Sorting

Dirty tricks to sort faster than *O(n log n)*

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Sorting

- We can sort any kind of element for which we have a similarity or distance measure between any two elements (subject to triangle inequality property*)
- Traditional sorting algorithms: bubble sort, merge sort, quicksort
- Dirty tricks: pigeonhole sort, bucket sort can often sort in O(n)
- Really dirty trick: nested bucket sort
- What's the fastest we could ever sort *n* numbers?
  - It depends on whether we're stuck using comparisons only
- Sorting notebook https://github.com/parrt/msds689/blob/master/notes/sorting.ipynb

*https://en.wikipedia.org/wiki/Triangle_inequality

Bubble sort

- $O(n^2)$
- *Stable*: order of equal elements doesn't change
- **Idea**: look for out-of-order elements and then keep swapping until nothing changes

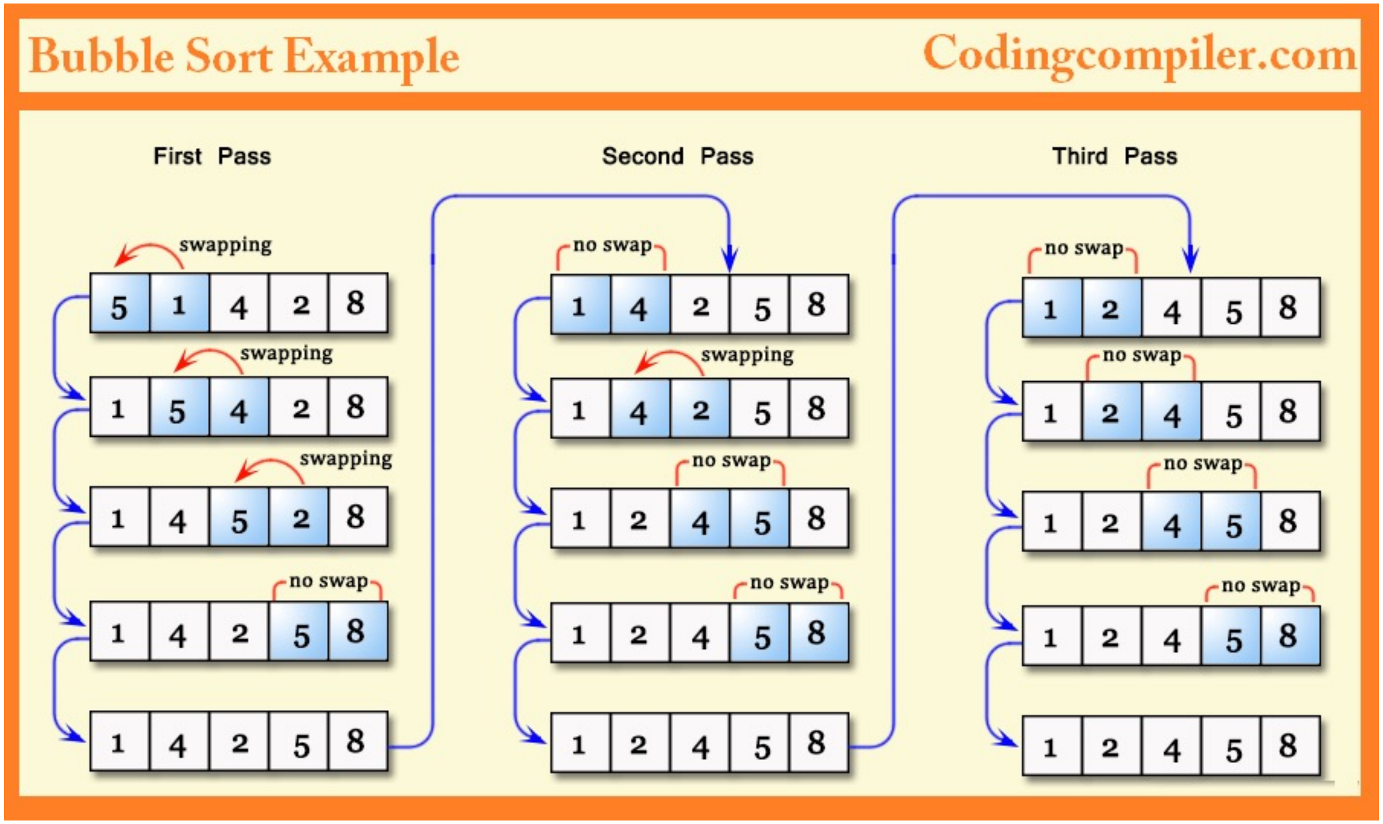
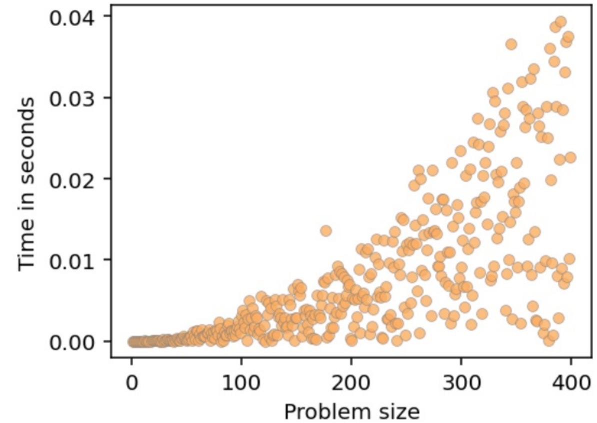


