Searching

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Common searching/membership strategies

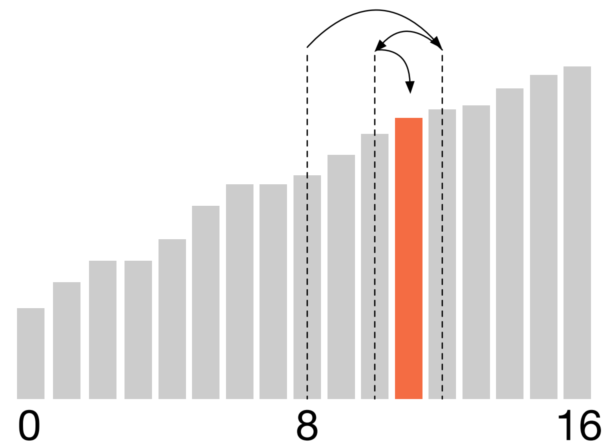
- *O(mn)* **linear**: scan data structure looking for element(s)
- *O(mlog(n))* **binary search**: if in array and sorted, split recursively in half
- *O(mlog(n))* **binary search tree**: subtree to left has elements less than current node and subtree to right has elements greater than
- *O(m)* **hash table**: function maps key to bucket, linear search in bucket; recall search index project from MSDS692; for word search, not arbitrary string search in document(s)
- *O(m)* **state machines** (graphs)

m = length of search string, *n* vocab size, unit of work = character comparison

Binary search (review sort of)

- If we know data is sorted, we can search much faster than linearly
- Means we don't have to examine every element even worst-case

```
def binsearch(a,x):
    left = 0; right = len(a)-1
    while left<=right:
        mid = (left + right)//2
        if a[mid]==x: return mid
        if x < a[mid]: right = mid-1
        else: left = mid+1
    return -1
```



See http://interactivepython.org/runestone/static/pythonds/SortSearch/TheBinarySearch.html

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Compare to (tail-)recursive version

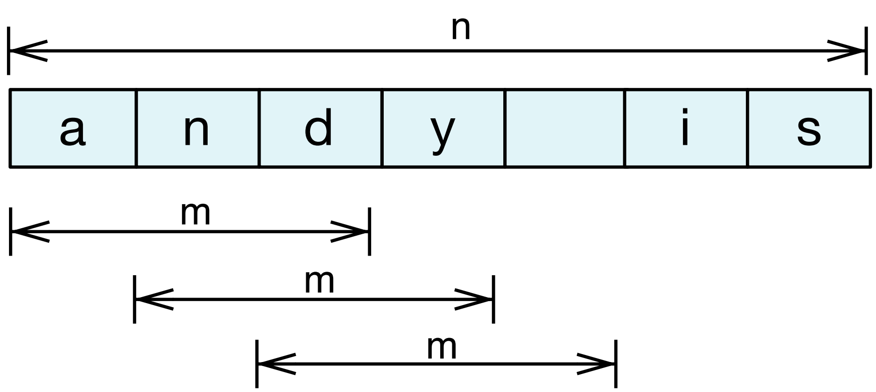
```
def binsearch(a,x,left,right):
    if left > right: return -1
    mid = (left + right)//2
    if a[mid]==x: return mid
    if x < a[mid]:
        return binsearch(a,x,left,mid-1)
    else:
        return binsearch(a,x,mid+1,right)
```

```
left = 0; right = len(a)-1
while left<=right:
    mid = (left + right)//2
    if a[mid]==x: return mid
    if x < a[mid]: right = mid-1
    else: left = mid+1
```

Bracket region with element

String matching

- **Problem**: Given a document of length n characters and a string of length m, find an occurrence or all occurrences
- Brute force algorithm is O(nm)
- Theoretical best-case algorithm exists for O(n + m)
- **Exercise**: Describe brute force algorithm; why is it ”slow”?



