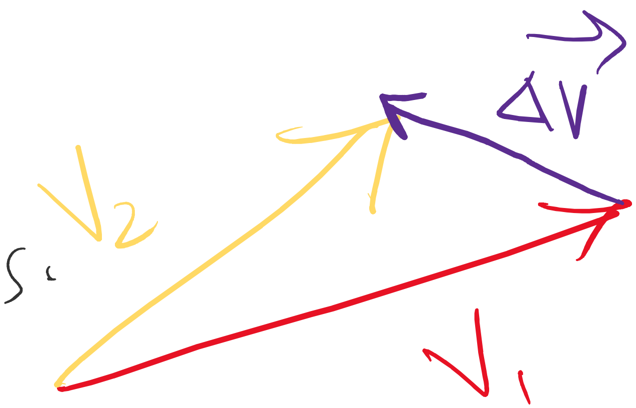
Curved paths

Projectile and Circular Motions

Time Evolution of Velocity vector

$$\underbrace{\overrightarrow{v_2} = \overrightarrow{v_1} + \overrightarrow{\Delta v}}_{\text{vector sum for velocity vectors.}}$$

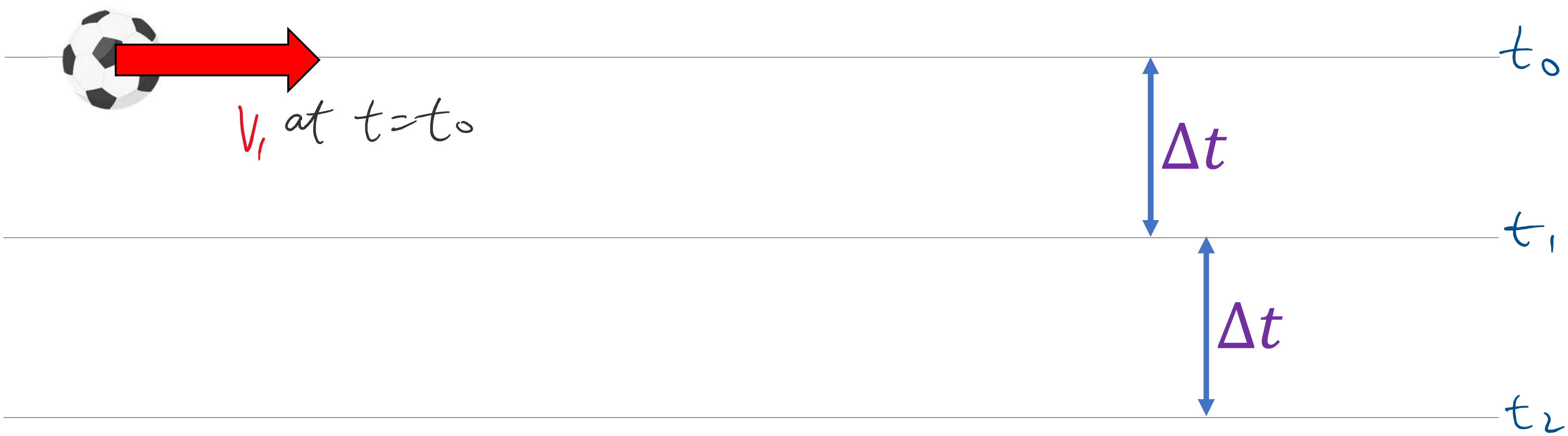


These velocity vectors exist at different points in space and time, however to workout a difference we place them on single point (vertex)

Time Evolution of Velocity vector

$$\overrightarrow{v_2} = \overrightarrow{v_1} + \overrightarrow{\Delta v} = \overrightarrow{v_1} + \vec{a}\Delta t$$

