EEE4018: Advanced Control Theory - Project Report

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Title of the Project:

Design of a PID controller using Root locus method

Description with a block diagram:

PID controller

- Systems that feed the error forward to the plant (or actual system) are called proportional control systems.
- Systems that feed the integral of the error to the plant are called integral controlsystems.
- Systems that feed the derivative of the error to the plant are called derivative control systems.
- PID stands for 'Proportional-plus-integral-plus-derivative'.
- Combination of PD controller and PI controller.
- Implemented using active components (for example, op-amps) as active networks.
- **PD controller:** The sum of a differentiator and a pure gain, also called an *ideal derivative compensator*. Its purpose is to improve the transient response of a system.

- **PI controller:** The sum of an integrator and a pure gain, also called an *ideal integral compensator*. Its purpose is to **reduce the steady-state error to zero**.
- Hence, the purpose of *PID controller* is to improve both the transient response and the steady state response (zero steady-state error).
- A PID controller is shown in Figure 1.

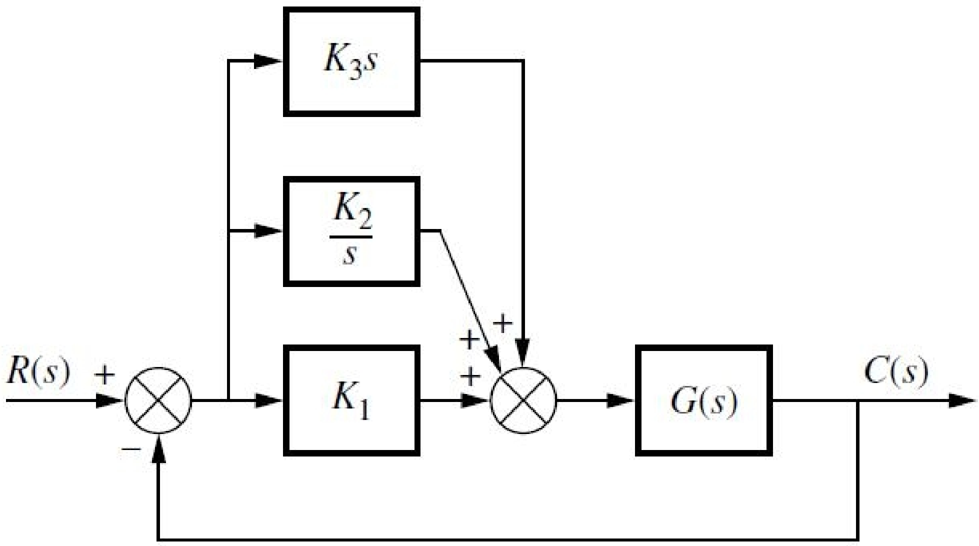

Figure 1: PID controller

Its transfer function is given by

$$G_c(s) = K_1 + \frac{K_2}{s} + K_3 s = \frac{K_1 s + K_2 + K_3 s^2}{s} = \frac{K_3\left(s^2 + \frac{K_1}{K_3}s + \frac{K_2}{K_3}\right)}{s} \qquad \textbf{(Equation (1))}$$

which has two zeros plus a pole at the origin. One zero and the pole at the origin can be designed as the ideal integral compensator; the other zero can be designed as the ideal derivative compensator.

- While improving both the steady-state response and transient response, both can be improved independently.
- For an approach, we can improve the steady-state error first and then follow with the design to improve the transient response. A disadvantage of this approach is that the improvement in transient

response in some cases yields some decay in the improvement of the steady-state error, which was designed first.

- In other case, the improvement in transient response yields further improvement in steady-state errors. Thus, a system can be over designed with respect to steady-state errors (Over design is usually not a problem unless it affects cost or produces other design problems).
- Here, I am going to design controller for improving the transient response first and then design for reducing the steady-state error.
- First, the *PD controller* has to be designed to improve the transient response. Then, we have to design the *PI controller* to obtain the required steady-state error.

Design of PID controller

The following steps summarize the design procedure for a PID controller:

1. Evaluate the performance of the uncompensated system to determine how much improvement in
transient response is required.
2. Design the PD controller to meet the transient response specifications. The design includes the
zero location and the loop gain.
3. Simulate the system to be sure all requirements have been met.
4. Redesign if the simulation shows that requirements have not been met.
5. Design the PI controller to yield the required steady-state error.
6. Determine the gains $K1$, $K2$, and $K3$ in Figure 1.
7. Simulate the system to be sure all requirements have been met.
8. Redesign if the simulation shows that requirements have not been met

Goals of Project

Or Goal in this project is to design a PID controller for a Transfer function to the desired values specified by the user .We will also show what changes are produced in the system with the addition of the designed controllers in the system by comparing the results the open loop transfer function to the transfer function obtained at the end of designed controller

Modelling with necessary equations:

Given the system of Figure 2, design a PID controller so that the system can operate with a peak time that is two-thirds that of the uncompensated system at 20% overshoot and with zero steady-state error for a step input.

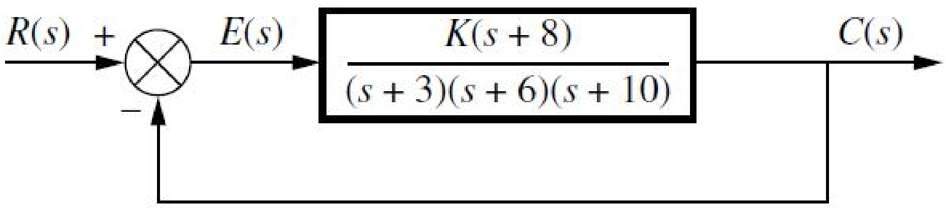

Figure 2: Uncompensated feedback control system

Step 1: First we evaluate the performance of the uncompensated system in MATLAB.

