&&ch[fa[a]][1] != a; }

inline void update(int x) { val[x] = (val[ch[x][0]]+val[ch[x][1]]).merge(x); }
inline void pushdown(int x)
{
    if (rev[x])
    {
        int &lc = ch[x][0],&rc = ch[x][1];
        swap(lc,rc);
        if (lc) rev[lc] ^= 1;
        if (rc) rev[rc] ^= 1;
        rev[x] = false;
    }
}

inline void rotate(int x)
{
    int y = fa[x],z = fa[y],l = ch[y][1] == x,r = l^1;
    if (!isroot(y)) ch[z][ch[z][1] == y] = x; fa[x] = z;
    if (ch[x][r]) fa[ch[x][r]] = y; ch[y][l] = ch[x][r];
    fa[y] = x; ch[x][r] = y; update(y); update(x);
}
inline void splay(int x)
{
    int top = 0,i;
    for (i = x;!isroot(i);i = fa[i]) stk[++top] = i; stk[++top] = i;
    while (top) pushdown(stk[top--]);
    while (!isroot(x))
    {
        int y = fa[x],z = fa[y];
        if (!isroot(y))
        {
            if ((ch[y][0] == x)^(ch[z][0] == y)) rotate(x);
```

```
34              else rotate(y);
35          }
36          rotate(x);
37      }
38  }
39
40  inline int access(int x)
41  {
42      int t = 0;
43      for (t = 0;x;t = x,x = fa[x])
44          splay(x),ch[x][1] = t,update(x);
45      return t;
46  }
47  inline int evert(int x) { int t; rev[t = access(x)] ^= 1; return t; }
48  inline int find(int x)
49  {
50      x = access(x);
51      while (pushdown(x),ch[x][0]) x = ch[x][0];
52      return x;
53  }
54  inline void cut(int x,int y)
55  {
56      evert(x); access(y); splay(y);
57      if (ch[y][0] != x||ch[x][1] != 0) return;
58      ch[y][0] = fa[x] = 0; update(x); update(y);
59  }
60  inline void link(int x,int y) { fa[evert(x)] = y; }
61
62
63  // Magic Forest
64  #include<algorithm>
65  #include<cstring>
66  #include<iostream>
67  #include<cstdio>
68  #include<cstdlib>
69  using namespace std;
70
71  const int maxn = 200010,inf = 1<<29;
72  int N,M,A[maxn],B[maxn],fa[maxn],ch[maxn][2];
73  int stk[maxn],ans = inf; bool rev[maxn];
74
75  struct Value
76  {
77      int ma,mb,id;
78      inline Value(int _ma = 0,int _mb = 0,int _id = 0):ma(_ma),mb(_mb),id(_id) {}
79      friend inline Value operator +(const Value &a,const Value &b)
80      {
81          Value ret = Value(max(a.ma,b.ma),max(a.mb,b.mb),a.id);
82          if (B[a.id] < B[b.id]) ret.id = b.id;
```

```
83          return ret;
84      }
85      inline Value merge(int i)
86      {
87          Value ret = Value(max(ma,A[i]),max(mb,B[i]),id);
88          if (B[i] > B[id]) ret.id = i;
89          return ret;
90      }
91  }val[maxn];
92
93  inline int gi()
94  {
95      char ch; int ret = 0,f = 1;
96      do ch = getchar(); while (!(ch >= '0'&&ch <= '9')&&ch != '-');
97      if (ch == '-') f = -1,ch = getchar();
98      do ret = ret*10+ch-'0',ch = getchar(); while (ch >= '0'&&ch <= '9');
99      return ret*f;
100 }
101
102 struct Edge
103 {
104     int x,y,a,b;
105     inline Edge(int _x = 0,int _y = 0,int _a = 0,int _b = 0):x(_x),y(_y),a(_a),b(_b) {}
106     inline void read() { x = gi(),y = gi(),a = gi(),b = gi(); }
107     friend inline bool operator <(const Edge &s,const Edge &t) { return s.a < t.a; }
108 }edge[maxn];
109
110 inline bool isroot(int a) { return ch[fa[a]][0] != a&&ch[fa[a]][1] != a; }
111
112 inline void update(int x) { val[x] = (val[ch[x][0]]+val[ch[x][1]]).merge(x); }
113 inline void pushdown(int x)
114 {
115     if (rev[x])
116     {
117         int &lc = ch[x][0],&rc = ch[x][1];
118         swap(lc,rc);
119         if (lc) rev[lc] ^= 1;
120         if (rc) rev[rc] ^= 1;
121         rev[x] = false;
122     }
123 }
124
125 inline void rotate(int x)
126 {
127     int y = fa[x],z = fa[y],l = ch[y][1] == x,r = l^1;
128     if (!isroot(y)) ch[z][ch[z][1] == y] = x; fa[x] = z;
129     if (ch[x][r]) fa[ch[x][r]] = y; ch[y][l] = ch[x][r];
130     fa[y] = x; ch[x][r] = y; update(y); update(x);
131 }
```

```
132 inline void splay(int x)
133 {
134     int top = 0,i;
135     for (i = x;!isroot(i);i = fa[i]) stk[++top] = i; stk[++top] = i;
136     while (top) pushdown(stk[top--]);
137     while (!isroot(x))
138     {
139         int y = fa[x],z = fa[y];
140         if (!isroot(y))
141         {
142             if ((ch[y][0] == x)^(ch[z][0] == y)) rotate(x);
143             else rotate(y);
144         }
145         rotate(x);
146     }
147 }
148
149 inline int access(int x)
150 {
151     int t = 0;
152     for (t = 0;x;t = x,x = fa[x])
153         splay(x),ch[x][1] = t,update(x);
154     return t;
155 }
156 inline int evert(int x) { int t; rev[t = access(x)] ^= 1; return t; }
157 inline int find(int x)
158 {
159     x = access(x);
160     while (pushdown(x),ch[x][0]) x = ch[x][0];
161     return x;
162 }
163 inline void cut(int x,int y)
164 {
165     evert(x); access(y); splay(y);
166     if (ch[y][0] != x||ch[x][1] != 0) return;
167     ch[y][0] = fa[x] = 0; update(x); update(y);
168 }
169 inline void link(int x,int y) { fa[evert(x)] = y; }
170
171 inline Value query(int x,int y) { evert(x); return val[access(y)]; }
172
173 int main()
174 {
175     // freopen("D.in","r",stdin);
176     N = gi(),M = gi();
177     for (int i = 1;i <= M;++i) edge[i].read();
178     sort(edge+1,edge+M+1);
179     for (int i = 0;i <= N;++i)
180         A[i] = B[i] = -inf,val[i] = Value(A[i],B[i],i);
```

```
    for (int i = 1;i <= M;++i)
        A[i+N] = edge[i].a,B[i+N] = edge[i].b,val[i+N] = Value(A[i+N],B[i+N],i+N);
    for (int i = 1;i <= M;++i)
    {
        if (edge[i].x == edge[i].y) continue;
        if (find(edge[i].x) == find(edge[i].y))
        {
            Value res = query(edge[i].x,edge[i].y); int id = res.id-N;
            if (edge[i].b < edge[id].b)
            {
                cut(edge[id].x,id+N),cut(edge[id].y,id+N);
                link(edge[i].x,i+N),link(edge[i].y,i+N);
            }
        }
        else link(edge[i].x,i+N),link(i+N,edge[i].y);
        if (find(1) == find(N))
        {
            Value res = query(1,N);
            ans = min(ans,res.ma+res.mb);
        }
    }
    if (ans == inf) ans = -1;
    printf("%d\n",ans);
    return 0;
}
```

3.7 Merge Split Treap

```
// jisuanke17123
// Warning: 给指针赋值时，不要赋 this，因为 this 是临时变量的地址
#include<sys/timeb.h>
#include<queue>
#include<algorithm>
#include<cstring>
#include<iostream>
#include<cstdio>
#include<cstdlib>
#include<set>
using namespace std;

typedef long long ll;
const int maxn = 1000010;
int N;

inline int rand(int n) { int x = rand(); if (x < 0) x = -x; return x%n+1; }

struct Node
{
    int size,key,val; Node *mn,*ch[2];
```

```
22      inline Node *update()
23      {
24          mn = this; size = 1;
25          if (ch[0])
26          {
27              size += ch[0]->size;
28              if (ch[0]->mn->val < mn->val) mn = ch[0]->mn;
29          }
30          if (ch[1])
31          {
32              size += ch[1]->size;
33              if (ch[1]->mn->val < mn->val) mn = ch[1]->mn;
34          }
35          return this;
36      }
37      inline Node() = default;
38      inline Node(int v,Node *_mn):size(1),key(rand()),val(v),mn(_mn) { ch[0] = ch[1] = NULL; }
39  }pool[maxn*100/4],*root[maxn],*cur;
40  struct Status
41  {
42      int l,r; ll val;
43      inline Status() = default;
44      inline Status(int _l,int _r,ll _val):l(_l),r(_r),val(_val) {}
45      friend inline bool operator <(const Status &a,const Status &b) { return a.val > b.val; }
46  };
47
48  inline int sz(const Node *x) { if (x == NULL) return 0; else return x->size; }
49
50  inline Node *newnode(int v = 0) { *cur = Node(v,cur); return cur++; }
51
52  Node *insert(Node *p,Node *q)
53  {
54      if (p == NULL&&q == NULL) return NULL;
55      if (p == NULL||q == NULL) return p?p:q;
56      Node *u = NULL;
57      if (rand(sz(p)+sz(q)) < sz(p))
58          u = p,u->ch[1] = insert(u->ch[1],q);
59      else u = q,u->ch[0] = insert(p,u->ch[0]);
60      return u->update();
61  }
62
63  Node *merge(Node *p,Node *q)
64  {
65      if (p == NULL&&q == NULL) return NULL;
66      if (p == NULL||q == NULL) return p?p:q;
67      Node *u = newnode();
68      if (rand(sz(p)+sz(q)) < sz(p))
69          *u = *p,u->ch[1] = merge(u->ch[1],q);
70      else *u = *q,u->ch[0] = merge(p,u->ch[0]);
```

```
71      return u->update();
72  }
73
74  Node *split(Node *u,int l,int r)
75  {
76      if (l > r||u == NULL) return 0;
77      Node *x = NULL;
78      if (l == 1&&r == sz(u))
79      {
80          x = newnode(); *x = *u;
81          return x->update();
82      }
83      int lsz = sz(u->ch[0]);
84      if (r <= lsz) return split(u->ch[0],l,r);
85      if (l > lsz+1) return split(u->ch[1],l-lsz-1,r-lsz-1);
86      x = newnode(); *x = *u;
87      x->ch[0] = split(u->ch[0],l,lsz);
88      x->ch[1] = split(u->ch[1],1,r-lsz-1);
89      return x->update();
90  }
91
92  inline int gi()
93  {
94      char ch; int ret = 0,f = 1;
95      do ch = getchar(); while (!(ch >= '0'&&ch <= '9')&&ch != '-');
96      if (ch == '-') f = -1,ch = getchar();
97      do ret = ret*10+ch-'0',ch = getchar(); while (ch >= '0'&&ch <= '9');
98      return ret*f;
99  }
100
101 int get_pos(Node *rt,Node *mn)
102 {
103     if (rt == mn) return sz(rt->ch[0]);
104     else if (rt->ch[0]&&rt->ch[0]->mn == mn)
105         return get_pos(rt->ch[0],mn);
106     else return sz(rt->ch[0])+1+get_pos(rt->ch[1],mn);
107 }
108 inline pair <int,int> Qmin(Node *rt,int l,int r)
109 {
110     if (l > r) return make_pair(-1,-1);
111     Node *v = split(rt,l,r);
112     auto ret = make_pair(v->mn->val,get_pos(v,v->mn)+l);
113     return ret;
114 }
115 inline int get(Node *u,int x) { return split(u,x,x)->val; }
116
117 inline void work(Node *rt,int k)
118 {
119     int n = sz(rt);
```

```
120     set <int> S; S.insert(0); S.insert(n+1);
121     priority_queue <Status> heap;
122     auto tmp = Qmin(rt,1,n);
123     heap.push(Status(tmp.second,tmp.second,tmp.first));
124     while (k--)
125     {
126         auto now = heap.top(); heap.pop();
127         printf("%lld\n",now.val);
128         if (now.l == now.r)
129         {
130             S.insert(now.l);
131             auto it = S.find(now.l);
132             int pre = *(--it);
133             int nxt = *++(++it);
134             auto ls = Qmin(rt,pre+1,now.l-1);
135             auto rs = Qmin(rt,now.l+1,nxt-1);
136             if (ls.first != -1)
137                 heap.push(Status(ls.second,ls.second,ls.first));
138             if (rs.first != -1)
139                 heap.push(Status(rs.second,rs.second,rs.first));
140         }
141         if (now.r < n)
142         {
143             int inc = get(rt,now.r+1);
144             ++now.r; now.val += (ll)inc;
145             heap.push(now);
146         }
147     }
148 }
149
150 inline void init() { N = 0; cur = pool; }
151
152 int main()
153 {
154     struct timeb ttt; ftime(&ttt);
155     srand(ttt.millitm+ttt.time*1000);
156     for (int T = gi();T--;)
157     {
158         init();
159         for (int Q = gi();Q--;)
160         {
161             int opt = gi();
162             if (opt == 1)
163             {
164                 root[++N] = NULL;
165                 for (int n = gi();n--;)
166                     root[N] = insert(root[N],newnode(gi()));
167             }
168             else if (opt == 2)
```

```
169             {
170                 root[++N] = NULL;
171                 int x = gi(),l1 = gi(),r1 = gi(),y = gi(),l2 = gi(),r2 = gi();
172                 Node *ls = split(root[x],l1,r1);
173                 Node *rs = split(root[y],l2,r2);
174                 root[N] = merge(ls,rs);
175             }
176             else
177             {
178                 int x = gi(),k = gi();
179                 work(root[x],k);
180             }
181         }
182         // cerr << cur - pool << endl;
183     }
184     return 0;
185 }
186
187 // By zky. To be rewritten.
188 const int mo=1e9+7;
189 int rnd(){
190     static int x=1;
191     return x=(x*23333+233);
192 }
193 int rnd(int n){
194     int x=rnd();
195     if(x<0)x=-x;
196     return x%n+1;
197 }
198 struct node{
199     int siz,key;
200     int val;
201     LL sum;
202     node *c[2];
203     node* rz(){
204         sum=val;siz=1;
205         if(c[0])sum+=c[0]->sum,siz+=c[0]->siz;
206         if(c[1])sum+=c[1]->sum,siz+=c[1]->siz;
207         return this;
208     }
209     node(){}
210     node(int v){
211         siz=1;key=rnd();
212         val=v;sum=v;
213         c[0]=c[1]=0;
214     }
215
216 }pool[maxn*8],*root,*cur=pool,*old_root,*stop;
217 node *newnode(int v=0){
```

```
218     *cur=node(v);
219     return cur++;
220 }
221 node *old_merge(node *p,node *q){
222     if(!p&&!q)return 0;
223     node *u=0;
224     if(!p||!q)return u=p?p->rz():(q?q->rz():0);
225     if(rnd(sz(p)+sz(q))<sz(p)){
226         u=p;
227         u->c[1]=old_merge(u->c[1],q);
228     }else{
229         u=q;
230         u->c[0]=old_merge(p,u->c[0]);
231     }
232     return u->rz();
233 }
234 node *merge(node *p,node *q){
235     if(!p&&!q)return 0;
236     node *u=newnode();
237     if(!p||!q)return u=p?p->rz():(q?q->rz():0);
238     if(rnd(sz(p)+sz(q))<sz(p)){
239         *u=*p;
240         u->c[1]=merge(u->c[1],q);
241     }else{
242         *u=*q;
243         u->c[0]=merge(p,u->c[0]);
244     }
245     return u->rz();
246 }
247 node *split(node *u,int l,int r){
248     if(l>r||!u)return 0;
249     node *x=0;
250     if(l==1&&r==sz(u)){
251         x=newnode();
252         *x=*u;
253         return x->rz();
254     }
255     int lsz=sz(u->c[0]);
256     if(r<=lsz)
257         return split(u->c[0],l,r);
258     if(l>lsz+1)
259         return split(u->c[1],l-lsz-1,r-lsz-1);
260     x=newnode();
261     *x=*u;
262     x->c[0]=split(u->c[0],l,lsz);
263     x->c[1]=split(u->c[1],1,r-lsz-1);
264     return x->rz();
265 }
```

3.8 Modui Algorithm on Tree

```cpp
// 询问树上路径元素 mex, inc dec 复杂度不对, 需要用线段树/set(带 log) 或者分块 (修改 O(1))
// 若包括 lca, 每组询问需要把 lca 补 (inc) 上去。
#include<cstdio>
#include<cstdlib>
#include<algorithm>
#include<cstring>
#include<iostream>
using namespace std;

const int Size = 337,maxn = 200010;
int N,Q,cnt,nxt[maxn],side[maxn],len[maxn],toit[maxn],f[maxn][20],key[maxn],timestamp;
int dep[maxn],L[maxn],R[maxn],dfn[maxn],ans[maxn],exist[maxn],show[maxn],res;

inline void add(int a,int b,int c) { nxt[++cnt] = side[a]; side[a] = cnt; toit[cnt] = b;
↪  len[cnt] = c; }
inline void ins(int a,int b,int c) { add(a,b,c); add(b,a,c); }

void dfs(int now)
{
    dfn[L[now] = ++timestamp] = now;
    for (int i = 1;(1<<i) <= dep[now];++i) f[now][i] = f[f[now][i-1]][i-1];
    for (int i = side[now];i;i = nxt[i])
    {
        if (toit[i] == f[now][0]) continue;
        f[toit[i]][0] = now; key[toit[i]] = len[i];
        dep[toit[i]] = dep[now]+1;
        dfs(toit[i]);
    }
    dfn[R[now] = ++timestamp] = now;
}

inline int jump(int a,int b) { for (int i = 0;b;b >>= 1,++i) if (b&1) a = f[a][i]; return a; }
inline int lca(int a,int b)
{
    if (dep[a] < dep[b]) swap(a,b);
    a = jump(a,dep[a]-dep[b]);
    if (a == b) return a;
    for (int i = 0;i >= 0;)
    {
        if (f[a][i] == f[b][i]) --i;
        else a = f[a][i],b = f[b][i],++i;
    }
    return f[a][0];
}

struct Node
{
    int a,b,c,id;
```

```
48      Node() = default;
49      Node(int _a,int _b,int _c = 0,int _id = 0):a(_a),b(_b),c(_c),id(_id) {}
50      inline void read(int i)
51      {
52          id = i; scanf("%d %d",&a,&b); c = lca(a,b);
53          if (c == a||c == b) { if (a != c) swap(a,b); a = L[c]+1; b = L[b]; }
54          else { if (L[a] > L[b]) swap(a,b); a = R[a]; b = L[b]; }
55      }
56      friend inline bool operator <(const Node &x,const Node &y) { return x.b < y.b; }
57  }query[maxn];
58
59  inline bool cmp(const Node &x,const Node &y) { return x.a < y.a; }
60
61  inline void inc(int id)
62  {
63      if (key[id] >= maxn) return;
64      ++exist[key[id]];
65      while (exist[res]) ++res;
66  }
67  inline void dec(int id)
68  {
69      if (key[id] >= maxn) return;
70      --exist[key[id]];
71      if (key[id] < res&&!exist[key[id]]) res = key[id];
72  }
73
74  inline void work()
75  {
76      int l = 1,r = 0;
77      for (int i = 1;i <= Q;++i)
78      {
79          while (r < query[i].b)
80          {
81              show[dfn[++r]]++;
82              if (show[dfn[r]] == 2) dec(dfn[r]); else inc(dfn[r]);
83          }
84          while (l > query[i].a)
85          {
86              show[dfn[--l]]++;
87              if (show[dfn[l]] == 2) dec(dfn[l]); else inc(dfn[l]);
88          }
89          while (r > query[i].b)
90          {
91              if (show[dfn[r]] == 1) dec(dfn[r]); else inc(dfn[r]);
92              show[dfn[r--]]--;
93          }
94          while (l < query[i].a)
95          {
96              if (show[dfn[l]] == 1) dec(dfn[l]); else inc(dfn[l]);
```

```
            show[dfn[l++]]--;
        }
        ans[query[i].id] = res;
    }
}

int main()
{
    freopen("F.in","r",stdin);
    scanf("%d %d",&N,&Q);
    for (int i = 1;i < N;++i)
    {
        int a,b,c;
        scanf("%d %d %d",&a,&b,&c);
        ins(a,b,c);
    }
    dfs(1);
    for (int i = 1;i <= Q;++i) query[i].read(i);
    sort(query+1,query+Q+1,cmp);
    for (int i = 1,j;i <= Q;i = j)
    {
        for (j = i;j <= Q&&query[j].a-query[i].a <= Size;++j);
        sort(query+i+1,query+j);
    }
    work();
    for (int i = 1;i <= Q;++i) printf("%d\n",ans[i]);
    return 0;
}
```

3.9 Modui Algorithm without Deletion

```
// r 单调右移，l 只会在 sqrt(N) 中移动，保证每次 undo 的复杂度可行即可。
// CodeForces 620F
#include<vector>
#include<algorithm>
#include<cstring>
#include<iostream>
#include<cstdio>
#include<cstdlib>
using namespace std;

const int maxn = 1000010,len = 200,inf = 1<<29;
int N,M,pre[maxn],A[maxn],ans[maxn];

inline int gi()
{
    char ch; int ret = 0,f = 1;
    do ch = getchar(); while (!(ch >= '0'&&ch <= '9')&&ch != '-');
    if (ch == '-') f = -1,ch = getchar();
```

```
19      do ret = ret*10+ch-'0',ch = getchar(); while (ch >= '0'&&ch <= '9');
20      return ret*f;
21  }
22
23  inline void upd(int &a,int b) { if (a < b) a = b; }
24
25  struct Trie
26  {
27      int nxt[maxn][2],val[maxn],root,cnt; vector < pair<int,int> > vec;
28      inline int newnode() { val[++cnt] = inf; memset(nxt[cnt],0,8); return cnt; }
29      inline void init() { val[0] = inf; cnt = 0; root = newnode(); }
30
31      inline int query(int key,int num)
32      {
33          int now = root,ret = 0;
34          for (int i = 19;i >= 0;--i)
35          {
36              int dir = !(num&(1<<i));
37              if (val[nxt[now][dir]] <= key)
38                  ret |= (1<<i),now = nxt[now][dir];
39              else now = nxt[now][dir^1];
40          }
41          return ret;
42      }
43
44      inline void insert(int key,int num,int mode) { insert(root,19,key,num,mode); }
45      inline void insert(int &now,int dep,int key,int num,int mode)
46      {
47          if (!now) now = newnode();
48          if (dep < 0)
49          {
50              if (mode) vec.push_back(make_pair(num,val[now]));
51              val[now] = min(val[now],key); return;
52          }
53          insert(nxt[now][(num&(1<<dep)) > 0],dep-1,key,num,mode);
54          val[now] = min(val[nxt[now][0]],val[nxt[now][1]]);
55      }
56
57      inline void change(int now,int dep,int num,int v)
58      {
59          if (dep < 0) { val[now] = v; return; }
60          change(nxt[now][(num&(1<<dep)) > 0],dep-1,num,v);
61          val[now] = min(val[nxt[now][0]],val[nxt[now][1]]);
62      }
63
64      inline void undo()
65      {
66          reverse(vec.begin(),vec.end());
67          for (auto x:vec) change(root,19,x.first,x.second);
```

```
68          vec.clear();
69      }
70  }tree1,tree2;
71
72  struct Query
73  {
74      int l,r,id;
75      inline void read(int i) { l = gi(),r = gi(),id = i; }
76      friend inline bool operator <(const Query &a,const Query &b) { return a.l < b.l; }
77  }query[maxn];
78  inline bool cmp(const Query &a,const Query &b) { return a.r < b.r; }
79
80  inline void work(int l,int r)
81  {
82      int lim = query[r].l;
83      sort(query+l,query+r+1,cmp);
84      tree1.init(); tree2.init();
85      for (int i = l;i <= r;++i)
86      {
87          if (query[i].r <= lim)
88          {
89              for (int j = query[i].l;j <= query[i].r;++j)
90              {
91                  tree1.insert(A[j],pre[A[j]-1],false);
92                  tree2.insert(-A[j],pre[A[j]],false);
93                  upd(ans[query[i].id],tree1.query(A[j],pre[A[j]]));
94                  upd(ans[query[i].id],tree2.query(-A[j],pre[A[j]-1]));
95              }
96              tree1.init(),tree2.init();
97          }
98          else
99          {
100             int pos = lim,mx = 0;
101             for (;i <= r;++i)
102             {
103                 while (pos < query[i].r)
104                 {
105                     ++pos;
106                     tree1.insert(A[pos],pre[A[pos]-1],false);
107                     tree2.insert(-A[pos],pre[A[pos]],false);
108                     upd(mx,tree1.query(A[pos],pre[A[pos]]));
109                     upd(mx,tree2.query(-A[pos],pre[A[pos]-1]));
110                 }
111                 upd(ans[query[i].id],mx);
112                 for (int j = lim;j >= query[i].l;--j)
113                 {
114                     tree1.insert(A[j],pre[A[j]-1],true);
115                     tree2.insert(-A[j],pre[A[j]],true);
116                     upd(ans[query[i].id],tree1.query(A[j],pre[A[j]]));
```