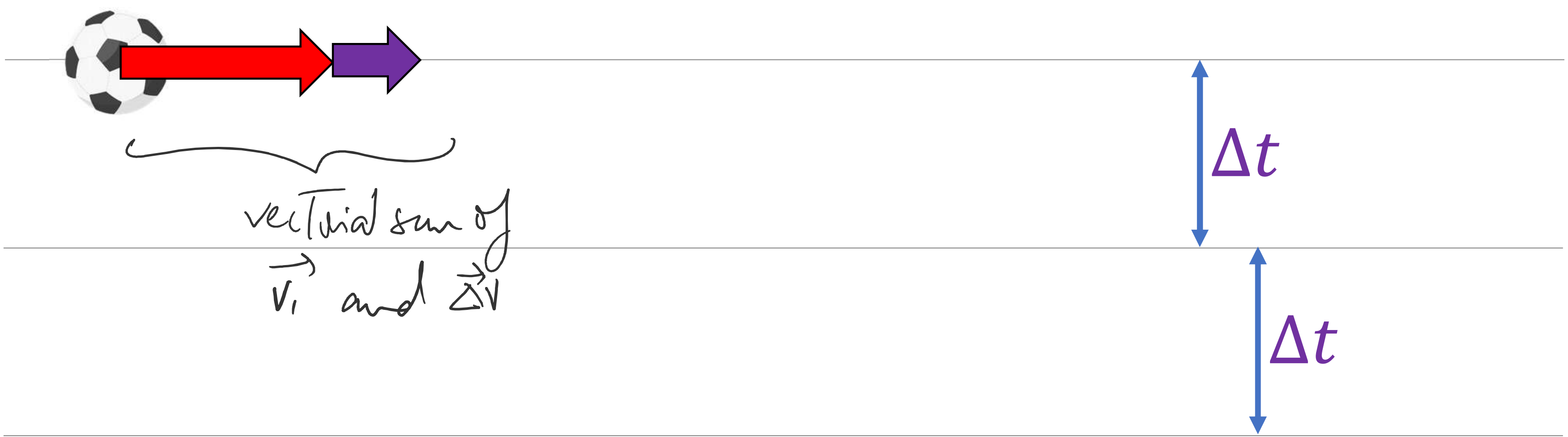
<u>One dimension</u> (constant acceleration, particle frame) → camera will move with the particle



Time Evolution of Velocity vector

$$\overrightarrow{v_2} = \overrightarrow{v_1} + \overrightarrow{\Delta v} = \overrightarrow{v_1} + \vec{a}\Delta t$$

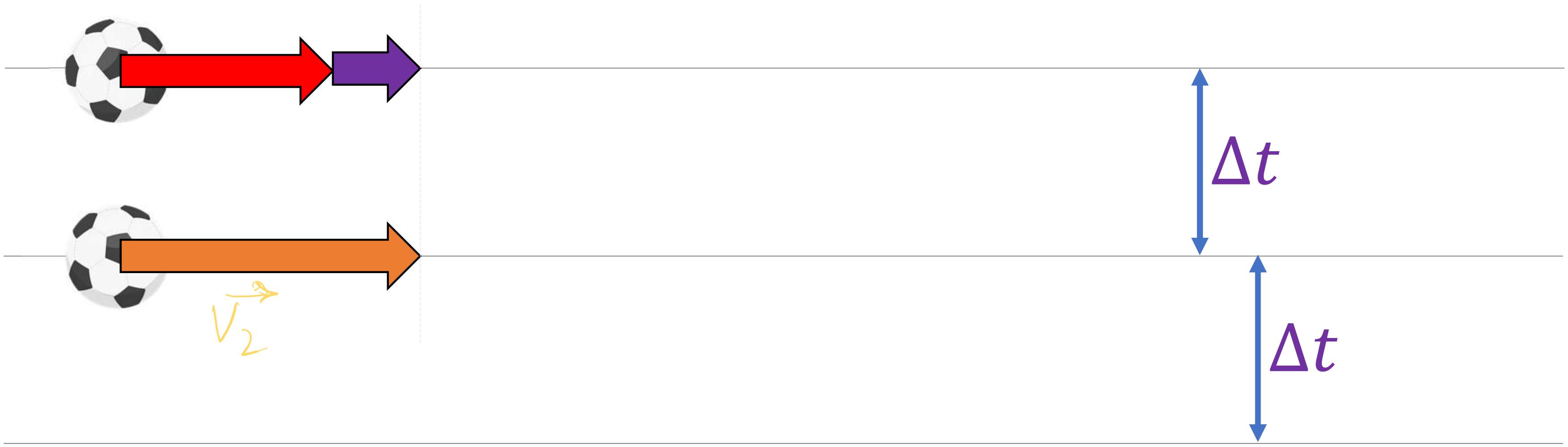
<u>One dimension</u> (constant acceleration , particle frame)