Hash searches

- First, note that two equal strings have same hash code so we can compare int codes quickly even for huge strings
- Rabin-Karp* algorithm uses hash function to speed up but is still O(nm) worst-case; works for any substring not just words
- **Idea**: h = hash search string s; compute hash for doc[i:i+m] and compare to h; if same, compare s to doc[i:i+m], return if found; move i from 0 to n-m
- Key is to avoid comparing strings unless the hash codes match, but usually hash computation costs same as comparing strings

*https://en.wikipedia.org/wiki/Rabin%E2%80%93Karp_algorithm

Rabin-Karp (almost)

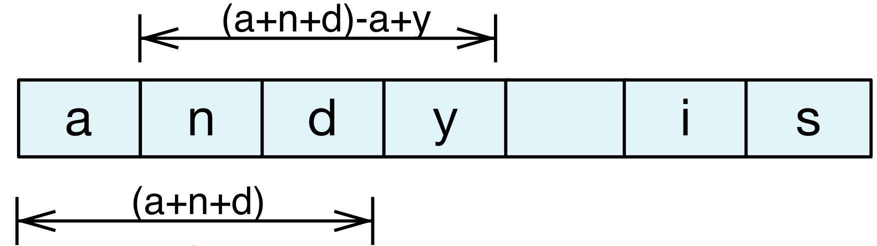
```
def search(doc, s) -> int:
    n = len(doc); m = len(s)
    hs = hash(s)
    for i in range(0,n-m+1):
        hdoc = hash(doc[i:i+m]) # slow O(m)
        if hdoc==hs: # fast
            if s==doc[i:i+m]: # slow
                return i
    return -1
```

```
def hash(s:str)->int:
    return sum(ord(c) for c in s)
```

Additive hashcode is important here

See https://github.com/parrt/msds689/blob/master/notes/searching.ipynb

Find "and"



More details

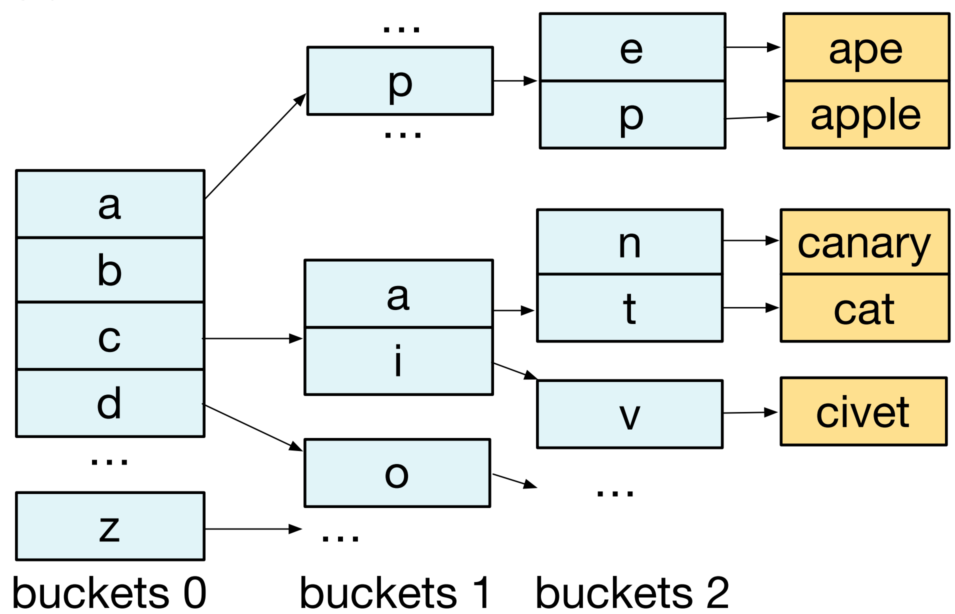
- Naïve hash(doc[i:i+m]) is O(m) for each i=1..n, so use rolling hash to reuse partial hash function computations:
  - next hash is old hash minus doc[i] plus doc[i+m]
  - drop old one off, add in new char (see improved search() in notebook):
    `hdoc = hdoc - ord(doc[i]) + ord(doc[i+m])  # roll it!`
- What about finding all occurrences of s in doc?
- Can check for k strings as we go along not just 1 using O(1) hashtable for each of k strings
- Algorithm is O(nm) since a weak hash function could cause us to compare s at each position

Is this the best we can do?