```
                    upd(ans[query[i].id],tree2.query(-A[j],pre[A[j]-1]));
                }
                tree1.undo(); tree2.undo();
            }
            break;
        }
    }
}

int main()
{
    // freopen("A.in","r",stdin);
    for (int i = 1;i <= 1000000;++i) pre[i] = pre[i-1]^i;
    N = gi(); M = gi();
    for (int i = 1;i <= N;++i) A[i] = gi();
    for (int i = 1;i <= M;++i) query[i].read(i);
    sort(query+1,query+M+1);
    for (int i = 1,j;i <= M;i = j)
    {
        for (j = i;j <= M&&query[j].l-query[i].l <= len;++j);
        work(i,j-1);
    }
    for (int i = 1;i <= M;++i) printf("%d\n",ans[i]);
    return 0;
}
```

3.10 President Tree

```
inline void build(int &now,int l,int r)
{
    now = ++cnt; if (l == r) return;
    int mid = (l+r)>>1;
    build(tree[now].ch[0],l,mid); build(tree[now].ch[1],mid+1,r);
}

inline void ins(int &now,int ref,int l,int r,int key)
{
    now = ++cnt; tree[now] = tree[ref];
    if (l == r) { ++tree[now].sum; return; }
    int mid = (l+r) >> 1;
    if (key <= mid) ins(tree[now].ch[0],tree[ref].ch[0],l,mid,key);
    else ins(tree[now].ch[1],tree[ref].ch[1],mid+1,r,key);
    tree[now].sum = tree[tree[now].ch[0]].sum+tree[tree[now].ch[1]].sum;
}
```

3.11 Splay

```
//splay

```

```
3  inline int find(int rk)
4  {
5      for (int now = root;;)
6      {
7          if (rk == size[ch[now][0]]+1) return now;
8          else if (rk > size[ch[now][0]]+1)
9              rk -= size[ch[now][1]]+1,now = ch[now][1];
10         else now = ch[now][0];
11     }
12     return 0;
13 }
14
15 inline int upperbound(int x)
16 {
17     int ret = 0;
18     for (int now = root;now;)
19     {
20         if (key[now] > x) ret = now,now = ch[now][0];
21         else now = ch[now][1];
22     }
23     return ret;
24 }
25 inline int lowerbound(int x)
26 {
27     int ret = 0;
28     for (int now = root;now;)
29     {
30         if (key[now] >= x) ret = now,now = ch[now][0];
31         else now = ch[now][1];
32     }
33     return ret;
34 }
35
36 inline void rotate(int x)
37 {
38     int y = fa[x],z = fa[y],l = ch[y][0] != x,r = l^1;
39     if (z) ch[z][ch[z][0] != y] = x; fa[x] = z;
40     if (ch[x][r]) fa[ch[x][r]] = y;
41     ch[y][l] = ch[x][r]; fa[y] = x; ch[x][r] = y;
42     update(y); update(x);
43 }
44 inline void splay(int x,int aim) //aim is x's father.
45 {
46     int top = 0;
47     for (int i = x;i;i = fa[i]) stack[++top] = i;
48     while (top) pushdown(stack[top--]);
49     while (fa[x] != aim)
50     {
51         int y = fa[x],z = fa[y];
```

```
52          if (z != aim)
53          {
54              if ((ch[y][0] == x)^(ch[z][0] == y)) rotate(x);
55              else rotate(y);
56          }
57          rotate(x);
58      }
59      if (!aim) root = x;
60  }
61
62  // 维修数列
63  #include<cassert>
64  #include<queue>
65  #include<algorithm>
66  #include<cstring>
67  #include<iostream>
68  #include<cstdio>
69  #include<cstdlib>
70  using namespace std;
71
72  const int maxn = 500010,inf = 1<<29;
73  int N,M,root,cnt,arr[maxn],tag[maxn],key[maxn],fa[maxn],ch[maxn][2],lb[maxn],rb[maxn];
74  int wb[maxn],sum[maxn],size[maxn],stk[maxn]; bool rev[maxn]; char cmd[20]; queue <int> team;
75
76  inline int gi()
77  {
78      char ch; int ret = 0,f = 1;
79      do ch = getchar(); while (!(ch >= '0'&&ch <= '9')&&ch != '-');
80      if (ch == '-') f = -1,ch = getchar();
81      do ret = ret*10+ch-'0',ch = getchar(); while (ch >= '0'&&ch <= '9');
82      return ret*f;
83  }
84
85  inline int newnode(int x = 0)
86  {
87      int ret;
88      if (!team.empty())
89          ret = team.front(),team.pop();
90      else ret = ++cnt;
91      key[ret] = sum[ret] = lb[ret] = rb[ret] = wb[ret] = x;
92      rev[ret] = false; tag[ret] = inf; size[ret] = 1;
93      return ret;
94  }
95
96  inline void pushdown(int now)
97  {
98      int lc = ch[now][0],rc = ch[now][1];
99      if (rev[now])
100     {
```

```
101         if (lc)
102         {
103             swap(ch[lc][0],ch[lc][1]);
104             swap(lb[lc],rb[lc]); rev[lc] ^= 1;
105         }
106         if (rc)
107         {
108             swap(ch[rc][0],ch[rc][1]);
109             swap(lb[rc],rb[rc]); rev[rc] ^= 1;
110         }
111         rev[now] = false;
112     }
113     if (tag[now] != inf)
114     {
115         if (lc)
116         {
117             key[lc] = tag[lc] = tag[now]; sum[lc] = tag[lc]*size[lc];
118             if (tag[lc] > 0) lb[lc] = rb[lc] = wb[lc] = sum[lc];
119             else lb[lc] = rb[lc] = wb[lc] = tag[lc];
120         }
121         if (rc)
122         {
123             key[rc] = tag[rc] = tag[now]; sum[rc] = tag[rc]*size[rc];
124             if (tag[rc] > 0) lb[rc] = rb[rc] = wb[rc] = sum[rc];
125             else lb[rc] = rb[rc] = wb[rc] = tag[rc];
126         }
127         tag[now] = inf;
128     }
129 }
130
131 inline void update(int now)
132 {
133     // pushdown(now);
134     int lc = ch[now][0],rc = ch[now][1];
135     size[now] = size[lc]+size[rc]+1;
136     sum[now] = sum[lc]+sum[rc]+key[now];
137     if (lc&&rc)
138     {
139         lb[now] = max(lb[lc],max(sum[lc]+key[now],sum[lc]+key[now]+lb[rc]));
140         rb[now] = max(rb[rc],max(sum[rc]+key[now],sum[rc]+key[now]+rb[lc]));
141         wb[now] = max(wb[lc],wb[rc]); wb[now] = max(wb[now],key[now]);
142         wb[now] = max(wb[now],rb[lc]+key[now]); wb[now] = max(wb[now],lb[rc]+key[now]);
143         wb[now] = max(wb[now],rb[lc]+key[now]+lb[rc]);
144     }
145     else if (lc)
146     {
147         lb[now] = max(lb[lc],sum[lc]+key[now]);
148         rb[now] = max(key[now],key[now]+rb[lc]);
149         wb[now] = max(wb[lc],key[now]);
```

```
150         wb[now] = max(wb[now],rb[lc]+key[now]);
151     }
152     else if (rc)
153     {
154         rb[now] = max(rb[rc],sum[rc]+key[now]);
155         lb[now] = max(key[now],key[now]+lb[rc]);
156         wb[now] = max(wb[rc],key[now]);
157         wb[now] = max(wb[now],lb[rc]+key[now]);
158     }
159     else sum[now] = lb[now] = rb[now] = wb[now] = key[now];
160 }
161
162 inline int build(int l,int r)
163 {
164     int mid = (l+r) >> 1,ret = newnode(arr[mid]);
165     if (l < mid) ch[ret][0] = build(l,mid-1),fa[ch[ret][0]] = ret;
166     if (r > mid) ch[ret][1] = build(mid+1,r),fa[ch[ret][1]] = ret;
167     update(ret); return ret;
168 }
169
170 inline void init()
171 {
172     root = newnode(); ch[root][1] = newnode(); fa[2] = 1;
173     for (int i = 1;i <= N;++i) arr[i] = gi();
174     ch[2][0] = build(1,N); fa[ch[2][0]] = 2;
175     update(2); update(1);
176 }
177
178 inline int find(int rk)
179 {
180     for (int now = root;;)
181     {
182         pushdown(now);
183         if (rk == size[ch[now][0]]+1) return now;
184         else if (rk > size[ch[now][0]]+1)
185             rk -= size[ch[now][0]]+1,now = ch[now][1];
186         else now = ch[now][0];
187     }
188     return 0;
189 }
190
191 inline void rotate(int x)
192 {
193     int y = fa[x],z = fa[y],l = ch[y][0] != x,r = l^1;
194     if (z) ch[z][ch[z][0] != y] = x;
195     fa[x] = z; fa[y] = x; fa[ch[x][r]] = y;
196     ch[y][l] = ch[x][r]; ch[x][r] = y;
197     update(y); update(x);
198 }
```

```
199 inline void splay(int x,int aim)
200 {
201     int top = 0;
202     for (int i = x;i;i = fa[i]) stk[++top] = i;
203     while (top) pushdown(stk[top--]);
204     while (fa[x] != aim)
205     {
206         int y = fa[x],z = fa[y];
207         if (z != aim)
208         {
209             if ((ch[y][0] == x)^(ch[z][0] == y)) rotate(x);
210             else rotate(y);
211         }
212         rotate(x);
213     }
214     if (!aim) root = x;
215 }
216
217 inline void Delete(int &now)
218 {
219     if (!now) return;
220     Delete(ch[now][0]);
221     Delete(ch[now][1]);
222     team.push(now); now = 0;
223 }
224
225 inline void print()
226 {
227     for (int i = 1;i <= cnt;++i)
228         printf("%d:%d %d\n",i,ch[i][0],ch[i][1]);
229     for (int i = 1;i <= cnt;++i)
230         printf("%d:%d\n",i,fa[i]);
231
232 }
233
234 inline void laydown(int now)
235 {
236     if (!now) return;
237     pushdown(now);
238     laydown(ch[now][0]);
239     printf("%d ",key[now]);
240     laydown(ch[now][1]);
241     update(now);
242 }
243
244 int main()
245 {
246     //freopen("C.in","r",stdin);
247     N = gi(); M = gi(); init();
```

```
248     while (M--)
249     {
250         scanf("%s",cmd);
251         if (cmd[0] == 'I')
252         {
253             int pos = gi(),a = find(pos+1),b = find(pos+2); N = gi();
254             for (int i = 1;i <= N;++i) arr[i] = gi();
255             splay(a,0); splay(b,a);
256             ch[b][0] = build(1,N); fa[ch[b][0]] = b;
257             update(b); update(a);
258         }
259         else if (cmd[0] == 'D')
260         {
261             int pos = gi(); N = gi();
262             int a = find(pos),b = find(pos+N+1);
263             splay(a,0); splay(b,a);
264             Delete(ch[b][0]); update(b); update(a);
265         }
266         else if (cmd[0] == 'M'&&cmd[2] == 'K')
267         {
268             int pos = gi(); N = gi();
269             int a = find(pos),b = find(pos+N+1);
270             splay(a,0); splay(b,a);
271             key[ch[b][0]] = tag[ch[b][0]] = gi(); sum[ch[b][0]] = tag[ch[b][0]]*size[ch[b][0]];
272             if (tag[ch[b][0]] > 0) lb[ch[b][0]] = rb[ch[b][0]] = wb[ch[b][0]] = sum[ch[b][0]];
273             else lb[ch[b][0]] = rb[ch[b][0]] = wb[ch[b][0]] = tag[ch[b][0]];
274             update(b); update(a);
275         }
276         else if (cmd[0] == 'R')
277         {
278             int pos = gi(); N = gi();
279             int a = find(pos),b = find(pos+N+1);
280             splay(a,0); splay(b,a);
281             rev[ch[b][0]] ^= 1;
282             swap(ch[ch[b][0]][0],ch[ch[b][0]][1]);
283             swap(lb[ch[b][0]],rb[ch[b][0]]);
284             update(b); update(a);
285         }
286         else if (cmd[0] == 'G')
287         {
288             int pos = gi(); N = gi();
289             int a = find(pos),b = find(pos+N+1);
290             splay(a,0); splay(b,a);
291             printf("%d\n",sum[ch[b][0]]);
292         }
293         else
294         {
295             splay(1,0); splay(2,1);
296             printf("%d\n",wb[ch[2][0]]);
```

```
297             }
298         }
299         return 0;
300     }
```

Chapter 4

Graph Theory

4.1 2-Sat

```
// bzoj 1823
#include<stack>
#include<iostream>
#include<cstring>
#include<cstdio>
#include<cstdlib>
using namespace std;

#define maxn 210
#define maxm 2010
int n,m,cnt,side[maxn],next[maxm],toit[maxm],dfn[maxn],id[maxn];
int tot,low[maxn],d[maxn],DFN;
stack <int> S; bool vis[maxn];

inline void init()
{
    DFN = tot = cnt = 0; memset(vis,false,2*(n+4));
    memset(side,0,8*(n+4)); memset(dfn,0,8*(n+4));
}

inline void add(int a,int b) { next[++cnt] = side[a]; side[a] = cnt; toit[cnt] = b; }

inline void dfs(int now)
{
    S.push(now); dfn[now] = low[now] = ++DFN;
    for (int i = side[now];i;i = next[i])
    {
        if (vis[toit[i]]) continue;
        if (!dfn[toit[i]]) dfs(toit[i]);
        low[now] = min(low[toit[i]],low[now]);
    }
    if (low[now] == dfn[now])
    {
```

```
        ++tot;
        while (S.top() != now) id[S.top()] = tot,vis[S.top()] = true,S.pop();
        id[S.top()] = tot,vis[S.top()] = true,S.pop();
    }
}

int main()
{
    freopen("1823.in","r",stdin);
    freopen("1823.out","w",stdout);
    int T; scanf("%d",&T);
    while (T--)
    {
        scanf("%d %d\n",&n,&m);
        init();
        while (m--)
        {
            char c1,c2; int a,b; bool o1,o2;
            scanf("%c%d %c%d\n",&c1,&a,&c2,&b);
            o1 = c1 == 'h'; o2 = c2 == 'h';
            add((o1^1)*n+a,o2*n+b);
            add((o2^1)*n+b,o1*n+a);
        }
        int i;
        for (i = 1;i <= n<<1;++i) if (!dfn[i]) dfs(i);
        for (i = 1;i <= n;++i) if (id[i] == id[i+n]) { printf("BAD\n"); break; }
        if (i <= n) continue;
        printf("GOOD\n");
    }
    fclose(stdin); fclose(stdout);
    return 0;
}
```

4.2 Bridges and Cut Vertices

```
// 求割边和割点
const int maxn = 200010;
int N,M,cnt,Ts,dfn[maxn],low[maxn],nxt[maxn];
int toit[maxn],side[maxn];
bool bridge[maxn],cut[maxn];

inline void dfs(int now,int fa)
{
    dfn[now] = low[now] = ++Ts; int child = 0;
    for (int i = side[now];i;i = nxt[i])
    {
        if (toit[i] == fa) continue;
        if (!dfn[toit[i]])
        {
```

```
            dfs(toit[i],now); ++child;
            low[now] = min(low[now],low[toit[i]]);
            if (low[toit[i]] > dfn[now]) bridge[i] = true;
            if (low[toit[i]] >= dfn[now]) cut[now] = true;
        }
        else low[now] = min(low[now],dfn[toit[i]]);
    }
    if (!fa&&child == 1) cut[now] = false;
}
```

4.3 Cost Flow

```
int side[maxv],nxt[maxe],toit[maxe],cost[maxe],pre[maxv];
int cap[maxv],arr[maxv],dis[maxv]; bool in[maxv];
int source,sink;

inline void add(int a,int b,int c,int d) { nxt[++cnt] = side[a]; side[a] = cnt; toit[cnt] = b;
↪  cap[cnt] = c; cost[cnt] = d; }
inline void ins(int a,int b,int c,int d) { add(a,b,c,d); add(b,a,0,-d); }

inline bool spfa(int &Flow,int &Cost)
{
    queue <int> team; team.push(source);
    memset(dis,0x7,4*(sink+5));
    dis[source] = 0; in[source] = true;
    arr[source] = inf; arr[sink] = 0;
    while (!team.empty())
    {
        int now = team.front(); team.pop();
        for (int i = side[now];i;i = nxt[i])
        {
            if (!cap[i]) continue;
            if (dis[toit[i]] > dis[now]+cost[i])
            {
                arr[toit[i]] = min(cap[i],arr[now]); pre[toit[i]] = i;
                dis[toit[i]] = dis[now]+cost[i];
                if (!in[toit[i]]) in[toit[i]] = true,team.push(toit[i]);
            }
        }
        in[now] = false;
    }
    if (!arr[sink]) return false;
    Flow += arr[sink];
    for (int now = sink,i;now != source;now = toit[i^1])
    {
        i = pre[now]; Cost += cost[pre[now]]*arr[sink];
        cap[i] -= arr[sink]; cap[i^1] += arr[sink];
    }
    return true;
```

```
}
```

4.4 Difference Constraints

```
// DFS 判负环，相当于用栈跑 SPFA，只判负环比队列快
inline bool SPFA(int n,int source)
{
    for(int i = 1;i <= n; i++)
        dis[i] = inf,vis[i] = false,arr[i] = 0;
    arr[source] = 1; dis[source] = 0; vis[source] = true;
    stack <int> stk; stk.push(source);
    while(!stk.empty())
    {
        int now = stk.top(); stk.pop(); vis[now] = false;
        for (int i = side[now];i;i = nxt[i])
            if (dis[toit[i]] > dis[now]+len[i])
            {
                dis[toit[i]] = dis[now]+len[i];
                if (!vis[toit[i]])
                {
                    if (++arr[toit[i]] > N) return false;
                    vis[toit[i]] = true;
                    team.push(toit[i]);
                }
            }
    }
    return true;
}

// bzoj2330
#include<iostream>
#include<stack>
#include<queue>
#include<cstdio>
#include<cstdlib>
using namespace std;

#define maxn 100010
#define maxm 200010
int cnt,side[maxn],next[maxm],toit[maxm],cost[maxm],d[maxn];
int nside[maxn],nnext[maxm],ntoit[maxm],ncost[maxm],m,tot;
int dfn[maxn],low[maxn],sum[maxn],id[maxn],arr[maxn],n;
bool vis[maxn]; stack <int> S;

inline void add(int a,int b,int c)
{
    next[++cnt] = side[a]; side[a] = cnt;
    toit[cnt] = b; cost[cnt] = c;
}
```

```
46
47  inline void ins(int a,int b,int c)
48  {
49      nnext[++cnt] = nside[a]; nside[a] = cnt;
50      ntoit[cnt] = b; ncost[cnt] = c; ++d[b];
51  }
52
53  inline void dfs(int now)
54  {
55      S.push(now); dfn[now] = low[now] = ++cnt;
56      for (int i = side[now];i;i = next[i])
57          if (!vis[toit[i]])
58          {
59              if (!dfn[toit[i]]) dfs(toit[i]);
60              low[now] = min(low[toit[i]],low[now]);
61          }
62      if (low[now] == dfn[now])
63      {
64          ++tot;
65          while (S.top() != now) id[S.top()] = tot,vis[S.top()] = true,S.pop();
66          id[S.top()] = tot,vis[S.top()] = true,S.pop();
67      }
68  }
69
70  inline bool rebuild()
71  {
72      cnt = 0;
73      for (int i = 1;i <= n;++i)
74          for (int j = side[i];j;j = next[j])
75          {
76              if (id[toit[j]] == id[i]) sum[id[i]] += cost[j];
77              else ins(id[i],id[toit[j]],cost[j]);
78          }
79      for (int i = 1;i <= tot;++i) if (sum[i]) return false;
80      return true;
81  }
82
83  inline void topsort()
84  {
85      queue <int> team;
86      for (int i = 1;i <= tot;++i) if (!d[i]) team.push(i),arr[i] = 1;
87      while (!team.empty())
88      {
89          int now = team.front(); team.pop();
90          for (int i = nside[now];i;i = nnext[i])
91          {
92              arr[ntoit[i]] = max(arr[now]+ncost[i],arr[ntoit[i]]);
93              if (!--d[ntoit[i]]) team.push(ntoit[i]);
94          }
```

```
    }
}

int main()
{
    freopen("2330.in","r",stdin);
    freopen("2330.out","w",stdout);
    scanf("%d %d",&n,&m);
    for (int i = 1;i <= m;++i)
    {
        int x,a,b; scanf("%d %d %d",&x,&a,&b);
        if (x == 1) add(a,b,0),add(b,a,0);
        else if (x == 2) add(a,b,1);
        else if (x == 3) add(b,a,0);
        else if (x == 4) add(b,a,1);
        else add(a,b,0);
    }
    cnt = 0; for (int i = n;i;--i) if (!dfn[i]) dfs(i);
    if (!rebuild()) printf("-1"),exit(0);
    topsort();
    long long ans = 0;
    for (int i = 1;i <= n;++i) ans += (long long)arr[id[i]];
    printf("%lld",ans);
    fclose(stdin); fclose(stdout);
    return 0;
}
```

4.5 Dinic Algorithm

```
// dinic
int source,sink,cnt = 1;
int d[maxv],side[maxv],cur[maxv],nxt[maxe],toit[maxe],cap[maxe]; bool in[maxv];

inline void add(int a,int b,int c) { nxt[++cnt] = side[a]; side[a] = cnt; toit[cnt] = b;
↪  cap[cnt] = c; }
inline void ins(int a,int b,int c) { add(a,b,c); add(b,a,0); }

inline bool bfs()
{
    queue <int> team; team.push(source); d[source] = 0;
    memset(in,false,tot+10); in[source] = true; team.push(source);
    while (!team.empty())
    {
        int now = team.front(); team.pop(); cur[now] = side[now];
        for (int i = side[now];i;i = nxt[i])
        {
            if (!cap[i]) continue;
            if (!in[toit[i]])
                in[toit[i]] = true,d[toit[i]] = d[now]+1,team.push(toit[i]);
```

```
        }
    }
    return in[sink];
}

inline int dfs(int now,int f)
{
    if (now == sink||!f) return f;
    int used = 0,w;
    for (int &i = cur[now];i;i = nxt[i])
        if (cap[i]&&d[toit[i]] == d[now]+1)
        {
            w = dfs(toit[i],min(cap[i],f-used));
            used += w; cap[i] -= w; cap[i^1] += w;
            if (used == f) break;
        }
    return used;
}

inline int dinic(int S,int T)
{
    source = S; sink = T; int ret = 0;
    while (bfs()) ret += dfs(source,inf);
    return ret;
}
```

4.6 Dominator Tree

```
//建出来的树点的编号 i 在原图中是 redfn[i]
int
 ↪  N,M,Ts,cnt,side[maxn],nxt[maxn],toit[maxn],dfn[maxn],redfn[maxn],idom[maxn],best[maxn],semi[maxn];
int ans[maxn],anc[maxn],fa[maxn],child[maxn],size[maxn]; vector <int>
 ↪  prod[maxn],bucket[maxn],son[maxn];

inline void init()
{
    cnt = 1; memset(side,0,sizeof side); memset(ans,0,sizeof ans);
    for (int i = 0;i <= N;++i) prod[i].clear(),bucket[i].clear(),son[i].clear();
}

inline void add(int a,int b) { nxt[++cnt] = side[a]; side[a] = cnt; toit[cnt] = b; }

inline int gi()
{
    char ch; int ret = 0,f = 1;
    do ch = getchar(); while (!(ch >= '0'&&ch <= '9')&&ch != '-');
    if (ch == '-') f = -1,ch = getchar();
    do ret = ret*10+ch-'0',ch = getchar(); while (ch >= '0'&&ch <= '9');
    return ret*f;
```

```
}

inline void dfs(int now)
{
    dfn[now] = ++Ts; redfn[Ts] = now;
    anc[Ts] = idom[Ts] = child[Ts] = size[Ts] = 0;
    semi[Ts] = best[Ts] = Ts;
    for (int i = side[now];i;i = nxt[i])
    {
        if (!dfn[toit[i]])
            dfs(toit[i]),fa[dfn[toit[i]]] = dfn[now];
        prod[dfn[toit[i]]].push_back(dfn[now]);
    }
}

inline void compress(int now)
{
    if (anc[anc[now]] != 0)
    {
        compress(anc[now]);
        if (semi[best[now]] > semi[best[anc[now]]])
            best[now] = best[anc[now]];
        anc[now] = anc[anc[now]];
    }
}

inline int eval(int now)
{
    if (!anc[now]) return now;
    else
    {
        compress(now);
        return semi[best[anc[now]]] >= semi[best[now]]?best[now]:best[anc[now]];
    }
}

inline void link(int v,int w)
{
    int s = w;
    while (semi[best[w]] < semi[best[child[w]]])
    {
        if (size[s]+size[child[child[s]]] >= 2*size[child[s]])
            anc[child[s]] = s,child[s] = child[child[s]];
        else size[child[s]] = size[s],s = anc[s] = child[s];
    }
    best[s] = best[w]; size[v] += size[w];
    if (size[v] < 2*size[w]) swap(s,child[v]);
    while (s) anc[s] = v,s = child[s];
}
```