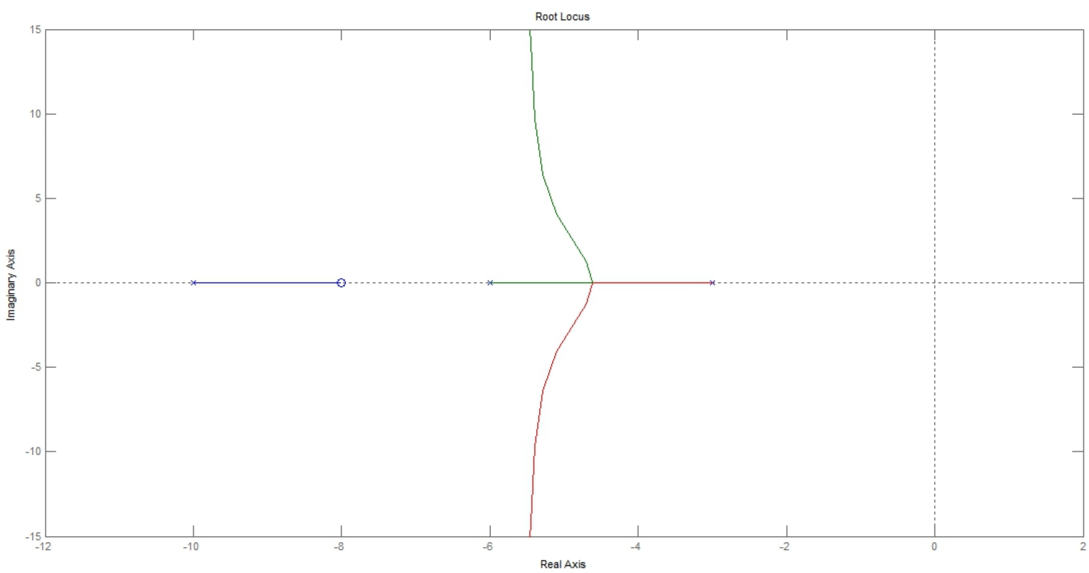

Figure 3: Root locus for uncompensated system

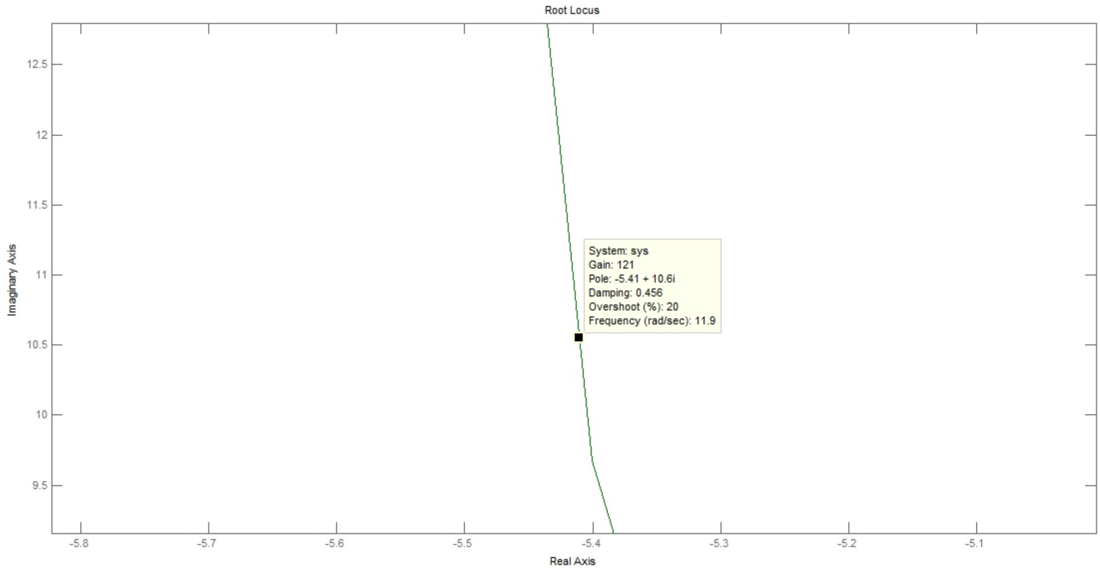

Figure 4: Root locus for uncompensated system (zoomed for finding 20% overshoot)

Searching for the 20% overshoot in Figure 4 (zoomed graph of Figure 3), we find the dominant poles at $-5.41 \pm 10.6\,j$ with a gain of 121, and the corresponding damping ratio $\zeta = 0.456$. Now, the step response of the uncompensated system with the gain value of 121 is shown in Figure 5.

