Forward Kinematics:

- Algebraic Calculation

$$T_i^{i-1} = \begin{bmatrix} \cos(\theta_i) & -\sin(\theta_i)\cdot\cos(\alpha_i) & \sin(\theta_i)\cdot\sin(\alpha_i) & a_i\cdot\cos(\theta_i) \\ \sin(\theta_i) & \cos(\theta_i)\cdot\cos(\alpha_i) & -\cos(\theta_i)\cdot\sin(\alpha_i) & a_i\cdot\sin(\theta_i) \\ 0 & \sin(\alpha_i) & \cos(\alpha_i) & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_1^0 = \begin{bmatrix} \cos(\theta_1) & 0 & \sin(\theta_1) & L1\cdot\cos(\theta_1) \\ \sin(\theta_1) & 0 & -\cos(\theta_1) & L1\cdot\sin(\theta_1) \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_2^1 = \begin{bmatrix} \cos(\theta_2) & 0 & -\sin(\theta_2) & L2\cdot\cos(\theta_2) \\ \sin(\theta_2) & 0 & \cos(\theta_2) & L2\cdot\sin(\theta_2) \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_3^2 = \begin{bmatrix} \cos(\theta_3) & 0 & -\sin(\theta_3) & L3\cdot\cos(\theta_3) \\ \sin(\theta_3) & 0 & \cos(\theta_3) & L3\cdot\sin(\theta_3) \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_3^0 = T_1^0\cdot T_2^1\cdot T_3^2 = \begin{bmatrix} T_{11} & T_{12} & T_{13} & P_x \\ T_{21} & T_{22} & T_{23} & P_y \\ T_{31} & T_{32} & 0 & P_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$T_{11} = cos(\theta_2 + \theta_3)\cdot cos(\theta_1)$

$T_{12} = -\sin(\theta_2 + \theta_3)\cdot\cos(\theta_1)$

$T_{13} = sin(\theta_1)$

$T_{21} = cos(\theta_2 + \theta_3)\cdot sin(\theta_1)$

$T_{22} = -sin(\theta_2 + \theta_3)\cdot sin(\theta_1)$

$T_{23} = -cos(\theta_1)$

$T_{31} = sin(\theta_2 + \theta_3)$

$T_{32} = cos(\theta_2 + \theta_3)$

$P_x = cos(\theta_1)\cdot(L3\cdot cos(\theta_2 + \theta_3) + L2\cdot cos(\theta_2) + L1)$

$P_y = sin(\theta_1)\cdot(L3\cdot cos(\theta_2 + \theta_3) + L2\cdot cos(\theta_2) + L1)$

$P_z = L3\cdot sin(\theta_2 + \theta_3) + L2\cdot sin(\theta_2)$

- Numerical Calculation:

$$T_1^0 = \begin{bmatrix} \cos(\theta_1) & 0 & -\sin(\theta_1) & 43 \cdot \cos(\theta_1) \\ \sin(\theta_1) & 0 & \cos(\theta_1) & 43 \cdot \sin(\theta_1) \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_2^1 = \begin{bmatrix} \cos(\theta_2) & -\sin(\theta_2) & 0 & 60 \cdot \cos(\theta_2) \\ \sin(\theta_2) & \cos(\theta_2) & 0 & 60 \cdot \sin(\theta_2) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_3^2 = \begin{bmatrix} \cos(\theta_3) & -\sin(\theta_3) & 0 & 104 \cdot \cos(\theta_3) \\ \sin(3) & \cos(\theta_3) & 0 & 104 \cdot \sin(\theta_3) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_3^0 = T_1^0 \cdot T_2^1 \cdot T_3^2 = \begin{bmatrix} T_{11} & T_{12} & T_{13} & P_x \\ T_{21} & T_{22} & T_{23} & P_y \\ T_{31} & T_{32} & 0 & P_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$T_{11} = cos(\theta_2 + \theta_3) \cdot cos(\theta_1)$

$T_{12} = -\sin(\theta_2 + \theta_3) \cdot \cos(\theta_1)$

$T_{13} = sin(\theta_1)$

$T_{21} = cos(\theta_2 + \theta_3) \cdot sin(\theta_1)$

$T_{22} = -sin(\theta_2 + \theta_3) \cdot sin(\theta_1)$

$T_{23} = -cos(\theta_1)$

$T_{31} = sin(\theta_2 + \theta_3)$

$T_{32} = cos(\theta_2 + \theta_3)$

$P_x = cos(\theta_1) \cdot (104 \cdot cos(\theta_2 + \theta_3) + 60 \cdot cos(\theta_2) + 43)$

$P_y = sin(\theta_1) \cdot (104 \cdot cos(\theta_2 + \theta_3) + 60 \cdot cos(\theta_2) + 43)$