```

inline void lengauer_tarjan()
{
    memset(dfn,0,sizeof dfn); memset(fa,-1,sizeof fa); Ts = 0;
    dfs(N); fa[1] = 0;
    for (int w = Ts;w > 1;--w)
    {
        for (auto x:prod[w])
        {
            int u = eval(x);
            if (semi[w] > semi[u]) semi[w] = semi[u];
        }
        bucket[semi[w]].push_back(w);
        link(fa[w],w); if (!fa[w]) continue;
        for (auto x:bucket[fa[w]])
        {
            int u = eval(x);
            if (semi[u] < fa[w]) idom[x] = u;
            else idom[x] = fa[w];
        }
        bucket[fa[w]].clear();
    }
    for (int w = 2;w <= Ts;++w)
        if (idom[w] != semi[w])
            idom[w] = idom[idom[w]];
    idom[1] = 0;
    for (int i = Ts;i > 1;--i)
    {
        if (fa[i] == -1) continue;
        son[idom[i]].push_back(i);
    }
}

// 例题：询问 i 号点到 N 号点所有必经点编号和
#include<algorithm>
#include<cstring>
#include<iostream>
#include<cstdio>
#include<cstdlib>
using namespace std;

const int maxn = 100010;
int
 ↪  N,M,Ts,cnt,side[maxn],nxt[maxn],toit[maxn],dfn[maxn],redfn[maxn],idom[maxn],best[maxn],semi[maxn];
int ans[maxn],anc[maxn],fa[maxn],child[maxn],size[maxn]; vector <int>
 ↪  prod[maxn],bucket[maxn],son[maxn];

inline void init()
{
```

```
116     cnt = 1; memset(side,0,sizeof side); memset(ans,0,sizeof ans);
117     for (int i = 0;i <= N;++i) prod[i].clear(),bucket[i].clear(),son[i].clear();
118 }
119
120 inline void add(int a,int b) { nxt[++cnt] = side[a]; side[a] = cnt; toit[cnt] = b; }
121
122 inline int gi()
123 {
124     char ch; int ret = 0,f = 1;
125     do ch = getchar(); while (!(ch >= '0'&&ch <= '9')&&ch != '-');
126     if (ch == '-') f = -1,ch = getchar();
127     do ret = ret*10+ch-'0',ch = getchar(); while (ch >= '0'&&ch <= '9');
128     return ret*f;
129 }
130
131 inline void dfs(int now)
132 {
133     dfn[now] = ++Ts; redfn[Ts] = now;
134     anc[Ts] = idom[Ts] = child[Ts] = size[Ts] = 0;
135     semi[Ts] = best[Ts] = Ts;
136     for (int i = side[now];i;i = nxt[i])
137     {
138         if (!dfn[toit[i]])
139             dfs(toit[i]),fa[dfn[toit[i]]] = dfn[now];
140         prod[dfn[toit[i]]].push_back(dfn[now]);
141     }
142 }
143
144 inline void compress(int now)
145 {
146     if (anc[anc[now]] != 0)
147     {
148         compress(anc[now]);
149         if (semi[best[now]] > semi[best[anc[now]]])
150             best[now] = best[anc[now]];
151         anc[now] = anc[anc[now]];
152     }
153 }
154
155 inline int eval(int now)
156 {
157     if (!anc[now]) return now;
158     else
159     {
160         compress(now);
161         return semi[best[anc[now]]] >= semi[best[now]]?best[now]:best[anc[now]];
162     }
163 }
164
```

```
165 inline void link(int v,int w)
166 {
167     int s = w;
168     while (semi[best[w]] < semi[best[child[w]]])
169     {
170         if (size[s]+size[child[child[s]]] >= 2*size[child[s]])
171             anc[child[s]] = s,child[s] = child[child[s]];
172         else size[child[s]] = size[s],s = anc[s] = child[s];
173     }
174     best[s] = best[w]; size[v] += size[w];
175     if (size[v] < 2*size[w]) swap(s,child[v]);
176     while (s) anc[s] = v,s = child[s];
177 }
178
179 inline void lengauer_tarjan()
180 {
181     memset(dfn,0,sizeof dfn); memset(fa,-1,sizeof fa); Ts = 0;
182     dfs(N); fa[1] = 0;
183     for (int w = Ts;w > 1;--w)
184     {
185         for (auto x:prod[w])
186         {
187             int u = eval(x);
188             if (semi[w] > semi[u]) semi[w] = semi[u];
189         }
190         bucket[semi[w]].push_back(w);
191         link(fa[w],w); if (!fa[w]) continue;
192         for (auto x:bucket[fa[w]])
193         {
194             int u = eval(x);
195             if (semi[u] < fa[w]) idom[x] = u;
196             else idom[x] = fa[w];
197         }
198         bucket[fa[w]].clear();
199     }
200     for (int w = 2;w <= Ts;++w)
201         if (idom[w] != semi[w])
202             idom[w] = idom[idom[w]];
203     idom[1] = 0;
204     for (int i = Ts;i > 1;--i)
205     {
206         if (fa[i] == -1) continue;
207         son[idom[i]].push_back(i);
208     }
209 }
210
211 inline void get_ans(int now)
212 {
213     ans[redfn[now]] += redfn[now];
```

```
    for (auto x:son[now])
        ans[redfn[x]] += ans[redfn[now]],get_ans(x);
}

int main()
{
    //freopen("I.in","r",stdin);
    while (scanf("%d %d",&N,&M) != EOF)
    {
        init();
        for (int i = 1,a,b;i <= M;++i)
            a = gi(),b = gi(),add(a,b);
        lengauer_tarjan(); get_ans(1);
        for (int i = 1;i <= N;++i)
            printf("%d%c",ans[i]," \n"[i == N]);
    }
    return 0;
}
```

4.7 Hungary

```
//匈牙利算法
//Version1
inline bool find(int x)
{
    if (cor[x]) return false;
    for (int i = side[x];i;i = next[i]) if (!used[toit[i]])
    {
        used[toit[i]] = true;
        if (!cho[toit[i]]||find(cho[toit[i]]))
        {
            cho[toit[i]] = x; map[x] = toit[i];
             return true;
        }
    }
    return false;
}

inline void hungry()
{
    for (int i = 1;i <= p;++i)
        memset(used,false,sizeof(used)),find(i);
    for (int i = 1;i <= m;++i)
    {
        memset(used,false,sizeof(used)),cho[map[i]] = 0;
        find(i),cor[i] = true;
    }
}
//Version2
```

```
29  inline int find(int x)
30  {
31      for (int i = 1;i <= n;++i)
32          if (f[x][i]&&!used[i])
33          {
34              used[i] = true;
35              if (!cho[i]||find(cho[i])) { cho[i] = x; return true; }
36          }
37      return false;
38  }
39
40  inline int hungry()
41  {
42      int ret = 0;
43      for (int i = 1;i <= n;++i)
44      {
45          memset(used,false,sizeof(used));
46          if (find(i)) ret++;
47      }
48      return ret;
49  }
```

4.8 Isap Algorithm

```
1   // isap: 有毒
2   inline void bfs()
3   {
4       queue <int> team; memcpy(cur,side,4*(N+1));
5       team.push(sink); d[sink] = 1; in[sink] = true;
6       while (!team.empty())
7       {
8           int now = team.front(); team.pop(); nd[d[now]]++;
9           for (int i = side[now];i;i = nxt[i])
10              if (cap[i^1] && !in[toit[i]])
11                  in[toit[i]] = true,d[toit[i]] = d[now]+1,team.push(toit[i]);
12      }
13      for (int i = 1;i <= N;++i) if (!in[i]) nd[d[i] = N+1]++;
14  }
15  inline int isap()
16  {
17      int res = 0,now = source,ca = inf;
18      bfs();
19      while (d[source] <= N)
20      {
21          if (now == sink)
22          {
23              while (now != source)
24              {
25                  cap[pre[now]] -= ca; cap[pre[now]^1] += ca;
```

```
                now = toit[pre[now]^1];
            }
            res += ca; ca = inf;
        }
        bool flag = false; arr[now] = ca;
        for (int i = cur[now];i;i = nxt[i])
            if (cap[i]&&d[toit[i]]==d[now]-1)
            {
                cur[now] = pre[toit[i]] = i; ca = min(ca,cap[i]);
                now = toit[i]; flag = true; break;
            }
        if (flag) continue; if (!--nd[d[now]]) break; int arg = N;
        for (int i = side[now];i;i = nxt[i])
            if (cap[i]&&d[toit[i]] < arg) arg = d[toit[i]];
        ++nd[d[now]=arg+1]; cur[now] = side[now];
        if (now != source) ca = arr[now = toit[pre[now]^1]];
    }
    return res;
}

int source;         // 源点
int sink;           // 汇点
int p[max_nodes];   // 可增广路上的上一条弧的编号
int num[max_nodes]; // 和 t 的最短距离等于 i 的节点数量
int cur[max_nodes]; // 当前弧下标
int d[max_nodes];   // 残量网络中节点 i 到汇点 t 的最短距离
bool visited[max_nodes];

// 预处理，反向 BFS 构造 d 数组
bool bfs()
{
    memset(visited, 0, sizeof(visited));
    queue<int> Q;
    Q.push(sink);
    visited[sink] = 1;
    d[sink] = 0;
    while (!Q.empty()) {
        int u = Q.front();
        Q.pop();
        for (iterator_t ix = G[u].begin(); ix != G[u].end(); ++ix) {
            Edge &e = edges[(*ix)^1];
            if (!visited[e.from] && e.capacity > e.flow) {
                visited[e.from] = true;
                d[e.from] = d[u] + 1;
                Q.push(e.from);
            }
        }
    }
    return visited[source];
```

```
75  }
76
77  // 增广
78  int augment()
79  {
80      int u = sink, df = __inf;
81      // 从汇点到源点通过 p 追踪增广路径，df 为一路上最小的残量
82      while (u != source) {
83          Edge &e = edges[p[u]];
84          df = min(df, e.capacity - e.flow);
85          u = edges[p[u]].from;
86      }
87      u = sink;
88      // 从汇点到源点更新流量
89      while (u != source) {
90          edges[p[u]].flow += df;
91          edges[p[u]^1].flow -= df;
92          u = edges[p[u]].from;
93      }
94      return df;
95  }
96
97  int max_flow()
98  {
99      int flow = 0;
100     bfs();
101     memset(num, 0, sizeof(num));
102     for (int i = 0; i < num_nodes; i++) num[d[i]]++;
103     int u = source;
104     memset(cur, 0, sizeof(cur));
105     while (d[source] < num_nodes) {
106         if (u == sink) {
107             flow += augment();
108             u = source;
109         }
110         bool advanced = false;
111         for (int i = cur[u]; i < G[u].size(); i++) {
112             Edge& e = edges[G[u][i]];
113             if (e.capacity > e.flow && d[u] == d[e.to] + 1) {
114                 advanced = true;
115                 p[e.to] = G[u][i];
116                 cur[u] = i;
117                 u = e.to;
118                 break;
119             }
120         }
121         if (!advanced) { // retreat
122             int m = num_nodes - 1;
123             for (iterator_t ix = G[u].begin(); ix != G[u].end(); ++ix)
```

```
                if (edges[*ix].capacity > edges[*ix].flow)
                    m = min(m, d[edges[*ix].to]);
            if (--num[d[u]] == 0) break; // gap 优化
            num[d[u] = m+1]++;
            cur[u] = 0;
            if (u != source)
                u = edges[p[u]].from;
        }
    }
    return flow;
}
//By mxh
#define maxn 1010
const int INF=1<<30;
int n,m;
int S,T;
struct Edge
{
    int v,flow,next;
}    e[510010];
int g[maxn],tot=1;//tot 初值必须赋为 1
void addedge(int x,int y,int flow)
{
    e[++tot].v=y;e[tot].flow=flow;e[tot].next=g[x];g[x]=tot;
    e[++tot].v=x;e[tot].flow=0;e[tot].next=g[y];g[y]=tot;
}
int w[maxn],hash[maxn],d[maxn];
int que[maxn],pre1[maxn],pre2[maxn],p[maxn];
bool vis[maxn];
int maxflow()
{
    for (int i=1;i<=n;i++)    hash[i]=0,d[i]=0,vis[i]=false;
    for (int i=1;i<=n;i++)    p[i]=g[i];
    //hash[0]=n;
    int l,r;
    l=r=1;
    que[1]=T;hash[0]=1;vis[T]=true;
    while (l<=r)
    {
        int u=que[l++];
        for (int i=g[u];i;i=e[i].next)
        if ((i&1) && !vis[e[i].v])
        {
            que[++r]=e[i].v;
            vis[e[i].v]=true;
            d[e[i].v]=d[u]+1;
            hash[d[e[i].v]]++;
        }
    }
```

```
    for (int i=1;i<=n;i++)
    if (!vis[i])    d[i]=n,hash[n]++;
    int flow=INF;
    int ans=0;
    int u=S;
    while (d[S]<n)
    {
        w[u]=flow;
        bool bo=true;
        for (int i=p[u];i;i=e[i].next)
        if (e[i].flow && d[e[i].v]==d[u]-1)
        {
            flow=min(flow,e[i].flow);
            p[u]=i;
            pre1[e[i].v]=u;
            pre2[e[i].v]=i;
            u=e[i].v;
            bo=false;
            if (u==T)
            {
                ans+=flow;
                while (u!=S)
                {
                    e[pre2[u]].flow-=flow;
                    e[pre2[u]^1].flow+=flow;
                    u=pre1[u];
                }
                flow=INF;
            }
            break;
        }
        if (!bo)    continue;
        int minx=n,pos=0;
        for (int i=g[u];i;i=e[i].next)
        if (e[i].flow && d[e[i].v]<minx)    minx=d[e[i].v],pos=i;
        p[u]=pos;
        hash[d[u]]--;
        if (hash[d[u]]==0)    break;
        d[u]=minx+1;
        hash[d[u]]++;
        if (u!=S)    u=pre1[u],flow=w[u];
    }
    return ans;
}
```

4.9 Kuhn-Munkres Algorithm

```
// Truly O(n^3), 最大权匹配
// 邻接矩阵，不能连的边设为-INF，求最小权匹配时边权取负，但不能连的还是 -INF，使用时先对 1 -> n 调用 hungary() ，再
```

```
3  struct KM
4  {
5      int w[maxn][maxn],lx[maxn],ly[maxn],match[maxn],way[maxn],slack[maxn];
6      bool used[maxn];
7
8      inline void init()
9      {
10         for (int i = 1;i <= N;++i)
11             match[i] = lx[i] = ly[i] = way[i] = 0;
12     }
13
14     inline void hungary(int x)
15     {
16         match[0] = x; int j0 = 0;
17         for (int j = 0;j <= N;++j)
18             slack[j] = inf,used[j] = false;
19         do
20         {
21             used[j0] = true;
22             int i0 = match[j0],delta = inf,j1 = 0;
23             for (int j = 1;j <= N;++j)
24                 if (!used[j])
25                 {
26                     int cur = -w[i0][j]-lx[i0]-ly[j];
27                     if (cur < slack[j])
28                         slack[j] = cur,way[j] = j0;
29                     if (slack[j] < delta)
30                         delta = slack[j],j1 = j;
31                 }
32             for (int j = 0;j <= N;++j)
33             {
34                 if (used[j]) lx[match[j]] += delta,ly[j] -= delta;
35                 else slack[j] -= delta;
36             }
37             j0 = j1;
38         }
39         while (match[j0]);
40         do
41         {
42             int j1 = way[j0];
43             match[j0] = match[j1];
44             j0 = j1;
45         }
46         while (j0);
47     }
48
49     inline void work() { for (int i = 1;i <= N;++i) hungary(i); }
50
51     inline int get_ans()
```

```
52      {
53          int sum = 0;
54          for (int i = 1;i <= N;++i)
55          {
56              // if (w[match[i]][i] == -inf) ; //无解
57              if (match[i] > 0) sum += w[match[i]][i];
58          }
59          return sum;
60      }
61  }km;
62  //最小权匹配
63  struct KM
64  {
65      int w[maxn][maxn],lx[maxn],ly[maxn],match[maxn],way[maxn],slack[maxn]; bool used[maxn];
66
67      inline void init()
68      {
69          for (int i = 1;i <= N;++i)
70              match[i] = lx[i] = ly[i] = way[i] = 0;
71      }
72
73      inline void hungary(int x)
74      {
75          match[0] = x; int j0 = 0;
76          for (int j = 0;j <= N;++j)
77              slack[j] = -inf,used[j] = false;
78          do
79          {
80              used[j0] = true;
81              int i0 = match[j0],delta = -inf,j1 = 0;
82              for (int j = 1;j <= N;++j)
83                  if (!used[j])
84                  {
85                      int cur = -w[i0][j]-lx[i0]-ly[j];
86                      if (cur > slack[j]) slack[j] = cur,way[j] = j0;
87                      if (slack[j] > delta) delta = slack[j],j1 = j;
88                  }
89              for (int j = 0;j <= N;++j)
90              {
91                  if (used[j]) lx[match[j]] += delta,ly[j] -= delta;
92                  else slack[j] -= delta;
93              }
94              j0 = j1;
95          }
96          while (match[j0]);
97          do
98          {
99              int j1 = way[j0];
100             match[j0] = match[j1];
```

```
            j0 = j1;
        }
        while (j0);
    }

    inline void work() { for (int i = 1;i <= N;++i) hungary(i); }

    inline int get_ans()
    {
        int sum = 0;
        for (int i = 1;i <= N;++i)
        {
            // if (w[match[i]][i] == inf) ; // 无解
            if (match[i] > 0) sum += w[match[i]][i];
        }
        return sum;
    }
}km;
```

4.10 Maximal Matching in General Graphs

```
// 接口 int matching(), 返回最大匹配数,G 为邻接矩阵
inline void push(int x)
{
    team.push(x); check[x] = true;
    if (!treec[x]) tra[++cnt] = x,treec[x] = true;
}
inline int root(int x) { return f[x]?f[x] = root(f[x]):x; }

inline void clear()
{
    for (int i = 1,j;i <= cnt;++i)
    {
        j = tra[i]; father[j] = 0,f[j] = 0;
        check[j] = treec[j] = false;
    }
}

inline int lca(int u,int v)
{
    int len = 0;
    for (;u;u = father[match[u]])
        pathc[path[++len] = u = root(u)] = true;
    for (;;v = father[match[v]])
        if (pathc[v = root(v)]) break;
    for (int i = 1;i <= len;++i)
        pathc[path[i]] = false;
    return v;
}
```

```
29
30  inline void reset(int u,int p)
31  {
32      for (int v;root(u) != p;)
33      {
34          if (!check[v = match[u]]) push(v);
35          if (!f[u]) f[u] = p; if (!f[v]) f[v] = p;
36          u = father[v]; if (root(u) != p) father[u] = v;
37      }
38  }
39
40  inline void flower(int u,int v)
41  {
42      int p = lca(u,v);
43      if (root(u) != p) father[u] = v;
44      if (root(v) != p) father[v] = u;
45      reset(u,p); reset(v,p);
46  }
47
48  inline bool find(int x)
49  {
50      while (!team.empty()) team.pop();
51      cnt = 0; push(x);
52      while (!team.empty())
53      {
54          int i = team.front(); team.pop();
55          for (int j = 1;j <= N;++j)
56              if (G[i][j]&&root(i) != root(j)&&match[j] != i)
57              {
58                  if (match[j]&&father[match[j]]) flower(i,j);
59                  else if (!father[j])
60                  {
61                      father[tra[++cnt] = j] = i; treec[j] = true;
62                      if (match[j]) push(match[j]);
63                      else
64                      {
65                          for (int k = i,l = j,p;k;l = p,k = father[l])
66                              p = match[k],match[k] = l,match[l] = k;
67                          return true;
68                      }
69                  }
70              }
71      }
72      return false;
73  }
74
75  inline int matching()
76  {
77      memset(father,0,sizeof father);  memset(f,0,sizeof f); memset(path,0,sizeof path);
```

```
78      memset(tra,0,sizeof tra); memset(match,0,sizeof match); memset(check,false,sizeof check);
79      memset(treec,false,sizeof treec); memset(pathc,false,sizeof pathc);
80      int ret = cnt = 0;
81      for (int i = 1;i <= N;++i)
82      {
83          if (match[i]) continue;
84          if (find(i)) ++ret; clear();
85      }
86      return ret;
87  }
```

4.11 Maximal Weighted Matching in General Graphs

```
1   // 接口 int matching(), 返回最大匹配数,G 为邻接矩阵
2   inline void push(int x)
3   {
4       team.push(x); check[x] = true;
5       if (!treec[x]) tra[++cnt] = x,treec[x] = true;
6   }
7   inline int root(int x) { return f[x]?f[x] = root(f[x]):x; }
8
9   inline void clear()
10  {
11      for (int i = 1,j;i <= cnt;++i)
12      {
13          j = tra[i]; father[j] = 0,f[j] = 0;
14          check[j] = treec[j] = false;
15      }
16  }
17
18  inline int lca(int u,int v)
19  {
20      int len = 0;
21      for (;u;u = father[match[u]])
22          pathc[path[++len] = u = root(u)] = true;
23      for (;;v = father[match[v]])
24          if (pathc[v = root(v)]) break;
25      for (int i = 1;i <= len;++i)
26          pathc[path[i]] = false;
27      return v;
28  }
29
30  inline void reset(int u,int p)
31  {
32      for (int v;root(u) != p;)
33      {
34          if (!check[v = match[u]]) push(v);
35          if (!f[u]) f[u] = p; if (!f[v]) f[v] = p;
36          u = father[v]; if (root(u) != p) father[u] = v;
```

```
    }
}

inline void flower(int u,int v)
{
    int p = lca(u,v);
    if (root(u) != p) father[u] = v;
    if (root(v) != p) father[v] = u;
    reset(u,p); reset(v,p);
}

inline bool find(int x)
{
    while (!team.empty()) team.pop();
    cnt = 0; push(x);
    while (!team.empty())
    {
        int i = team.front(); team.pop();
        for (int j = 1;j <= N;++j)
            if (G[i][j]&&root(i) != root(j)&&match[j] != i)
            {
                if (match[j]&&father[match[j]]) flower(i,j);
                else if (!father[j])
                {
                    father[tra[++cnt] = j] = i; treec[j] = true;
                    if (match[j]) push(match[j]);
                    else
                    {
                        for (int k = i,l = j,p;k;l = p,k = father[l])
                            p = match[k],match[k] = l,match[l] = k;
                        return true;
                    }
                }
            }
    }
    return false;
}

inline int matching()
{
    memset(father,0,sizeof father);  memset(f,0,sizeof f); memset(path,0,sizeof path);
    memset(tra,0,sizeof tra); memset(match,0,sizeof match); memset(check,false,sizeof check);
    memset(treec,false,sizeof treec); memset(pathc,false,sizeof pathc);
    int ret = cnt = 0;
    for (int i = 1;i <= N;++i)
    {
        if (match[i]) continue;
        if (find(i)) ++ret; clear();
    }
```

```
    return ret;
}
```

4.12 Maximum Cardinality Search

```
// BZOJ 1006
#include<algorithm>
#include<queue>
#include<cstdio>
#include<cstdlib>
#include<set>
using namespace std;

#define maxn 10010
#define maxc 510
#define maxm 1000010
int tot,n,m,cnt,color[maxn][maxc],label[maxn],all;
int side[maxn],next[maxm*2],toit[maxm*2],per[maxn];
bool in[maxn];
struct node
{
    int key,ord;
    friend bool operator < (node a,node b) {return a.key > b.key; }
};
multiset <node> S;

inline void add(int a,int b)
{
    next[++cnt] = side[a]; side[a] = cnt; toit[cnt] = b;
}

inline void ins(int a,int b){add(a,b); add(b,a);}

inline void mcs()
{
    int i,u;
    for (i = 1;i <= n;++i) S.insert((node){0,i});
    while (all < n)
    {
        u = (*S.begin()).ord; S.erase(S.begin()); if (in[u]) continue;
        in[u] = true; per[++all] = u;
        for (i = side[u];i;i = next[i])
            if (!in[toit[i]])
            {
                label[toit[i]]++;
                S.insert((node){label[toit[i]],toit[i]});
            }
    }
}
```

```
45
46  inline void paint()
47  {
48      int p,i,j,t;
49      for (p = 1;p <= n;++p)
50      {
51          i = per[p];
52          for (j = 1;j <= tot;++j)
53              if (!color[i][j]) {t = j; break; }
54          if (j == tot + 1) t = ++tot;
55          for (j = side[i];j;j = next[j])
56              color[toit[j]][t] = true;
57      }
58  }
59
60  int main()
61  {
62      freopen("1006.in","r",stdin);
63      freopen("1006.out","w",stdout);
64      scanf("%d %d",&n,&m);
65      for (int i = 1;i <= m;++i)
66      { int a,b; scanf("%d %d",&a,&b); ins(a,b); }
67      mcs();
68      paint();
69      printf("%d",tot);
70      fclose(stdin); fclose(stdout);
71      return 0;
72  }
```

4.13 Network Flow with Lower Bound

1. **无源汇有上下界可行流**

   设原来源点为 $Source$，汇点是 $Sink$。新建一个超级源 $SuperSource$ 和超级汇 $SuperSink$。对于原网络中的每一条边 $u \to v$，上界 U，下界 L，将它拆分为三条边：

   (1) $u \to SuperSink$，容量为 L。

   (2) $SuperSource \to v$，容量为 L。

   (3) $u \to v$，容量为 $U - L$。

   最后添加边 $Sink \to Source$，容量为 $+\infty$。在新建的网络上，计算从 $SuperSource$ 到 $SuperSink$ 的最大流。若每条从 $SuperSource$ 发出的边都满流，说明存在可行流，否则不。每条边实际流量为容量下界 + 附加流中它的流量。

2. **有源汇有上下界可行流**

   在“无源汇有上下界可行流”建图上，新增一条 $Sink \to Source$ 的边，容量为 $+\infty$ 即可。

3. **有源汇有上下界最大流**

在“有源汇有上下界可行流”建图上，先判断是否存在可行流，若存在可行流，拆掉 $Sink \to Source$ 的边后，接着在图中 $Source \to Sink$ 最大流增广加上原可行流即为最大流答案。(若存在可行流，去掉下界后最大流即为原图有源汇有上下界最大流)

4. **有源汇有上下界最小流**

在“有源汇有上下界可行流”建图上，先判断是否存在可行流，若存在可行流，拆掉 $Sink \to Source$ 的边后，用可行流减去在图中 $Sink \to Source$ 增广的最大流即为最小流答案。

在实现时，可以吧 $SuperSource$ 连向同一节点的多条边合成一条（容量合并。从同一节点指向 $SuperSink$ 的多条边也应合并。

对于费用流，只需要改变将网络流算法改成费用流算法。对于原网络中的每一条边 $u \to v$，上界 U，下界 L，费用 c，将它拆分为三条边：

(1) $u \to SuperSink$，容量为 L，费用 c。

(2) $SuperSource \to v$，容量为 L，费用 0。

(3) $u \to v$，容量为 $U - L$，费用 c。

4.14 Point Biconnected Component