- Can we do better than this O(nm) or even O(n+m) algorithms?
- Yes, if we prepare a proper side data structure beforehand once for O(n), and we search for words instead of arbitrary strings. **How**?
- First, consider a hash table, which is O(1) for n words, But, relies on good hash function for good distribution and we still must search buckets of average size k; that means O(1) is really hiding O(mk)
- If we are counting string compares not chars then O(mk) = O(1)
- Constant on that complexity can be kind of high
- I claim we can search for any string in doc in O(m); how is this possible?!

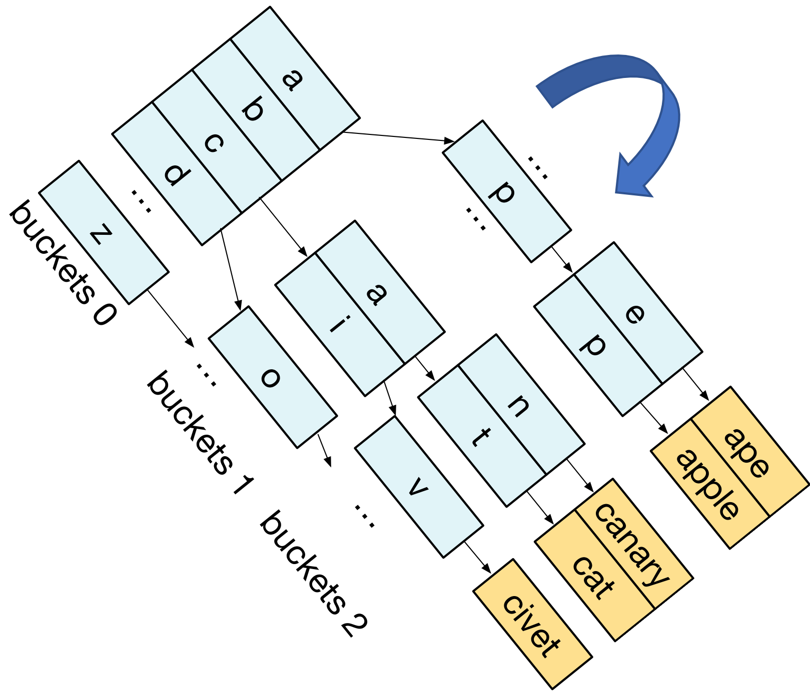
Revisit recursive bucket sort

- Break up doc into words, make nested bucket structure as we saw before
- Add deeper buckets if buckets get too big
- To find word s, use s[i] to navigate and find final "leaf" with list of words w/same prefix
- The index says how to navigate
- How long does it take to find s for n=len(doc), m=len(s), k= average bucket size?
  T(n,m,k) = m+k*avgwordsize

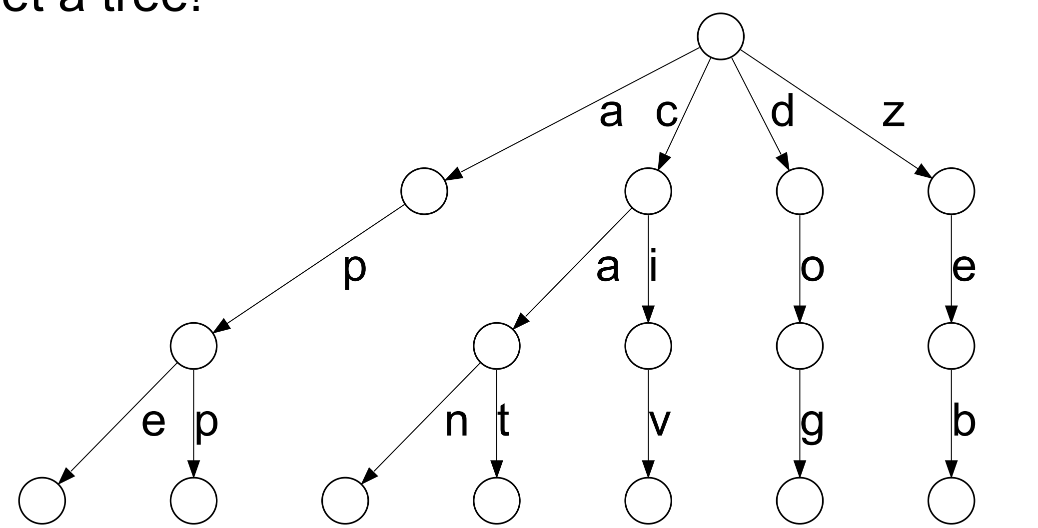


Can we do better than that?

