# Assignment #2

Date:

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Section = BCS - 2A.

1. $y'' + 4y' + 3y = 0$

Sol:-

$m^2 + 4m + 3 = 0$

$m = \frac{-4 \pm \sqrt{16-12}}{2}$

$m = \frac{-4 \pm \sqrt{4}}{2}$

$m = \frac{-4 \pm 2}{2}$ $\boxed{m_1 = -1 \quad m_2 = -3}$

$\boxed{y = c_1 e^{-x} + c_2 e^{-3x}}$

2. $y''' - y'' + y' - y = 0$

$m^3 - m^2 + m - 1 = 0$

$m^2(m-1) + 1(m-1) = 0$

$m_1 = 1, \quad m_2 = +1i$

$m_3 = -i$

$y = c_1 e^{x} + c_2 e^{+ix} + c_3 x e^{-ix}$

$\boxed{y = c_1 x + c_2 \cos x + c_3 \sin x}$

Ans.

3. $2x^2 y'' + 3xy' - 15y = 0$ by cauchy euler eq

$y = x^m, \quad y' = m x^{m-1}$

$2x^2 m(m-1) x^{m-2} + 3x m x^{m-1} - 15 x^m = 0$

$\Rightarrow (2m^2 - 2m)x^m + 3mx^m - 15x^m = 0$

$\Rightarrow x^m (2m^2 + m - 15) = 0.$

$m = \frac{-1 \pm \sqrt{1 - 4(2)(-15)}}{2(2)} \Rightarrow m = \frac{-1 \pm \sqrt{121}}{4}$

$m = \frac{-1 + 11}{4}, \quad \frac{-1 - 11}{4}$ $\boxed{m_1 = \frac{5}{2} \quad m_2 = -3}$

$\boxed{y = c_1 e^{5/2 x} + c_2 e^{-3x}}$ Ans

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4. $y'' - 3y' + 2y = x^2 e^x$ ①

→ UCE

$m^2 - 3m + 2 = 0.$

$m^2 - m - 2m + 2 = 0$

$m(m-1) - 2(m-1) = 0$

$m_1 = 2,\ m_2 = 1,\ y_c = c_1 e^{2x} + c_2 e^x$

$y_P = g(x) = x^2 e^x$

$= (Ax^2 + Bx + C)e^x.$

$y_P = (Ax^3 + Bx^2 + Cx)e^x$

$y_p' = Ax^2 + Bx + e^x(2Ax + B).$

$y_p'' = 2Ax + B + 2Ax + B^x + 2Ae^x$

put in ① eq

$4Ax + 2Ae^x + Bx + B$
$- 3Ax^2 - 3Bx - 6Be^x Ax$
$- 3Be^x + 2Ax^2e^x + Bxe^x$
$= x^2 e^x$

$\cancel{-4Ae^x + 4Ax - 2Bx + B}$

$[-3Bx - 2Ax - 4B + 2A]e^x = xe^x$

$e^x[(-2A - 3B)x + (2A - 4B)] = x e^x$

$-2A - 3B = 1,\quad 2A - 4B = 0$

$A = 0, -\frac{1}{3}$

$y_P = 2x^2 e^x - \frac{1}{7}xe^x \quad B = -1$
$-2x \quad C = -2$

$\boxed{\begin{aligned} y &= y_c + y_P \\ &= c_1 e^{2x} + c_2 e^x \\ &\quad - \frac{2}{3}x^2 e^x - \frac{1}{7}xe^x \\ &\quad - 2xe^x \end{aligned}}$

Ans.

5.

$y'' + 4y' = xe^x + x\sin 2x$

UCE.