```cpp
// Source: HackerRank - bonnie-and-clyde
#include<algorithm>
#include<vector>
#include<stack>
#include<iostream>
#include<cstdio>
#include<cstdlib>
using namespace std;

const int maxn = 400010;
int N,M,Q,cnt = 1,side[maxn],toit[maxn],nxt[maxn],f[maxn][25],father[maxn],low[maxn];
int tot,dep[maxn],dfn[maxn],nside[maxn],ntoit[maxn],nnxt[maxn]; bool cut[maxn];
stack <int> S; vector <int> bel[maxn],bcc[maxn]; bool vis[maxn];

inline int find(int a) { if (father[a] != a) father[a] = find(father[a]); return father[a]; }

inline void add(int a,int b) { nxt[++cnt] = side[a]; side[a] = cnt; toit[cnt] = b; }
inline void ins(int a,int b) { add(a,b); add(b,a); }
inline void nadd(int a,int b) { nnxt[++cnt] = nside[a]; nside[a] = cnt; ntoit[cnt] = b; }
inline void nins(int a,int b) { nadd(a,b); nadd(b,a); }

inline int gi()
{
    char ch; int ret = 0,f = 1;
    do ch = getchar(); while (!(ch >= '0'&&ch <= '9')&&ch != '-');
    if (ch == '-') f = -1,ch = getchar();
    do ret = ret*10+ch-'0',ch = getchar(); while (ch >= '0'&&ch <= '9');
    return ret*f;
```

```
29  }
30
31  inline void tj(int now,int fa)
32  {
33      dfn[now] = low[now] = ++cnt; int child = 0;
34      for (int i = side[now];i;i = nxt[i])
35      {
36          if (toit[i] == fa) continue;
37          if (!dfn[toit[i]])
38          {
39              S.push(i>>1); tj(toit[i],now); ++child;
40              low[now] = min(low[now],low[toit[i]]);
41              if (low[toit[i]] >= dfn[now])
42              {
43                  cut[now] = true; ++tot;
44                  while (true)
45                  {
46                      int t = S.top(); S.pop();
47                      bel[toit[t<<1]].push_back(tot);    bel[toit[t<<1|1]].push_back(tot);
48                      bcc[tot].push_back(toit[t<<1]); bcc[tot].push_back(toit[t<<1|1]);
49                      if (t == (i>>1)) break;
50                  }
51              }
52          }
53          else low[now] = min(low[now],dfn[toit[i]]);
54      }
55      if (!fa&&child == 1) cut[now] = false;
56  }
57
58  inline void build()
59  {
60      vector <int> cuts; cnt = 1;
61      for (int i = 1;i <= tot;++i)
62      {
63          sort(bcc[i].begin(),bcc[i].end());
64          bcc[i].erase(unique(bcc[i].begin(),bcc[i].end()),bcc[i].end());
65      }
66      for (int i = 1;i <= N;++i) if (cut[i]) cuts.push_back(i);
67      for (auto x:cuts)
68      {
69          sort(bel[x].begin(),bel[x].end());
70          bel[x].erase(unique(bel[x].begin(),bel[x].end()),bel[x].end());
71          ++tot; for (auto y:bel[x]) nins(tot,y);
72          bel[x].clear(); bel[x].push_back(tot); bcc[tot].push_back(x);
73      }
74  }
75
76  inline void dfs(int now)
77  {
```

```
78      vis[now] = true;
79      for (int i = 1;(1<<i) <= dep[now];++i) f[now][i] = f[f[now][i-1]][i-1];
80      for (int i = nside[now];i;i = nnxt[i])
81      {
82          if (vis[ntoit[i]]) continue; f[ntoit[i]][0] = now;
83          dep[ntoit[i]] = dep[now]+1; dfs(ntoit[i]);
84      }
85  }
86
87  inline int jump(int a,int b) { for (int i = 0;b;++i,b >>= 1) if (b&1) a = f[a][i]; return a; }
88  inline int lca(int a,int b)
89  {
90      if (dep[a] < dep[b]) swap(a,b);
91      a = jump(a,dep[a]-dep[b]); if (a == b) return a;
92      for (int i = 0;i >= 0;)
93      {
94          if (f[a][i] != f[b][i]) a = f[a][i],b = f[b][i],++i;
95          else --i;
96      }
97      return f[a][0];
98  }
99
100 inline bool check(int u,int v,int w)
101 {
102     if (find(u) != find(v)||find(v) != find(w)) return false;
103     if (u == w||v == w) return true; if (u == v) return false;
104     int uu = bel[u][0],vv = bel[v][0],ww = bel[w][0],su,sv;
105     if (uu == ww||vv == ww) return true;
106     if (lca(uu,ww) == ww) su = jump(uu,dep[uu]-dep[ww]-1); else su = f[ww][0];
107     if (lca(vv,ww) == ww) sv = jump(vv,dep[vv]-dep[ww]-1); else sv = f[ww][0];
108     if (su == sv)
109     {
110         if (!cut[w]) return false;
111         else
112         {
113             if (su == uu||sv == vv) return true; int ssu,ssv;
114             if (lca(su,uu) == su) ssu = jump(uu,dep[uu]-dep[su]-1); else ssu = f[su][0];
115             if (lca(sv,vv) == sv) ssv = jump(vv,dep[vv]-dep[sv]-1); else ssv = f[sv][0];
116             if (ssu == ssv) return false; else return true;
117         }
118     }
119     else return true;
120 }
121
122 int main()
123 {
124     freopen("J.in","r",stdin);
125     freopen("J.out","w",stdout);
126     N = gi(); M = gi(); Q = gi();
```

```
    for (int i = 1;i <= N;++i) father[i] = i;
    for (int i = 1,a,b;i <= M;++i)
    {
        ins(a = gi(),b = gi());
        a = find(a),b = find(b);
        if (a != b) father[a] = b;
    }
    cnt = 0; for (int i = 1;i <= N;++i) if (!dfn[i]) tj(i,0);
    build(); for (int i = 1;i <= N;++i) if (!vis[i]) dfs(i);
    while (Q--)
    {
        int u = gi(),v = gi(),w = gi();
        if (check(u,v,w)) puts("YES"); else puts("NO");
    }
    return 0;
}
```

4.15 Steiner Tree

```
/*
 *  Steiner Tree: 求，使得指定 K 个点连通的生成树的最小总权值
 *  st[i] 表示顶点 i 的标记值，如果 i 是指定集合内第 m(0<=m<K) 个点，则 st[i]=1<<m
 *  endSt=1<<K
 *  dptree[i][state] 表示以 i 为根，连通状态为 state 的生成树值
 */
inline void update(int &x,int y) { if (x == -1) x = y; else if (x > y) x = y; }
inline void spfa(int state)
{
    while (!team.empty())
    {
        int now = team.front(); team.pop();
        for (int i = side[now];i;i = nxt[i])
        {
            int v = toit[i];
            if (f[v][st[v]|state] == -1||f[v][st[v]|state] > f[now][state]+len[i])
            {
                f[v][st[v]|state] = f[now][state]+len[i];
                if ((st[v]|state) != state||vis[v][state]) continue;
                vis[v][state] = true; team.push(v);
            }
        }
        vis[now][state] = false;
    }
}
inline int work()
{
    endSt = 1<<(K<<1);
    memset(f,-1,sizeof(f)); memset(st,0,sizeof(st)); memset(dp,-1,sizeof(dp));
    memset(vis,false,sizeof(vis)); memset(side,0,sizeof(side));
```

```
    for (int i = 1;i <= K;++i) st[i] = 1<<(i-1);
    for (int i = 1;i <= K;++i) st[N-K+i] = 1<<(i+K-1);
    for (int i = 1;i <= N;++i) f[i][st[i]] = 0;
    for (int j = 1;j < endSt;++j)
    {
        for (int i = 1;i <= N;++i)
        {
            if (!st[i]||(st[i]&j))
                for (int sub = (j-1)&j;sub;sub = (sub-1)&j)
                {
                    int x = sub|st[i],y = (j-sub)|st[i];
                    if (f[i][x] != -1&&f[i][y] != -1)
                        update(f[i][j],f[i][x]+f[i][y]);
                }
            if (f[i][j] != -1) team.push(i),vis[i][j] = true;
        }
        spfa(j);
    }
}
```

4.16 Stoer Wagner Algorithm

```
int G[maxn][maxn],node[maxn],dis[maxn]; bool visit[maxn];

inline int solve(int n)
{
    if (n == 1) return inf;
    int answer = inf;
    for (int i = 0;i < n;++i) node[i] = i;
    while (n > 1)
    {
        int mx = 1;
        for (int i = 0;i < n;++i)
        {
            dis[node[i]] = G[node[0]][node[i]];
            if (dis[node[i]] > dis[node[mx]]) mx = i;
        }
        int prev = 0;
        memset(visit,false,sizeof visit);
        visit[node[0]] = true;
        for (int i = 1;i < n;++i)
        {
            if (i == n-1)
            {
                answer = min(answer,dis[node[mx]]);
                for (int k = 0;k < n;++k)
                    G[node[k]][node[prev]] = (G[node[prev]][node[k]] += G[node[k]][node[mx]]);
                node[mx] = node[--n];
            }
```

```
            visit[node[mx]] = true; prev = mx; mx = -1;
            for (int j = 1;j < n;++j)
                if (!visit[node[j]])
                {
                    dis[node[j]] += G[node[prev]][node[j]];
                    if (mx == -1||dis[node[mx]] < dis[node[j]]) mx = j;
                }
        }
    }
    return answer;
}
```

4.17 Strongly Connected Component

```
int dfn[maxn],low[maxn],timestamp;
stack <int> stk; vector <int> scc[maxn];
void tarjan(int now)
{
    dfn[now] = low[now] = ++timestamp;
    stk.push(now);
    for (int i = side[now];i;i = nxt[i])
    {
        if (!dfn[toit[i]])
            tarjan(toit[i]),low[now] = min(low[now],low[toit[i]]);
        else if (!bel[toit[i]]) low[now] = min(low[now],dfn[toit[i]]);
    }
    if (dfn[now] == low[now])
    {
        ++tot;
        while (stk.top() != now)
        {
            scc[tot].push_back(stk.top());
            bel[stk.top()] = tot; stk.pop();
        }
        scc[tot].push_back(stk.top());
        bel[stk.top()] = tot; stk.pop();
    }
}
```

4.18 Virtual Tree

```
int N,cnt,timestamp,dfn[maxn],f[maxn][25],side[maxn],H[maxn];
int dep[maxn],toit[maxn],nxt[maxn],last[maxn],cost[maxn],stk[maxn];
ll best[maxn],g[maxn];

inline void add(int a,int b,int c) { nxt[++cnt] = side[a]; side[a] = cnt; toit[cnt] = b;
↪  cost[cnt] = c; }
inline void ins(int a,int b,int c) { add(a,b,c); add(b,a,c); }

```

```
8  inline void nadd(int a,int b,int idc)
9  {
10     if (a == b) return;
11     if (last[a] != idc) side[a] = 0,last[a] = idc;
12     if (last[b] != idc) side[b] = 0,last[b] = idc;
13     nxt[++cnt] = side[a]; side[a] = cnt; toit[cnt] = b;
14 }
15
16 inline bool cmp(int a,int b) { return dfn[a] < dfn[b]; }
17
18 inline void dfs(int now)
19 {
20     dfn[now] = ++timestamp;
21     for (int i = 1;(1<<i) <= dep[now];++i)
22         f[now][i] = f[f[now][i-1]][i-1];
23     for (int i = side[now];i;i = nxt[i])
24         if (toit[i] != f[now][0])
25         {
26             best[toit[i]] = min(best[now],(ll)cost[i]);
27             dep[toit[i]] = dep[now]+1;
28             f[toit[i]][0] = now; dfs(toit[i]);
29         }
30 }
31
32 inline int jump(int a,int step) { for (int i = 0;step;step >>= 1,++i) if (step&1) a = f[a][i];
   ↪  return a; }
33 inline int lca(int a,int b)
34 {
35     if (dep[a] < dep[b]) swap(a,b);
36     a = jump(a,dep[a]-dep[b]);
37     if (a == b) return a;
38     for (int i = 0;i >= 0;)
39     {
40         if (f[a][i] != f[b][i])
41             a = f[a][i],b = f[b][i],++i;
42         else --i;
43     }
44     return f[a][0];
45 }
46
47 inline void work(int idc)
48 {
49     cnt = 0; int K = gi(),tot,top;
50     for (int i = 1;i <= K;++i) H[i] = gi();
51     sort(H+1,H+K+1,cmp); H[tot = 1] = H[1];
52     for (int i = 2;i <= K;++i) if (lca(H[tot],H[i]) != H[tot]) H[++tot] = H[i];
53     stk[top = 1] = 1;
54     for (int i = 1;i <= tot;++i)
55     {
```

```
56          int ans = lca(H[i],stk[top]);
57          while (true)
58          {
59              if (dep[ans] >= dep[stk[top-1]]) { nadd(ans,stk[top--],idc); break; }
60              nadd(stk[top-1],stk[top],idc); --top;
61          }
62          if (stk[top] != ans) stk[++top] = ans;
63          if (stk[top] != H[i]) stk[++top] = H[i];
64      }
65      while (--top) nadd(stk[top],stk[top+1],idc);
66      // dp(1); printf("%lld\n",g[1]);
67  }
```

4.19 Zhu-Liu Algorithm

```
1   struct Directed_MT
2   {
3       struct Edge
4       {
5           int u,v,w;
6           inline Edge() = default;
7           inline Edge(int _u,int _v,int _w):u(_u),v(_v),w(_w) {}
8       };
9       int n,m,vis[maxn],pre[maxn],id[maxn],in[maxn]; Edge edges[maxm];
10
11      inline void init(int _n) { n = _n; m = 0; }
12      inline void AddEdge(int u,int v,int w) { edges[m++] = Edge(u,v,w); }
13      inline int work(int root)
14      {
15          int ret = 0;
16          while (true)
17          {
18              // 初始化
19              for (int i = 0;i < n;++i) in[i] = inf+1;
20              for (int i = 0;i < m;++i)
21              {
22                  int u = edges[i].u,v = edges[i].v;
23                  // 找寻最小入边，删除自环
24                  if (edges[i].w < in[v]&&u != v)
25                      in[v] = edges[i].w,pre[v] = u;
26              }
27              // 如果没有最小入边，表示该点不连通，则最小树形图形成失败
28              for (int i = 0;i < n;++i)
29              {
30                  if (i == root) continue;
31                  if (in[i] == inf+1) return inf;
32              }
33              int cnt = 0; // 记录缩点
34              memset(id,-1,sizeof id); memset(vis,-1,sizeof vis);
```

```
            in[root] = 0;
            for (int i = 0;i < n;++i)
            {
                ret += in[i]; int v = i;
                // 找寻自环
                while (vis[v] != i&&id[v] == -1&&v != root)
                    vis[v] = i,v = pre[v];
                if (v != root&&id[v] == -1)
                {
                    // 这里不能从 i 开始找，因为 i 有可能不在自环内
                    for (int u = pre[v];u != v;u = pre[u]) id[u] = cnt;
                    id[v] = cnt++;
                }
            }
            // 如果没有自环了，表示最小树形图成功了
            if (!cnt) break;
            // 找到那些不是自环的，重新给那些点进行标记
            for (int i = 0;i < n;++i)
                if (id[i] == -1) id[i] = cnt++;
            for (int i = 0;i < m;++i)
            {
                int u = edges[i].u,v = edges[i].v;
                edges[i].v = id[v]; edges[i].u = id[u];
                if (id[u] != id[v]) edges[i].w -= in[v];
            }
            // 缩点完后，点的数量就变了
            n = cnt; root = id[root];
        }
        return ret;
    }
}MT;
```

4.20 ZKW Cost Flow

```
// To be written
bool spfa()
{
    memset(mark,0,sizeof(mark));
    memset(d,0x7,sizeof(d));
    d[T] = 0; mark[T] = 1;
    queue <int> team;
    team.push(T);
    while (!team.empty())
    {
        int now = team.front();
        team.pop();
        for (int i = head[now];i;i = e[i].next)
            if (e[i^1].v&&d[e[i].to] > d[now]-e[i].c)
            {
```

```
16                  d[e[i].to] = d[now]-e[i].c;
17                  if (!mark[e[i].to])
18                  {
19                      mark[e[i].to] = true;
20                      team.push(e[i].to);
21                  }
22              }
23          mark[now] = false;
24      }
25      if (d[0] > 10000000) return false;
26      return true;
27  }
28
29  int dfs(int x,int f)
30  {
31      if (x == T)
32      {
33          mark[T] = 1;
34          return f;
35      }
36      int used = 0,w;
37      mark[x] = true;
38      for (int i = head[x];i;i = e[i].next)
39          if (!mark[e[i].to]&&e[i].v&&d[x]-e[i].c==d[e[i].to])
40          {
41              w = f - used;
42              w = dfs(e[i].to,small(e[i].v,w));
43              ans += w*e[i].c;
44              e[i].v -= w;
45              e[i^1].v += w;
46              used += w;
47              if (used == f) return f;
48          }
49      return used;
50  }
51
52  void zkw()
53  {
54      while (spfa())
55      {
56          mark[T] = 1;
57          while (mark[T])
58          {
59              memset(mark,0,sizeof(mark));
60              dfs(0,inf);
61          }
62      }
63  }
```

Chapter 5

Number Theory

5.1 Baby Step Giant Step

```cpp
// To Be Verified
// 求出最小的 t 使得 X^t = Y mod mod
inline int bsgs(int X,int Y,int mod)
{
    int m = ceil(sqrt(mod+0.5)),mul = 1,res = 1;
    if (Y == 1) return 0;
    hash.clear(); hash[Y] = 0;
    for (int i = 1;i <= m;++i)
    {
        mul = ((ll)mul*(ll)X)%mod;
        if (mul == Y) return i;
        hash[(ll)Y*(ll)mul%mod] = i;
    }
    res = mul;
    for (int i = 2;(i-1)*m <= mod;++i)
    {
        res = (ll)res*(ll)mul%mod;
        if (hash.find(res) != hash.end()) return i*m-hash[res];
    }
    return -1;
}
```

5.2 Chinese Remainder Theorem

```cpp
//快速乘
inline ll qsc(ll a,ll b,ll mod)
{
    ll ret = 0; a %= mod,b %= mod;
    for (;b;b >>= 1)
    {
        if (b&1)
        {
```

```
            ret += a;
            if (ret >= mod) ret -= mod;
        }
        a += a; if (a >= mod) a -= mod;
    }
    return ret;
}

inline ll msm(ll a,ll b,ll mod)
{
    ll ret = 1;
    for (;b;b >>= 1,a = qsc(a,a,mod)) if (b&1) ret = qsc(ret,a,mod);
    return ret;
}

inline ll crt()
{
    ll lcm = 1,ret = 0;
    for (int i = 1;i <= K;++i) lcm *= (ll)P[i];
    for (int i = 1;i <= K;++i)
    {
        ll tm = lcm/P[i];
        ll inv = msm(tm,P[i]-2,P[i]);
        ret = (ret+qsj(qsj(tm,inv,lcm),res[i],lcm))%lcm;
    }
    return ret;
}
```

5.3 Extended Euclidean Algorithm

```
//By yxj
inline ll exgcd(ll a,ll b,ll c) //ax mod b = c
{
    if (a == 0) return -1;
    else if (c % a == 0) return c/a;
    ll t = exgcd(b % a,a,((-c % a)+a)%a);
    if (t == -1) return -1;
    return (t*b+c)/a;
}

//Input:a,b,&x,&y,ax+by = gcd(a,b)
//Output:gcd(a,b)
inline int exgcd(int a,int b,int &x,int &y)
{
    if (!b) { x = 1,y = 0; return a; }
    else
    {
        int r = exgcd(b,a%b,y,x);
        y -= x*(a/b); return r;
```

```
    }
}
```

5.4 Linearly Sieve

```
//欧拉函数
inline void ready()
{
    phi[1] = 1;
    for (int i = 2;i < maxn;++i)
    {
        if (!exist[i]) phi[i] = i-1,prime[++tot] = i;
        for (int j = 1;j <= tot;++j)
        {
            if (i*prime[j] >= maxn) break;
            exist[i*prime[j]] = true;
            if (i % prime[j] == 0)
                { phi[i*prime[j]] = phi[i]*prime[j]; break; }
            else phi[i*prime[j]] = phi[i]*phi[prime[j]];
        }
    }
}
//莫比乌斯函数
inline void ready()
{
    mu[1] = 1;
    for (int i = 2;i <= 50000;++i)
    {
        if (!exist[i]) { prime[++tot] = i; mu[i] = -1; }
        for (int j = 1;j <= tot&&prime[j]*i <= 50000;++j)
        {
            exist[i*prime[j]] = true;
            if (i % prime[j] == 0) { mu[i*prime[j]] = 0; break; }
            mu[i*prime[j]] = -mu[i];
        }
    }
}
```

5.5 N-Power Residue

```
//Input:p,N,a p is a prime
//Output:the solutions to equation x^N=a(mod p) in [0,p-1]
inline vector <int> residue(int p,int N,int a)
{
    int g = PrimitiveRoot(p); ll m = bsgs(g,a,p);
    vector <int> ret;
    if (!a) { ret.push_back(0); return ret; }
    if (m == -1) return ret;
    ll A = N,B = p-1,C = m,x,y,d = exgcd(A,B,x,y);
```

```
    if (C % d) return ret;
    x *= (C / d)%B;
    ll delta = B / d;
    for (int i = 0;i < d;++i)
    {
        x += delta; if (x >= B) x -= B;
        ret.push_back((int)qsm(g,x,p));
    }
    sort(ret.begin(),ret.end());
    ret.erase(unique(ret.begin(),ret.end()),ret.end());
    return ret;
}
```

5.6 Number Theoretic Transformation

```
// The First Version
struct node
{
    int a[maxn*2],len;
    inline void NTT(int loglen,int len,int on)
    {
        for (int i = 0,j,t,p;i < len;++i)
        {
            for (j = 0,t = i,p = 0;j < loglen;++j,t >>= 1)
                p <<= 1,p |= t&1;
            if (p > i) swap(a[p],a[i]);
        }
        for (int s = 1,k = 2;s <= loglen;++s,k <<= 1)
        {
            int wn; if (on) wn = e[s]; else wn = ine[s];
            for (int i = 0;i < len;i += k)
            {
                int w = 1;
                for (int j = 0;j < (k >> 1);++j,w = (ll)wn*w%rhl)
                {
                    int u = a[i+j],v = (ll)w*a[i+j+(k>>1)]%rhl;
                    a[i+j] = u+v; if (a[i+j] >= rhl) a[i+j] -= rhl;
                    a[i+j+(k>>1)] = u-v;
                    if (a[i+j+(k>>1)] < 0) a[i+j+(k>>1)] += rhl;
                }
            }
        }
        if (!on)
        {
            int inv = qsm(len,rhl-2,rhl);
            for (int i = 0;i < len;++i) a[i] = (ll)a[i]*inv%rhl;
        }
    }
    friend inline bool operator *(node x,node y)
```

```
35      {
36          int loglen = 0,len;
37          for (;(1<<loglen)<x.len+y.len;++loglen); len = 1<<loglen;
38          x.NTT(loglen,len,1); y.NTT(loglen,len,1);
39          for (int i = 0;i < (1<<loglen);++i) x.a[i] = (ll)x.a[i]*y.a[i]%rhl;
40          x.NTT(loglen,len,0);
41      }
42  };
43
44  int main()
45  {
46      for (int i = 1;i < 20;++i)
47          e[i] = qsm(gg,(rhl-1)>>i,rhl),ine[i] = qsm(e[i],rhl-2,rhl);
48  }
49
50  // The Second Version
51  typedef long long ll;
52  ll e[20],ine[20];
53
54  inline ll qsm(ll a,int b,int c)
55  {
56      ll ret = 1;
57      for (;b;b >>= 1,(a *= a) %= c) if (b&1) (ret *= a) %= c;
58      return ret;
59  }
60
61  inline void NTT(ll *a,int loglen,int len,int on)
62  {
63      for (int i = 0,j,t,p;i < len;++i)
64      {
65          for (j = 0,t = i,p = 0;j < loglen;++j,t >>= 1)
66              p <<= 1,p |= t&1;
67          if (p > i) swap(a[p],a[i]);
68      }
69      for (int s = 1,k = 2;s <= loglen;++s,k <<= 1)
70      {
71          int wn; if (on) wn = e[s]; else wn = ine[s];
72          for (int i = 0;i < len;i += k)
73          {
74              int w = 1;
75              for (int j = 0;j < (k >> 1);++j,w = (ll)wn*w%lhh)
76              {
77                  int u = a[i+j],v = (ll)w*a[i+j+(k>>1)]%lhh;
78                  a[i+j] = u+v; if (a[i+j] >= lhh) a[i+j] -= lhh;
79                  a[i+j+(k>>1)] = u-v;
80                  if (a[i+j+(k>>1)] < 0) a[i+j+(k>>1)] += lhh;
81              }
82          }
83      }
```