Introducing “*Tries*” or *Prefix Trees*



First step, convert buckets to nodes and rotate: we get a tree!



Words are edge labels on path from root to leaves

(was in a “big internet company” interview)

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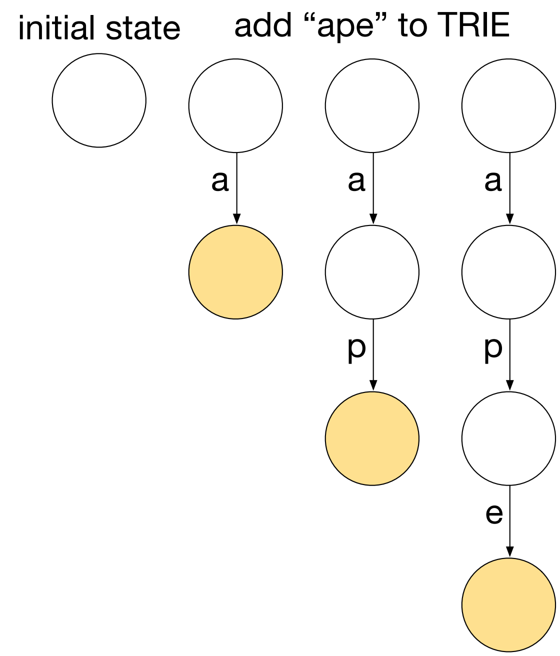
Adding string s to TRIE

```
class TrieNode:
    def __init__(self):
        self.edges = {}
```



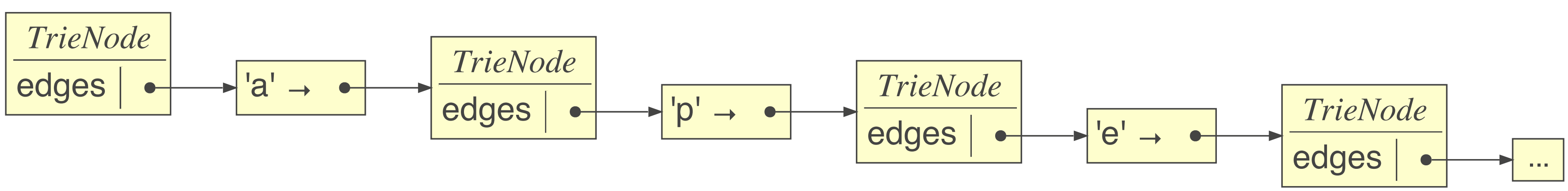
- TRIE can hold a big set of words and we can search for a word superfast
- Like bucket sort but add nested buckets for entire length of each string: pigeonhole!
- Note: We’re not sorting so order of edges is not important; can use dict()
- Starting at the root, add edge labeled with s[0] pointing to new node
- Traverse edge root.edges[s[0]] to child and add subtree for s[1:] to that child
- Recurse until out of chars in string s
- Adding one s is O(m) since we must add edge for each char

Implementation

```
class TrieNode:
    def __init__(self):
        self.edges = {}
```

- add(root, “ape”)

```
def add(p:TrieNode, s:str, i=0) -> None:
    if i>=len(s): return
    if s[i] not in p.edges:
        p.edges[s[i]] = TrieNode()
    add(p.edges[s[i]], s, i+1)
```



