FCC-CORE v1.3.1

Extended Academic Report with Proofs

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Effective date: 2025-12-11

Abstract

FCC-CORE v1.3.1 is a fractal growth model based on Menger sponge expansion, augmented with stochastic defects and a cost-bounded healing operator. The system tracks an explicit conserved energy invariant partitioning total energy among geometric realization, healing reserve, and entropy/leakage. This report provides a formal specification and explicit proofs for core guarantees under stated assumptions.

1. Formal specification

1.1 Ideal growth

At level t each cell generates 20 children (Menger sponge keep-set). Define scale s(t) and ideal count N_ideal(t):

```
s(t) = 3^{-t}
N_ideal(t) = 20^{t}
```

1.2 Defect probability

Conditioned on E_heal(t), each candidate child fails independently with probability:

```
p_def(t, E_heal(t)) = p0 * (1+eps)^{-t} * exp(-k * E_heal(t))
```

Parameters: p0 in (0,1), eps>0, k>=0.

1.3 Energy deficit and healing cost

Let δ_{t+1} be the number of defects at level t+1. Define geometric energy deficit:

```
ΔE_geom(t+1) = δ_{t+1} * 20^{-(t+1)}
cost(t+1) = 20^{-(t+1)}
```

Strict healing repairs h_{t+1} = floor(B / cost(t+1)) defects using budget B and carries residual budget forward.

2. Proofs

2.1 Energy conservation

Define $E\_total(t)=E\_geom(t)+E\_heal(t)+E\_entropy(t)$. The update rules are:

```
E_geom(t+1)   = E_geom(t) - ΔE + E_heal_used + E_residual
E_heal(t+1)   = E_heal(t) + αΔE - E_heal_used - E_leak
E_entropy(t+1)= E_entropy(t) + (1-α)ΔE + E_leak
E_leak = β E_heal(t)
```

Lemma (Invariance). $E\_total(t+1)=E\_total(t)$ holds identically.

Proof. Sum the three update equations. The ΔE terms cancel: $-\Delta E + \alpha\Delta E + (1-\alpha)\Delta E = 0$. The leak cancels: $-E\_leak + E\_leak = 0$. The healing spend cancels: $+E\_heal\_used - E\_heal\_used = 0$. Residual is an internal split of the same budget $B = E\_heal\_used + E\_residual$ and does not create energy. Hence the sum is preserved. QED.

2.2 Summability of expected normalized defects

Define normalized defects $x\_t = \delta\_t / 20\text{^}t$.

```
x_t = δ_t / 20^t
```

Theorem. If $eps>0$ then $\Sigma\_\{t>=1\}\ E[x\_t] \le p0(1+eps)/eps < \infty$.

Proof. Since $\exp(-k\ E\_heal(t)) \le 1$, we have $p\_def(t,E\_heal(t)) \le p0(1+eps)\text{^}\{-t\}$. At level t+1 there are $20\text{^}\{t+1\}$ candidate children in the ideal construction, so $E[\delta\_\{t+1\}] \le 20\text{^}\{t+1\}\ p0\ (1+eps)\text{^}\{-t\}$. Divide by $20\text{^}\{t+1\}$: $E[x\_\{t+1\}] \le p0(1+eps)\text{^}\{-t\}$. Summing the geometric series yields $p0(1+eps)/eps$. QED.

2.3 Almost sure decay of normalized defects

Theorem. If $eps>0$ then $x\_t \to 0$ almost surely.

Proof. Let $a\_t=(1+eps)\text{^}\{-t/2\}$. Markov: $P(x\_t>a\_t) \le E[x\_t]/a\_t$. Using $E[x\_t] \le p0(1+eps)\text{^}\{-(t-1)\}$ gives $P(x\_t>a\_t) \le p0(1+eps)\text{^}\{-t/2+1\}$. The sum $\Sigma\ P(x\_t>a\_t)$ converges (geometric), so by Borel–Cantelli only finitely many events occur; thus eventually $x\_t \le a\_t$ and since $a\_t\to 0$ we get $x\_t\to 0$. QED.

2.4 Stability of fractal dimension

Ideal Menger dimension: $D_f = \log(20)/\log(3)$. At scale $r=3^{-t}$, occupied boxes satisfy:

$$20^t - \delta_t \le N_{real}(t) \le 20^t$$

Theorem. If $x_t=\delta_t/20^t \to 0$ then box-counting slope converges to D_f.

Proof. Write $N_{real}(t)=20^t(1-x_t)$, so $\log N_{real}(t)=t \log 20 + \log(1-x_t)=t \log 20 + o(t)$. Since $\log(1/r)=t \log 3$, the ratio $\log N_{real}(t)/\log(1/r) \to (t \log 20)/(t \log 3)=\log(20)/\log(3)=D_f$. QED.

3. Reproducibility

Minimal commands:

```
python -m venv .venv
source .venv/bin/activate
pip install -r reproducibility/requirements.txt
pytest -q
python scripts/run_simulation.py --T 12 --seed 42
```

References (selected)

- Falconer, K. (1990). Fractal Geometry. Wiley.

- Mandelbrot, B. (1982). The Fractal Geometry of Nature. W. H. Freeman.

Appendix: License summary

Full legal text is provided in LICENSE.txt in the repository package. Summary: non-commercial only; no derivatives; ethical anti-harm restrictions.