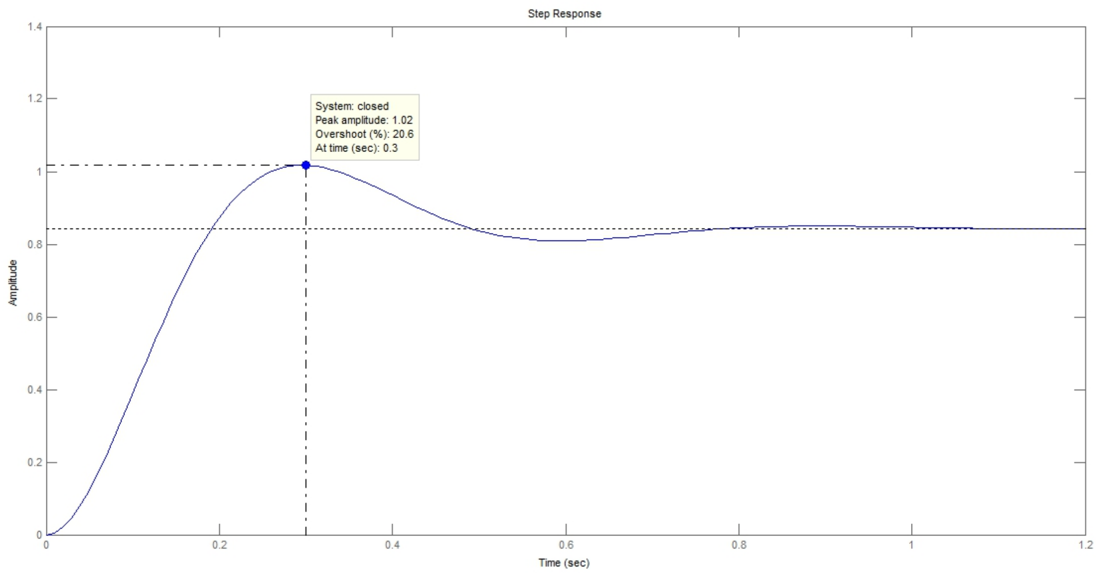

Figure 5: Step response of the uncompensated system with K = 121

From Figure 5, the peak time is observed as **0.3 seconds**. (Theoretically, the peak time can be calculated from the imaginary part of the dominant pole (that is, 10.6*j*). We know that the dominant poles are given by the roots $-\zeta w_n \pm j w_d$ where $w_d = w_n \sqrt{1-\zeta^2}$ Now, the peak time (Tp) is given by Tp= $\frac{\pi}{w_d} = \frac{\pi}{10.6}$ =**0.264 seconds**

Step 2: Next, a PD controller has to be designed to reduce the peak time to two-thirds of that of the uncompensated system, that is, (2 / 3) × 0.3 seconds = 0.2 seconds (theoretical value = 0.1976 seconds). The dominant poles for the desired peak time of 0.1976 seconds (theoretical value) can be obtained by theoretical means. From the peak time relation, the imaginary part of the desired dominant pole, w_d is obtained as $w_d = \frac{\pi}{0.197} = = \mathbf{15.9}$

Now, for finding the real part of the desired dominant pole, a line has to be drawn to the left of the imaginary axis in s-plane with respect to the origin at an angle,

$\theta = \cos^{-1}(\zeta) = \cos^{-1}(0.456)$ = **62.87** as shown in Figure 6. From that, the real part of the desired dominant pole can be obtained as follows:

$$\tan(62.87) = \frac{15.9}{\zeta w_n} \quad \Rightarrow \quad \zeta w_n = \frac{15.9}{\tan(62.87)} = \mathbf{8.147}$$

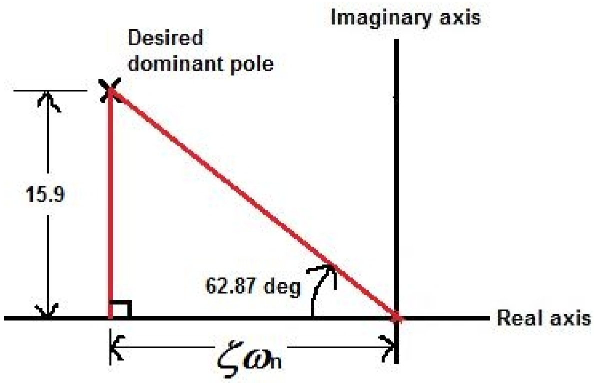

Figure 6: Right angled triangle formed in s-plane to find the dominant pole

Therefore, the dominant poles for the desired transient response specification of peak time = 0.1976 seconds are **–8.147±15.9j**. Now, we start the design of PD controller by finding the location of the controller's zero using the root locus property as shown in Figure 7 by connecting the dominant pole to all the uncompensated poles and zeros.

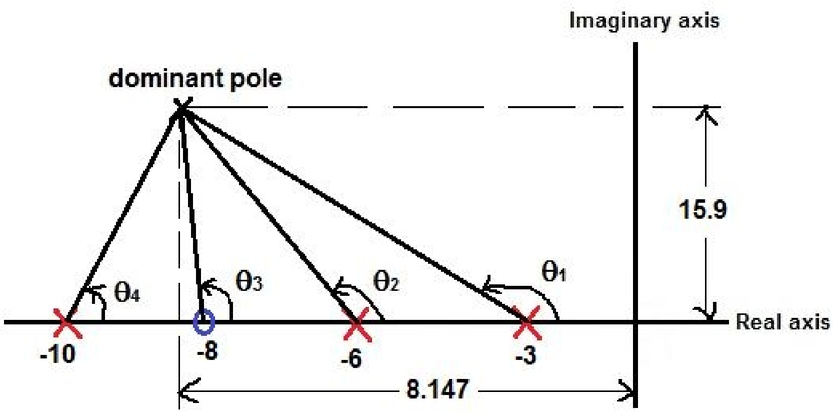

Figure 7: Angles formed between the dominant pole and all other poles and zeros

The angles formed between the dominant pole and all other poles and zeros can be obtained as follows:

θ_1=180 -$\{tan^{-1}[\frac{15.9}{8.147-3}]\}$ = 107.94°

θ_2=180 -$\{tan^{-1}[\frac{15.9}{8.147-6}]\}$ = 97.69°

θ_3=180 -$\{tan^{-1}[\frac{15.9}{8.147-8}]\}$ = 90.53

tan θ_4 =$\frac{15.9}{10-8.147}$=8.5807 ⇒ θ_4 =83.35

Now, the angle contribution required from the PD controller zero (zc) in order to make the root locus to pass through the desired dominant pole can be obtained as

Angle contribution = 180° – (sum of angles from the dominant pole to all other poles) + (sum of angles from the dominant pole to all other zeros)