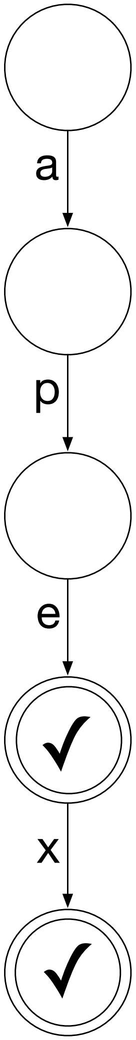
Note that nodes have no values, edges contain the letters

Words that are prefixes of other words

- What about when we have two words "**ape**" and "**apex**"?
- "ape" stops before being a leaf, so we must mark as accept state, which is sometimes called a stop state

```
class TrieNode:
    def __init__(self):
        self.isword = False # set to true if accept state
        self.edges = {}
```

```
def add(p:TrieNode, s:str, i=0) -> None:
    if i>=len(s): p.isword=True; return
    if s[i] not in p.edges:
        p.edges[s[i]] = TrieNode()
    add(p.edges[s[i]], s, i+1)
```

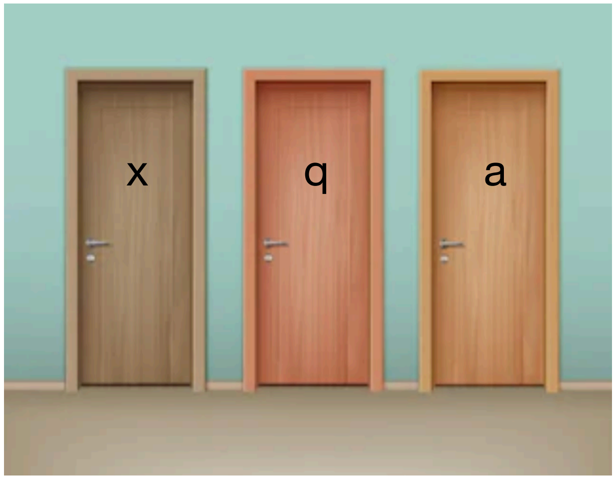


Searching a Trie

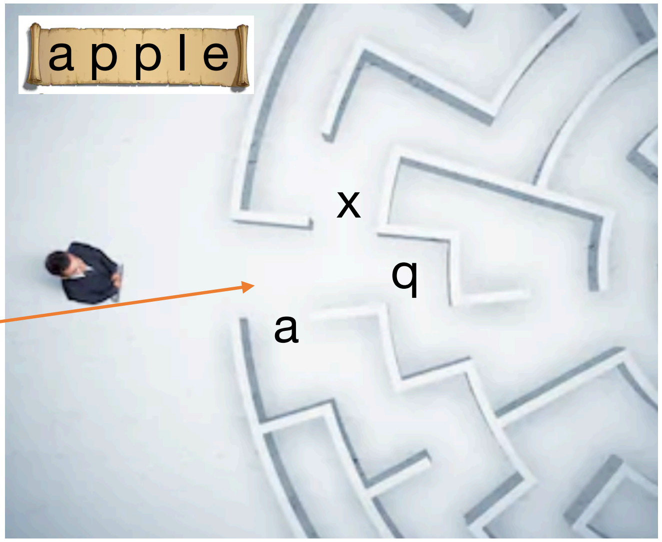
(with analogies)

- Return true if s is prefix of word in Trie or full word in Trie
- Note that the search depends on len(s) NOT num words n in the vocabulary!!!

choose door based upon current letter

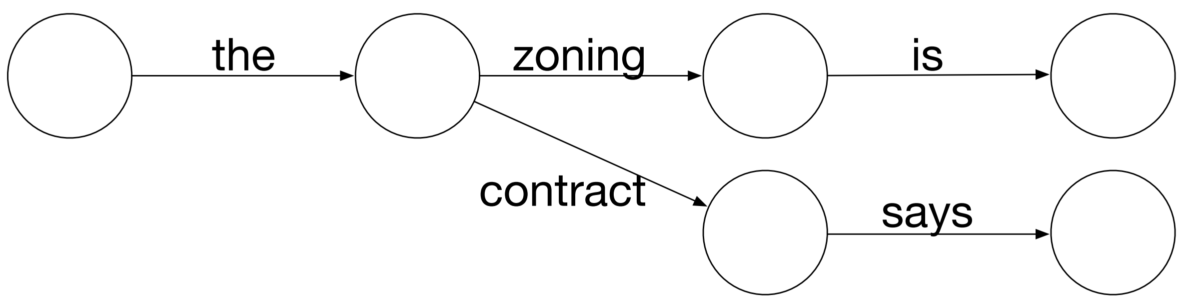



```
def search(root:TrieNode, s:str, i=0) -> bool:
    p = root
    while p is not None:
        if i>=len(s): return True
        if s[i] not in p.edges: return False
        p = p.edges[s[i]]
        i += 1
    return True
```