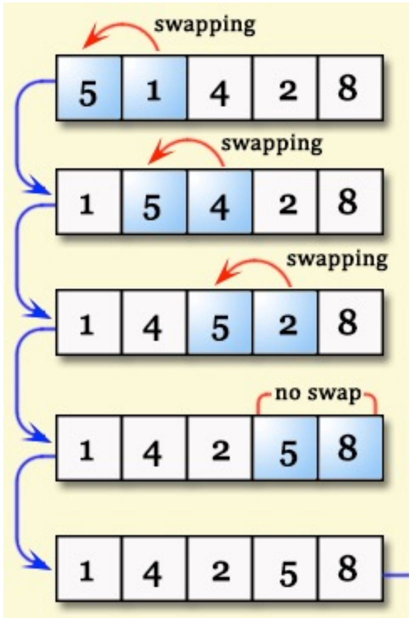
Image credit https://codingcompiler.com/bubble-sort-program-in-c-using-function/

Bubble sort in Python

```python
changed=True
second_to_last_idx = len(A)-2
while changed:
  changed=False
  for i in range(second_to_last_idx+1):
    if A[i] > A[i+1]:
      A[i], A[i+1] = A[i+1], A[i]
      changed=True
```

Why is this $O(n^2)$?
(hint: What is worst case order in array?)





Sorting notebook https://github.com/parrt/msds689/blob/master/notes/sorting.ipynb

Merge sort (review)

- Faster than bubblesort: *O(n log n)*
- Simpler too, if you are comfortable with recursion
- It's stable
- Not in-place, uses lots of extra storage (sort halves)
- **Idea**: split currently active region in half, sorting both the left and right subregions, then merge two sorted subregions
- Eventually, the regions are so small we can sort in constant time; i.e., sorting 2 nums is easy
- Merging two sorted lists can be done in linear time

Quicksort, another divide and conquer sort

- $O(n^2)$ worst-case behavior but O(*n log n*) typical behavior
- **Idea**: pick pivot, partition so elements left of pivot are less than pivot and elements right are greater (not sorting here); recursively partition the left and right until small enough to sort trivially
- Picks a pivot element, rather than just split in half like mergesort
- Faster than bubble because it moves elements more than just one spot in the array
- Quicksort is in-place whereas merge sort makes lots of temporary arrays, which can get expensive
- Quicksort is mostly faster than merge sort due to the constant in front of the complexity (memory allocation, hardware efficiencies, …)