```
84      if (!on)
85      {
86          int inv = qsm(len,lhh-2,lhh);
87          for (int i = 0;i < len;++i) a[i] = a[i]*inv%lhh;
88      }
89  }
90
91  struct Polynomial
92  {
93      int len; ll array[maxn<<2];
94      inline Polynomial(int _len = 0):len(_len) {}
95      inline Polynomial(ll a[],int n):len(n) { for (int i = 0;i < n;++i) array[i] = a[i]; }
96      inline ll operator [](int n) const { return array[n]; }
97      inline ll &operator [](int n) { return array[n]; }
98      inline void set(int n) { len = n; }
99      inline void set(int n,ll a[]) { len = n; for (int i = 0;i < n;++i) array[i] = a[i]; }
100     inline void extend(int key)
101     {
102         for (int i = len;i < (1<<key);++i)
103             array[i] = 0;
104     }
105     inline void cut(int key) { len = key; }
106     inline void transform(int loglen,int on) { NTT(array,loglen,1<<loglen,on); }
107 }; //变量只能定义在全局，不然会 re
108
109 inline Polynomial multiply(Polynomial &pa,Polynomial &ret) // self-multiply
110 {
111     int loglen = 0;
112     while ((1<<loglen) < (pa.len<<1)-1) ++loglen;
113     pa.extend(1<<loglen); pa.transform(loglen,1);
114     for (int i = 0;i < (1<<loglen);++i) ret[i] = pa[i]*pa[i]%lhh;
115     ret.transform(loglen,0); ret.cut((pa.len<<1)-1);
116     return ret;
117 }
118 inline Polynomial multiply(Polynomial &pa,Polynomial &pb,Polynomial &ret)
119 {
120     int loglen = 0;
121     while ((1<<loglen) < (pa.len+pb.len-1)) ++loglen;
122     pa.extend(1<<loglen); pa.transform(loglen,1);
123     pb.extend(1<<loglen); pb.transform(loglen,1);
124     for (int i = 0;i < (1<<loglen);++i) ret[i] = pa[i]*pb[i]%lhh;
125     ret.transform(loglen,0); ret.cut(pa.len+pb.len-1);
126     return ret;
127 }
128
129 int main()
130 {
131     for (int i = 1;i < 20;++i)
132         e[i] = qsm(g,(lhh-1)>>i,lhh),ine[i] = qsm(e[i],lhh-2,lhh);
```

```
}
```

5.7 Pollard Rho Algorithm

```
const int prime[] = {0,2,3,5,7,11,13,17,19,23,29,31};

inline ll mul(ll a,ll b,ll p) { return (a*b-((ll)((ld)a/p*b+1e-3)*p)+p)%p; }

inline bool check(ll m)
{
    if (m <= 2) return m == 2;
    ll tmp = m-1; int t = 0;
    while (!(tmp&1)) ++t,tmp >>= 1;
    for (int i = 1;i <= 10;++i)
    {
        int a = prime[i];
        if (a == m) return true;
        ll w = qsm(a,tmp,m);
        for (int it = 1;it <= t;++it)
        {
            ll pf = mul(w,w,m);
            if (pf == 1&&(w != 1&&w != m-1)) return false;
            w = pf;
        }
        if (w != 1) return false;
    }
    return true;
}
inline void rho(ll m)
{
    if (check(m)) { fac[++nn] = m; return; }
    while (true)
    {
        ll X = (ll)rand()*rand()%(m-1)+1,Y = X;
        ll c = (ll)rand()*rand()%(m-1)+1; int i,j;
        for (i = j = 2;;++i)
        {
            X = (mul(X,X,m)+c) % m;
            ll d = __gcd(abs(X-Y),m);
            if (1 < d&&d < m) { rho(d),rho(m/d); return; }
            if (X == Y) break; if (i == j) Y = X,j <<= 1;
        }
    }
}
inline void factor(ll m) { nn = 0; if (m > 1) rho(m); sort(fac+1,fac+nn+1); }


//__int128 Version
typedef __int128 int128;
```

```
46  inline int128 mul(int128 a,int128 b,int128 mod)
47  {
48      int128 ret = 0; a %= mod,b %= mod;
49      for (;b;b >>= 1)
50      {
51          if (b&1)
52          {
53              ret += a;
54              if (ret >= mod) ret -= mod;
55          }
56          a += a; if (a >= mod) a -= mod;
57      }
58      return ret;
59  }
60
61  inline int128 qsm(int128 a,int128 b,int128 mod)
62  {
63      int128 ret = 1;
64      for (;b;b >>= 1,a = mul(a,a,mod)) if (b&1) ret = mul(ret,a,mod);
65      return ret;
66  }
67
68  inline void ready()
69  {
70      for (int i = 2;i <= 100;++i)
71      {
72          if (prime[i]) continue; prime[++tot] = i;
73          for (int j = i*i;j <= 100;j += i) prime[j] = 1;
74      }
75  }
76
77  inline int128 gi()
78  {
79      int128 ret = 0; char ch;
80      do ch = getchar(); while (!(ch >= '0'&&ch <= '9'));
81      do ret = ret*10+ch-'0',ch = getchar(); while (ch >= '0'&&ch <= '9');
82      return ret;
83  }
84
85  inline int128 gcd(int128 a,int128 b) { if (b == 0) return a; return gcd(b,a%b); }
86
87  inline int128 Abs(int128 a) { if (a < 0) return -a; return a; }
88
89  inline bool check(int128 m)
90  {
91      if (m <= 2) return m == 2;
92      int128 tmp = m-1; int t = 0;
93      while (!(tmp&1)) ++t,tmp >>= 1;
94      for (int i = 1;i <= tot;++i)
```

```
    {
        int a = prime[i];
        if (a == m) return true;
        int128 w = qsm(a,tmp,m);
        for (int it = 1;it <= t;++it)
        {
            int128 pf = mul(w,w,m);
            if (pf == 1&&(w != 1&&w != m-1)) return false;
            w = pf;
        }
        if (w != 1) return false;
    }
    return true;
}
inline void rho(int128 m)
{
    if (check(m)) { fac[++nn] = m; return; }
    while (true)
    {
        int128 X = (int128)rand()*(int128)rand()%(m-1)+1,Y = X;
        int128 c = (int128)rand()*(int128)rand()%(m-1)+1; int i,j;
        for (i = j = 2;;++i)
        {
            X = (mul(X,X,m)+c)%m;
            int128 d = gcd(Abs(X-Y),m);
            if (1 < d&&d < m) { rho(d),rho(m/d); return; }
            if (X == Y) break; if (i == j) Y = X,j <<= 1;
        }
    }
}

inline void factor(int128 m) { nn = 0; if (m > 1) rho(m); sort(fac+1,fac+nn+1); }
```

5.8 Primitive Root

```
//Input:A prime p
//Output:p's primitive root
vector <ll> a;

inline g_test(ll g,ll p)
{
    for (ll i = a.size()-1;i >= 0;--i)
        if (qsm(g,(p-1)/a[i],p) == 1) return 0;
    return 1;
}

inline ll PrimitiveRoot(ll p)
{
    ll tmp = p - 1;
```

```
    for (ll i = 2;i <= tmp/i;++i)
    {
        if (!(tmp % i))
        {
            a.push_back(i);
            while (!(tmp%i)) tmp /= i;
        }
        if (tmp != 1) a.push_back(tmp);
    }
    for (ll g = 1;;++g) if (g_test(g,p)) return g;
}
```

5.9 Quadratic Residue

```
//判断是否存在 x, 使得 x^2=a mod n, 存在返回最小 x, 否则返回-1
inline int modsqr(int a,int n)
{
    int b,k,i,x;
    if (n == 2) return a & 1;
    if (qsm(a,(n-1)>>1,n) == 1)
    {
        if (n % 4 == 3) x = qsm(a,(n+1)>>2,n);
        else
        {
            for (b = 1;qsm(b,(n-1)>>1,n) == 1;++b);
            i = (n-1)>>1; k = 0;
            do
            {
                i >>= 1,k >>= 1;
                if (!((qsm(a,i,n)*(ll)qsm(b,k,n)+1)%n)) k += ((n-1)>>1);
            }
            while (!(i&1));
            x = (qsm(a,(i+1)>>1,n)*(ll)qsm(b,k>>1,n)) % n;
        }
        if ((x << 1) > n) x = n-x;
        return x;
    }
    return -1;
}
```

5.10 Single Variable Modulus Linear Equation

```
//Input:a,b,n
//Output:All the solutions in [0,n) to the equation ax=b(mod n)
inline vector <ll> LineModEquation(ll a,ll b,ll n)
{
    ll x,y,d = exgcd(a,n,x,y); vector <ll> ans;
    if (!(b % d))
    {
```

```
8           x %= n; x += n; x % n;
9           ans.push_back(x*(b/d)%n);
10          for (ll i= 1;i < d;++i) ans.push_back((ans[0]+i*n/d)%n);
11          //若找最小的，直接就是 (ans[0]%(n/d))
12      }
13      return ans;
14  }
```

Chapter 6

Numerical Algorithms

6.1 Counting Integral Points under Straight Line

```
// \sum_{i = 0}^{n-1} (a+bi)/m
inline ll count(ll n,ll a,ll b,ll m)
{
    if (!b) return n*(a/m);
    else if (a >= m) return n*(a/m)+count(n,a%m,b,m);
    else if (b >= m) return (n-1)*n/2*(b/m)+count(n,a,b%m,m);
    else return count((a+b*n)/m,(a+b*n)%m,m,b);
}
```

6.2 Evaluation of Expression

```
#include<bitset>
#include<stack>
#include<iostream>
#include<cstdio>
#include<cstdlib>
using namespace std;

const int maxn = 200010;
int T,N,M,pri[256],match[maxn]; bitset <maxn> A,B; char s[maxn];

inline int gi()
{
    char ch; int ret = 0,f = 1;
    do ch = getchar(); while (!(ch >= '0'&&ch <= '9')&&ch != '-');
    if (ch == '-') f = -1,ch = getchar();
    do ret = ret*10+ch-'0',ch = getchar(); while (ch >= '0'&&ch <= '9');
    return ret*f;
}

inline bitset <maxn> calc(int l,int r)
{
```

```
    if (l > r) return bitset <maxn>();
    while (match[l] == r) ++l,r--;
    if (l == r) { if (s[l] == 'A') return A; else return B; }
    int cur = 0; pair <int,int> mn(1<<30,0);
    for (int i = l;i <= r;++i)
    {
        if (s[i] == '(') cur += 10;
        else if (s[i] == ')') cur -= 10;
        else if (pri[s[i]])
            if (make_pair(cur+pri[s[i]],i) < mn)
                mn = make_pair(cur+pri[s[i]],i);
    }
    int pos = mn.second; auto L = calc(l,pos-1),R = calc(pos+1,r);
    if (s[pos] == '+') return L|R;
    else if (s[pos] == '*') return L&R;
    else return ~R;
}

int main()
{
    freopen("H.in","r",stdin);
    freopen("H.out","w",stdout);
    pri['+'] = 1; pri['*'] = 2; pri['-'] = 3;
    while (++T)
    {
        N = gi(),M = gi(); if (!N) break;
        A.reset(); B.reset(); printf("Case %d: ",T);
        for (int K = gi();K--;) A[gi()] = 1;
        for (int K = gi();K--;) B[gi()] = 1;
        scanf("%s",s+1); stack <int> S;
        for (int i = 1;i <= M;++i) match[i] = -1;
        for (int i = 1;i <= M;++i)
        {
            if (s[i] == '(') S.push(i);
            else if (s[i] == ')')
            {
                int t = S.top(); S.pop();
                match[match[t] = i] = t;
            }
        }
        int tot = 0; auto ans = calc(1,M);
        for (int i = 1;i <= N;++i) tot += ans[i]; printf("%d",tot);
        for (int i = 1;i <= N;++i) if (ans[i]) printf(" %d",i); putchar('\n');
    }
    return 0;
}
```

6.3 Fast Fourier Transformation

```
// The First Version
struct Vir
{
    double re,im;
    inline Vir(double _re = 0,double _im = 0):re(_re),im(_im) {}
    friend inline Vir operator*(const Vir &a,const Vir &b) { return
    ↪  Vir(a.re*b.re-a.im*b.im,a.re*b.im+a.im*b.re); }
    friend inline Vir operator+(const Vir &a,const Vir &b) { return Vir(a.re+b.re,a.im+b.im); }
    friend inline Vir operator-(const Vir &a,const Vir &b) { return Vir(a.re-b.re,a.im-b.im); }
    friend inline Vir operator/(const Vir &a,double r) { return Vir(a.re/r,a.im/r); }
}pa[maxn],pb[maxn];

inline void fft(Vir *a,int loglen,int len,int on)
{
    for (register int i = 0,j,t,p;i < len;++i)
    {
        for (p = j = 0,t = i;j < loglen;++j,t >>= 1)
            p <<= 1,p |= (t&1);
        if (p > i) swap(a[p],a[i]);
    }
    for (register int m = 2,s = 1;s <= loglen;++s,m <<= 1)
    {
        register Vir w(cos(2*pi*on/m),sin(2*pi*on/m));
        for (int i = 0;i < len;i += m)
        {
            register Vir wn(1,0);
            for (register int j = 0;j < (m>>1);++j,wn = wn*w)
            {
                register Vir u = a[i+j],v = wn*a[i+j+(m>>1)];
                a[i+j] = u+v; a[i+j+(m>>1)] = u-v;
            }
        }
    }
    if (on == -1) for (int i = 0;i < len;++i) a[i] = a[i]/len;
}

inline void work()
{
    int loglen = 0,len;
    while ((1<<loglen) < l) ++loglen; len = 1 << loglen;
    fft(pa,loglen,len,1); fft(pb,loglen,len,1);
    for (int i = 0;i < len;++i) pa[i] = pa[i]*pb[i];
    fft(pa,loglen,len,-1);
}

//The Second Version
const double pi = acos(-1.0);
struct Complex
```

```
48  {
49      double re,im;
50      inline Complex() = default;
51      inline Complex(double _re,double _im):re(_re),im(_im) {}
52      friend inline Complex operator*(const Complex &a,const Complex &b) { return
    ↪  Complex(a.re*b.re-a.im*b.im,a.re*b.im+a.im*b.re); }
53      friend inline Complex operator+(const Complex &a,const Complex &b) { return
    ↪  Complex(a.re+b.re,a.im+b.im); }
54      friend inline Complex operator-(const Complex &a,const Complex &b) { return
    ↪  Complex(a.re-b.re,a.im-b.im); }
55      friend inline Complex operator/(const Complex &a,double r) { return Complex(a.re/r,a.im/r);
    ↪  }
56  };
57
58  inline void FFT(Complex *a,int loglen,int len,int on)
59  {
60      for (register int i = 0,j,t,p;i < len;++i)
61      {
62          for (p = j = 0,t = i;j < loglen;++j,t >>= 1)
63              p <<= 1,p |= (t&1);
64          if (p > i) swap(a[p],a[i]);
65      }
66      for (register int m = 2,s = 1;s <= loglen;++s,m <<= 1)
67      {
68          register Complex w(cos(2*pi*on/m),sin(2*pi*on/m));
69          for (int i = 0;i < len;i += m)
70          {
71              register Complex wn(1,0);
72              for (register int j = 0;j < (m>>1);++j,wn = wn*w)
73              {
74                  register Complex u = a[i+j],v = wn*a[i+j+(m>>1)];
75                  a[i+j] = u+v; a[i+j+(m>>1)] = u-v;
76              }
77          }
78      }
79      if (on == -1) for (int i = 0;i < len;++i) a[i] = a[i]/len;
80  }
81
82  struct Polynomial
83  {
84      int len; Complex array[maxn<<2];
85      inline Polynomial(int _len = 0):len(_len) {}
86      inline Polynomial(Complex a[],int n):len(n) { for (int i = 0;i < n;++i) array[i] = a[i]; }
87      inline Complex operator [](int n) const { return array[n]; }
88      inline Complex &operator [](int n) { return array[n]; }
89      inline void set(int n) { len = n; }
90      inline void set(int n,Complex a[]) { len = n; for (int i = 0;i < n;++i) array[i] = a[i]; }
91      inline void extend(int key)
92      {
```

```
        for (int i = len;i < (1<<key);++i)
            array[i] = Complex(0,0);
    }
    inline void cut(int key) { len = key; }
    inline void transform(int loglen,int on) { FFT(array,loglen,1<<loglen,on); }
}; //变量只能定义在全局，不然会 re

inline Polynomial multiply(Polynomial &pa,Polynomial &ret) // self-multiply
{
    int loglen = 0;
    while ((1<<loglen) < (pa.len<<1)-1) ++loglen;
    pa.extend(1<<loglen); pa.transform(loglen,1);
    for (int i = 0;i < (1<<loglen);++i) ret[i] = pa[i]*pa[i];
    ret.transform(loglen,-1); ret.cut((pa.len<<1)-1);
    return ret;
}
inline Polynomial multiply(Polynomial &pa,Polynomial &pb,Polynomial &ret)
{
    int loglen = 0;
    while ((1<<loglen) < (pa.len+pb.len-1)) ++loglen;
    pa.extend(1<<loglen); pa.transform(loglen,1);
    pb.extend(1<<loglen); pb.transform(loglen,1);
    for (int i = 0;i < (1<<loglen);++i) ret[i] = pa[i]*pb[i];
    ret.transform(loglen,-1); ret.cut(pa.len+pb.len-1);
    return ret;
}
```

6.4 Fast Input and Output

```
// Input and Output of Int
// Be careful of Max_Int and Min_Int
inline int gi()
{
    char ch; int ret = 0,f = 1;
    do ch = getchar(); while (!(ch >= '0'&&ch <= '9')&&ch != '-');
    if (ch == '-') f = -1,ch = getchar();
    do ret = ret*10+ch-'0',ch = getchar(); while (ch >= '0'&&ch <= '9');
    return ret*f;
}

inline void pi(int a)
{
    if (!a) putchar('0');
    if (a < 0) a = -a,putchar('-');
    int num[10],n = 0;
    while (a) num[n++] = a%10,a /= 10;
    for (int i = n-1;i >= 0;--i) putchar('0'+num[i]);
}
```

6.5 Fraction Class

```
typedef long long ll;
struct Fraction
{
    ll num,den;
    inline Fraction(ll a = 0,ll b = 1)
    {
        if (den < 0) a = -a,b = -b;
        assert(b != 0); ll g = gcd(abs(a),b);
        num = a/g; den = b/g;
    }
    friend inline Fraction operator +(const Fraction &a,const Fraction &b) const { return Fraction(a.num*b.den+b.num*a.den,a.den*b.den); }
    friend inline Fraction operator -(const Fraction &a,const Fraction &b) const { return Fraction(a.num*b.den-b.num*a.den,a*den*b.den); }
    friend inline Fraction operator *(const Fraction &a,const Fraction &b) const { return Fraction(a.num*b.num,a*den*b.den); }
    friend inline Fraction operator /(const Fraction &a,const Fraction &b) const { return Fraction(a.num*b.den,a*den*b.num); }
    friend inline bool operator <(const Fraction &a,const Fraction &b) const { return a.num*b.den < a.den*b.num; }
    friend inline bool operator <(const Fraction &a,const Fraction &b) const { return a.num==b.num&&a.den==b.den; }
};
```

6.6 Gray Code

```
//0-2^n-1 的格雷码
inline vector <int> GrayCreat(int n)
{
    vector <int> res;
    for (int i = 0;i < (1<<n);++i) res.push_back(i^(i>>1));
    return res;
}
```

6.7 Numerical Integration

```
//self-adapt simpson
inline long double simpson(long double l,long double r,long double mid,long double Cl,long double Cr,long double Cm)
{
    long double tCl = calc((l+mid)/2),tCr = calc((mid+r)/2);
    long double ans=(r-l)*(Cl+Cr+4*Cm)/6,lans=(mid-l)*(Cl+Cm+4*tCl)/6,rans=(r-mid)*(Cr+Cm+4*tCr)/6;
    if (r-l <= 1e-3&&fabs(lans+rans-ans)<eps) return ans;
    // if (dep > lim&&fabs(lans+rans-ans)<eps) return ans;
    else return simpson(l,mid,(l+mid)/2,Cl,Cm,tCl)+simpson(mid,r,(mid+r)/2,Cm,Cr,tCr);
}
```

```
10
11  //romberg---To Be Verified
12  template <class T>
13  inline double romberg(const T &f,double a,double b,double eps = 1e-8)
14  {
15      vector <double> t; double h = b-a,last,cur;
16      int k = 1,i = 1;
17      t.push_back(h*(f(a)+f(b))/2);
18      do
19      {
20          last = t.back(); cur = 0; double x = a+h/2;
21          for (int j = 0;j < k;++j) cur += f(x),x += h;
22          cur = (t[0]+h*cur)/2;
23          double k1 = 4.0/3,k2 = 1.0/3;
24          for (int j = 0;j < i;++j)
25          {
26              double temp = k1*cur-k2*t[j];
27              t[j] = cur; cur = temp; k2 /= 4*k1-k2; k1 = k2+1;
28          }
29          t.push_back(cur); k *= 2; h /= 2; ++i;
30      }
31      while (fabs(last - cur) > eps);
32      return t.back();
33  }
```

6.8 Simplex

6.8.1 Description

有 n 个实数变量 $x_1, x_2, \ldots, x_n$ 和 m 条约束，其中第 i 条约束形如 $\sum_{i=1}^{n} a_{i,j} x_j \le b_i$。

此外这 n 个变量需要满足非负性限制，$x_j \ge 0$。

在满足上述所有条件的情况下，你需要指定每个变量 x_j 的取值，使得目标函数 $F = \sum_{j=1}^{n} c_j x_j$ 的值最大。

6.8.2 Input

第一行三个正整数 n, m, t。其中 $t \in \{0, 1\}$。

第二行有 n 个整数 $c_1, c_2, \ldots, c_n$，整数间均用一个空格分隔。

接下来 m 行，每行代表一条约束，其中第 i 行有 $n+1$ 个整数 $a_{i1}, a_{i2}, \ldots, a_{in}, b_i$，整数间均用一个空格分隔。

6.8.3 Output

如果不存在满足所有约束的解，仅输出一行 “Infeasible”。

如果对于任意的 M，都存在一组解使得目标函数的值大于 M，仅输出一行 “Unbounded”。

否则，第一行输出一个实数，表示目标函数的最大值 F。

如果 $t = 1$，那么你还需要输出第二行，用空格隔开的 n 个非负实数，表示此时 $x_1, x_2, \ldots, x_n$ 的取值，如有多组方案请任意输出其中一个。

6.8.4 Code

```
// uoj 179
#include<iostream>
#include<cstdio>
#include<cstdlib>
using namespace std;

#define maxn (30)
#define eps (1e-8)

int N,M,op,tot,q[maxn],idx[maxn],idy[maxn]; double a[maxn][maxn],A[maxn];

inline void pivot(int x,int y)
{
    swap(idy[x],idx[y]);
    double tmp = a[x][y]; a[x][y] = 1/a[x][y];
    for (int i = 0;i <= N;++i) if (y != i) a[x][i] /= tmp;
    tot = 0; for (int i = 0;i <= N;++i) if (i != y&&(a[x][i] > eps||a[x][i] < -eps)) q[++tot] =
    ↪  i;
    for (int i = 0;i <= M;++i)
    {
        if ((x == i)||(a[i][y] < eps&&a[i][y] > -eps)) continue;
        for (int j = 1;j <= tot;++j) a[i][q[j]] -= a[x][q[j]]*a[i][y];
        a[i][y] = -a[i][y]/tmp;
    }
}

int main()
{
    freopen("179.in","r",stdin);
    freopen("179.out","w",stdout);
    scanf("%d %d %d",&N,&M,&op); srand(233);
    for (int i = 1;i <= N;++i) scanf("%lf",a[0]+i);
    for (int i = 1;i <= M;++i)
    {
        for (int j = 1;j <= N;++j) scanf("%lf",a[i]+j);
        scanf("%lf",a[i]);
    }
    for (int i = 1;i <= N;++i) idx[i] = i;
    for (int i = 1;i <= M;++i) idy[i] = i+N;
    while (true)
    {
        int x = 0,y = 0;
        for (int i = 1;i <= M;++i) if (a[i][0] < -eps&&((!x)||(rand()&1))) x = i; if (!x) break;
        for (int i = 1;i <= N;++i) if (a[x][i] < -eps&&((!y)||(rand()&1))) y = i; if (!y) return
    ↪  puts("Infeasible"),0;
```