Time Evolution of Velocity vector

$$\overrightarrow{v_2} = \overrightarrow{v_1} + \overrightarrow{\Delta v} = \overrightarrow{v_1} + \vec{a}\Delta t$$

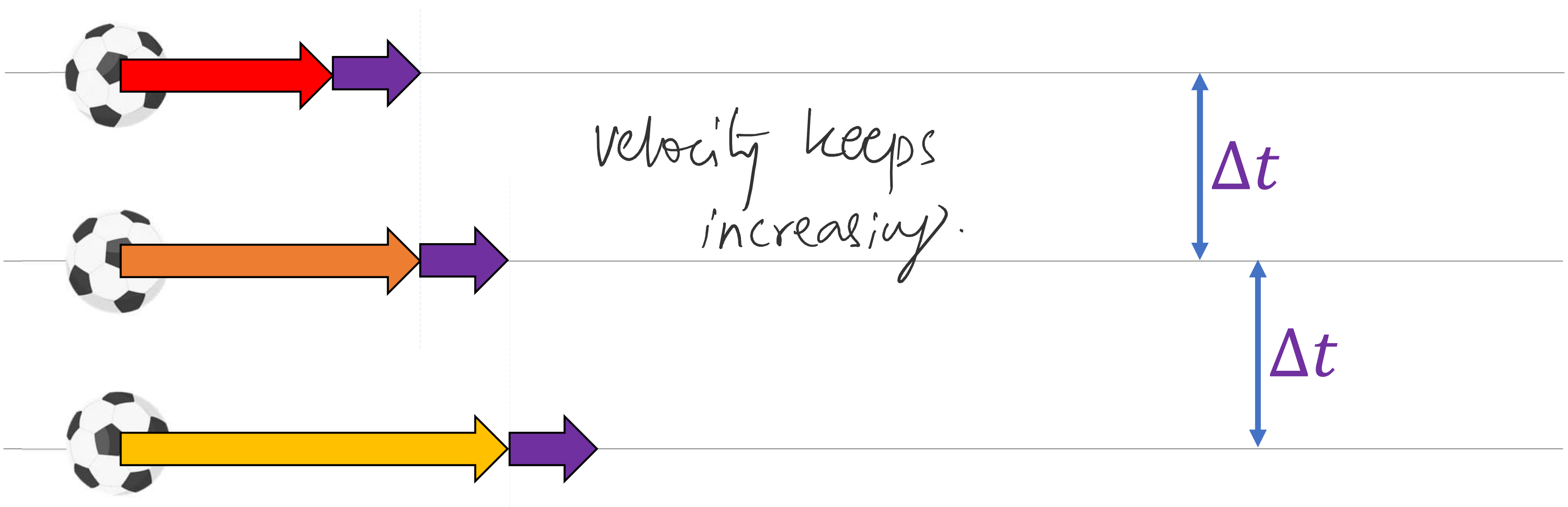
<u>One dimension</u> (constant acceleration , particle frame)



Time Evolution of Velocity vector

$$\overrightarrow{v_2} = \overrightarrow{v_1} + \overrightarrow{\Delta v} = \overrightarrow{v_1} + \vec{a}\Delta t$$