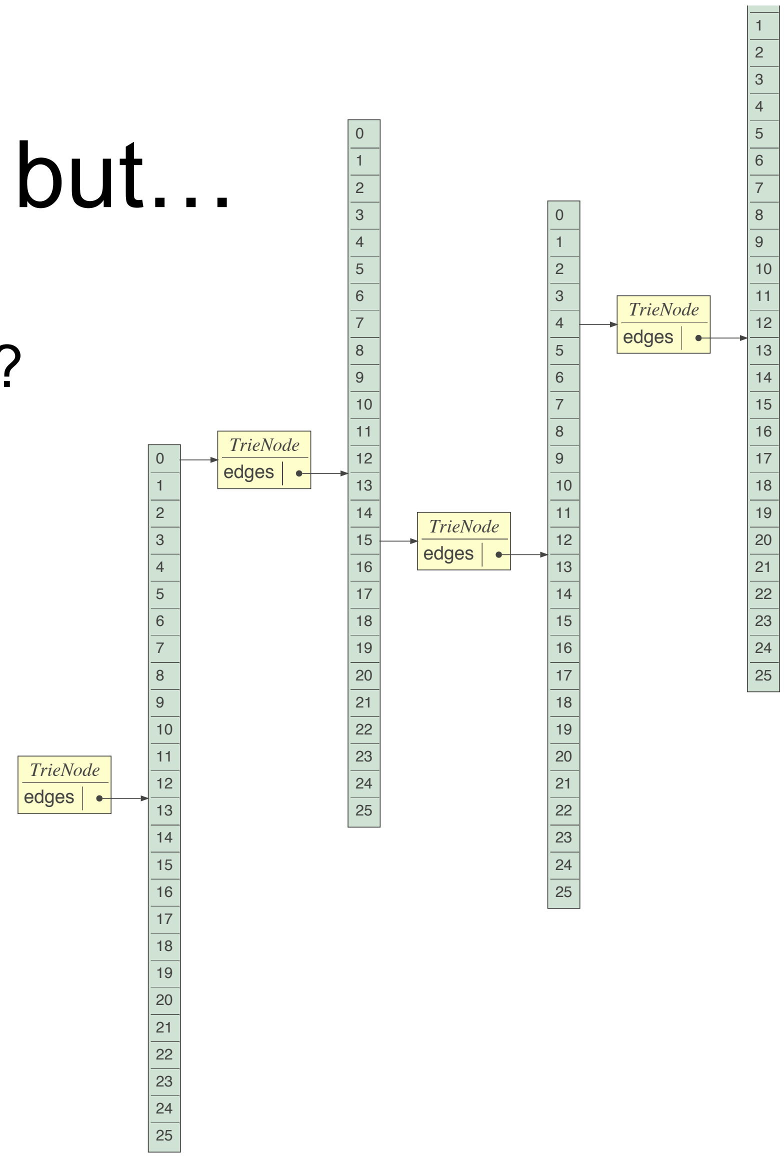
Can search for word sequences too

- TRIE remembers set of sentences not words, in this case
- Tokenize document into words then add sentence sequence to TRIE or just bigrams, trigrams etc…



Edge dictionaries are O(1) but…

- self.edges = {} using general hashtable; can we do faster version of self.edges['x']?
- Yes, use array access via perfect hash function f(c) = ord(c) - ord('a')
- But we use 26 slots even for one edge
- How can we reduce memory costs?
    - Many nodes will have just one outgoing edge so we can optimize for that case with single pointer instead of an array
    - Switch to 26-element edge array if we need more than one edge