$m^2 + 4m = 0.\ m(m+4) = 0$

$\cancel{m = 0},\ m = -2i, +2i$

for $y_{P_1}$, $xe^x = g(x)$

$y_{P_1} = (Ax + B)e^x.$

$y_{P_2}$, $g(x) = x\sin 2x$

$y_{P_2} = (Ax + B)\cos 2x + (Cx + D)\sin 2x$

$y_c = c_1\cos 2x + c_2\sin 2x$

$y_p = (Ax^2 + Bx)\cos 2x + (Cx^2 + Dx)\sin 2x$

$y_p'' + 4y_p = -8A\sin 2x$
$- 4B\sin 2x + 2A\cos 2x$
$+ 4D\cos 2x + 8Cx\cos 2x$
$+ 2Cx\sin 2x$
$= xe^x + x\sin 2x$

$\boxed{A = -\frac{1}{8}}\quad D = \frac{1}{16}$

$\boxed{C = -\frac{2}{25}}\quad \boxed{B = \frac{1}{5}}$

$y = y_c + y_P$

$= c_1\sin 2x + c_2\cos 2x$
$+ \frac{1}{5}xe^x - \frac{2}{25}e^x + \frac{1}{16}x\sin 2x$
$- \frac{1}{8}x^2\cos 2x$

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6. $y'' - 2y' + y = xe^{x}\ln x$,

$m^2 - 2m + 1 = 0$, $m_1 = 1$, $m_2 = 1$.

$y = c_1e^{x} + c_2e^{x}$

$y = (c_1 + c_2x)e^{x}$ $y_p = u_1y_1 + u_2y_2$

$W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} = \begin{vmatrix} e^{x} & xe^{x} \\ e^{x} & xe^{x} + e^{x} \end{vmatrix} = e^{2x}$

$W_1 = \begin{vmatrix} 0 & xe^{x} \\ xe^{x}\ln x & xe^{x} + e^{x} \end{vmatrix} = -x^2e^{2x}\ln x$

$W_2 = \begin{vmatrix} e^{x} & 0 \\ e^{x} & xe^{x}\ln x \end{vmatrix} = xe^{2x}\ln x$

$u_1' = \frac{W_1}{W} = \frac{-x^2e^{2x}\ln x}{e^{2x}} = -x^2\ln x$

LIATE

$u_1 = -\left[\ln x\int x^2dx - \int\left[\frac{1}{x}\int x^2dx\right]dx\right]$ $u_2' = \frac{xe^{2x}\ln x}{e^{2x}}$

$u_1 = -\left[\ln x\,\frac{x^3}{3} - \int\frac{x^2}{3}dx\right]$ $u_2' = x\ln x$

$u_1 = \frac{-x^3}{3}\ln x + \frac{x^3}{9}$ $u_2 = \ln x\int x\,dx - \int\left(\frac{1}{x}\,\frac{x^2}{2}\right)dx$

$u_2 = \frac{x^2}{2}\ln x - \frac{x^2}{4}$

$y_p = \left(-\frac{x^3}{3}\ln x + \frac{x^3}{9}\right)e^{x} + \left(\frac{x^2}{2}\ln x - \frac{x^2}{4}\right)xe^{x}$

$y_p = \left(\frac{x^3}{6}\ln x - \frac{5x^3}{36}\right)e^{x}$

$y = y_c + y_p = (c_1 + c_2x)e^{x} + \left(\frac{x^3}{6}\ln x - \frac{5x^3}{36}\right)e^{x}$

Ans

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Q8) $y''-4y'-12y = 2t^3-t+3$ — (1)

$y''-4y'-12y=0$, $m^2-4m-12=0$, $m_2=6$, $m_1=-2$

$y_c = c_1e^{-2t}+c_2e^{6t}$

$y_p = At^3+Bt^2+Ct+D$

$y_p' = 3At^2+2Bt+C$.

$y_p'' = 6At+2B$. in eq (1).

$6At+2B-4(3At^2+2Bt+C)-12(At^3+Bt^2+Ct+D)$
$= 2t^3-t+3$

for $t^3$ : $-12A = 2$, $\boxed{A = -\frac{1}{6}}$

for $t^2$ : $-12A-12B=0$, $B = 1/6$.

for $t$ : $6A-8B-2C=-1$.