```
44          pivot(x,y);
45      }
46      while (true)
47      {
48          int x = 0,y = 0; double mn = 1e15;
49          for (int i = 1;i <= N;++i) if (a[0][i] > eps) { y = i; break; } if (!y) break;
50          for (int i = 1;i <= M;++i) if (a[i][y] > eps && a[i][0]/a[i][y] < mn) mn =
    ↪  a[i][0]/a[i][y],x = i; if (!x) return puts("Unbounded"),0;
51          pivot(x,y);
52      }
53      printf("%.8lf\n",-a[0][0]); if (!op) return 0;
54      for (int i = 1;i <= M;++i) if (idy[i] <= N) A[idy[i]] = a[i][0];
55      for (int i = 1;i <= N;++i) printf("%.8lf ",A[i]);
56      fclose(stdin); fclose(stdout);
57      return 0;
58  }
```

6.9 Solutions of Equation of Higher Order

```
1   // vector <double> solve(vector <double> coef,int n)
2   // coef 方程的系数; n 方程的系数
3   // 输出所有实数解
4   const double EPS = 1e-15,inf = 1e12;
5
6   inline int sign(double x) { return x < -EPS?-1:x > EPS; }
7
8   inline double get(const vector <double> &coef,double x)
9   {
10      double e = 1,s = 0;
11      for (int i = 0;i < coef.size();++i) s += coef[i]*e,e *= x;
12      return s;
13  }
14
15  inline double find(const vector <double> &coef,int n,double lo,double hi)
16  {
17      double sign_lo,sign_hi;
18      if ((sign_lo = sign(get(coef,lo)))== 0) return lo;
19      if ((sign_hi = sign(get(coef,hi)))== 0) return hi;
20      if (sign_lo*sign_hi > 0) return inf;
21      for (int step = 0;step < 100&&hi-lo > EPS;++step)
22      {
23          double m = (lo+hi)/2; int sign_mid = sign(get(coef,m));
24          if (sign_mid == 0) return m;
25          else if (sign_lo*sign_mid < 0) hi = m;
26          else lo = m;
27      }
28      return (lo+hi)/2;
29  }
30
```

```
31 inline vector <double> solve(const vector <double> &coef,int n)
32 {
33     vector <double> ret;
34     if (n == 1)
35     {
36         if (sign(coef[1])) ret.push_back(-coef[0]/coef[1]);
37         return ret;
38     }
39     vector <double> dcoef(n);
40     for (int i = 0;i < n;++i) dcoef[i] = coef[i+1]*(i+1);
41     vector <double> droot = solve(dcoef,n-1);
42     droot.insert(droot.begin(),-inf);
43     droot.push_back(inf);
44     for (int i = 0;i+1 < droot.size();++i)
45     {
46         double tmp = find(coef,n,droot[i],droot[i+1]);
47         if (tmp < inf) ret.push_back(tmp);
48     }
49     return ret;
50 }
```

Chapter 7

String Algorithms

7.1 Aho-Corasick Automaton

```
// ac 自动机
inline int newnode()
{
    memset(nxt[L],-1,sizeof(nxt[L]));
    return ++L-1;
}
inline void init() { L = 0; root = newnode(); }
inline void insert()
{
    int len = strlen(buf),now = root;
    for (int i = 0;i < len;++i)
    {
        if (nxt[now][buf[i]-'0'] == -1)
            nxt[now][buf[i]-'0'] = newnode();
        now = nxt[now][buf[i]-'0'];
    }
    end[now] = true;
}
inline void build()
{
    int now = root; queue <int> team;
    fail[root] = root;
    for (int i = 0;i < 10;++i)
    {
        if (nxt[now][i] == -1) nxt[now][i] = root;
        else fail[nxt[now][i]] = root,team.push(nxt[now][i]);
    }
    while (!team.empty())
    {
        now = team.front(); team.pop();
        for (int i = 0;i < 10;++i)
        {
            if (nxt[now][i] == -1)
```

```
                nxt[now][i] = nxt[fail[now]][i];
            else
            {
                fail[nxt[now][i]] = nxt[fail[now]][i];
                team.push(nxt[now][i]);
            }
        }
    }
}
```

7.2 Extended Knuth-Morris-Pratt Algorithm

```
// To Be Rewritten
// extend[i] 表示 T 与 S[i,n-1] 的最长公共前缀
const int maxn=100010;   //字符串长度最大值
int next[maxn],ex[maxn]; //ex 数组即为 extend 数组
//预处理计算 next 数组
void GETNEXT(char *str)
{
    int i=0,j,po,len=strlen(str);
    next[0]=len;//初始化 next[0]
    while(str[i]==str[i+1]&&i+1<len)//计算 next[1]
    i++;
    next[1]=i;
    po=1;//初始化 po 的位置
    for(i=2;i<len;i++)
    {
        if(next[i-po]+i<next[po]+po)//第一种情况，可以直接得到 next[i] 的值
        next[i]=next[i-po];
        else//第二种情况，要继续匹配才能得到 next[i] 的值
        {
            j=next[po]+po-i;
            if(j<0)j=0;//如果 i>po+next[po]，则要从头开始匹配
            while(i+j<len&&str[j]==str[j+i])//计算 next[i]
            j++;
            next[i]=j;
            po=i;//更新 po 的位置
        }
    }
}
//计算 extend 数组
void EXKMP(char *s1,char *s2)     // s1 is S, s2 is T
{
    int i=0,j,po,len=strlen(s1),l2=strlen(s2);
    GETNEXT(s2);//计算 T 串的 next 数组
    while(s1[i]==s2[i]&&i<l2&&i<len)//计算 ex[0]
    i++;
    ex[0]=i;
    po=0;//初始化 po 的位置
```

```
    for(i=1;i<len;i++)
    {
        if(next[i-po]+i<ex[po]+po)//第一种情况，直接可以得到 ex[i] 的值
        ex[i]=next[i-po];
        else//第二种情况，要继续匹配才能得到 ex[i] 的值
        {
            j=ex[po]+po-i;
            if(j<0)j=0;//如果 i>ex[po]+po 则要从头开始匹配
            while(i+j<len&&j<l2&&s1[j+i]==s2[j])//计算 ex[i]
            j++;
            ex[i]=j;
            po=i;//更新 po 的位置
        }
    }
}
```

7.3 Knuth-Morris-Pratt Algorithm

```
// To Be Verified
void cal_next(char *str, int *next, int len)
{
    int i,j;
    next[0] = -1;
    for (int i = 1; i < len; i++)
    {
        j = next[i - 1];
        while (str[j+1] != str[i]&&(j >= 0)) j = next[j];
        if (str[i] == str[j+1]) next[i] = j + 1;
        else next[i] = -1;
    }
}

int KMP(char *str,int slen, char *ptr,int plen,int *next)
{
    int s_i = 0,p_i = 0;
    while (s_i < slen&&p_i < plen)
    {
        if (str[s_i] == ptr[p_i]) s_i++,p_i++;
        else
        {
            if (!p_i) s_i++;
            else p_i = next[p_i-1] + 1;
        }
    }
    return (p_i == plen)?(s_i - plen):-1;
}
```

7.4 Manacher Algorithm

```
// Correct but to Be Rewritten
inline void ready()
{
    for (int i = 1;i <= 2*l1+1;++i)
        { if (i & 1) bac[i] = '#'; else bac[i] = s[i>>1]; }
    bin[0] = 1;
    for (int i = 1;i <= l1;++i)
        hash[i] = hash[i-1]*37+s[i]-'A'+1,bin[i] = 37*bin[i-1];
}

inline void manacher()
{
    rad[1] = 1;int best = 1;
    for (int i = 2;i <= 2*l1+1;++i)
    {
        int j;
        if (best+rad[best]-1<i) j = 1;
        else j = min(rad[2*best-i],best+rad[best]-i)+1;
        while (i-j+1&&i+j-1<=2*l1+1&&bac[i-j+1]==bac[i+j-1])
        {
            if (bac[i+j-1] != '#')
            {
                ull h = (hash[(i+j-1)>>1]-hash[((i-j+1)>>1)-1]*bin[j]);
                if (!exist1[h%rhl1]||!exist2[h%rhl2]||!exist3[h%rhl3])
                {
                    exist1[h%rhl1]=exist2[h%rhl2]=exist3[h%rhl3] = true;
                    ++tot,have[tot][0] = (i-j+1)>>1;
                    have[tot][1] = (i+j-1)>>1;
                }
            }
            ++j;
        }
        rad[i] = j-1;
        if (i+rad[i]>best+rad[best]) best = i;
    }
}
```

7.5 Palindrome Automaton

```
// Correct but to Be Rewritten
struct pat
{
    int next[maxn][26],fail[maxn],cnt[maxn],len[maxn],s[maxn],last,n,p;
    inline int newnode(int l) { cnt[p] = 0; len[p] = 1; return p++; }
    inline void init() { last = n = p = 0; newnode(0); newnode(-1); s[0] = -1; fail[0] = 1; }
    inline int getfail(int x) { while (s[n-len[x]-1] != s[n]) x = fail[x]; return x; }
    inline void add(int c)
```

```
    {
        c -= 'a'; s[++n] = c; int cur = getfail(last);
        if (!next[cur][c])
        {
            int now = newnode(len[cur]+2);
            fail[now] = next[getfail(fail[cur])][c];
            next[cur][c] = now;
        }
        last = next[cur][c]; cnt[last]++;
    }
}
```

7.6 Suffix Array

```
// 记得最后填一个字符集中没有的字符
inline void build(char *buf,int *Sa,int *Rank,int *Height,int n,int now,int m)
{
    int i,j,k,*x = t1,*y = t2;
    memset(c,0,4*m);
    for (i = 0;i < n;++i) c[x[i] = buf[i]-'A']++;
    for (i = 1;i < m;++i) c[i] += c[i-1];
    for (i = n-1;i >= 0;--i) Sa[--c[x[i]]] = i;
    for (k = 1;k < n;k <<= 1)
    {
        int p = 0;
        for (i = n-k;i < n;++i) y[p++] = i;
        for (i = 0;i < n;++i) if (Sa[i] >= k) y[p++] = Sa[i] - k;
        memset(c,0,4*m);
        for (i = 0;i < n;++i) c[x[y[i]]]++;
        for (i = 1;i < m;++i) c[i] += c[i-1];
        for (i = n-1;i >= 0;--i) Sa[--c[x[y[i]]]] = y[i];
        swap(x,y); p = 1; x[Sa[0]] = 0;
        for (i = 1;i < n;++i)
            x[Sa[i]] = y[Sa[i-1]] == y[Sa[i]]&&y[Sa[i-1]+k] == y[Sa[i]+k]?p-1:p++;
        if (p >= n) break; m = p;
    }
    for (i = 0;i < n;++i) Rank[Sa[i]] = i;
    for (i = k = 0;i < n;++i)
    {
        if (k) --k; if (!Rank[i]) continue;
        j = Sa[Rank[i]-1];
        while (i+k<n&&j+k<n&&buf[i+k]==buf[j+k]) ++k;
        Height[Rank[i]] = k;
    }
}
```

7.7 Suffix Automaton

```
// Correct but to Be Rewritten
struct SAM
{
    int tot,tail,cnt,p,np,q,nq,sz[maxn],arr[maxn],step[maxn],tran[maxn][26],parent[maxn];
    inline SAM() { tail = tot = 1; }
    inline void insert(int c)
    {
        p = tail; np = tail = ++tot; step[np] = step[p]+1;
        for (;!tran[p][c];p = parent[p]) tran[p][c] = np;
        if (!p) parent[np] = 1;
        else
        {
            q = tran[p][c];
            if (step[p]+1 == step[q]) parent[np] = q;
            else
            {
                nq = ++tot; step[nq] = step[p]+1;
                memcpy(tran[nq],tran[q],104);
                parent[nq] = parent[q]; parent[np] = parent[q] = nq;
                for (;tran[p][c] == q;p = parent[p]) tran[p][c] = nq;
            }
        }
        sz[np] = 1;
    }

    inline void dfs(int now)
    {
        if (vis[now]) return; vis[now] = true;
        for (int i = 0;i < 26;++i)
            if (tran[now][i]) dfs(tran[now][i]),arr[now] += arr[tran[now][i]];
        arr[now] += sz[now];
    }

    inline void build()
    {
        if (!mode) for (int i = 1;i <= tot;++i) sz[i] = 1;
        else
        {
            for (int i = 2;i <= tot;++i) ++d[parent[i]];
            queue <int> team; for (int i = 1;i <= tot;++i) if (!d[i]) team.push(i);
            while (!team.empty())
            {
                int now = team.front(); team.pop();
                sz[parent[now]] += sz[now];
                if (!--d[parent[now]]) team.push(parent[now]);
            }
        }
        sz[1] = 0; dfs(1);
```

```
49      }
50      inline void work()
51      {
52          int now = 1,l = 0,rank = 0; memset(s,0,N+1);
53          if (K > arr[1]) puts("-1");
54          else
55          {
56              while (true)
57              {
58                  rank += sz[now]; if (rank >= K) break;
59                  for (int i = 0;i < 26;++i)
60                  {
61                      if (rank+arr[tran[now][i]] < K) rank += arr[tran[now][i]];
62                      else { s[++l] = 'a'+i; now = tran[now][i]; break; }
63                  }
64              }
65              printf("%s",s+1);
66          }
67      }
68  }sam;
```

Chapter 8

Others

8.1 Calculation of Date

```
//ya 年 ma 月 da 日与 yb 年 mb 月 db 日相差几天
const int days = 365,s[] = {0,31,28,31,30,31,30,31,31,30,31,30,31};
inline bool isleap(int y)
{
    if ((!(y%400)||y%100)&&!(y%4)) return true;
    return false;
}

inline int leap(int y)
{
    if (!y) return 0;
    return y/4-y/100+y/400;
}

inline int calc(int day,int mon,int year)
{
    int res = (year-1)*days+leap(year-1);
    for (int i = 1;i < mon;++i) res += s[i];
    if (isleap(year)&&mon > 2) res++;
    res += day; return res;
}

inline int count_day(int da,int ma,int ya,int db,int mb,int yb)
{
    int resa = calc(da,ma,ya);
    int resb = calc(db,mb,yb);
    return abs(resa-resb);
}
```

8.2 Java Hints

8.2.1 Code Example

```java
// Code 1
import java.io.*;
import java.math.*;
import java.util.*;

public class Main
{
    final static int lhh = 998244353,maxn = 1655;
    static long jc[] = new long [maxn];

    static int calc(BigInteger N)
    {
        if (N.compareTo(BigInteger.ONE) <= 0) return 0;
        // System.out.println(N);
        int l = 2,r = 1650,mid;
        while (l <= r)
        {
            mid = (l+r)>>1;
            if (((BigInteger.valueOf(mid)).pow(mid)).compareTo(N) <= 0) l = mid+1;
            else r = mid-1;
        }
        // System.out.println(l+" "+r);
         int ret = (int)jc[r]-1,d = l; if (ret < 0) ret += lhh;
        int digit[] = new int[d]; BigInteger _d = BigInteger.valueOf(l);
        for (int i = 0;i < d;++i)
        {
            digit[i] = N.mod(_d).intValue();
            N = N.divide(_d);
        }
        // for (int i = d-1;i >= 0;--i) System.out.print(digit[i]);
        // System.out.println();
        boolean cho[] = new boolean[d],safe = false,exist = true;
        Arrays.fill(cho,false);
        int per[] = new int [d];
        for (int i = d-1;i >= 0;--i)
        {
            int cur = -1;
            if (safe)
            {
                int down = i == d-1?1:0;
                for (int j = d-1;j >= down;--j)
                {
                    if (cho[j] == true) continue;
                    cur = j; break;
                }
                if (cur == -1) { exist = false; break; }
```

```
47                  cho[cur] = true; per[i] = cur;
48              }
49              else
50              {
51                  if (cho[digit[i]] == true)
52                  {
53                      // System.out.println(i+":"+digit[i]);
54                      while (i < d)
55                      {
56                          cur = -1;
57                          int down = i == d-1?1:0;
58                          for (int j = digit[i]-1;j >= down;--j)
59                          {
60                              if (cho[j] == true) continue;
61                              cur = j; break;
62                          }
63                          // System.out.println(cur+":"+i);
64                          if (cur == -1) { ++i; if (i < d) cho[per[i]] = false; }
65                          else { cho[cur] = true; per[i] = cur; break; }
66                      }
67
68                      if (cur == -1) { exist = false; break; }
69                      safe = true;
70                  }
71                  else
72                  {
73                      if (digit[i] == 0&&i == d-1) { exist = false; break; }
74                      per[i] = digit[i];
75                      cho[digit[i]] = true;
76                  }
77              }
78          }
79          // for (int i = d-1;i >= 0;--i) System.out.print(per[i]);
80          // System.out.println();
81          if (!exist) return ret;
82          for (int i = d-1;i >= 0;--i)
83          {
84              int tmp = per[i];
85              for (int j = d-1;j > i;--j)
86                  if (per[j] < per[i]) --tmp;
87              ret += jc[i]*tmp%lhh;
88              if (ret >= lhh) ret -= lhh;
89          }
90          ret++; if (ret >= lhh) ret -= lhh;
91          ret -= jc[d-1]; if (ret < 0) ret += lhh;
92          // System.out.println(ret);
93          return ret;
94      }
95
```

```java
    public static void main(String args[])
    {
        jc[0] = 1;
        for (int i = 1;i <= 1650;++i)
            jc[i] = jc[i-1]*(long)i%lhh;
        Scanner cin = new Scanner(System.in);
        int T = cin.nextInt();

        while (T-- > 0)
        {
            BigInteger l = cin.nextBigInteger(),r = cin.nextBigInteger();
            int ans = calc(r)-calc(l.subtract(BigInteger.ONE));
            if (ans < 0) ans += lhh; System.out.println(ans);
        }
        // calc(BigInteger.valueOf(123455));
    }
}

//Code 2
import java.io.*;
import java.util.*;
import java.math.*;
public class Main
{
    static BigDecimal ratio[] = new BigDecimal[110];
    public static void main(String[] args)
    {
        Scanner cin = new Scanner(System.in);
        int T = cin.nextInt();
        for (int Case = 1;Case <= T;++Case)
        {
            int N = cin.nextInt();
            for (int i = 1;i <= N;++i)
            {
                String S = cin.next();
                String[] str = S.split(":");
                BigDecimal a = new BigDecimal(str[0]),b = new BigDecimal(str[1]);
                ratio[i] = a.divide(a.add(b),30,BigDecimal.ROUND_HALF_EVEN);
            }
            Arrays.sort(ratio,1,N+1);
            BigDecimal res = new BigDecimal(0),_1 = new BigDecimal(1); int ans = 0;
            for (int i = 1;i <= N;++i)
            {
                res = res.add(ratio[i]);
                if (res.compareTo(_1) < 0) ans = i;
                else break;
            }
            System.out.println("Case #"+Case+": "+ans);
        }
```

```
145     }
146 }
147
148 // Code 3
149 import java.math.*;
150 import java.util.*;
151 public class Main
152 {
153     static BigInteger d,ret,temp,yy;
154     static int n,dd;
155     static boolean mark = true;
156     static BigInteger[] a = new BigInteger[20];
157     public static void main(String[] args)
158     {
159         Scanner in = new Scanner (System.in);
160         n = in.nextInt();
161         temp = BigInteger.ONE;
162         ret = BigInteger.ZERO;
163         for (int i = 0;i < n;++i)
164         {
165             int k = in.nextInt();
166             a[i] = BigInteger.valueOf(k);
167             d = temp.gcd(a[i]);
168             temp = temp.multiply(a[i]).divide(d);
169         }
170         for (int i = 1;i < (1<<n);++i)
171         {
172             mark = false; yy = BigInteger.ONE;
173             for (int j = 0;j < n;++j) if (((1 << j) & i) > 0) { mark = !mark; d = a[j].gcd(yy);
    ↪  yy = yy.multiply(a[j]).divide(d); }
174             if (mark) ret = ret.add(temp.divide(yy));
175             else ret = ret.subtract(temp.divide(yy));
176         }
177         d = ret.gcd(temp);
178         System.out.println(ret.divide(d));
179         System.out.println(temp.divide(d));
180     }
181 }
182
183 // Code 4
184 import java.io.*;
185 import java.math.*;
186 import java.util.*;
187
188 public class Main
189 {
190     public static String reverse(String str) { return new
    ↪  StringBuffer(str).reverse().toString(); }
191
```

```
192     public static void main(String args[])
193     {
194         Scanner cin = new Scanner(System.in);
195         int T = cin.nextInt(); BigInteger zero = BigInteger.valueOf(0);
196         while (T-- > 0)
197         {
198             int base1 = cin.nextInt(),base2 = cin.nextInt();
199             String S = cin.next(); int len = S.length();
200             System.out.println(base1+" "+S);
201             BigInteger res = BigInteger.valueOf(0),b1 = BigInteger.valueOf(base1),b2 =
    ↪  BigInteger.valueOf(base2);
202             for (int i = 0;i < len;++i)
203             {
204                 res = res.multiply(b1);
205                 int rep = 0;
206                 if (S.charAt(i) >= '0'&&S.charAt(i) <= '9') rep = S.charAt(i)-'0';
207                 else if (S.charAt(i) >= 'A'&&S.charAt(i) <= 'Z') rep = 10+S.charAt(i)-'A';
208                 else rep = 36+S.charAt(i)-'a';
209                 res = res.add(BigInteger.valueOf(rep));
210             }
211             String ret = new String();
212             // System.out.println(res);
213             if (res.compareTo(zero) == 0) ret += '0';
214             else
215                 while (res.compareTo(zero) > 0)
216                 {
217                     long val = res.remainder(b2).longValue();
218                     // System.out.println(val);
219                     if (val < 10) ret += (char)(val+'0');
220                     else if (val < 36) ret += (char)(val+'A'-10);
221                     else ret += (char)(val+'a'-36);
222                     res = res.divide(b2);
223                 }
224             System.out.println(base2+" "+reverse(ret)+"\n");
225         }
226     }
227 }
```

8.2.2 Class Reference

BigDecimal Class

java.math

Class BigDecimal

java.lang.Object
  java.lang.Number
    java.math.BigDecimal

All Implemented Interfaces:

Serializable, Comparable<BigDecimal>

```
public class BigDecimal
extends Number
implements Comparable<BigDecimal>
```

Immutable, arbitrary-precision signed decimal numbers. A `BigDecimal` consists of an arbitrary precision integer *unscaled value* and a 32-bit integer *scale*. If zero or positive, the scale is the number of digits to the right of the decimal point. If negative, the unscaled value of the number is multiplied by ten to the power of the negation of the scale. The value of the number represented by the `BigDecimal` is therefore `(unscaledValue × 10-scale)`.

The `BigDecimal` class provides operations for arithmetic, scale manipulation, rounding, comparison, hashing, and format conversion. The `toString()` method provides a canonical representation of a `BigDecimal`.

The `BigDecimal` class gives its user complete control over rounding behavior. If no rounding mode is specified and the exact result cannot be represented, an exception is thrown; otherwise, calculations can be carried out to a chosen precision and rounding mode by supplying an appropriate `MathContext` object to the operation. In either case, eight *rounding modes* are provided for the control of rounding. Using the integer fields in this class (such as `ROUND_HALF_UP`) to represent rounding mode is largely obsolete; the enumeration values of the `RoundingMode enum`, (such as `RoundingMode.HALF_UP`) should be used instead.

When a `MathContext` object is supplied with a precision setting of 0 (for example, `MathContext.UNLIMITED`), arithmetic operations are exact, as are the arithmetic methods which take no `MathContext` object. (This is the only behavior that was supported in releases prior to 5.) As a corollary of computing the exact result, the rounding mode setting of a `MathContext` object with a precision setting of 0 is not used and thus irrelevant. In the case of divide, the exact quotient could have an infinitely long decimal expansion; for example, 1 divided by 3. If the quotient has a nonterminating decimal expansion and the operation is specified to return an exact result, an `ArithmeticException` is thrown. Otherwise, the exact result of the division is returned, as done for other operations.

When the precision setting is not 0, the rules of `BigDecimal` arithmetic are broadly compatible with selected modes of operation of the arithmetic defined in ANSI X3.274-1996 and ANSI X3.274-1996/AM 1-2000 (section 7.4). Unlike those standards, `BigDecimal` includes many rounding modes, which were mandatory for division in `BigDecimal` releases prior to 5. Any conflicts between these ANSI standards and the `BigDecimal` specification are resolved in favor of `BigDecimal`.

Since the same numerical value can have different representations (with different scales), the rules of arithmetic and rounding must specify both the numerical result and the scale used in the result's representation.

In general the rounding modes and precision setting determine how operations return results with a limited number of digits when the exact result has more digits (perhaps infinitely many in the case of division) than the number of digits returned. First, the total number of digits to return is specified by the `MathContext`'s `precision` setting; this determines the result's *precision*. The digit count starts from the leftmost nonzero digit of the exact result. The rounding mode determines how any discarded trailing digits affect the returned result.

For all arithmetic operators , the operation is carried out as though an exact intermediate result were first calculated and then rounded to the number of digits specified by the precision setting (if necessary), using the selected rounding mode. If the exact result is not returned, some digit positions of the exact result are discarded. When rounding increases the magnitude of the returned result, it is possible for a new digit position to be created by a carry propagating to a leading "9" digit. For example, rounding the value 999.9 to three digits rounding up would be numerically equal to one thousand, represented as 100×10^1. In such cases, the new "1" is the leading digit position of the returned result.

Besides a logical exact result, each arithmetic operation has a preferred scale for representing a result. The preferred scale for each operation is listed in the table below.

Preferred Scales for Results of Arithmetic Operations

Operation	Preferred Scale of Result
Add	max(addend.scale(), augend.scale())
Subtract	max(minuend.scale(), subtrahend.scale())
Multiply	multiplier.scale() + multiplicand.scale()
Divide	dividend.scale() - divisor.scale()

These scales are the ones used by the methods which return exact arithmetic results; except that an exact divide may have to use a larger scale since the exact result may have more digits. For example, `1/32` is `0.03125`.

Before rounding, the scale of the logical exact intermediate result is the preferred scale for that operation. If the exact numerical result cannot be represented in `precision` digits, rounding selects the set of digits to return and the scale of the result is reduced from the scale of the intermediate result to the least scale which can represent the `precision` digits actually returned. If the exact result can be represented with at most `precision` digits, the representation of the result with the scale closest to the preferred scale is returned. In particular, an exactly representable quotient may be represented in fewer than `precision` digits by removing trailing zeros and decreasing the scale. For example, rounding to three digits using the floor rounding mode,