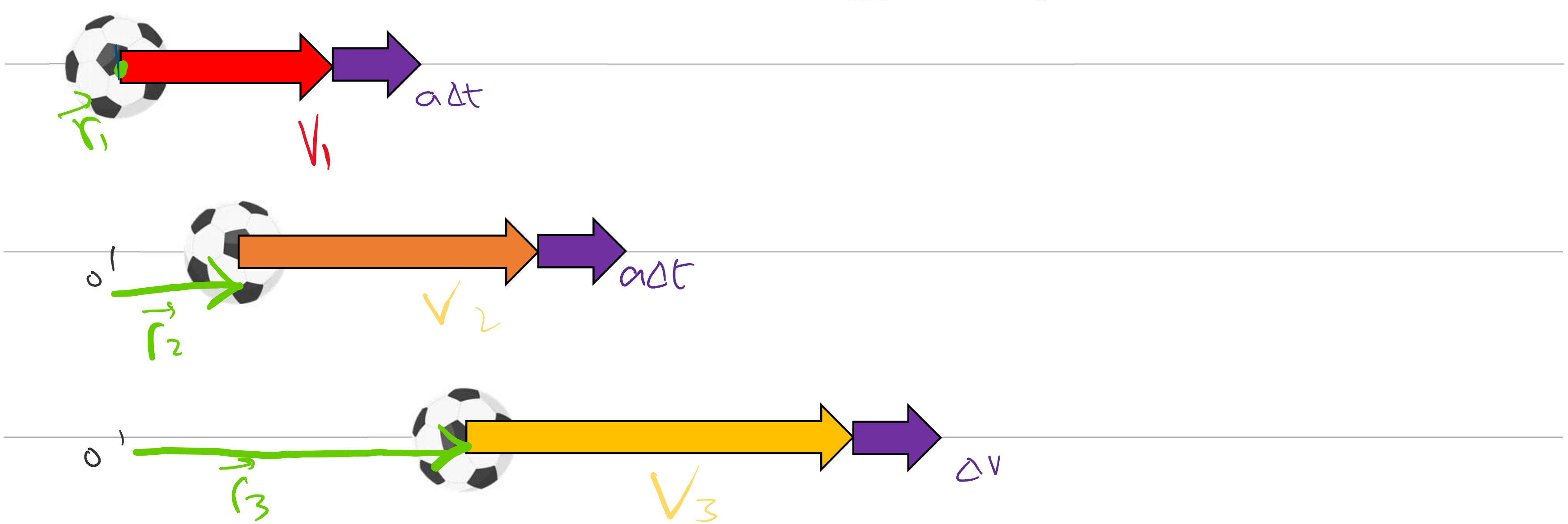
<u>One dimension</u> (constant acceleration , particle frame)



Time Evolution of Velocity vector

$$\overrightarrow{v_2} = \overrightarrow{v_1} + \overrightarrow{\Delta v} = \overrightarrow{v_1} + \vec{a}\Delta t$$

<u>One dimension</u> (constant acceleration) Static camera situation

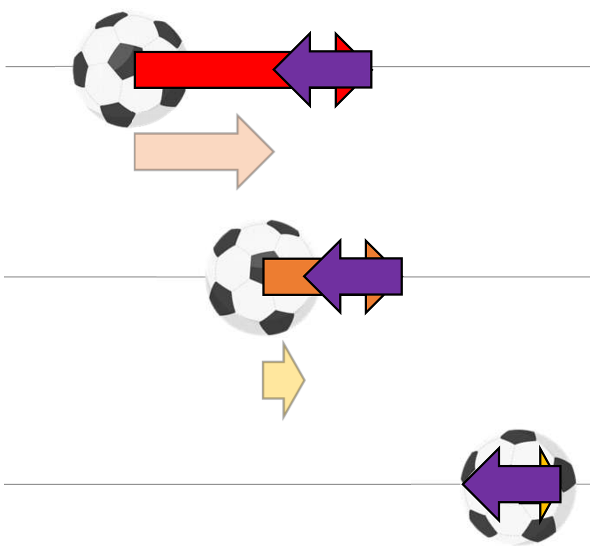


Time Evolution of Velocity vector

$$\overrightarrow{v_2} = \overrightarrow{v_1} + \overrightarrow{\Delta v} = \overrightarrow{v_1} + \vec{a}\Delta t$$

One dimension (constant acceleration)