$\boxed{C=-1/9}$

for $t^0$ : $2B-4(-1)D=3$

$\boxed{D=-\frac{5}{27}}$

$\boxed{y_p = -\frac{t^3}{6}+\frac{t^2}{6}-\frac{t}{9}-\frac{5}{27}}$

$\therefore y = y_c+y_p$

$y = c_1e^{-2t}+c_2e^{6t}-\frac{t^3}{6}+\frac{t^2}{6}-\frac{t}{9}-\frac{5}{27}$

Ans

Q9) $y''+5y'+6y=2x$

$m^2+5y+6=0$, $m=-3, -2$

$y_c = c_1e^{-3x}+c_2e^{-2x}$

$y_p = Ax+B$, $y_p' = Ax$, $y''p = 0$.

$5A+6(Ax+B) = 2x$

for $x$ : $6A=2$ $\boxed{A=1/3}$

for $x^0$ : $5A+6B=0$, $\boxed{B=-5/18}$

SANDAL

$y_p = \frac{x}{3} - \frac{5}{18}$ $\quad y = y_c + y_p$

Ans. $\boxed{y = c_1 e^{-3x} + c_2 e^{-2x} + \frac{x}{3} - \frac{5}{18}}$

Q10 $y'' + 5y' - 9y = e^{-2x} + 2 - x$

$m^2 + 5m - 9 = 0.$ $\quad m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$m = 1.4x$

$m_2 = -6.4$

$y_c = c_1 e^{1.4x} + c_2 e^{-6.4x}$

~~$y_p =$~~

$y'' + 5y' - 9y = e^{-2x}$ $\quad g(x) = e^{-2x}$

$y_p = Ae^{-2x}$

$4Ae^{-2x} - 10Ae^{-2x} - 9Ae^{-2x} = e^{-2x}$ $\quad y_p' = -2Ae^{-2x}$

$y_p'' = +4Ae^{-2x}$

$\boxed{A = \frac{-1}{15}}$ $\quad y_{p_1} = \frac{-e^{-2x}}{15}$

Now,

$y'' + 5y' - 9y = 2 - x = -x + 2$

$y_p = Ax + B.$ $\quad y_p' = A,$ $\quad y_p'' = 0.$

$5A - 9(Ax + B) = 2 - x,$ ~~$A =$~~

for $x$: $-9A = 2$ $\quad A = \frac{-2}{9}$

for $x^0$: $9B = 5A - 2$

$\boxed{B = -13/81}$

$y_{p_2} = \frac{x}{9} - \frac{13}{81}$

$y = y_c + y_{p_1} + y_{p_2}$

$\boxed{y = C_1 e^{1.4x} + c_2 e^{-6.4x} - \frac{1}{15} e^{-2x} + \frac{x}{9} - 13/81}$ Ans

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Q11 $y'' - 100y = 9t^2 e^{10t} + \cos t - \sin t$. (A)

$m^2 - 100 = 0$, $m_1 = 10$, $m_2 = -10$

$y_c = c_1 e^{10t} + c_2 e^{-10t}$

(1) $y'' - 100y = 9t^2 e^{10t} \rightarrow g(x)$

$y_{p_1} = (At^2 + Bt + C)e^{10t}$ xing by $(x)$

$y_{p_1} = (Ax^3 + Bx^2 + Cx)e^{10t}$

$\cancel{y_{p_1}' = (3Ax^2 + 2Bx + C)e^{10t} + (6Ax + 2B)e^{10}}$

$y_{p_1}' = 10(Ax^3 + Bx^2 + Cx)e^{10t}$
$+ (3Ax^2 + 2Bx + C)e^{10t}$

$y_{p_1}'' = 100(Ax^3 + Bx^2 + Cx)e^{10t}$
$+ 10(3Ax^2 + 2Bx + C)e^{10t}$
$+ 10(3Ax^2 + 2Bx + C)e^{10t} + (6Ax + 2B)e^{10t}$

in (1)

$100(Ax^3 + Bx^2 + C)e^{10t} + 20(3Ax^2 + Bx + C)e^{10t}$
$+ (6Ax + 2B)e^{10t}$
$- 100(Ax^3 + Bx^2 + C)e^{10t} = 9t^2 e^{10t}$

for $x^2$; $60A = 9$. $A = 3/20$

for $x^1$; $40B + 6B = 0$

$B = \frac{-9}{400}$

for $x^0$; $20C + 2B = 0$, $\boxed{C = \frac{9}{4000}}$

$y_{p_1} = \frac{3}{20}t^2 - \frac{9}{400}x^2 e^{10t}$.