In-place quicksort in notebook https://github.com/parrt/msds689/blob/master/notes/sorting.ipynb

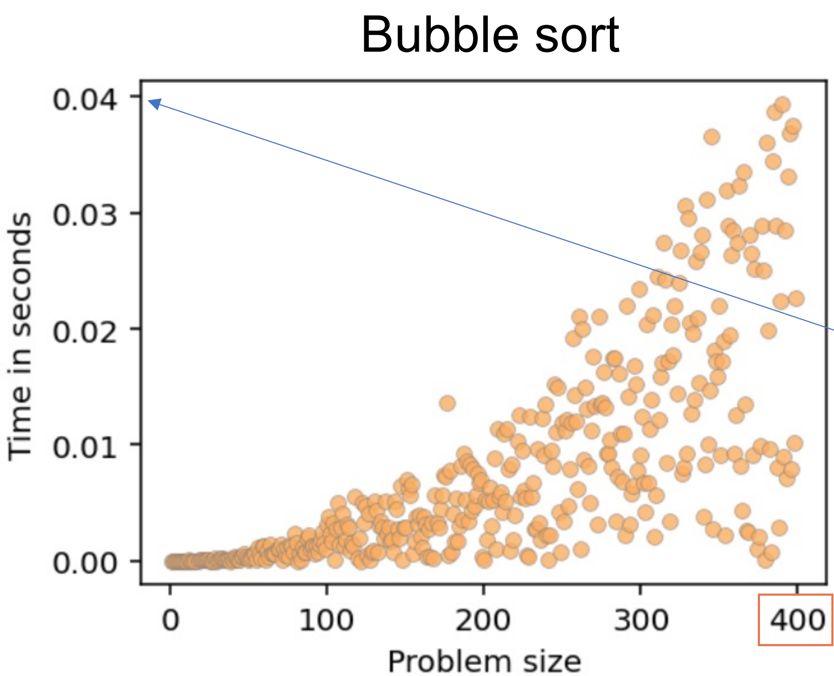
Quicksort algorithm

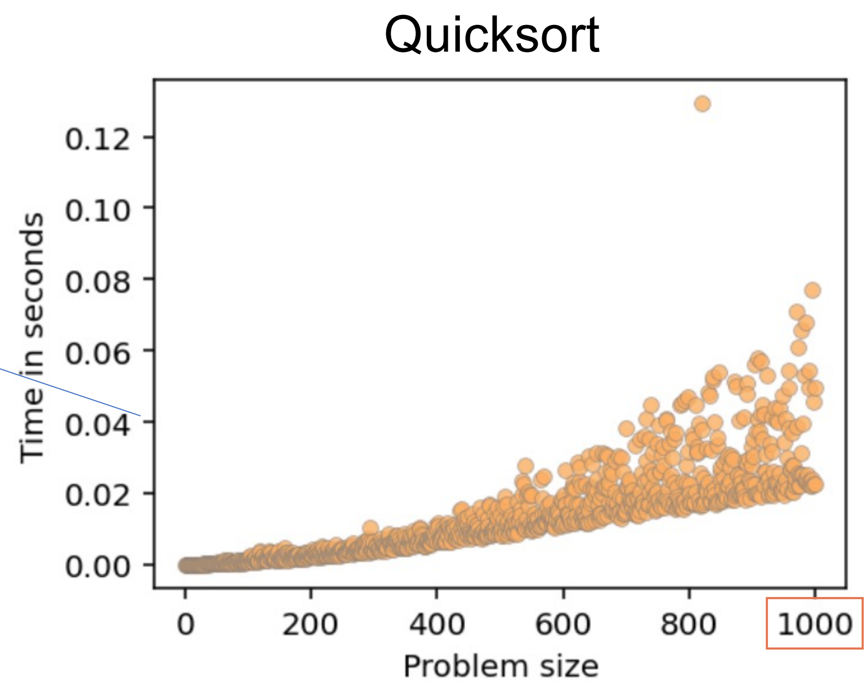
```
def qsort(A, lo=0, hi=len(A)-1):
    if lo >= hi:
       return
    pivot_idx = partition(A,lo,hi)
    qsort(A, lo, pivot_idx-1)
    qsort(A, pivot_idx+1, hi)
```

```
# many ways to do this; here’s a slow O(n) one
# breaks idea of in-place for qsort
def partition(A,lo,hi):
  pivot = A[hi]  # pick last element as pivot
  left = [a for a in A if a<pivot]
  right = [a for a in A if a>pivot]
  A[lo:hi+1] = left+[pivot]+right # copy back
  return len(left) # return index of pivot
```