=180-(θ_1+θ_2+θ_4) + θ_3 = 180 – (107.94+97.69+83.35) + 90.53 = **–18.45°**

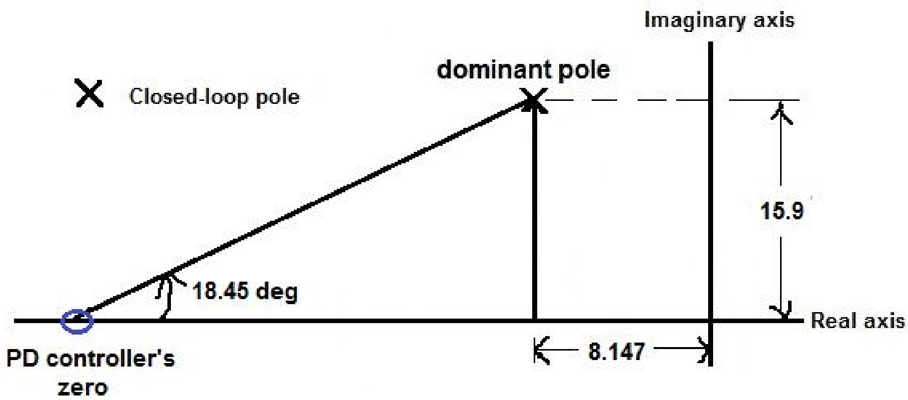

Figure 8: Calculating the PD compensator zero (z_c)

Using the geometry shown in Figure 8,

$$\frac{15.9}{zc - 8.147} = \tan(18.45°)$$

from which the location of the PD compensator zero, zc, is found to be **–55.8**.

Therefore, the transfer function of the designed PD controller is given by

$$GPD\,(s) = K\,(s + 55.8)$$

Now, the loop gain K for the PD-compensated system can be determined by either manual calculations or from root locus graph of the PD-compensated system. The complete root locus of the PD compensated system $GPD\,(s)\,G(s)$ is shown in Figure 9.

From the root locus graph obtained from MATLAB simulation (zoomed in Figure 10), it is evident that the dominant pole for the PD-compensated system with 20% overshoot shows **–8.17±15.9*j*** with a loop gain of **5.39**. Now, we can check the loop gain value through manual calculations.

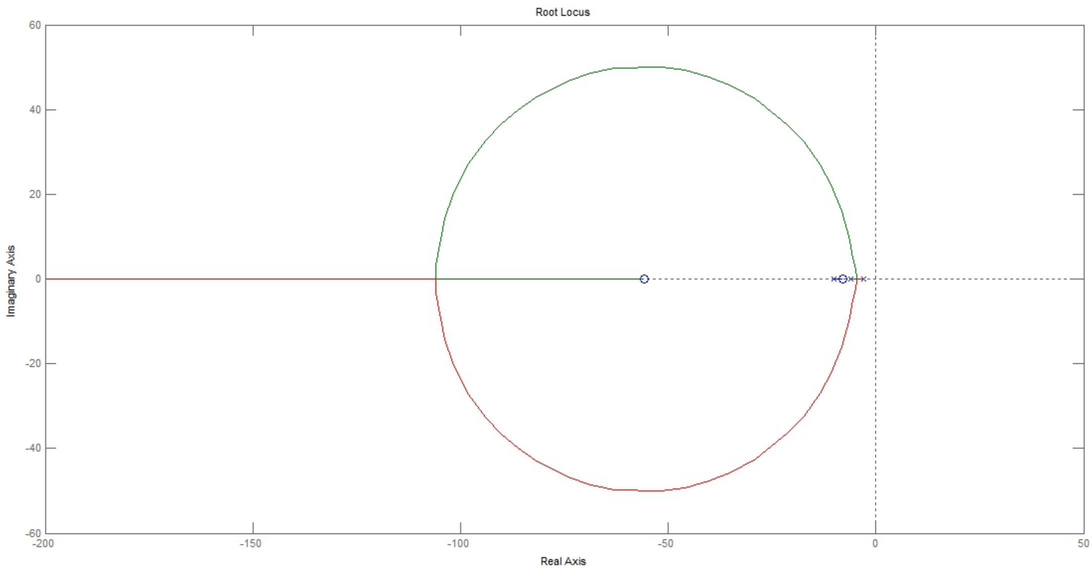

Figure 9: Root locus for PD-compensated system

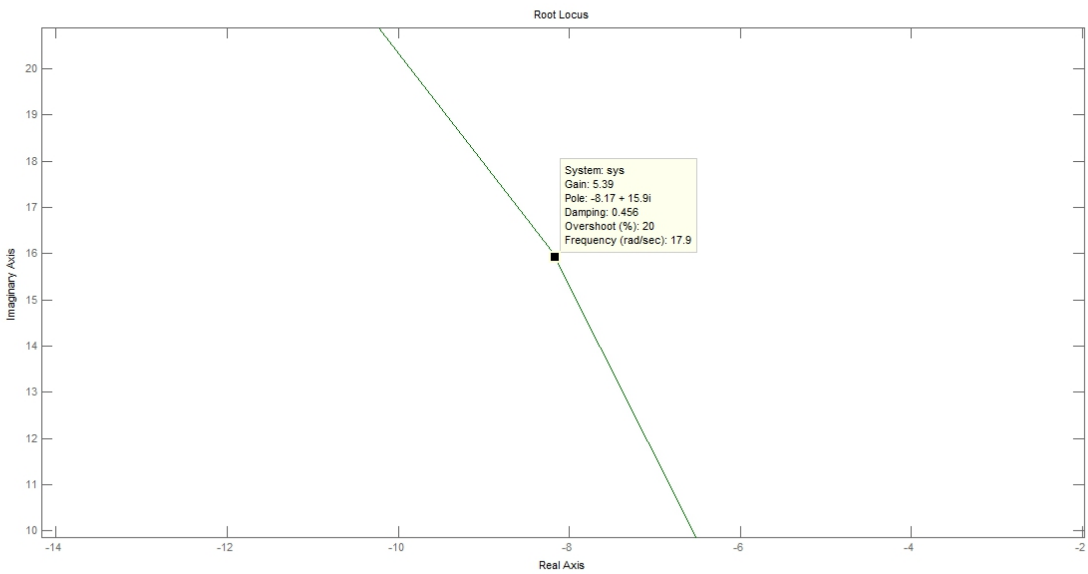

Figure 10: Root locus for PD-compensated system

(zoomed for finding the gain for 20% overshoot and dominant pole location)