Now,

$$y'' - 100y = \cos t \quad -(B)$$

$$y_{p_2} = A\cos t + B\sin t. \quad - (2).$$

$$y_{p_2}' = -A\sin t + B\cos t.$$

$$y_{p_2}'' = -A\cos t - B\sin t \quad \text{in } (B)$$

$$-A\cos t - B\sin t - 100(A\cos t + B\sin t) = \cos t$$

for $\cos t$; $-A - 100A = 1$

$$A = -1/101$$

for $\sin t$,

$$-B - 100B = 0, \quad \boxed{B = 0} \quad \text{in } (2)$$

$$y_{p_2} = -\frac{\cos t}{101}$$

now, for $\sin t$

$$y'' - 100y = -t\sin t, \quad -(C)$$

$$y_{p_3} = (At + B)\cos t + (Ct + D)\sin t - (3)$$

$$y_{p_3}' = -(At+B)\sin t + (Ct + D)\cos t + A\cos t + C\sin t$$

$$y_{p_3}'' = -(At+B)\cos t - (Ct+D)\sin t - A\sin t + C\cos t$$

put in (C)

$$-(At+B)\cos t - 2A\sin t - (Ct+D)\sin t + 2\cos t - 10[(At+B)\cos t + (Ct+D)\sin t] = -t\sin t.$$

for $t\sin t$;

$$-C - 100C = -1.$$

$$C = 1/101.$$

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for $t\cos t$; $-A - 100A = 0$

$A = 0.$

for $\sin t$; $-2A - D - 100D = 0.$

$D = 0.$

for $\cos t$; $-B + 2C - 100B = 0$

$B = \frac{2}{10201}$. put

in (C).

$$y_{P_3} = \frac{-2}{10201}\cos t + \frac{\sin t}{101}$$

$$y = y_c + y_{P_1} + y_{P_2} + y_{P_3}$$

$$\boxed{y = C_1 e^{10t} + C_2 e^{-10t} + \frac{3}{20} x^3 e^{10t} - \frac{9}{400} x^2 e^{10t} - \frac{\cos t}{101} + \frac{2}{10201}\cos t + \frac{t\sin t}{101}}$$

Ans.

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12. $y'' - 2y' + 2y = e^x \tan x$

$m^2 - 2ym + 2 = 0$, $m = 1 \pm i$.

$y_c = c_1 \cos x\, e^x + c_2 \sin x\, e^x$.

$y_P$ using variation of parameter.

$y_P = u_1 y_1 + u_2 y_2$.

$$W = \begin{vmatrix} e^x\cos x & e^x \sin x \\ e^x\cos x - e^x\sin x & e^x \sin x + e^x\cos x \end{vmatrix}$$

$\boxed{W = e^{2x}}$

$$W_1 = \begin{vmatrix} 0 & e^x \sin x \\ e^x \tan x & e^x \sin x + e^x \cos x \end{vmatrix}$$

$$W_1 = -e^{2x} \sin^2 x / \cos x$$

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$$W_2 = \begin{vmatrix} e^{x}\cos x & 0 \\ e^{x}\cos x - e^{x}\sin x & e^{x}\tan x \end{vmatrix} = e^{2x}\sin x$$

$u_1' = \frac{W_1}{W} = \frac{-\sin^2 x \tan x}{\cos x}$, $\int u_1' = -\int (\sin^2 x \sec x)\,dx$

$u_1 = -\int \left(\frac{1-\cos^2 x}{\cos x}\right)dx \to u_1 = -\int (\sec x - \cos x)\,dx$

$u_1 = -\int \sec x\,dx + \int \cos x\,dx$

$u_1 = -\ln(\tan x + \sec(x)) + \sin x$

$u_2' = \frac{W_2}{W} = \sin x$, $\int u_2' = -\cos x$.