Video on partitioning: https://www.youtube.com/watch?v=MZaf_9IZCrc

Compare bubble, quicksort





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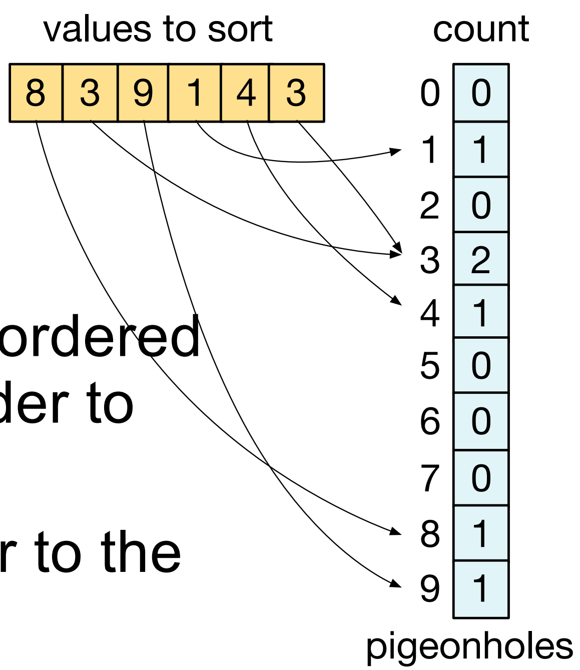
So much for traditional sorts

- Theory says we can't beat O(n log n)…
- …for generic elements and doing comparisons
- But, what if we know the elements are ints or strings or floats?
- What if we know something about the values?
- E.g., what if we know the elements are ints in range 0..99?
- How can we sort those numbers in less than O(n log n)?

Pigeonhole sort



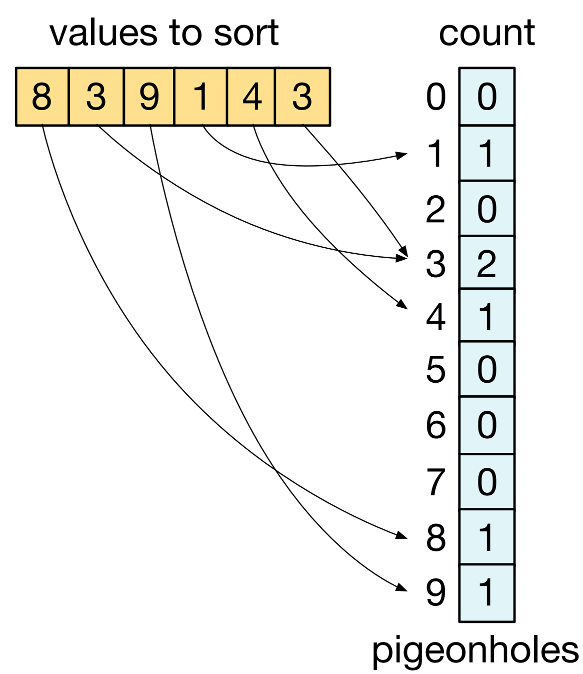
- **Idea**: Map each key to unique pigeonhole in an ordered range of holes; then just walk pigeonholes in order to get sorted elements
- Works best when the range of keys, *m*, is similar to the number of elements, n; why is that?
- T(n,m) = n + m
- This should smack of perfect hashing to you!