Exercise: find all words starting with prefix

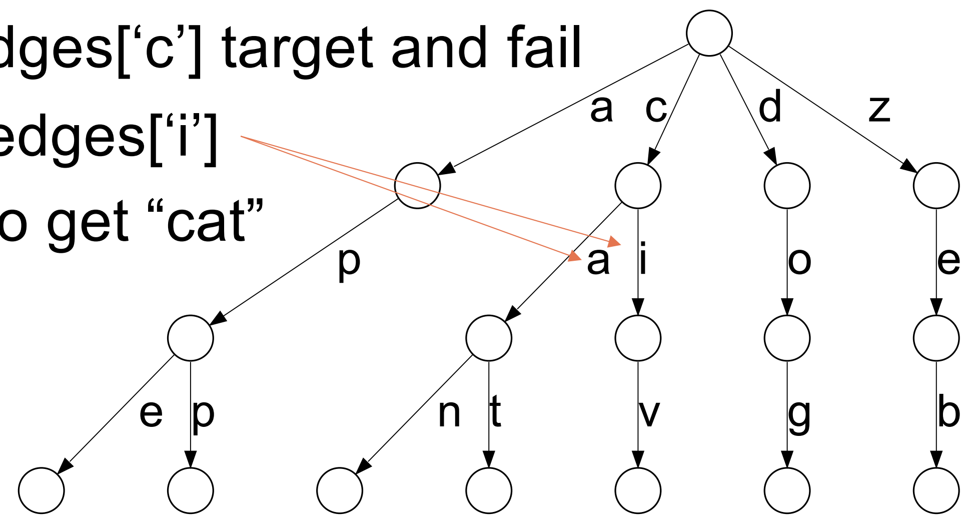
- Create a trie again from the word list
- Write a function that prints all words in trie that begin with a specific prefix like “app”; it should get “apple”, “application”, …
- How would this work?
- Trace prefix into trie, reaching specific non-leaf node p; find all reachable leaves; track string as recursion parameter for each path; print the string when you reach a leaf

Exercise: How to build a suffix tree?

- Simple: create trie from reversed strings or modify add() method to walk backwards through string

Exercise: Given misspelled words off by 1 letter only, find all possible words

- Trace word into trie until no edge exists for s[i]; this is node p
- Get list of words reachable from each node targeted by p starting with s[i+1]
- E.g., “cxt” would get to p=root.edges[‘c’] target and fail
- Find “t” from p.edges[‘a’] and p.edges[‘i’]
- We find only “t” matches via ‘a’ to get “cat”



Summary

- Lots of ways to search beyond linear and binary search
- String searching has some really efficient solutions such as Rabin-Karp; idea is to compare hash codes before doing string comparisons and do a rolling hash for the document substrings
- If we are willing to build a graph data structure in O(nm), the TRIE is pretty hard to beat complexity and performance; looking up a word in the TRIE is O(m) for m-character string!
- TRIE is just a nested pigeonhole sort turned into a graph
- Useful as prefix and suffix trees; can find misspelled words

Coding exercises next…

Exercise: Brute force dictionary search

- Load words from **/usr/share/dict/words** file (one per line) into list
- Search for each word in list of words; what is complexity?
- This takes almost 5 minutes on my fast computer. Ugh
- For 50k works, takes 13s (still brutally slow)

```
A
a
aa
aal
aalii
aam
Aani
aardvark
aardwolf
Aaron
Aaronic
Aaronical
…
```

See solution https://github.com/parrt/msds689/blob/master/notes/code/trie.py

Exercise: Build Trie from dictionary of words

- From searching notebook, get Trie implementation
- Add each word to a trie, which takes about 7s on my machine
- Search the trie for each of 235,886 words; takes 0.70s for me!!
- Rejoice in your new super powers
- Cool interview question/task:
  How can you do fast spell checking on big documents?

See solution https://github.com/parrt/msds689/blob/master/notes/code/trie.py