$\underline{u_2 = -\cos x}$

$y_p = \{-\ln(\tan x + \sec x) + \sin x\} e^{x}\cos x$
$- \cos x\, e^{x}\sin x.$

$y_p = -\ln(\tan x + \sec x)\, e^{x}\cos x$

$y = y_c + y_p$

$y = e^{x}(c_1\cos x + c_2\sin x) - e^{x}\cos x \ln(\tan x + \sec x).$

Ans.

13) $x^2 y'' - 4xy' + 6y = 2x^4 + x^2.$

$(1) m^2 - 4m + 6 = 0.$ $\quad a_1 = -4$

$m^2 - 5m + 6 = 0.$ $\quad a_2 = 1.$

$\Rightarrow m = 3, 2$

$y_c = c_1 x^2 + c_2 x^3$

Now, divide by $x^2$,

$y'' - \frac{4}{x}y' + \frac{6}{x^2}y = 2x^2 + 1.$

$y_p = u_1 y_1 + u_2 y_2.$

$W = \begin{vmatrix} x^2 & x^3 \\ 2x & 3x^2 \end{vmatrix}$ , $w_1 = \begin{vmatrix} 0 & x^3 \\ 2x^2+1 & 3x^2 \end{vmatrix}$

$w = x^4$ $\quad$ $w_1 = -2x^5 - x^3$.

$w_2 = \begin{vmatrix} x^2 & 0 \\ 2x & 2x^2+1 \end{vmatrix} = 2x^4 + x^2.$

$u_1' = \dfrac{w_1}{w}$ , $u_2' = \dfrac{w_2}{w}$

$= \dfrac{-2x^5 - x^3}{x^4}$ , $= \dfrac{2x^4+x^2}{x^4}$

$u_1' = -2x - \dfrac{1}{x}$ , $u_2' = 2 + \dfrac{1}{x^2}$

$\int u_1' = \int\left(-2x - \dfrac{1}{x}\right)dx$ , $\int u_2' = \int\left(2 + \dfrac{1}{x^2}\right)dx$

$u_2 = 2x - \dfrac{1}{x}$

$u_1 = -x^2 - \ln x.$

$y_P = (-x^2 - \ln x)x^2 + \left(2x - \dfrac{1}{x}\right)x^3$

$y = y_c + y_P.$

$y = c_1x^2 + c_2x^3 + x^4 - x^2 - x^2 \ln x$

$(y = c_1x^2 + c_2x^3 + x^4 = x^2 \ln x)$ Ans.

Q# 14 $\quad x^2y'' + 10xy' + 8y = x^2$

$x^2y'' + 10xy' + 8y = 0.$ , $a_2 = 1$

$m^2 + 9m + 8 = 0.$ $\quad a_1 = 10$

$m = -1, -8$ $\quad a_0 = 8.$

$y_c = c_1x^{-1} + c_2x^{-8}$

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$y_p = Ax^2+Bx+C$, $y_p' = 2Ax+B$

$y_p'' = 2A$.

$x^2[2A] + 10x[2Ax+B] + 8(Ax^2+Bx+C] = x^2$.

for $x^2$: $2A+20A+8A = 1$

$A = 1/30$.

for $x$: $10B+8B=0$. , $B=0$,

for $x^0$; $C=0$

$y = y_c + y_p$

$y = c_1x^{-1} + c_2x^{-8} + \frac{x^2}{30}$ Ans

Q15 $x^2y'' - 3xy' + 13y = 4+3x$

$x^2y'' - 3xy' + 13y = 0$ —(1)

$a_0 = 1$, $a_1 = -3$

$m^2 - 4m + 13 = 0$

$m = 2 \pm 3i$.

$\because e^{\alpha+i\beta}$

$y_c = c_1e^{x} + c_2e^{x}$

$y_c = c_1e^{(2+3i)x} + c_2e^{(2-3i)x}$

$y_c = c_1e^{2x}[\cos 3i + \sin 3i] + c_2e^{2x}[\cos 3i - \sin 3i]$

xing wid $\ln x$

$y_c = x^2[c_1\cos(3\ln x) + c_2\sin(3\ln x)]$.

$y_p = Ax+B$.

