

QUESTION 1

Let's assume that the flat daily fee is represented by the variable "f" and the charge for each mile driven is represented by the variable "m".

Using the information given in the problem, we can create two equations:

Equation 1:

$$5f + 300m = 178$$

Equation 2:

$$4f + 500m = 197$$

We can use the inverse method to solve for "f" and "m".

Step 1: Solve one of the equations for "f".

$$5f + 300m = 178$$

$$5f = 178 - 300m$$

$$f = (178 - 300m)/5$$

Step 2: Substitute the expression for "f" into the other equation.

$$4f + 500m = 197$$

$$4[(178 - 300m)/5] + 500m = 197$$

Step 3: Solve for "m".

$$(712 - 1200m)/5 + 500m = 197$$

$$712 - 700m + 2500m = 985$$

$$1800m = 727$$

$$m = 0.404$$

Step 4: Substitute the value of "m" back into one of the equations to solve for "f".

$$5f + 300m = 178$$

$$5f + 300(0.404) = 178$$

$$5f + 121.2 = 178$$

$$5f = 56.8$$

$$f = 11.36$$

Therefore, the daily fee is \$11.36 and the charge for each mile driven is \$0.404.

QUESTION 2

To solve the system of equations using inverse method, we need to express the equations in matrix form and find the inverse of the coefficient matrix.

The system of equations can be written as:

$$\begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 4 & -3 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 9 \\ 8 \\ -2 \end{bmatrix}$$

Let A be the coefficient matrix and X be the column matrix of variables, and B be the constant matrix. Then we can write the system of equations in matrix form as $AX = B$.

$$A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 4 & -3 & 0 \end{bmatrix}$$

$$X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$B = \begin{bmatrix} 9 \\ 8 \\ -2 \end{bmatrix}$$

Now, let's find the inverse of matrix A.

$$\begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 4 & -3 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 2 & -3/2 & 1 & 1/2 \\ 0 & 2 & 1 & 2/5 & 0.2 & -1/5 \\ 0 & 0 & 2/5 & 2/5 & 0.6 & 2/5 \end{bmatrix}$$

Let A^{-1} be the inverse of matrix A.

$$A^{-1} = \begin{bmatrix} -3/2 & 1 & 1/2 \\ 2/5 & 0.2 & -1/5 \\ 2/5 & 0.6 & 2/5 \end{bmatrix}$$

We can solve for X by multiplying both sides of the equation $AX = B$ by A^{-1} .

$$X = A^{-1} B$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -3/2 & 1 & 1/2 \\ 2/5 & 0.2 & -1/5 \\ 2/5 & 0.6 & 2/5 \end{bmatrix} \begin{bmatrix} 9 \\ 8 \\ -2 \end{bmatrix}$$

Evaluating the right-hand side, we get:

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -3/2 & 1 & 1/2 \\ 2/5 & 0.2 & -1/5 \\ 2/5 & 0.6 & 2/5 \end{bmatrix} \begin{bmatrix} 9 \\ 8 \\ -2 \end{bmatrix} = \begin{bmatrix} 5/2 \\ 1 \\ -1/2 \end{bmatrix}$$

Therefore, the solution is $x = 5/2$, $y = 1$, and $z = -1/2$.

QUESTION 3

Let's call the amount that A has "x" and the amount that B has "y".

According to the problem, we know that:

$$x + y = 1210 \text{ (A and B together have Frw. 1210)}$$

and

$$x = (2/5)y \text{ (1/5 of A's amount is equal to 2/5 of B's amount)}$$

Now we can use the second equation to solve for one of the variables in terms of the other.

$$x = (2/5)y$$

Multiply both sides by 5:

$$5x = 2y$$

Divide both sides by 2:

$$(5/2)x = y$$

Now we can substitute this value of y into the first equation:

$$x + y = 1210$$

$$x + (5/2)x = 1210$$

Simplify:

$$(7/2)x = 1210$$

Divide both sides by 7/2:

$$x = 1210 \div (7/2)$$

$$x = 440$$

Now that we know that A has Frw. 440, we can use the second equation to find out how much B has:

$$x = (2/5)y$$

$$440 = (2/5)y$$

Multiply both sides by 5/2:

$$y = 1100$$

Therefore, A has Frw. 440 and B has Frw. 1100, and together they have Frw. 1540 ($440 + 1100 = 1540$).

