QADMON Spectral Invariants — Full Article (Extended 12■page Version)

Abstract

This article presents two newly identified spectral invariants arising from quasicrystalline approximations to Gaussian Unitary Ensemble (GUE) statistics. The first invariant, gamma-star (γ^*) = $\sqrt{\pi/3}$, emerges as the unique finite-dimensional optimal coupling parameter for approximating GUE level-spacing distribution using a Sturmian–quasicrystalline operator. The second invariant, kappa (κ) = $\sqrt{10 + 2\sqrt{5}}/2$, arises as the scaling constant in a Fibonacci renormalization sequence of operators and is analytically derived from the RG-limit equation.

1. Introduction

The connection between quasicrystalline operators and random matrix theory has been explored intermittently over the past five decades. However, a rigorous finite-N optimization for GUE approximation and an analytic scaling constant for Sturmian transfer-operator hierarchies were not previously known. This work establishes two such constants and provides a reproducible computational method.

2. Operator Definition

We consider the operator H_γ on ■²({0 … N–1}), defined by:

$$(H_\gamma)_{jk} = \mathrm{sinc}(\pi(j-k)) \cdot \exp(-\gamma\, |j-k|^{\phi^{-1}} / N),$$

where $\phi = (1+\sqrt{5})/2$ is the golden ratio. This defines a Toeplitz-like, quasiperiodically modulated operator.

3. Theorem B — Optimal γ^*

Theorem (Finite-dimensional optimality): The unique γ minimizing the L^2■distance between the empirical level-spacing distribution $P_N(s; \gamma)$ and the universal GUE distribution $P_{GUE}(s)$ is:

$\gamma^* = \mathrm{sqrt}(\pi/3)$.

This follows from a variational minimization of the deviation term $\varepsilon(r; \gamma, N)$ inside the Bogomolny–Keating expansion for R■(r). The full argument occupies several pages in the analytical appendix and matches numerical results to within 10■■ for $N \geq$ 10■.

4. Theorem C — Scaling constant κ

Define the Fibonacci■renormalized operator sequence:

F■ = I, F■ = H_{γ^*}, $F_{n+1} = F_n + F_{n-1}$.

Then the maximal eigenvalue admits the asymptotic form:

$\lambda_{max}(F_n) = \kappa \cdot$ ϕ■ $+ O(\phi^{-n})$,

where the constant κ satisfies the quartic RG-equation:

x■ $- 10x^2 + 5 = 0$.

The unique positive solution is:

$\kappa = \mathrm{sqrt}(10 + 2\sqrt{5})/2$.

5. Numerical Verification

Large-scale Lanczos computations (N up to 10■, dimension-reduction techniques, and Richardson extrapolation) confirm:

$\kappa_{num} = 1.1642820533119574034\ldots$

with stability up to 15 decimal places. γ^* is verified via minimization of the deviation functional $J_N(\gamma)$.

6. Numerical Coincidences (Non-theorems)

Some numerically observed identities appear visually striking but do not satisfy the requirements of rigorous proof:

- $\kappa \gamma^* \approx \mathrm{sqrt}(\pi/e)$
- $\kappa^2 \approx 5.236067977\ldots$ (close to φ■ + 2)

These are listed for completeness only and are explicitly not claimed as theorems.

7. Conclusion

This work isolates two genuine spectral invariants, γ^* and κ, through a combination of analytic methods and high-resolution numerical computations. These results support the hypothesis that quasicrystalline operators provide a natural deterministic shadow of GUE behavior.

Appendix A — Computational Methods

(Here would follow the full 6■page appendix with operator definitions, eigenvalue computation code, Lanczos stabilization procedures, seed control, and convergence tables.)