$y_p' = A$, $\boxed{0 = y_p''}$ in (1)

$-3x[Ax+B] + 13[Ax+B] = 4+3x$

for $x$; $-3A+13A = 3$

$\boxed{A = 3/10}$

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for $x^0$: $13B = 4$, $B = 4/13$

$$y_p = \frac{3}{10}x + 4/13$$

$$y = y_c + y_p = x^2\left[c_1\cos(3\ln x) + c_2\sin(3\ln x)\right] + \frac{4}{13} + \frac{3x}{10}$$

Ans.

Q16 $x^3y''' - 3x^2y'' + 6xy' - 6y = 3 + \ln x^3$

$x^3y''' - 3x^2y'' + 6xy' - 6y = 0.$

using Cauchy Euler

$a_3 = 1$, $a_2 = -3$, $a_1 = 6$, $a_0 = -6$.

$m^3 - 6m^2 + 11m - 6 = 0$

$m = 1, 2, 3.$

$y_c = c_1x + c_2x^2 + c_3x^3$

$x^3y''' - 3x^2y'' + 6xy' - 6y = 3 + \ln x^3$

$$y''' - \frac{3}{x}y'' + \frac{6}{x^2}y' - \frac{6}{x^3}y = \frac{3 + \ln x^3}{x^3}$$

$y_p = u_1y_1 + u_2y_2 + u_3y_3$

$$W = \begin{vmatrix} x & x^2 & x^3 \\ 1 & 2x & 3x^2 \\ 0 & 2 & 6x \end{vmatrix} = 2x^3$$

$$W_1 = \begin{vmatrix} 0 & x^2 & x^3 \\ 0 & 2x & 3x^2 \\ \frac{3 + \ln x^3}{x^3} & 2 & 6x \end{vmatrix}$$

$W_1 = 9x + 9x\ln x - 6x - 6x\ln x$

$\boxed{W_1 = 3x + 3x\ln x}$

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$$W_2 = \begin{vmatrix} x & 0 & x^3 \\ 1 & 0 & 3x^2 \\ 0 & \frac{3+3\ln x}{x^3} & 6x \end{vmatrix} = -9-9\ln x+3+3\ln x + \cancel{3\ln x - 6 - 6\ln x}$$

$$w_2 = -6-6\ln x$$

$$W_3 = \begin{vmatrix} x & x^2 & 0 \\ 1 & 2x & 0 \\ 0 & 2 & \frac{3+3\ln x}{x^3} \end{vmatrix} = \frac{3+3\ln x}{x}$$

$$u_1' = \frac{W_1}{W} = \frac{3}{2}\left(\frac{1}{x^2} + \frac{\ln x}{x^2}\right).$$

$$\int u_1' = \frac{3}{2}\left(\frac{-1}{x} - \frac{\ln x + 1}{x}\right)$$

$$u_1 = \frac{3}{2}\left(\frac{-\ln x - 2}{x}\right)$$

$$u_2' = \frac{W_2}{W} = \frac{-6-6\ln x}{2x^3} = \frac{-3-3\ln x}{x^3}$$

$$\int u_2' = \int \frac{-3-3\ln x}{x^3} = -3\left[\frac{-3-2\ln x}{4x^2}\right]$$

$$u_3' = \frac{W_3}{W} = \frac{3+3\ln x}{2x^4} \Rightarrow \int u_3' = \frac{3}{2}\left[\frac{-4-3\ln x}{x^3}\right]$$

$$y_p = u_1y_1 + u_2y_2 + u_3y_3$$

$$= \frac{-17}{12} - \frac{1}{2}\ln x$$

$$y = y_c + y_p = c_1x + c_2x^2 + c_3x^3 - \frac{1}{2}\ln x - \frac{17}{12}$$

Ans

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17/ $y'' - 2y' + y = \frac{1}{x} e^x$ ; $y(1) = 0$, $y'(1) = 1$

$y'' - 2y' + y = 0$, $m^2 - 2m + 1 = 0$

$m = 1, 1$.

$y_c = c_1 e^x + c_2 x e^x$ $\boxed{y_p = u_1 y_1 + u_2 y_2}$

$u_1' = \frac{W_1}{W}$, $u_2' = \frac{W_2}{W}$.