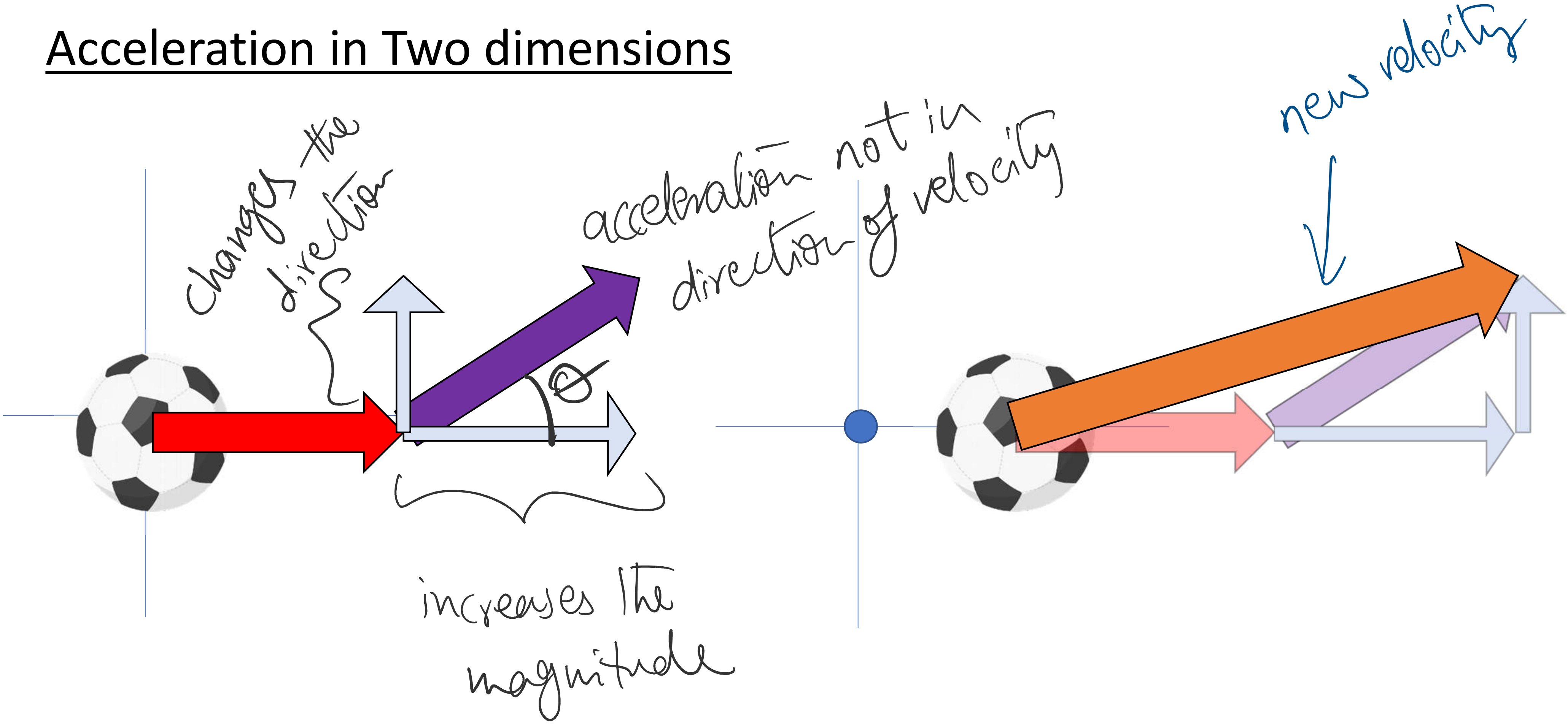
(Acceleration in negative direction)