Determination of loop gain K for the PD-compensated system:
We know that the location of the dominant pole is a closed-loop pole location. And the closed loop poles can be obtained directly from the characteristic equation for various values of K. Here, we know the dominant pole as –8.147±15.9j and the corresponding K value can be obtained as follows:The characteristic equation for a system is given by 1+G(s) H(s) = 0. Since the given system in Figure 1 is unity feedback system, H(s) = 1. Therefore, the characteristic equation is simply 1+G(s) = 0. The transfer function G(s) of the PD compensated system (including the loop gain K) is given by

$$G(s) = \frac{K(s+8)(s+55.8)}{(s+3)(s+6)(s+10)}$$

From characteristic equation, $1+\frac{K(s+8)(s+55.8)}{(s+3)(s+6)(s+10)} = 0 \quad \Rightarrow \quad \frac{K(s+8)(s+55.8)}{(s+3)(s+6)(s+10)} = -1$

$$\Rightarrow \quad K = \left.\frac{-((s+3)(s+6)(s+10))}{(s+8)(s+55.8)}\right|_{s=-8.147+15.9j} = \left.\frac{-(s^3+19s^2+108s+180)}{s^2+63.8s+446.4}\right|_{s=-8.147+15.9j}$$

$$= 5.3735 - 0.0001\,j$$

Therefore, $|K| = \sqrt{(5.3735)^2 + (0.0001)^2}$ **= 5.3735** (remember, K = 5.39 from simulation)

Steps 3 and 4: Now, we have to check the PD-compensator design with loop gain of 5.39 in MATLAB simulation. The step response of the PD-compensated system is shown in Figure 11.

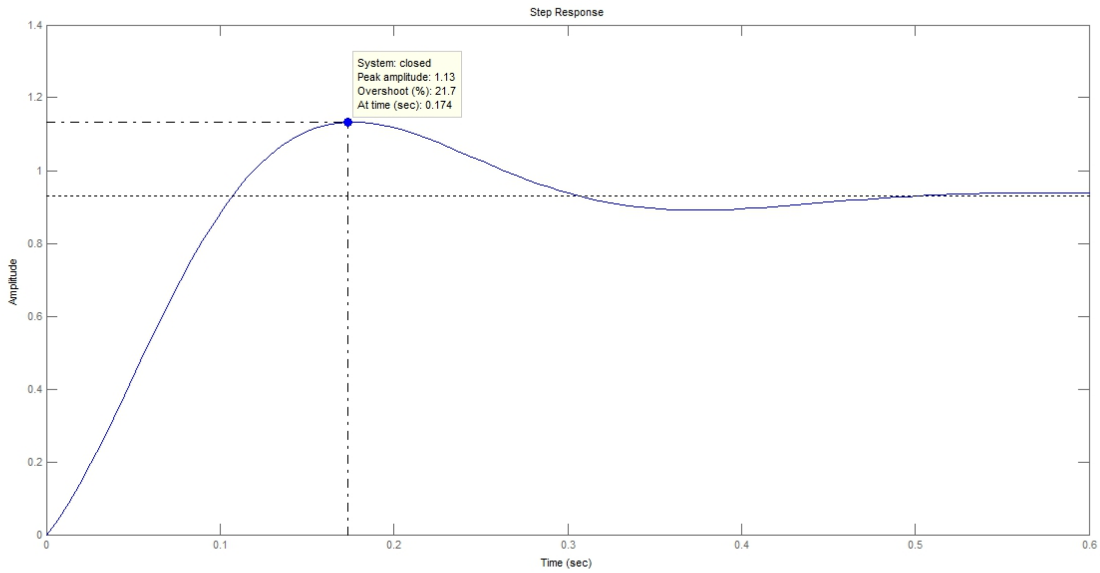

Figure 11: Step response of the PD-compensated system with K = 5.39

In Figure 11, the peak time is observed as **0.174 seconds** (desired value = 0.1976 seconds) with the loop gain value of 5.39. Thus, it is evident from the simulation that the PD-compensated system satisfies the peak time requirement better than the desired one and operating with overshoot of nearly 22% (somewhat acceptable) and the result is satisfactory. The comparison of step responses of uncompensated system and PD-compensated system is shown in Figure 12.