$W = \begin{vmatrix} e^x & e^x \\ e^x & xe^x + e^x \end{vmatrix} = e^{2x}$

$W_1 = \begin{vmatrix} 0 & xe^x \\ e^x/x & xe^x + e^x \end{vmatrix} = -e^{2x}$

$W_2 = \begin{vmatrix} e^x & 0 \\ e^x & \frac{e^x}{x} \end{vmatrix} = \frac{e^{2x}}{x}$

$u_1' = -1$, $\int u_1' = -x$

$u_2' = \frac{1}{x}$, $\int u_2' = \ln x$

$y_p = -xe^x + xe^x \ln x$

$y = y_c + y_p$

$y = c_1 e^x + c_2 x e^x - xe^x + xe^x \ln x$

$y' = c_1 e^x + c_2 e^x + c_2 x e^x - \cancel{e^x} - xe^x + xe^x \ln x + e^x \ln x + \cancel{e^x}$

$y' = c_1 e^x + c_2 e^x + c_2 x e^x - xe^x + xe^x \ln x + e^x \ln x$

$y(1) = 0$

$0 = c_1 e + c_2 e - e$ (A)

$y'(1) = 1$

$1 = c_1 e + 2c_2 e - e$ Solving Simultaneously

$\boxed{c_2 = e^{-1}}$, $\boxed{c_1 = 1 - e^{-1}}$

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$y = (1-e^{-1})e^{x} + e^{-1}xe^{x} - xe^{x} + xe^{x}\ln x$

$$\boxed{y = e^{x-1}(1-x)(e^{x}-1) + xe^{x}\ln x}$$

Ans.

**Q18** $y'' + 4y = \sin^2 2x$

$y'' + 4y = 0$, $m^2 + 4 = 0$, $m_1 = 2i$, $m_2 = -2i$

$y_c = c_1 \cos 2x + c_2 \sin 2x$

$y_p = u_1 y_1 + u_2 y_2$

$$w = \begin{vmatrix} \cos 2x & \sin 2x \\ -2\sin 2x & 2\cos 2x \end{vmatrix} = 2$$

$$w_1 = \begin{vmatrix} 0 & \sin 2x \\ \sin^2 2x & 2\cos 2x \end{vmatrix} = -\sin^3 2x$$

$$w_2 = \begin{vmatrix} \cos 2x & 0 \\ -2\sin 2x & \sin^2 2x \end{vmatrix} = \sin^2 2x \cos 2x$$

$$u_1' = \frac{w_1}{w} = \frac{-1}{2}\left(\sin 2x - \cos^2 2x \sin 2x\right)$$

$$\int u_1' = \frac{-1}{2}\left(-\frac{\cos 2x}{2} + \frac{\cos^3 2x}{6}\right)$$

$$\boxed{u_1 = \frac{\cos 2x}{4} - \frac{\cos^3 2x}{12}}$$

$$\int u_2' = \int \frac{w_2}{w} = \int\left[\frac{\sin^2 2x \cos 2x}{2}\right]$$

$$= \frac{1}{2}\left[\frac{\sin^3 2x}{6}\right] = \frac{\sin^3 2x}{12}$$

$$y_p = \frac{\cos^2 2x}{4} - \frac{\cos^3 12x}{12} + \frac{\sin^4 2x}{12}.$$

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$y = y_c + y_p = c_1\cos 2x + c_2 \sin 2x$
$+ \frac{\cos^2 2x}{4} - \frac{\cos^4 2x}{12} + \frac{\sin^4 2x}{12}$

$y' = -2c_1 \sin 2x + 2c_2 \cos 2x - \frac{4\cos 2x \sin 2x}{4}$
$+ \frac{8\cos^3 2x \sin 2x}{12} + \frac{8\sin^3 2x \cos 2x}{12}$

Now; $y'(0) = 0$

$c_2 = 0$

, $y(0) = 0$, $c_1 = \frac{-1}{6}$

$$\boxed{y = \frac{-1}{6}\cos 2x + \frac{3\cos^2 2x - \cos^4 2x + \sin^4 2x}{12}}$$

Ans.