https://en.wikipedia.org/wiki/Pigeonhole_sort

Pigeonhole sort algorithm

```
# fill holes
size = max(A) + 1
holes = [0] * size
for a in A:
    holes[a] += 1

# pull out in order
A_ = []
for i in range(0,size):
   A_.extend([i] * holes[i])
```



See sorting notebook

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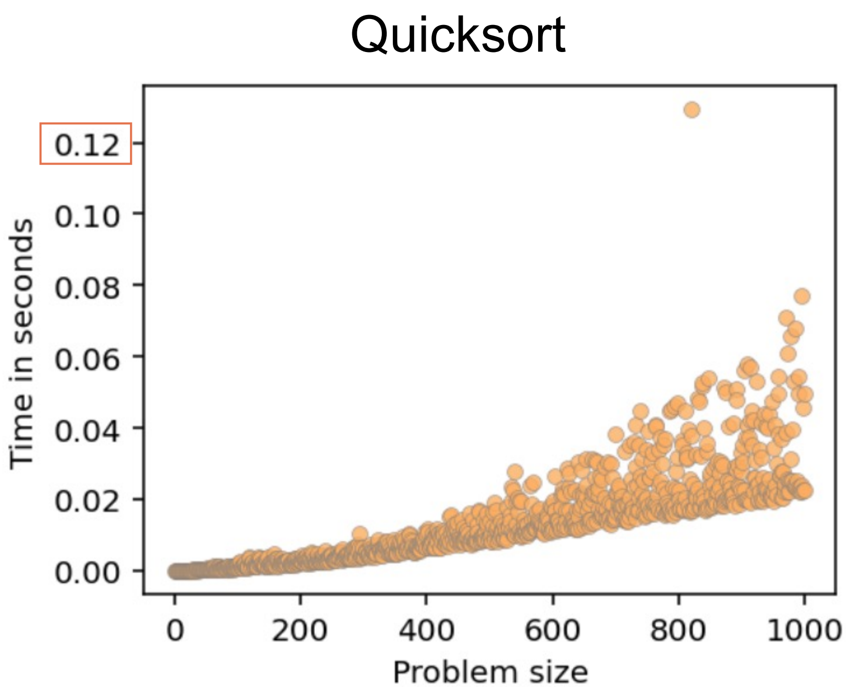
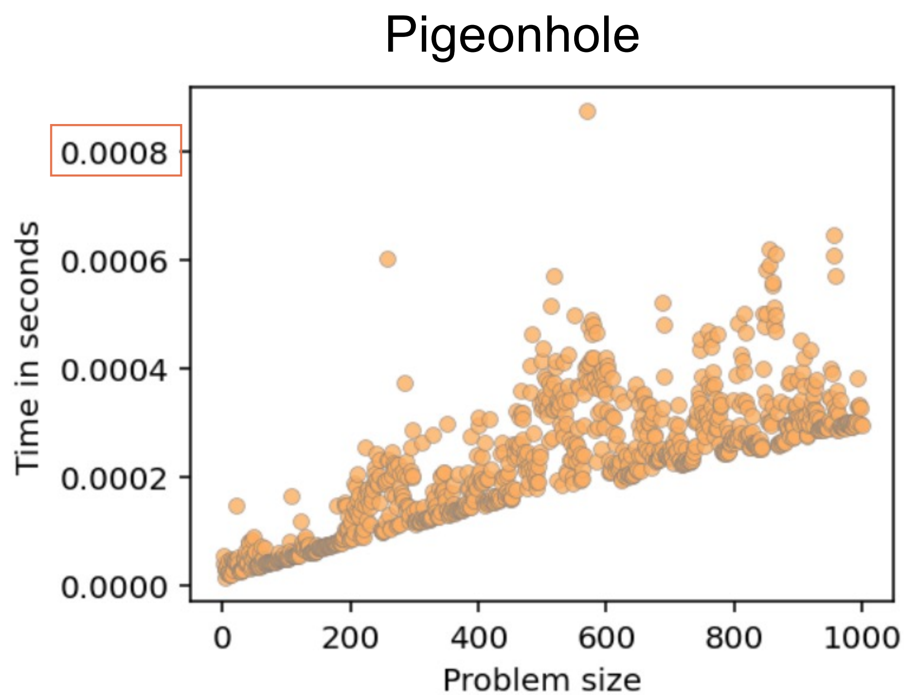
Compare quicksort, pigeonhole