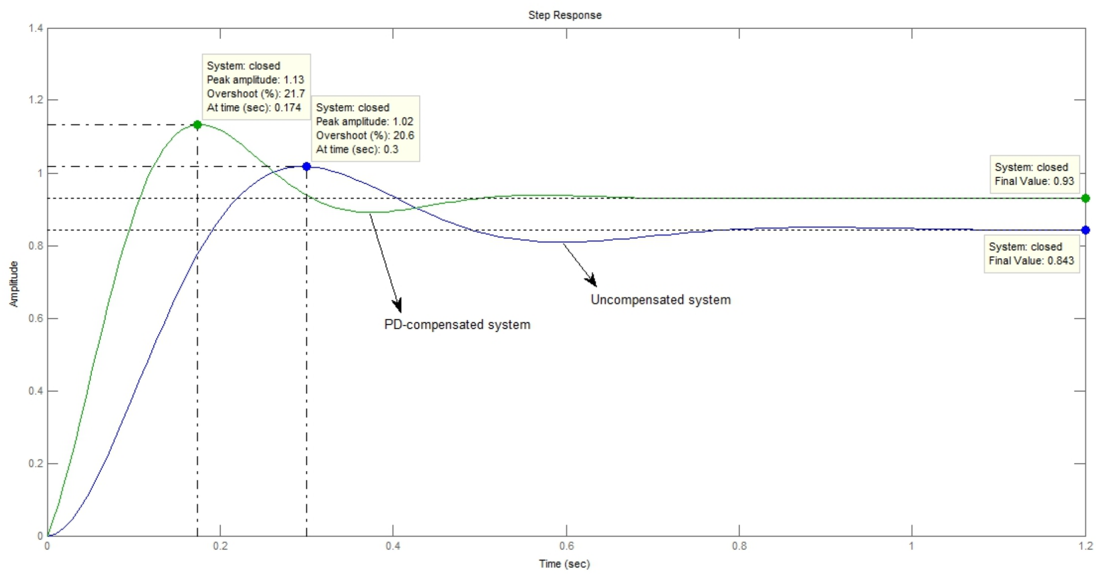

Figure 13: Comparison of step response for uncompensated system and PD-compensated system

Step 5: Even though the PD-compensated system reduces the steady-state error over the uncompensated system, the given requirement is zero steady-state error. So, we need to design the ideal integral compensator (or PI controller) to reduce the steady-state error to zero for a step input. Any ideal integral compensator zero will work, as long as the **zero is placed close to the origin**. Here, I am choosing the ideal integral compensator zero to be 0.1, and thus the transfer function of the PI controller is given by

$$G_{PI} = \frac{s + 0.1}{s}$$

Now, the loop gain *K* for the PID-compensated system (combination of PD and PI) can be determined by either manual calculations or from root locus graph of the PID-compensated system. The complete root locus of the PID-compensated system is shown in Figure 14. From the root locus graph obtained from MATLAB simulation (zoomed in Figure 15), it is evident that the dominant pole for the PID compensated system with 20% overshoot shows **–8.1±15.8*j*** with a loop gain of **5.32**.

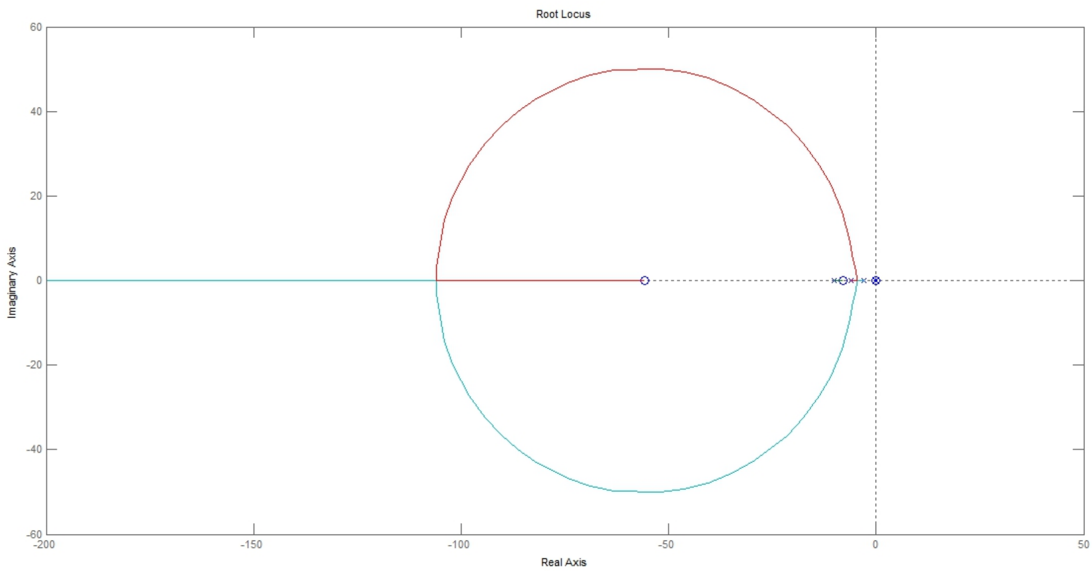

Figure 14: Root locus for PID-compensated system

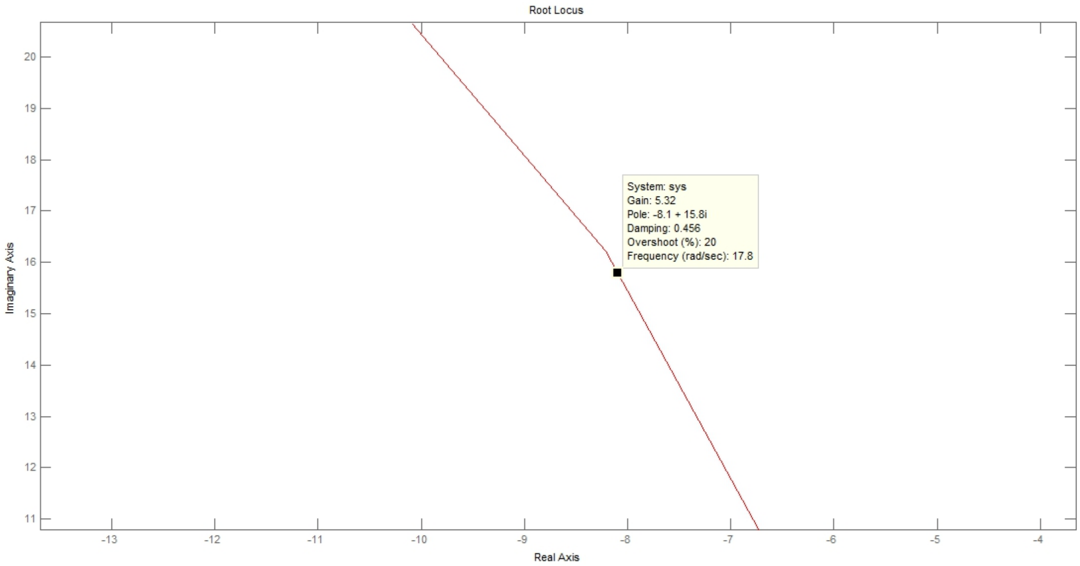

Figure 15: Root locus for PID-compensated system

(zoomed for finding the gain for 20% overshoot and dominant pole location)