$P_z = 104 \cdot sin(\theta_2 + \theta_3) + 60 \cdot sin(\theta_2)$

Inverse Kinematics

- Theta1:

$$\frac{P_y}{P_x} = \frac{\sin(\theta_1) \cdot (L3 \cdot \cos(\theta_2 + \theta_3) + L2 \cdot \cos(\theta_2) + L1)}{\cos(\theta_1) \cdot (L3 \cdot \cos(\theta_2 + \theta_3) + L2 \cdot \cos(\theta_2) + L1)}$$

$$\frac{P_y}{P_x} = \frac{\sin(\theta_1)}{\cos(\theta_1)}$$

$$\boxed{\therefore \theta_1 = atan2\left(\frac{P_y}{P_x}\right)}$$

- Theta3:

$$(T_1^0)^{-1} \cdot T_3^0 = \begin{bmatrix} \cos(\theta_1) & \sin(\theta_1) & 0 & -L1 \\ 0 & 0 & 1 & 0 \\ \sin(\theta_1) & -\cos(\theta_1) & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} T_{11} & T_{12} & T_{13} & P_x \\ T_{21} & T_{22} & T_{23} & P_y \\ T_{31} & T_{32} & 0 & P_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} r_{11} & r_{12} & r_{13} & P_x \cdot \cos(\theta_1) + P_y \cdot \sin(\theta_1) - L1 \\ r_{21} & T_{22} & rs_{23} & P_z \\ r_{31} & r_{32} & r_{33} & P_x \cdot \sin(\theta_1) - P_y \cdot \cos(\theta_1) \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \cos(\theta_2 + \theta_3) & -\sin(\theta_2 + \theta_3) & 0 & L3 \cdot \cos(\theta_2 + \theta_3) + L2 \cdot \cos(\theta_2) \\ \sin(\theta_2 + \theta_3) & \cos(\theta_2 + \theta_3) & 0 & L3 \cdot \sin(\theta_2 + \theta_3) + L2 \cdot \sin(\theta_2) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\left(P_x \cdot \cos(\theta_1) + P_y \cdot \sin(\theta_1) - L1\right)^2 + P_z^2 = (L3 \cdot \cos(\theta_2 + \theta_3) + L2 \cdot \cos(\theta_2))^2 + (L3 \cdot \sin(\theta_2 + \theta_3) + L2 \cdot \sin(\theta_2))^2$$

$$\boxed{\theta_3 = arcos\left(\frac{\left(P_x \cdot \cos(\theta_1) + P_y \cdot \sin(\theta_1) - L1\right)^2 - L2^2 - L3^2}{2 \cdot L2 \cdot L3}\right)}$$

- Theta2:

$$(T_1^0)^{-1} \cdot T_3^0 = \begin{bmatrix} \cos(\theta_1) & \sin(\theta_1) & 0 & -L1 \\ 0 & 0 & 1 & 0 \\ \sin(\theta_1) & -\cos(\theta_1) & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} T_{11} & T_{12} & T_{13} & P_x \\ T_{21} & T_{22} & T_{23} & P_y \\ T_{31} & T_{32} & 0 & P_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} r_{11} & r_{12} & r_{13} & P_x \cdot \cos(\theta_1) + P_y \cdot \sin(\theta_1) - L1 \\ r_{21} & T_{22} & rs_{23} & P_z \\ r_{31} & r_{32} & r_{33} & P_x \cdot \sin(\theta_1) - P_y \cdot \cos(\theta_1) \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \cos(\theta_2 + \theta_3) & -\sin(\theta_2 + \theta_3) & 0 & L3 \cdot \cos(\theta_2 + \theta_3) + L2 \cdot \cos(\theta_2) \\ \sin(\theta_2 + \theta_3) & \cos(\theta_2 + \theta_3) & 0 & L3 \cdot \sin(\theta_2 + \theta_3) + L2 \cdot \sin(\theta_2) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\frac{P_x \cdot \cos(\theta_1) + P_y \cdot \sin(\theta_1) - L1}{P_z} = \frac{L3 \cdot \cos(\theta_2 + \theta_3) + L2 \cdot \cos(\theta_2)}{L3 \cdot \sin(\theta_2 + \theta_3) + L2 \cdot \sin(\theta_2)}$$

$$\boxed{\theta_2 = arctan2\left(\frac{P_z \cdot (L2 + L3 \cdot \sin(\theta_3)) - (L3 \cdot \sin(\theta_3)) \cdot (P_x \cdot \cos(\theta_1) + P_y \cdot \sin(\theta_1) - L1)}{P_z \cdot (L3 \cdot \sin(\theta_3)) - (L2 + L3 \cdot \sin(\theta_3)) \cdot (P_x \cdot \cos(\theta_1) + P_y \cdot \sin(\theta_1) - L1)}\right)}$$