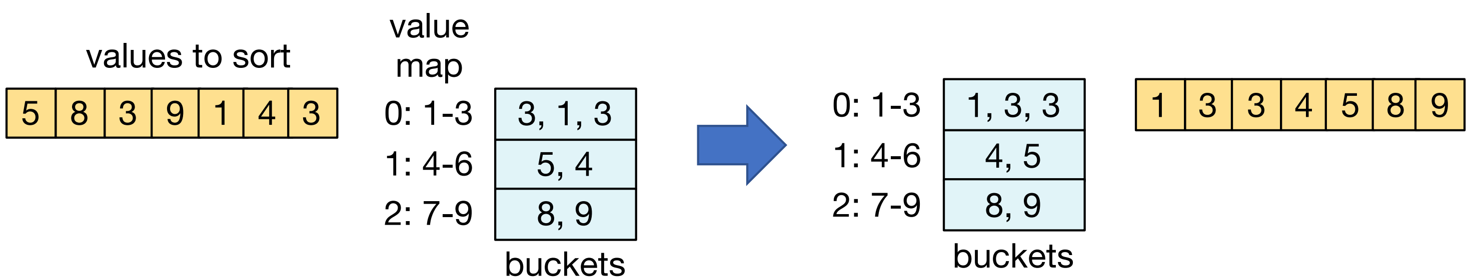


Issue with pigeonhole sort

- Super fast and simple but…
- What do we do when *m* >> *n*? E.g., sort 2 numbers, 5 and 5 million. Takes $T(n,m) = n + m = 5 + 5{,}000{,}000$
- How can we handle this case & generalize to work for floats too?
- Hint: compress *m* to some fixed number of buckets instead of range of numbers
- Now we have hash table but with special hash function

Bucket sort (also called bin sort)

- Idea: distribute *n* elements across *m* buckets, sort elements within buckets, then concatenate elements from buckets in order



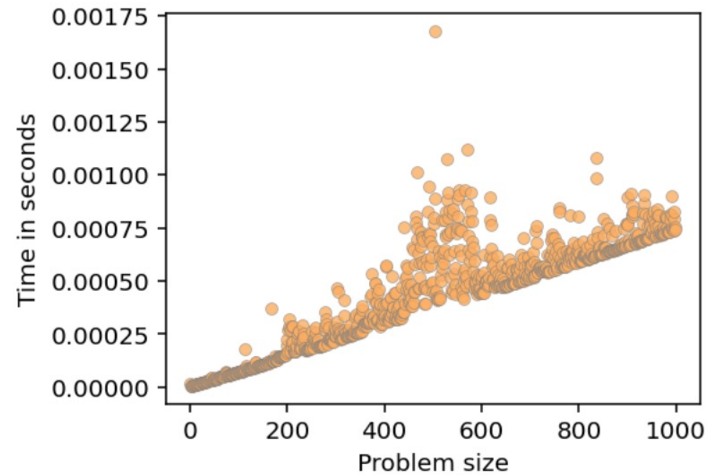
- Hash must preserve order of values!
- Similar to pigeonhole sort but pigeonhole has 1 key per bucket
- Best when there is even distribution of values like hash table
- Works for floats not just ints; see notebook for implementation