Manual calculations for finding the loop gain *K* for the PID-compensated system:

The characteristic equation is simply $1+G(s) = 0$. The transfer function $G(s)$ of the PID-compensated system (including the loop gain K) is given by

$$G(s) = \frac{K(s+0.1)(s+8)(s+55.8)}{s(s+3)(s+6)(s+10)} \qquad \textbf{(Equation (2))}$$

From characteristic equation, $1+\dfrac{K(s+0.1)(s+8)(s+55.8)}{s(s+3)(s+6)(s+10)}=0 \;\Rightarrow\; \dfrac{K(s+0.1)(s+8)(s+55.8)}{s(s+3)(s+6)(s+10)}=-1$

$$\Rightarrow \quad K = \left.\frac{-\left(s(s+3)(s+6)(s+10)\right)}{(s+0.1)(s+8)(s+55.8)}\right|_{s=-8.147+15.9j} = \left.\frac{-\left(s^4+19s^3+108s^2+180s\right)}{s^3+63.9s^2+452.78s+44.64}\right|_{s=-8.147+15.9j}$$

$$= 5.3871+0.0268j$$

Therefore, $|K| = \sqrt{(5.3871)^2+(0.0268)^2}$ = **5.3872** (remember, $K = 5.32$ from simulation)

Step 6: Now we determine the gains K_1, K_2, and K_3 in Figure 1. We have obtained the transfer function of the PID-compensated system and is given in Equation (2). From that equation, the transfer function of the PID controller can be obtained as

$$G_{PID}(s) = \frac{K(s+55.8)(s+0.1)}{s}, \quad K = 5.32$$

$$= \frac{5.32\left(s^2+55.9s+5.58\right)}{s} \qquad \textbf{(Equation (3))}$$

Matching equations (1) and (3), $\boldsymbol{K_1 = 297.4}$, $\boldsymbol{K_2 = 29.7}$, $\boldsymbol{K_3 = 5.32}$

Simulation Results

Steps 7 and 8: Now, we have to check the performance of PID controller design with loop gain of 5.32 in MATLAB simulation. The step response of the PID-compensated system is shown in Figure 16.

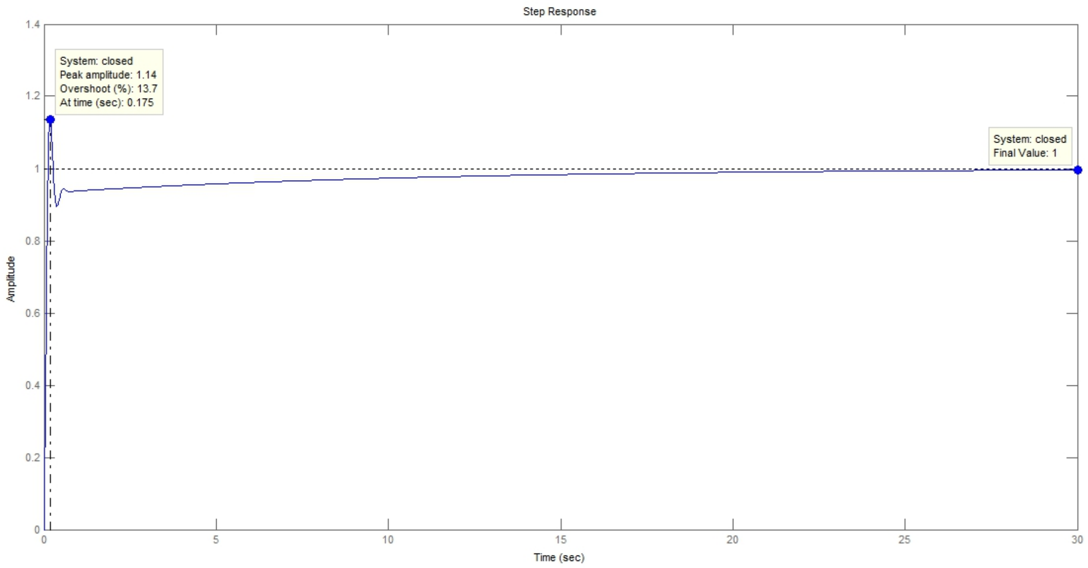

Figure 16: Step response of the PID-compensated system with $K = 5.32$

From Figure 16, it is evident that the PID-compensated system gives a peak time of **0.175 seconds** (which is lesser than the desired value of 0.1976 seconds) and a final value of **exactly 1**; hence the steady-state error is zero. Finally, with the designed PID controller, the given **specifications have been met**. The comparison of step responses of uncompensated system and PID-compensated system is shown in Figure 17 (Zoomed graphs are shown in Figures 18 and 19)