```
19/100 = 0.19 // integer=19, scale=2
```

but

```
21/110 = 0.190 // integer=190, scale=3
```

Note that for add, subtract, and multiply, the reduction in scale will equal the number of digit positions of the exact result which are discarded. If the rounding causes a carry propagation to create a new high-order digit position, an additional digit of the result is discarded than when no new digit position is created.

Other methods may have slightly different rounding semantics. For example, the result of the `pow` method using the specified algorithm can occasionally differ from the rounded mathematical result by more than one unit in the last place, one *ulp*.

Two types of operations are provided for manipulating the scale of a `BigDecimal`: scaling/rounding operations and decimal point motion operations. Scaling/rounding operations (`setScale` and `round`) return a `BigDecimal` whose value is approximately (or exactly) equal to that of the operand, but whose scale or precision is the specified value; that is, they increase or decrease the precision of the stored number with minimal effect on its value. Decimal point motion operations (`movePointLeft` and `movePointRight`) return a `BigDecimal` created from the operand by moving the decimal point a specified distance in the specified direction.

For the sake of brevity and clarity, pseudo-code is used throughout the descriptions of `BigDecimal` methods. The pseudo-code expression `(i + j)` is shorthand for "a `BigDecimal` whose value is that of the `BigDecimal` `i` added to that of the `BigDecimal` `j`." The pseudo-code expression `(i == j)` is shorthand for "`true` if and only if the `BigDecimal` `i` represents the same value as the `BigDecimal` `j`." Other pseudo-code expressions are interpreted similarly. Square brackets are used to represent the particular `BigInteger` and scale pair defining a `BigDecimal` value; for example [19, 2] is the `BigDecimal` numerically equal to 0.19 having a scale of 2.

Note: care should be exercised if `BigDecimal` objects are used as keys in a `SortedMap` or elements in a `SortedSet` since `BigDecimal`'s *natural ordering* is *inconsistent with equals*. See `Comparable`, `SortedMap` or `SortedSet` for more information.

All methods and constructors for this class throw `NullPointerException` when passed a `null` object reference for any input parameter.

See Also:

`BigInteger`, `MathContext`, `RoundingMode`, `SortedMap`, `SortedSet`, Serialized Form

Field Summary

Fields

Modifier and Type	Field and Description
`static BigDecimal`	**`ONE`** The value 1, with a scale of 0.
`static int`	**`ROUND_CEILING`** Rounding mode to round towards positive infinity.
`static int`	**`ROUND_DOWN`**

	Rounding mode to round towards zero.
static int	**ROUND_FLOOR** Rounding mode to round towards negative infinity.
static int	**ROUND_HALF_DOWN** Rounding mode to round towards "nearest neighbor" unless both neighbors are equidistant, in which case round down.
static int	**ROUND_HALF_EVEN** Rounding mode to round towards the "nearest neighbor" unless both neighbors are equidistant, in which case, round towards the even neighbor.
static int	**ROUND_HALF_UP** Rounding mode to round towards "nearest neighbor" unless both neighbors are equidistant, in which case round up.
static int	**ROUND_UNNECESSARY** Rounding mode to assert that the requested operation has an exact result, hence no rounding is necessary.
static int	**ROUND_UP** Rounding mode to round away from zero.
static BigDecimal	**TEN** The value 10, with a scale of 0.
static BigDecimal	**ZERO** The value 0, with a scale of 0.

Constructor Summary

Constructors

Constructor and Description
BigDecimal(**BigInteger** val) Translates a BigInteger into a BigDecimal.
BigDecimal(**BigInteger** unscaledVal, int scale) Translates a BigInteger unscaled value and an int scale into a BigDecimal.
BigDecimal(**BigInteger** unscaledVal, int scale, **MathContext** mc) Translates a BigInteger unscaled value and an int scale into a BigDecimal, with rounding according to the context settings.
BigDecimal(**BigInteger** val, **MathContext** mc) Translates a BigInteger into a BigDecimal rounding according to the context settings.
BigDecimal(char[] in) Translates a character array representation of a BigDecimal into a BigDecimal, accepting the same sequence of characters as the **BigDecimal(String)** constructor.
BigDecimal(char[] in, int offset, int len) Translates a character array representation of a BigDecimal into a BigDecimal, accepting the same sequence of characters as the **BigDecimal(String)** constructor, while allowing a sub-array to be specified.
BigDecimal(char[] in, int offset, int len, **MathContext** mc) Translates a character array representation of a BigDecimal into a BigDecimal, accepting the same sequence of characters as the **BigDecimal(String)** constructor, while allowing a sub-array to be specified and with rounding according to the context settings.
BigDecimal(char[] in, **MathContext** mc) Translates a character array representation of a BigDecimal into a BigDecimal, accepting the same sequence of characters as the **BigDecimal(String)** constructor and with rounding according to the context settings.
BigDecimal(double val) Translates a double into a BigDecimal which is the exact decimal representation of the double's binary floating-point value.
BigDecimal(double val, **MathContext** mc) Translates a double into a BigDecimal, with rounding according to the context settings.

BigDecimal(int val)
Translates an int into a BigDecimal.

BigDecimal(int val, **MathContext** mc)
Translates an int into a BigDecimal, with rounding according to the context settings.

BigDecimal(long val)
Translates a long into a BigDecimal.

BigDecimal(long val, **MathContext** mc)
Translates a long into a BigDecimal, with rounding according to the context settings.

BigDecimal(**String** val)
Translates the string representation of a BigDecimal into a BigDecimal.

BigDecimal(**String** val, **MathContext** mc)
Translates the string representation of a BigDecimal into a BigDecimal, accepting the same strings as the **BigDecimal(String)** constructor, with rounding according to the context settings.

Method Summary

Methods

Modifier and Type	Method and Description
BigDecimal	**abs**() Returns a BigDecimal whose value is the absolute value of this BigDecimal, and whose scale is this.scale().
BigDecimal	**abs**(**MathContext** mc) Returns a BigDecimal whose value is the absolute value of this BigDecimal, with rounding according to the context settings.
BigDecimal	**add**(**BigDecimal** augend) Returns a BigDecimal whose value is (this + augend), and whose scale is max(this.scale(), augend.scale()).
BigDecimal	**add**(**BigDecimal** augend, **MathContext** mc) Returns a BigDecimal whose value is (this + augend), with rounding according to the context settings.
byte	**byteValueExact**() Converts this BigDecimal to a byte, checking for lost information.
int	**compareTo**(**BigDecimal** val) Compares this BigDecimal with the specified BigDecimal.
BigDecimal	**divide**(**BigDecimal** divisor) Returns a BigDecimal whose value is (this / divisor), and whose preferred scale is (this.scale() - divisor.scale()); if the exact quotient cannot be represented (because it has a non-terminating decimal expansion) an ArithmeticException is thrown.
BigDecimal	**divide**(**BigDecimal** divisor, int roundingMode) Returns a BigDecimal whose value is (this / divisor), and whose scale is this.scale().
BigDecimal	**divide**(**BigDecimal** divisor, int scale, int roundingMode) Returns a BigDecimal whose value is (this / divisor), and whose scale is as specified.
BigDecimal	**divide**(**BigDecimal** divisor, int scale, **RoundingMode** roundingMode) Returns a BigDecimal whose value is (this / divisor), and whose scale is as specified.
BigDecimal	**divide**(**BigDecimal** divisor, **MathContext** mc) Returns a BigDecimal whose value is (this / divisor), with rounding according to the context settings.
BigDecimal	**divide**(**BigDecimal** divisor, **RoundingMode** roundingMode)

	Returns a BigDecimal whose value is (this / divisor), and whose scale is this.scale().
BigDecimal[]	**divideAndRemainder**(BigDecimal divisor) Returns a two-element BigDecimal array containing the result of divideToIntegralValue followed by the result of remainder on the two operands.
BigDecimal[]	**divideAndRemainder**(BigDecimal divisor, MathContext mc) Returns a two-element BigDecimal array containing the result of divideToIntegralValue followed by the result of remainder on the two operands calculated with rounding according to the context settings.
BigDecimal	**divideToIntegralValue**(BigDecimal divisor) Returns a BigDecimal whose value is the integer part of the quotient (this / divisor) rounded down.
BigDecimal	**divideToIntegralValue**(BigDecimal divisor, MathContext mc) Returns a BigDecimal whose value is the integer part of (this / divisor).
double	**doubleValue**() Converts this BigDecimal to a double.
boolean	**equals**(Object x) Compares this BigDecimal with the specified Object for equality.
float	**floatValue**() Converts this BigDecimal to a float.
int	**hashCode**() Returns the hash code for this BigDecimal.
int	**intValue**() Converts this BigDecimal to an int.
int	**intValueExact**() Converts this BigDecimal to an int, checking for lost information.
long	**longValue**() Converts this BigDecimal to a long.
long	**longValueExact**() Converts this BigDecimal to a long, checking for lost information.
BigDecimal	**max**(BigDecimal val) Returns the maximum of this BigDecimal and val.
BigDecimal	**min**(BigDecimal val) Returns the minimum of this BigDecimal and val.
BigDecimal	**movePointLeft**(int n) Returns a BigDecimal which is equivalent to this one with the decimal point moved n places to the left.
BigDecimal	**movePointRight**(int n) Returns a BigDecimal which is equivalent to this one with the decimal point moved n places to the right.
BigDecimal	**multiply**(BigDecimal multiplicand) Returns a BigDecimal whose value is (this × multiplicand), and whose scale is (this.scale() + multiplicand.scale()).
BigDecimal	**multiply**(BigDecimal multiplicand, MathContext mc) Returns a BigDecimal whose value is (this × multiplicand), with rounding according to the context settings.
BigDecimal	**negate**() Returns a BigDecimal whose value is (-this), and whose scale is this.scale().
BigDecimal	**negate**(MathContext mc) Returns a BigDecimal whose value is (-this), with rounding according to the context settings.
BigDecimal	**plus**() Returns a BigDecimal whose value is (+this), and whose scale is this.scale().
BigDecimal	**plus**(MathContext mc)

	Returns a `BigDecimal` whose value is `(+this)`, with rounding according to the context settings.
BigDecimal	**pow**(int n)
	Returns a `BigDecimal` whose value is $(this^n)$, The power is computed exactly, to unlimited precision.
BigDecimal	**pow**(int n, **MathContext** mc)
	Returns a `BigDecimal` whose value is $(this^n)$.
int	**precision**()
	Returns the *precision* of this `BigDecimal`.
BigDecimal	**remainder**(**BigDecimal** divisor)
	Returns a `BigDecimal` whose value is `(this % divisor)`.
BigDecimal	**remainder**(**BigDecimal** divisor, **MathContext** mc)
	Returns a `BigDecimal` whose value is `(this % divisor)`, with rounding according to the context settings.
BigDecimal	**round**(**MathContext** mc)
	Returns a `BigDecimal` rounded according to the `MathContext` settings.
int	**scale**()
	Returns the *scale* of this `BigDecimal`.
BigDecimal	**scaleByPowerOfTen**(int n)
	Returns a BigDecimal whose numerical value is equal to $(this * 10^n)$.
BigDecimal	**setScale**(int newScale)
	Returns a `BigDecimal` whose scale is the specified value, and whose value is numerically equal to this `BigDecimal`'s.
BigDecimal	**setScale**(int newScale, int roundingMode)
	Returns a `BigDecimal` whose scale is the specified value, and whose unscaled value is determined by multiplying or dividing this `BigDecimal`'s unscaled value by the appropriate power of ten to maintain its overall value.
BigDecimal	**setScale**(int newScale, **RoundingMode** roundingMode)
	Returns a `BigDecimal` whose scale is the specified value, and whose unscaled value is determined by multiplying or dividing this `BigDecimal`'s unscaled value by the appropriate power of ten to maintain its overall value.
short	**shortValueExact**()
	Converts this `BigDecimal` to a `short`, checking for lost information.
int	**signum**()
	Returns the signum function of this `BigDecimal`.
BigDecimal	**stripTrailingZeros**()
	Returns a `BigDecimal` which is numerically equal to this one but with any trailing zeros removed from the representation.
BigDecimal	**subtract**(**BigDecimal** subtrahend)
	Returns a `BigDecimal` whose value is `(this - subtrahend)`, and whose scale is `max(this.scale(), subtrahend.scale())`.
BigDecimal	**subtract**(**BigDecimal** subtrahend, **MathContext** mc)
	Returns a `BigDecimal` whose value is `(this - subtrahend)`, with rounding according to the context settings.
BigInteger	**toBigInteger**()
	Converts this `BigDecimal` to a `BigInteger`.
BigInteger	**toBigIntegerExact**()
	Converts this `BigDecimal` to a `BigInteger`, checking for lost information.
String	**toEngineeringString**()
	Returns a string representation of this `BigDecimal`, using engineering notation if an exponent is needed.
String	**toPlainString**()
	Returns a string representation of this `BigDecimal` without an exponent field.
String	**toString**()

	Returns the string representation of this `BigDecimal`, using scientific notation if an exponent is needed.
`BigDecimal`	**`ulp`**`()` Returns the size of an ulp, a unit in the last place, of this `BigDecimal`.
`BigInteger`	**`unscaledValue`**`()` Returns a `BigInteger` whose value is the *unscaled value* of this `BigDecimal`.
`static BigDecimal`	**`valueOf`**`(double val)` Translates a `double` into a `BigDecimal`, using the `double`'s canonical string representation provided by the **`Double.toString(double)`** method.
`static BigDecimal`	**`valueOf`**`(long val)` Translates a `long` value into a `BigDecimal` with a scale of zero.
`static BigDecimal`	**`valueOf`**`(long unscaledVal, int scale)` Translates a `long` unscaled value and an `int` scale into a `BigDecimal`.

Methods inherited from class java.lang.Number

`byteValue`, `shortValue`

Methods inherited from class java.lang.Object

`clone`, `finalize`, `getClass`, `notify`, `notifyAll`, `wait`, `wait`, `wait`

Field Detail

ZERO

```
public static final BigDecimal ZERO
```

The value 0, with a scale of 0.

Since:

1.5

ONE

```
public static final BigDecimal ONE
```

The value 1, with a scale of 0.

Since:

1.5

TEN

```
public static final BigDecimal TEN
```

The value 10, with a scale of 0.

Since:

1.5

BigInteger Class

java.math

Class BigInteger

java.lang.Object
  java.lang.Number
    java.math.BigInteger

All Implemented Interfaces:

Serializable, Comparable<BigInteger>

```
public class BigInteger
extends Number
implements Comparable<BigInteger>
```

Immutable arbitrary-precision integers. All operations behave as if BigIntegers were represented in two's-complement notation (like Java's primitive integer types). BigInteger provides analogues to all of Java's primitive integer operators, and all relevant methods from java.lang.Math. Additionally, BigInteger provides operations for modular arithmetic, GCD calculation, primality testing, prime generation, bit manipulation, and a few other miscellaneous operations.

Semantics of arithmetic operations exactly mimic those of Java's integer arithmetic operators, as defined in *The Java Language Specification*. For example, division by zero throws an `ArithmeticException`, and division of a negative by a positive yields a negative (or zero) remainder. All of the details in the Spec concerning overflow are ignored, as BigIntegers are made as large as necessary to accommodate the results of an operation.

Semantics of shift operations extend those of Java's shift operators to allow for negative shift distances. A right-shift with a negative shift distance results in a left shift, and vice-versa. The unsigned right shift operator (>>>) is omitted, as this operation makes little sense in combination with the "infinite word size" abstraction provided by this class.

Semantics of bitwise logical operations exactly mimic those of Java's bitwise integer operators. The binary operators (`and`, `or`, `xor`) implicitly perform sign extension on the shorter of the two operands prior to performing the operation.

Comparison operations perform signed integer comparisons, analogous to those performed by Java's relational and equality operators.

Modular arithmetic operations are provided to compute residues, perform exponentiation, and compute multiplicative inverses. These methods always return a non-negative result, between `0` and `(modulus - 1)`, inclusive.

Bit operations operate on a single bit of the two's-complement representation of their operand. If necessary, the operand is sign-extended so that it contains the designated bit. None of the single-bit operations can produce a BigInteger with a different sign from the BigInteger being operated on, as they affect only a single bit, and the "infinite word size" abstraction provided by this class ensures that there are infinitely many "virtual sign bits" preceding each BigInteger.

For the sake of brevity and clarity, pseudo-code is used throughout the descriptions of BigInteger methods. The pseudo-code expression `(i + j)` is shorthand for "a BigInteger whose value is that of the BigInteger `i` plus that of the BigInteger `j`." The pseudo-code expression `(i == j)` is shorthand for "`true` if and only if the BigInteger `i` represents the same value as the BigInteger `j`." Other pseudo-code expressions are interpreted similarly.

All methods and constructors in this class throw `NullPointerException` when passed a null object reference for any input parameter.

Since:

JDK1.1

See Also:

`BigDecimal`, Serialized Form

Field Summary

Fields

Modifier and Type	Field and Description
static BigInteger	ONE The BigInteger constant one.
static BigInteger	TEN The BigInteger constant ten.
static BigInteger	ZERO The BigInteger constant zero.

Constructor Summary

Constructors

Constructor and Description
BigInteger(byte[] val) Translates a byte array containing the two's-complement binary representation of a BigInteger into a BigInteger.
BigInteger(int signum, byte[] magnitude) Translates the sign-magnitude representation of a BigInteger into a BigInteger.
BigInteger(int bitLength, int certainty, Random rnd) Constructs a randomly generated positive BigInteger that is probably prime, with the specified bitLength.
BigInteger(int numBits, Random rnd) Constructs a randomly generated BigInteger, uniformly distributed over the range 0 to ($2^{numBits} - 1$), inclusive.
BigInteger(String val) Translates the decimal String representation of a BigInteger into a BigInteger.
BigInteger(String val, int radix) Translates the String representation of a BigInteger in the specified radix into a BigInteger.

Method Summary

Methods

Modifier and Type	Method and Description
BigInteger	abs() Returns a BigInteger whose value is the absolute value of this BigInteger.
BigInteger	add(BigInteger val) Returns a BigInteger whose value is (this + val).
BigInteger	and(BigInteger val) Returns a BigInteger whose value is (this & val).
BigInteger	andNot(BigInteger val) Returns a BigInteger whose value is (this & ~val).
int	bitCount() Returns the number of bits in the two's complement representation of this BigInteger that differ from its sign bit.
int	bitLength() Returns the number of bits in the minimal two's-complement representation of this BigInteger, *excluding* a sign bit.
BigInteger	clearBit(int n)

	Returns a BigInteger whose value is equivalent to this BigInteger with the designated bit cleared.
int	**compareTo**(**BigInteger** val) Compares this BigInteger with the specified BigInteger.
BigInteger	**divide**(**BigInteger** val) Returns a BigInteger whose value is (this / val).
BigInteger[]	**divideAndRemainder**(**BigInteger** val) Returns an array of two BigIntegers containing (this / val) followed by (this % val).
double	**doubleValue**() Converts this BigInteger to a double.
boolean	**equals**(**Object** x) Compares this BigInteger with the specified Object for equality.
BigInteger	**flipBit**(int n) Returns a BigInteger whose value is equivalent to this BigInteger with the designated bit flipped.
float	**floatValue**() Converts this BigInteger to a float.
BigInteger	**gcd**(**BigInteger** val) Returns a BigInteger whose value is the greatest common divisor of abs(this) and abs(val).
int	**getLowestSetBit**() Returns the index of the rightmost (lowest-order) one bit in this BigInteger (the number of zero bits to the right of the rightmost one bit).
int	**hashCode**() Returns the hash code for this BigInteger.
int	**intValue**() Converts this BigInteger to an int.
boolean	**isProbablePrime**(int certainty) Returns true if this BigInteger is probably prime, false if it's definitely composite.
long	**longValue**() Converts this BigInteger to a long.
BigInteger	**max**(**BigInteger** val) Returns the maximum of this BigInteger and val.
BigInteger	**min**(**BigInteger** val) Returns the minimum of this BigInteger and val.
BigInteger	**mod**(**BigInteger** m) Returns a BigInteger whose value is (this mod m).
BigInteger	**modInverse**(**BigInteger** m) Returns a BigInteger whose value is $(this^{-1} \bmod m)$.
BigInteger	**modPow**(**BigInteger** exponent, **BigInteger** m) Returns a BigInteger whose value is $(this^{exponent} \bmod m)$.
BigInteger	**multiply**(**BigInteger** val) Returns a BigInteger whose value is (this * val).
BigInteger	**negate**() Returns a BigInteger whose value is (-this).
BigInteger	**nextProbablePrime**() Returns the first integer greater than this BigInteger that is probably prime.
BigInteger	**not**() Returns a BigInteger whose value is (~this).
BigInteger	**or**(**BigInteger** val) Returns a BigInteger whose value is (this | val).
BigInteger	**pow**(int exponent)

	Returns a BigInteger whose value is $(this^{exponent})$.
static BigInteger	**probablePrime**(int bitLength, Random rnd) Returns a positive BigInteger that is probably prime, with the specified bitLength.
BigInteger	**remainder**(BigInteger val) Returns a BigInteger whose value is (this % val).
BigInteger	**setBit**(int n) Returns a BigInteger whose value is equivalent to this BigInteger with the designated bit set.
BigInteger	**shiftLeft**(int n) Returns a BigInteger whose value is (this << n).
BigInteger	**shiftRight**(int n) Returns a BigInteger whose value is (this >> n).
int	**signum**() Returns the signum function of this BigInteger.
BigInteger	**subtract**(BigInteger val) Returns a BigInteger whose value is (this - val).
boolean	**testBit**(int n) Returns true if and only if the designated bit is set.
byte[]	**toByteArray**() Returns a byte array containing the two's-complement representation of this BigInteger.
String	**toString**() Returns the decimal String representation of this BigInteger.
String	**toString**(int radix) Returns the String representation of this BigInteger in the given radix.
static BigInteger	**valueOf**(long val) Returns a BigInteger whose value is equal to that of the specified long.
BigInteger	**xor**(BigInteger val) Returns a BigInteger whose value is (this ^ val).

Methods inherited from class java.lang.Number

byteValue, shortValue

Methods inherited from class java.lang.Object

clone, finalize, getClass, notify, notifyAll, wait, wait, wait

Field Detail

ZERO

```
public static final BigInteger ZERO
```

The BigInteger constant zero.

Since:

1.2

ONE

MathContext Class

java.math

Class MathContext

java.lang.Object
  java.math.MathContext

All Implemented Interfaces:

Serializable

```
public final class MathContext
extends Object
implements Serializable
```

Immutable objects which encapsulate the context settings which describe certain rules for numerical operators, such as those implemented by the `BigDecimal` class.

The base-independent settings are:

1. `precision`: the number of digits to be used for an operation; results are rounded to this precision
2. `roundingMode`: a `RoundingMode` object which specifies the algorithm to be used for rounding.

Since:

1.5

See Also:

`BigDecimal`, `RoundingMode`, Serialized Form

Field Summary

Fields

Modifier and Type	Field and Description
`static MathContext`	**`DECIMAL128`** A `MathContext` object with a precision setting matching the IEEE 754R Decimal128 format, 34 digits, and a rounding mode of **`HALF_EVEN`**, the IEEE 754R default.
`static MathContext`	**`DECIMAL32`** A `MathContext` object with a precision setting matching the IEEE 754R Decimal32 format, 7 digits, and a rounding mode of **`HALF_EVEN`**, the IEEE 754R default.
`static MathContext`	**`DECIMAL64`** A `MathContext` object with a precision setting matching the IEEE 754R Decimal64 format, 16 digits, and a rounding mode of **`HALF_EVEN`**, the IEEE 754R default.
`static MathContext`	**`UNLIMITED`** A `MathContext` object whose settings have the values required for unlimited precision arithmetic.

Constructor Summary

Constructors

Constructor and Description
MathContext(int setPrecision) Constructs a new MathContext with the specified precision and the **HALF_UP** rounding mode.
MathContext(int setPrecision, **RoundingMode** setRoundingMode) Constructs a new MathContext with a specified precision and rounding mode.
MathContext(**String** val) Constructs a new MathContext from a string.

Method Summary

Methods

Modifier and Type	Method and Description
boolean	**equals**(**Object** x) Compares this MathContext with the specified Object for equality.
int	**getPrecision**() Returns the precision setting.
RoundingMode	**getRoundingMode**() Returns the roundingMode setting.
int	**hashCode**() Returns the hash code for this MathContext.
String	**toString**() Returns the string representation of this MathContext.

Methods inherited from class java.lang.Object

clone, finalize, getClass, notify, notifyAll, wait, wait, wait

Field Detail

UNLIMITED

public static final MathContext UNLIMITED

A MathContext object whose settings have the values required for unlimited precision arithmetic. The values of the settings are: precision=0 roundingMode=HALF_UP

DECIMAL32

public static final MathContext DECIMAL32

A MathContext object with a precision setting matching the IEEE 754R Decimal32 format, 7 digits, and a rounding mode of HALF_EVEN, the IEEE 754R default.

DECIMAL64

public static final MathContext DECIMAL64

A MathContext object with a precision setting matching the IEEE 754R Decimal64 format, 16 digits, and a rounding mode of HALF_EVEN, the IEEE 754R default.

DECIMAL128