See sorting notebook and https://en.wikipedia.org/wiki/Bucket_sort

Key bits of bucket sort algorithm

```
mx = max(A)
…
for a in A:
    a_normalized = a / mx # get into 0..1
    # spread across buckets
    i = int(a_normalized * (nbuckets-1))
    buckets[i].append(a)
…
for i in range(nbuckets):
    A_.extend( sorted(buckets[i]) )
```

Bucket sort worst-case analysis



- What is T(n,m) worst-case?
- What if all values are the same? All go into 1 bucket!
- Sorting one bucket at best costs us O(k log k) for bucket size k
- Bubblesort might be faster for small buckets but that's O(k^2) worst-case in theory
- Can use insertion sort is O(k^2) for adding to bucket or leave unsorted and sort later

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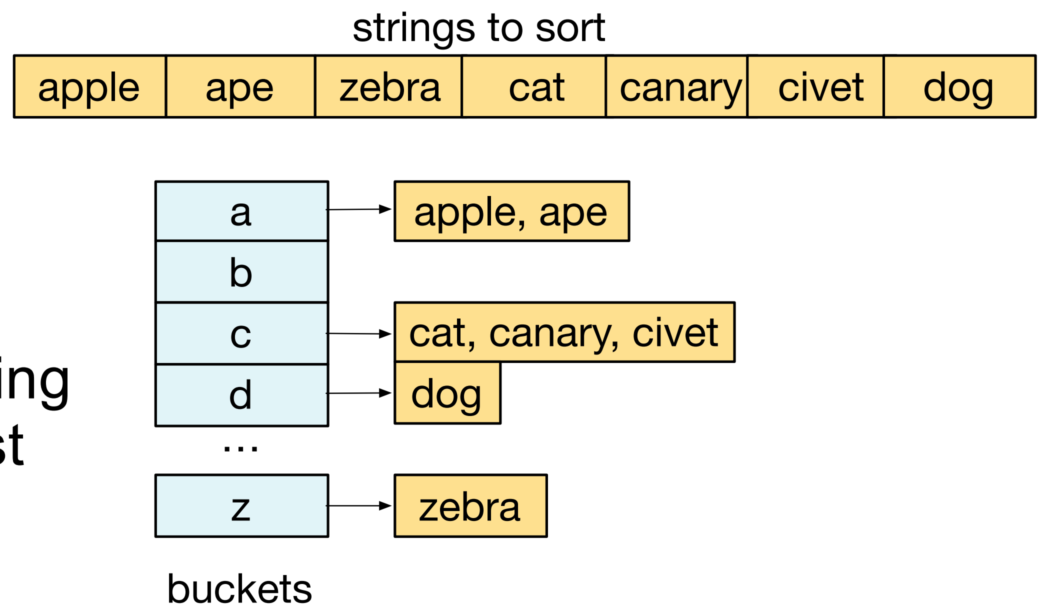
Bucket sort best-case analysis

- What does the best case or average case look like?
- Assume even distribution of elements across m buckets
- Choose m always so k=n/m is some small fixed constant size k
- Sort k elements m times (bubblesort O(k^2)), merge m sorted lists
- T(n,m,k=n/m) = m * k^2 + n
- Replace m=n/k:
  T(n,k) = n/k * k^2 + n = n*k + n = n(k+1)   (choose small k)
- That gives us O(n)