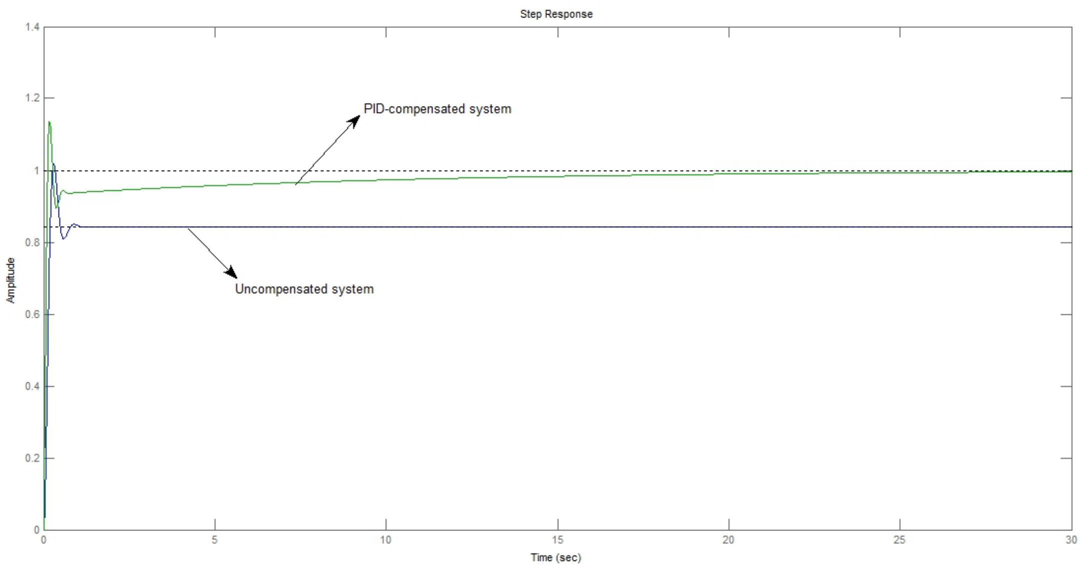

Figure 17: Comparison of step response for uncompensated system and PID-compensated system

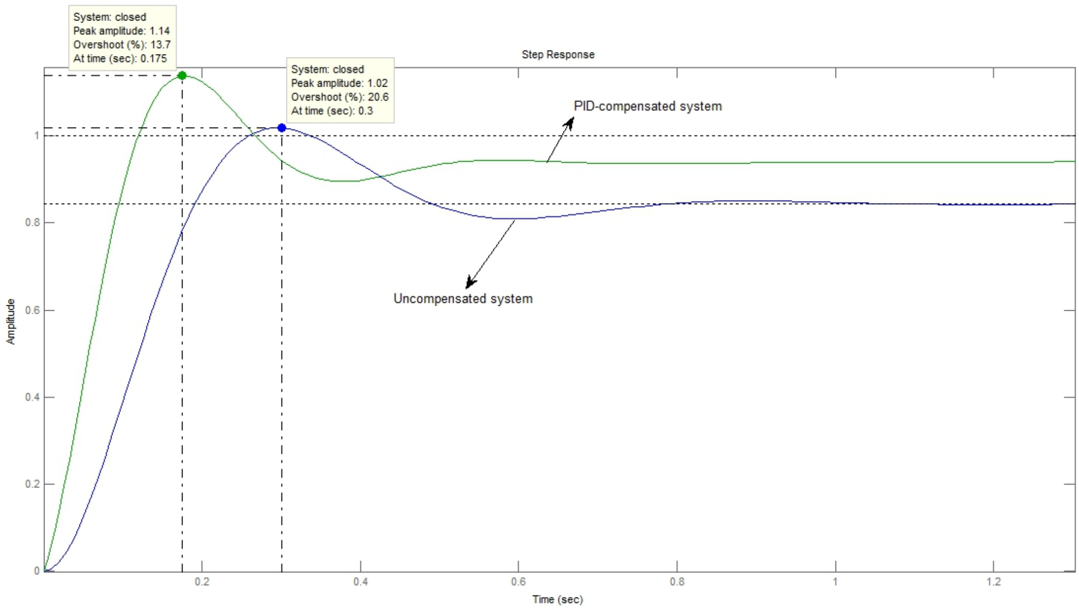

Figure 18: Comparison of peak time for uncompensated system and PID-compensated system

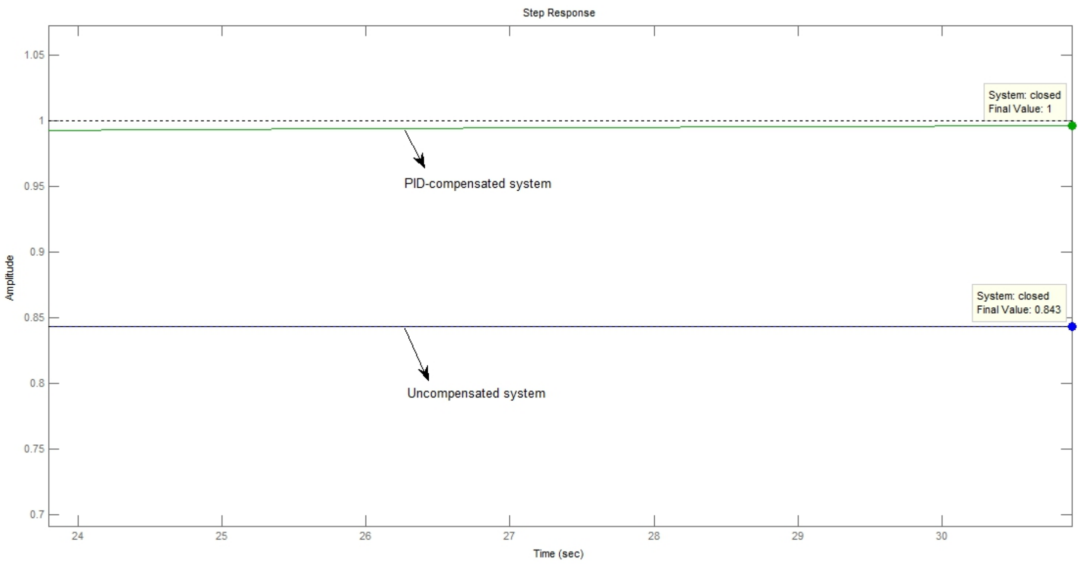

Figure 19: Comparison of steady-state error for uncompensated system and PID-compensated system

Simulation Tools used:

MATLAB R2020B