```
public static final MathContext DECIMAL128
```

A `MathContext` object with a precision setting matching the IEEE 754R Decimal128 format, 34 digits, and a rounding mode of `HALF_EVEN`, the IEEE 754R default.

Constructor Detail

MathContext

```
public MathContext(int setPrecision)
```

Constructs a new `MathContext` with the specified precision and the `HALF_UP` rounding mode.

Parameters:

`setPrecision` - The non-negative `int` precision setting.

Throws:

`IllegalArgumentException` - if the `setPrecision` parameter is less than zero.

MathContext

```
public MathContext(int setPrecision,
           RoundingMode setRoundingMode)
```

Constructs a new `MathContext` with a specified precision and rounding mode.

Parameters:

`setPrecision` - The non-negative `int` precision setting.

`setRoundingMode` - The rounding mode to use.

Throws:

`IllegalArgumentException` - if the `setPrecision` parameter is less than zero.

`NullPointerException` - if the rounding mode argument is `null`

MathContext

```
public MathContext(String val)
```

Constructs a new `MathContext` from a string. The string must be in the same format as that produced by the `toString()` method.

An `IllegalArgumentException` is thrown if the precision section of the string is out of range (`< 0`) or the string is not in the format created by the `toString()` method.

Parameters:

`val` - The string to be parsed

Throws:

`IllegalArgumentException` - if the precision section is out of range or of incorrect format

`NullPointerException` - if the argument is `null`

Method Detail

getPrecision

```
public int getPrecision()
```

Returns the `precision` setting. This value is always non-negative.

Returns:

an `int` which is the value of the `precision` setting

getRoundingMode

```
public RoundingMode getRoundingMode()
```

Returns the roundingMode setting. This will be one of `RoundingMode.CEILING`, `RoundingMode.DOWN`, `RoundingMode.FLOOR`, `RoundingMode.HALF_DOWN`, `RoundingMode.HALF_EVEN`, `RoundingMode.HALF_UP`, `RoundingMode.UNNECESSARY`, or `RoundingMode.UP`.

Returns:

a `RoundingMode` object which is the value of the `roundingMode` setting

equals

```
public boolean equals(Object x)
```

Compares this `MathContext` with the specified `Object` for equality.

Overrides:

`equals` in class `Object`

Parameters:

`x` - `Object` to which this `MathContext` is to be compared.

Returns:

`true` if and only if the specified `Object` is a `MathContext` object which has exactly the same settings as this object

See Also:

`Object.hashCode()`, `HashMap`

hashCode

```
public int hashCode()
```

Returns the hash code for this `MathContext`.

Overrides:

`hashCode` in class `Object`

Returns:

hash code for this `MathContext`

See Also:

`Object.equals(java.lang.Object)`, `System.identityHashCode(java.lang.Object)`

toString

```
public String toString()
```

Returns the string representation of this `MathContext`. The `String` returned represents the settings of the `MathContext` object as two space-delimited words (separated by a single space character, `'\u0020'`, and with no leading or trailing white space), as follows:

1. The string `"precision="`, immediately followed by the value of the precision setting as a numeric string as if generated by the `Integer.toString` method.
2. The string `"roundingMode="`, immediately followed by the value of the `roundingMode` setting as a word. This word will be the same as the name of the corresponding public constant in the `RoundingMode` enum.

For example:

```
precision=9 roundingMode=HALF_UP
```

Additional words may be appended to the result of `toString` in the future if more properties are added to this class.

Overrides:

`toString` in class `Object`

Returns:

a `String` representing the context settings

Submit a bug or feature
For further API reference and developer documentation, see Java SE Documentation. That documentation contains more detailed, developer-targeted descriptions, with conceptual overviews, definitions of terms, workarounds, and working code examples.
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RoundingMode Class

java.math

Enum RoundingMode

java.lang.Object
  java.lang.Enum<RoundingMode>
    java.math.RoundingMode

All Implemented Interfaces:

Serializable, Comparable<RoundingMode>

```
public enum RoundingMode
extends Enum<RoundingMode>
```

Specifies a *rounding behavior* for numerical operations capable of discarding precision. Each rounding mode indicates how the least significant returned digit of a rounded result is to be calculated. If fewer digits are returned than the digits needed to represent the exact numerical result, the discarded digits will be referred to as the *discarded fraction* regardless the digits' contribution to the value of the number. In other words, considered as a numerical value, the discarded fraction could have an absolute value greater than one.

Each rounding mode description includes a table listing how different two-digit decimal values would round to a one digit decimal value under the rounding mode in question. The result column in the tables could be gotten by creating a `BigDecimal` number with the specified value, forming a `MathContext` object with the proper settings (`precision` set to `1`, and the `roundingMode` set to the rounding mode in question), and calling `round` on this number with the proper `MathContext`. A summary table showing the results of these rounding operations for all rounding modes appears below.

Summary of Rounding Operations Under Different Rounding Modes

	Result of rounding input to one digit with the given rounding mode							
Input Number	UP	DOWN	CEILING	FLOOR	HALF_UP	HALF_DOWN	HALF_EVEN	UNNECESSARY
5.5	6	5	6	5	6	5	6	throw ArithmeticException
2.5	3	2	3	2	3	2	2	throw ArithmeticException
1.6	2	1	2	1	2	2	2	throw ArithmeticException
1.1	2	1	2	1	1	1	1	throw ArithmeticException
1.0	1	1	1	1	1	1	1	1
-1.0	-1	-1	-1	-1	-1	-1	-1	-1
-1.1	-2	-1	-1	-2	-1	-1	-1	throw ArithmeticException
-1.6	-2	-1	-1	-2	-2	-2	-2	throw ArithmeticException
-2.5	-3	-2	-2	-3	-3	-2	-2	throw ArithmeticException
-5.5	-6	-5	-5	-6	-6	-5	-6	throw ArithmeticException

This `enum` is intended to replace the integer-based enumeration of rounding mode constants in `BigDecimal` (`BigDecimal.ROUND_UP`, `BigDecimal.ROUND_DOWN`, etc.).

Since:

1.5

See Also:

`BigDecimal`, `MathContext`

Enum Constant Summary

Enum Constants

Enum Constant and Description
CEILING Rounding mode to round towards positive infinity.
DOWN Rounding mode to round towards zero.
FLOOR Rounding mode to round towards negative infinity.
HALF_DOWN Rounding mode to round towards "nearest neighbor" unless both neighbors are equidistant, in which case round down.
HALF_EVEN Rounding mode to round towards the "nearest neighbor" unless both neighbors are equidistant, in which case, round towards the even neighbor.
HALF_UP Rounding mode to round towards "nearest neighbor" unless both neighbors are equidistant, in which case round up.
UNNECESSARY Rounding mode to assert that the requested operation has an exact result, hence no rounding is necessary.
UP Rounding mode to round away from zero.

Method Summary

Methods

Modifier and Type	Method and Description
static RoundingMode	**valueOf**(int rm) Returns the RoundingMode object corresponding to a legacy integer rounding mode constant in **BigDecimal**.
static RoundingMode	**valueOf**(String name) Returns the enum constant of this type with the specified name.
static RoundingMode[]	**values**() Returns an array containing the constants of this enum type, in the order they are declared.

Methods inherited from class java.lang.Enum

clone, compareTo, equals, finalize, getDeclaringClass, hashCode, name, ordinal, toString, valueOf

Methods inherited from class java.lang.Object

getClass, notify, notifyAll, wait, wait, wait

Enum Constant Detail

UP

```
public static final RoundingMode UP
```

Rounding mode to round away from zero. Always increments the digit prior to a non-zero discarded fraction. Note that this rounding mode never decreases the magnitude of the calculated value.

Example:

Input Number	Input rounded to one digit with UP rounding
5.5	6
2.5	3
1.6	2
1.1	2
1.0	1
-1.0	-1
-1.1	-2
-1.6	-2
-2.5	-3
-5.5	-6

DOWN

```
public static final RoundingMode DOWN
```

Rounding mode to round towards zero. Never increments the digit prior to a discarded fraction (i.e., truncates). Note that this rounding mode never increases the magnitude of the calculated value.

Example:

Input Number	Input rounded to one digit with DOWN rounding
5.5	5
2.5	2
1.6	1
1.1	1
1.0	1
-1.0	-1
-1.1	-1
-1.6	-1
-2.5	-2
-5.5	-5

CEILING

```
public static final RoundingMode CEILING
```

Rounding mode to round towards positive infinity. If the result is positive, behaves as for `RoundingMode.UP`; if negative, behaves as for `RoundingMode.DOWN`. Note that this rounding mode never decreases the calculated value.

Example:

Input Number	Input rounded to one digit with CEILING rounding
5.5	6
2.5	3
1.6	2
1.1	2
1.0	1
-1.0	-1
-1.1	-1
-1.6	-1
-2.5	-2
-5.5	-5

FLOOR

```
public static final RoundingMode FLOOR
```

Rounding mode to round towards negative infinity. If the result is positive, behave as for `RoundingMode.DOWN`; if negative, behave as for `RoundingMode.UP`. Note that this rounding mode never increases the calculated value.

Example:

Input Number	Input rounded to one digit with FLOOR rounding
5.5	5
2.5	2
1.6	1
1.1	1
1.0	1
-1.0	-1
-1.1	-2
-1.6	-2
-2.5	-3
-5.5	-6

HALF_UP

```
public static final RoundingMode HALF_UP
```

Rounding mode to round towards "nearest neighbor" unless both neighbors are equidistant, in which case round up. Behaves as for `RoundingMode.UP` if the discarded fraction is ≥ 0.5; otherwise, behaves as for `RoundingMode.DOWN`. Note that this is the rounding mode commonly taught at school.

Example:

Input Number	Input rounded to one digit with HALF_UP rounding
5.5	6

2.5	3
1.6	2
1.1	1
1.0	1
-1.0	-1
-1.1	-1
-1.6	-2
-2.5	-3
-5.5	-6

HALF_DOWN

```
public static final RoundingMode HALF_DOWN
```

Rounding mode to round towards "nearest neighbor" unless both neighbors are equidistant, in which case round down. Behaves as for `RoundingMode.UP` if the discarded fraction is > 0.5; otherwise, behaves as for `RoundingMode.DOWN`.

Example:

Input Number	Input rounded to one digit with `HALF_DOWN` rounding
5.5	5
2.5	2
1.6	2
1.1	1
1.0	1
-1.0	-1
-1.1	-1
-1.6	-2
-2.5	-2
-5.5	-5

HALF_EVEN

```
public static final RoundingMode HALF_EVEN
```

Rounding mode to round towards the "nearest neighbor" unless both neighbors are equidistant, in which case, round towards the even neighbor. Behaves as for `RoundingMode.HALF_UP` if the digit to the left of the discarded fraction is odd; behaves as for `RoundingMode.HALF_DOWN` if it's even. Note that this is the rounding mode that statistically minimizes cumulative error when applied repeatedly over a sequence of calculations. It is sometimes known as "Banker's rounding," and is chiefly used in the USA. This rounding mode is analogous to the rounding policy used for `float` and `double` arithmetic in Java.

Example:

Input Number	Input rounded to one digit with `HALF_EVEN` rounding
5.5	6
2.5	2

1.6	2
1.1	1
1.0	1
-1.0	-1
-1.1	-1
-1.6	-2
-2.5	-2
-5.5	-6

UNNECESSARY

```
public static final RoundingMode UNNECESSARY
```

Rounding mode to assert that the requested operation has an exact result, hence no rounding is necessary. If this rounding mode is specified on an operation that yields an inexact result, an `ArithmeticException` is thrown.

Example:

Input Number	Input rounded to one digit with UNNECESSARY rounding
5.5	throw ArithmeticException
2.5	throw ArithmeticException
1.6	throw ArithmeticException
1.1	throw ArithmeticException
1.0	1
-1.0	-1
-1.1	throw ArithmeticException
-1.6	throw ArithmeticException
-2.5	throw ArithmeticException
-5.5	throw ArithmeticException

Method Detail

values

```
public static RoundingMode[] values()
```

Returns an array containing the constants of this enum type, in the order they are declared. This method may be used to iterate over the constants as follows:

```
for (RoundingMode c : RoundingMode.values())
    System.out.println(c);
```

Returns:

an array containing the constants of this enum type, in the order they are declared

valueOf

```
public static RoundingMode valueOf(String name)
```

Returns the enum constant of this type with the specified name. The string must match *exactly* an identifier used to declare an enum constant in this type. (Extraneous whitespace characters are not permitted.)

Parameters:

`name` - the name of the enum constant to be returned.

Returns:

the enum constant with the specified name

Throws:

`IllegalArgumentException` - if this enum type has no constant with the specified name

`NullPointerException` - if the argument is null

valueOf

```
public static RoundingMode valueOf(int rm)
```

Returns the `RoundingMode` object corresponding to a legacy integer rounding mode constant in `BigDecimal`.

Parameters:

`rm` - legacy integer rounding mode to convert

Returns:

`RoundingMode` corresponding to the given integer.

Throws:

`IllegalArgumentException` - integer is out of range

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String-Class

java.lang

Class String

java.lang.Object
    java.lang.String

All Implemented Interfaces:

Serializable, CharSequence, Comparable<String>

```
public final class String
extends Object
implements Serializable, Comparable<String>, CharSequence
```

The `String` class represents character strings. All string literals in Java programs, such as `"abc"`, are implemented as instances of this class.

Strings are constant; their values cannot be changed after they are created. String buffers support mutable strings. Because String objects are immutable they can be shared. For example:

```
String str = "abc";
```

is equivalent to:

```
char data[] = {'a', 'b', 'c'};
String str = new String(data);
```

Here are some more examples of how strings can be used:

```
System.out.println("abc");
String cde = "cde";
System.out.println("abc" + cde);
String c = "abc".substring(2,3);
String d = cde.substring(1, 2);
```

The class `String` includes methods for examining individual characters of the sequence, for comparing strings, for searching strings, for extracting substrings, and for creating a copy of a string with all characters translated to uppercase or to lowercase. Case mapping is based on the Unicode Standard version specified by the `Character` class.

The Java language provides special support for the string concatenation operator (+), and for conversion of other objects to strings. String concatenation is implemented through the `StringBuilder`(or `StringBuffer`) class and its `append` method. String conversions are implemented through the method `toString`, defined by `Object` and inherited by all classes in Java. For additional information on string concatenation and conversion, see Gosling, Joy, and Steele, *The Java Language Specification*.

Unless otherwise noted, passing a `null` argument to a constructor or method in this class will cause a `NullPointerException` to be thrown.

A `String` represents a string in the UTF-16 format in which *supplementary characters* are represented by *surrogate pairs* (see the section Unicode Character Representations in the `Character` class for more information). Index values refer to `char` code units, so a supplementary character uses two positions in a `String`.

The `String` class provides methods for dealing with Unicode code points (i.e., characters), in addition to those for dealing with Unicode code units (i.e., `char` values).

Since:

JDK1.0

See Also:

`Object.toString()`, `StringBuffer`, `StringBuilder`, `Charset`, Serialized Form

Field Summary

Fields

Modifier and Type	Field and Description
`static Comparator<String>`	**`CASE_INSENSITIVE_ORDER`** A Comparator that orders `String` objects as by `compareToIgnoreCase`.

Constructor Summary

Constructors

Constructor and Description
`String``()` Initializes a newly created `String` object so that it represents an empty character sequence.
`String``(byte[] bytes)` Constructs a new `String` by decoding the specified array of bytes using the platform's default charset.
`String``(byte[] bytes, Charset charset)` Constructs a new `String` by decoding the specified array of bytes using the specified **charset**.
`String``(byte[] ascii, int hibyte)` **Deprecated.** *This method does not properly convert bytes into characters. As of JDK 1.1, the preferred way to do this is via the `String` constructors that take a **`Charset`**, charset name, or that use the platform's default charset.*
`String``(byte[] bytes, int offset, int length)` Constructs a new `String` by decoding the specified subarray of bytes using the platform's default charset.
`String``(byte[] bytes, int offset, int length, Charset charset)` Constructs a new `String` by decoding the specified subarray of bytes using the specified **charset**.
`String``(byte[] ascii, int hibyte, int offset, int count)` **Deprecated.** *This method does not properly convert bytes into characters. As of JDK 1.1, the preferred way to do this is via the `String` constructors that take a **`Charset`**, charset name, or that use the platform's default charset.*
`String``(byte[] bytes, int offset, int length, String charsetName)` Constructs a new `String` by decoding the specified subarray of bytes using the specified charset.
`String``(byte[] bytes, String charsetName)` Constructs a new `String` by decoding the specified array of bytes using the specified **charset**.
`String``(char[] value)` Allocates a new `String` so that it represents the sequence of characters currently contained in the character array argument.
`String``(char[] value, int offset, int count)` Allocates a new `String` that contains characters from a subarray of the character array argument.
`String``(int[] codePoints, int offset, int count)` Allocates a new `String` that contains characters from a subarray of the **Unicode code point** array argument.
`String``(String original)` Initializes a newly created `String` object so that it represents the same sequence of characters as the argument; in other words, the newly created string is a copy of the argument string.
`String``(StringBuffer buffer)` Allocates a new string that contains the sequence of characters currently contained in the string buffer argument.
`String``(StringBuilder builder)`

Allocates a new string that contains the sequence of characters currently contained in the string builder argument.

Method Summary

Methods

Modifier and Type	Method and Description
char	**charAt**(int index) Returns the char value at the specified index.
int	**codePointAt**(int index) Returns the character (Unicode code point) at the specified index.
int	**codePointBefore**(int index) Returns the character (Unicode code point) before the specified index.
int	**codePointCount**(int beginIndex, int endIndex) Returns the number of Unicode code points in the specified text range of this String.
int	**compareTo**(String anotherString) Compares two strings lexicographically.
int	**compareToIgnoreCase**(String str) Compares two strings lexicographically, ignoring case differences.
String	**concat**(String str) Concatenates the specified string to the end of this string.
boolean	**contains**(CharSequence s) Returns true if and only if this string contains the specified sequence of char values.
boolean	**contentEquals**(CharSequence cs) Compares this string to the specified CharSequence.
boolean	**contentEquals**(StringBuffer sb) Compares this string to the specified StringBuffer.
static String	**copyValueOf**(char[] data) Returns a String that represents the character sequence in the array specified.
static String	**copyValueOf**(char[] data, int offset, int count) Returns a String that represents the character sequence in the array specified.
boolean	**endsWith**(String suffix) Tests if this string ends with the specified suffix.
boolean	**equals**(Object anObject) Compares this string to the specified object.
boolean	**equalsIgnoreCase**(String anotherString) Compares this String to another String, ignoring case considerations.
static String	**format**(Locale l, String format, Object... args) Returns a formatted string using the specified locale, format string, and arguments.
static String	**format**(String format, Object... args) Returns a formatted string using the specified format string and arguments.
byte[]	**getBytes**() Encodes this String into a sequence of bytes using the platform's default charset, storing the result into a new byte array.
byte[]	**getBytes**(Charset charset) Encodes this String into a sequence of bytes using the given **charset**, storing the result into a new byte array.
void	**getBytes**(int srcBegin, int srcEnd, byte[] dst, int dstBegin) **Deprecated.** *This method does not properly convert characters into bytes. As of JDK 1.1, the preferred way to do this is via the **getBytes()** method, which uses the platform's default charset.*
byte[]	**getBytes**(String charsetName)

	Encodes this `String` into a sequence of bytes using the named charset, storing the result into a new byte array.
`void`	**`getChars`**`(int srcBegin, int srcEnd, char[] dst, int dstBegin)` Copies characters from this string into the destination character array.
`int`	**`hashCode`**`()` Returns a hash code for this string.
`int`	**`indexOf`**`(int ch)` Returns the index within this string of the first occurrence of the specified character.
`int`	**`indexOf`**`(int ch, int fromIndex)` Returns the index within this string of the first occurrence of the specified character, starting the search at the specified index.
`int`	**`indexOf`**`(String str)` Returns the index within this string of the first occurrence of the specified substring.
`int`	**`indexOf`**`(String str, int fromIndex)` Returns the index within this string of the first occurrence of the specified substring, starting at the specified index.
`String`	**`intern`**`()` Returns a canonical representation for the string object.
`boolean`	**`isEmpty`**`()` Returns `true` if, and only if, **`length()`** is `0`.
`int`	**`lastIndexOf`**`(int ch)` Returns the index within this string of the last occurrence of the specified character.
`int`	**`lastIndexOf`**`(int ch, int fromIndex)` Returns the index within this string of the last occurrence of the specified character, searching backward starting at the specified index.
`int`	**`lastIndexOf`**`(String str)` Returns the index within this string of the last occurrence of the specified substring.
`int`	**`lastIndexOf`**`(String str, int fromIndex)` Returns the index within this string of the last occurrence of the specified substring, searching backward starting at the specified index.
`int`	**`length`**`()` Returns the length of this string.
`boolean`	**`matches`**`(String regex)` Tells whether or not this string matches the given **regular expression**.
`int`	**`offsetByCodePoints`**`(int index, int codePointOffset)` Returns the index within this `String` that is offset from the given `index` by `codePointOffset` code points.
`boolean`	**`regionMatches`**`(boolean ignoreCase, int toffset, String other, int ooffset, int len)` Tests if two string regions are equal.
`boolean`	**`regionMatches`**`(int toffset, String other, int ooffset, int len)` Tests if two string regions are equal.
`String`	**`replace`**`(char oldChar, char newChar)` Returns a new string resulting from replacing all occurrences of `oldChar` in this string with `newChar`.
`String`	**`replace`**`(CharSequence target, CharSequence replacement)` Replaces each substring of this string that matches the literal target sequence with the specified literal replacement sequence.
`String`	**`replaceAll`**`(String regex, String replacement)` Replaces each substring of this string that matches the given **regular expression** with the given replacement.
`String`	**`replaceFirst`**`(String regex, String replacement)` Replaces the first substring of this string that matches the given **regular expression** with the given replacement.
`String``[]`	**`split`**`(String regex)`

	Splits this string around matches of the given **regular expression**.
`String[]`	**`split`**`(String regex, int limit)` Splits this string around matches of the given **regular expression**.
`boolean`	**`startsWith`**`(String prefix)` Tests if this string starts with the specified prefix.
`boolean`	**`startsWith`**`(String prefix, int toffset)` Tests if the substring of this string beginning at the specified index starts with the specified prefix.
`CharSequence`	**`subSequence`**`(int beginIndex, int endIndex)` Returns a new character sequence that is a subsequence of this sequence.
`String`	**`substring`**`(int beginIndex)` Returns a new string that is a substring of this string.
`String`	**`substring`**`(int beginIndex, int endIndex)` Returns a new string that is a substring of this string.
`char[]`	**`toCharArray`**`()` Converts this string to a new character array.
`String`	**`toLowerCase`**`()` Converts all of the characters in this `String` to lower case using the rules of the default locale.
`String`	**`toLowerCase`**`(Locale locale)` Converts all of the characters in this `String` to lower case using the rules of the given `Locale`.
`String`	**`toString`**`()` This object (which is already a string!) is itself returned.
`String`	**`toUpperCase`**`()` Converts all of the characters in this `String` to upper case using the rules of the default locale.
`String`	**`toUpperCase`**`(Locale locale)` Converts all of the characters in this `String` to upper case using the rules of the given `Locale`.
`String`	**`trim`**`()` Returns a copy of the string, with leading and trailing whitespace omitted.
`static String`	**`valueOf`**`(boolean b)` Returns the string representation of the `boolean` argument.
`static String`	**`valueOf`**`(char c)` Returns the string representation of the `char` argument.
`static String`	**`valueOf`**`(char[] data)` Returns the string representation of the `char` array argument.
`static String`	**`valueOf`**`(char[] data, int offset, int count)` Returns the string representation of a specific subarray of the `char` array argument.
`static String`	**`valueOf`**`(double d)` Returns the string representation of the `double` argument.
`static String`	**`valueOf`**`(float f)` Returns the string representation of the `float` argument.
`static String`	**`valueOf`**`(int i)` Returns the string representation of the `int` argument.
`static String`	**`valueOf`**`(long l)` Returns the string representation of the `long` argument.
`static String`	**`valueOf`**`(Object obj)` Returns the string representation of the `Object` argument.

Methods inherited from class java.lang.Object

`clone`, `finalize`, `getClass`, `notify`, `notifyAll`, `wait`, `wait`, `wait`

8.3 Emacs Configuration-Competition

```
;; Default Font: Courier 10 Pitch Bold    Size: 15
;; Remember to set CUA-mode and save your options.
(global-set-key (kbd "M-n") 'forward-paragraph)
(global-set-key (kbd "M-p") 'backward-paragraph)
(global-linum-mode t)
(defun compile-cpp ()
  (interactive)
  (compile (format "g++ -o %s %s -g -lm -Wall -std=c++11"  (file-name-sans-extension
  ↪  (buffer-name))(buffer-name))))
(global-set-key (kbd "<f9>") 'compile-cpp)
(global-set-key (kbd "<f8>") 'gud-gdb)
(setq default-tab-width 4)
(setq c-basic-offset 4)
(global-set-key (kbd "RET") 'newline-and-indent)
```