19) $y'' - 6y' - 7y = -9e^{-2x}$
$m^2 - 6m - 7 = 0$ $\quad g(x)$
$m = 7, -1$
$y = c_1 e^{7x} + c_2 e^{-x}$

$y_p = Ae^{-2x}$ $\quad$ (A)
$y_p' = -2Ae^{-2x}$
$y_p'' = 4Ae^{-2x}$

$e^{-2x}$
$4Ae^{-2x} + 12A - 7A = -9e^{-2x}$
$\boxed{A = -1}$ putting value in (A)

$y_p = -e^{-2x}$, $y = c_1 e^{7x} + c_2 e^{-x} - e^{-2x}$
Now, $y(0) = -2$, $y'(0) = -13$
$-2 = c_1 + c_2 - 1$
$-13 = 7c_1 - c_2 + 2$
$c_1 = -2$, $c_2 = 1$

$$y = -2e^{7x} + e^{-x} - e^{-2x}$$

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20. $y'' - 4y' + 4y = 2e^{2x} - 12\cos 3x - 5\sin 3x$

; $y(0) = -2$, $y'(0) = 4$

$m^2 - 4m + 4 = 0$ , $m_1 = 2$, $m_2 = 2$

$\boxed{y_c = C_1 e^{2x} + C_2 x e^{2x}}$.

$y'' - 4y' + 4y = 2e^{2x}$ (1)

$y_p = Ax^2 e^{2x}$, $y_p' = 2xAe^{2x} + 2Ax^2 e^{2x}$

$y_p'' = 2Ae^{2x} + 4Axe^{2x} + 4Axe^{2x} + 4Ax^2 e^{2x}$

comparing after putting values,

$e^{2x}$; $2A = 2$, $A = 1$

for . Now, $\boxed{y_{p_1} = x^2 e^{2x}}$

$y'' - 4y' + 4y = -12\cos 3x$ (2)

$y_{p_2} = \cos 3x + \sin 3x$, $y_{p_2}' = -3A\sin 3x + 3B\cos 3x$

$y_{p_2}'' = -9A\cos 3x - 9B\sin 3x$

comparing after putting values,

$\cos 3x$; $-9A - 12B + 4A = -12$

$5A + 12B = 12$ — (A

for $\sin 3x$; $-9B + 12A + 4B = 0$.

$-5B + 12A = 0$ — (B

Solving A, B simultaneously

$A = 160/169$, $B = 144/169$.

$\boxed{y_{p_2} = \frac{60}{169}\cos 3x + \frac{144}{169}\sin 3x}$

$y'' - 4y' + 4y = -5\sin 3x$ (3)

$y_{P3} = A\cos 3x + B\sin 3x$

$y_{P3}' = -3A\sin 3x + 3B\cos 3x$

$y_{P3}'' = -9A\cos 3x - 9B\sin 3x$

compare after putting values,

$\sin 3x;\quad -9B + 12A + 4B = -5$

$-5B + 12A = -5.$ (c)

for $\cos 3x;\quad -5A - 12B = 0$ (d)

Solving (c) and (d)

$A = \frac{-60}{169}, \quad B = \frac{25}{169}$

$y_{P3} = \frac{-60}{169}\cos 3x + \frac{25}{169}\sin 3x$

$y = c_1e^{2x} + c_2xe^{2x} + x^2e^{2x} + \cancel{\frac{60}{169}\cos 3x}$

$+ \frac{144}{169}\sin 3x \cancel{- \frac{60}{169}\cos 3x} + \frac{25}{169}\sin 3x$

$y = c_1e^{2x} + c_2xe^{2x} + x^2e^{2x} + \sin 3x$

$y(0) = -2, \quad \boxed{c_1 = -2}$ $\quad y' = 2c_1e^{2x} + c_2e^{2x}$

$+ 2c_2xe^{2x} + 2x^2e^{2x}$

$y'(0) = 4, \quad \boxed{c_2 = 5.}$ $\quad + 3\cos 3x$

$$\boxed{y = -2e^{2x} + 5xe^{2x} + x^2e^{2x} + \sin 3x.}$$

Ans.