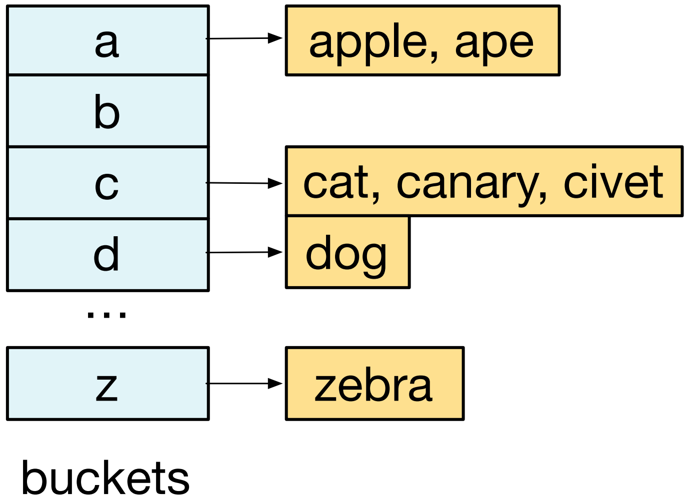
Bucket sort on strings

- Use first letter as bucket key
- Add strings to buckets
- Sort within bucket
- Walk a..z buckets, concatenating those sorted lists into single list
- See sorting notebook for implementation



Key bits of string bucket sort

```
for s in A:
    i = ord(s[0])-ord('a’)
    holes[i].append(s)
…
for i in range(ord('z')-ord('a') + 1):
    A_.extend( sorted(holes[i]) )
```

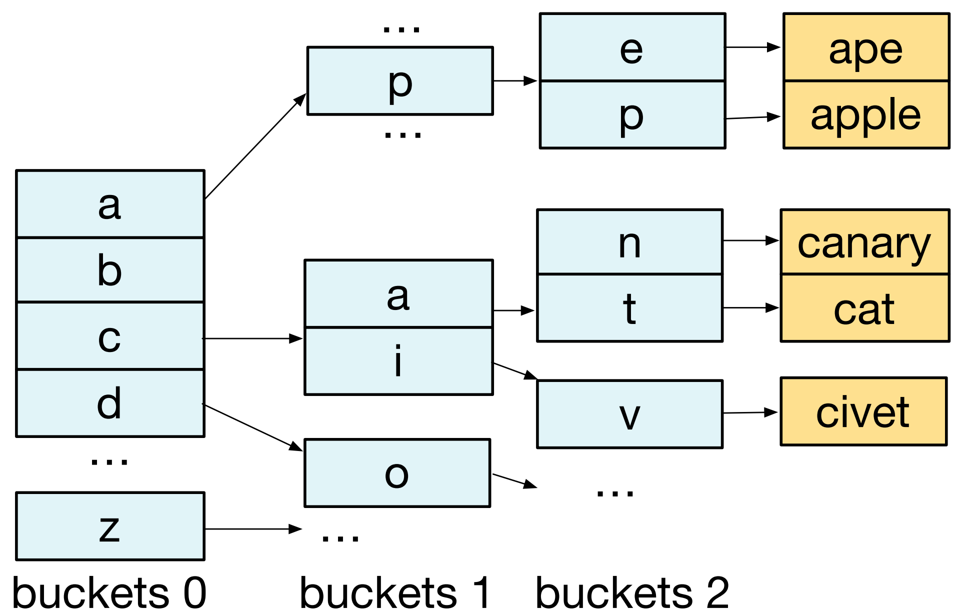


Exercise: What if all words start with same letter?

Nested or recursive string bucket sort

(Called TRIEs and we'll see again)

- Nested indexes based upon s[i]
- With nesting k deep, words are sorted uniquely to first k letters, giving nested bucket sort
- Nested dynamically to full len of string gives nested pigeonhole sort
- Walk all edges in alpha order to collect words in leaves



Summary

- If asked, sorting is O(n log n) (via comparisons)
- Divide and conquer, merge and quicksort, are primary algorithms
  - Mergesort merges two sorted halves recursively; takes extra memory
  - Quicksort partitions instead of sorting halves; works in-place (usually better)
- But, we can do better with pigeonhole sort, mapping each element to unique bucket based on the key; O(n)
- If mapping to unique bucket is hard, as with floating-point numbers, use bin/bucket sort like a hash table; O(n) if reasonably evenly distributed and enough buckets
- Use ord(char) for strings to bucket sort
- Use all letters in strings to get nested bucket sort (called a TRIE)