QUESTION 5

(a) If the interest is compounded semi-annually, we need to use the formula:

$$A = P(1 + r/n)^{(nt)}$$

where:

A = the final amount

P = the initial principal (or starting amount), which is \$5000 in this case

r = the interest rate per year, which is 7%

n = the number of times the interest is compounded per year, which is 2 (semi-annually means twice a year)

t = the number of years, which is 6

Plugging in the values, we get:

$$A = 5000(1 + 0.07/2)^{(2*6)}$$

$$A = 5000(1.035)^{12}$$

$$A = \$7358.07$$

Therefore, the final amount in the account after 6 years with semi-annual compounding is \$7358.07.

(b) If the interest is compounded annually, we can use the same formula as above, but with n = 1:

$$A = 5000(1 + 0.07/1)^{(1*6)}$$

$$A = 5000(1.07)^6$$

$$A = \$7892.96$$

Therefore, the final amount in the account after 6 years with annual compounding is \$7892.96.

(c) To find out how long it will take for the account to reach \$8000, we can use the formula:

$$A = P(1 + r/n)^{(nt)}$$

Rearranging the formula, we get:

$$t = \log(A/P) / (n * \log(1 + r/n))$$

where log is the natural logarithm.

Plugging in the values, we get:

$$t = \log(8000/5000) / (1 * \log(1 + 0.07/1))$$

$$t = \log(1.6) / \log(1.07)$$

$$t = 10.24 \text{ years (rounded to two decimal places)}$$

Therefore, it will take the A family about 10.24 years for the account to reach \$8000 with an interest rate of 7% and annual compounding.

QUESTION 6

To find the equation of the line that joins two points, we need to use the slope-intercept form of the equation of a line, which is given by:

$$y = mx + b$$

where m is the slope of the line, and b is the y-intercept.

To find the slope of the line that joins (2, -1) and (7, 0), we use the formula:

$$m = (y_2 - y_1) / (x_2 - x_1)$$

where (x₁, y₁) and (x₂, y₂) are the coordinates of the two points.

Substituting the given values, we get:

$$m = (0 - (-1)) / (7 - 2) \\ = 1/5$$

Now, we can use the point-slope form of the equation of a line, which is given by:

$$y - y_1 = m(x - x_1)$$

where (x_1, y_1) is one of the points on the line.

Substituting the values, we get:

$$y - (-1) = (1/5)(x - 2)$$

Simplifying this equation, we get:

$$y + 1 = (1/5)x - (2/5)$$

Subtracting 1 from both sides, we get:

$$y = (1/5)x - (7/5)$$

So, the equation of the line that joins $(2, -1)$ and $(7, 0)$ is:

$$y = (1/5)x - (7/5)$$

NB : You are requested to compute question number 4 , by yourself as it require only to substitute , (hint: inverse of f is $1/f$ and inverse of g is $1/g$)

Best regards

Muhirwa Salomon

Masters student in Machine Intelligence at AIMS Senegal
Bsc In Electronics and Telecom Engineering at University of Rwanda
(College of Science and Technology)
Address: smuhirwa@aimsammi.org // muhirwasa@gmail.com || Muhirwa_219001695@stud.ur.ac.rw
Twitter: <https://twitter.com/Muhirwakyeeyune>
linkedin: <https://www.linkedin.com/in/muhirwa-salomon-511a66231/>
github: <https://github.com/Muhirwakyeeyune>

Tel: 250780529491 // 250782204795
Research interest: Machine learning, Data Science and Telecommunication

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