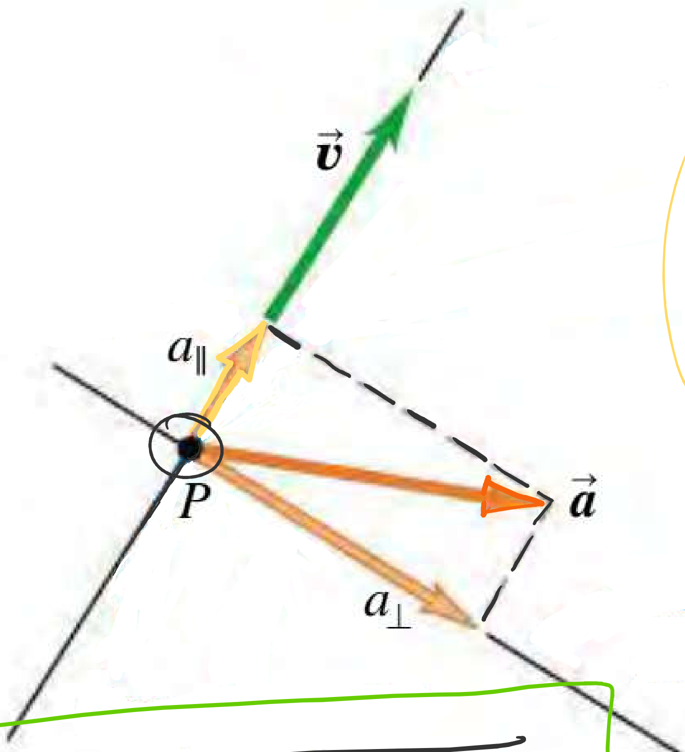
Vectors still add but resulting velocities are reducing in magnitude.

Velocity	Acceleration	Resulting Velocity
+ axis	+ axis	increases, in + axis
+ axis	− axis	decreases, remains in + axis untill zero
− axis	+ axis	decreases, remains in − axis untill zero
− axis	− axis	increases, in − axis

Acceleration in Two dimensions



Acceleration in Two dimensions



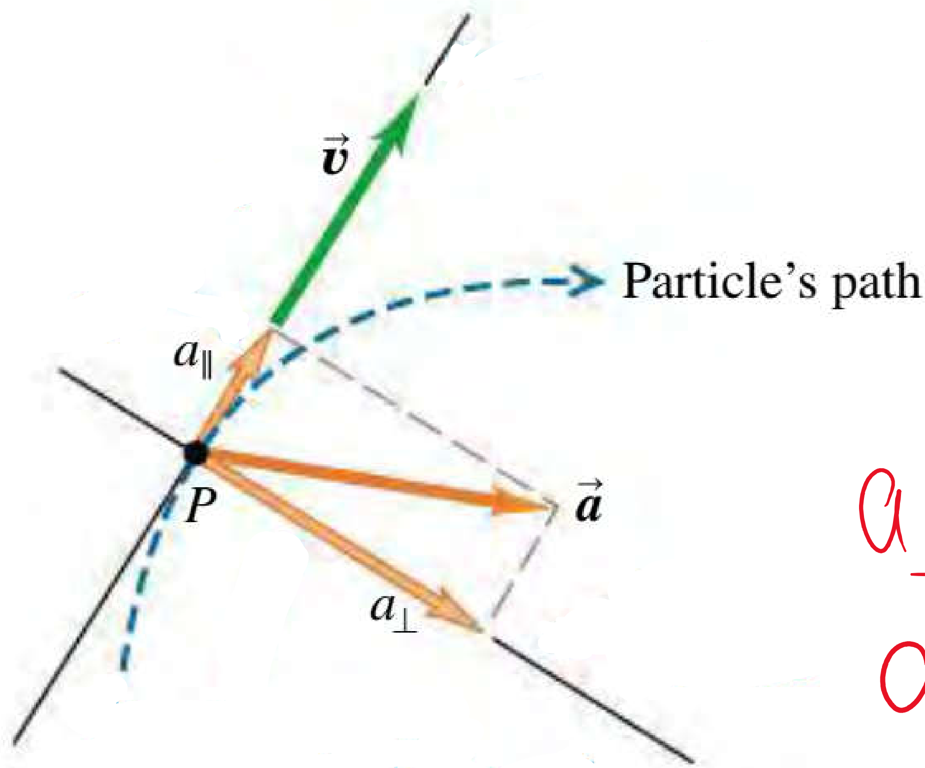
(Most useful representation when $\vec{V}$ and $\vec{a}$ do not point in same direction)

$a_\parallel$ — changes magnitude

$a_\perp$ — changes direction

$$\boxed{|\vec{a}| = \sqrt{a_\parallel^2 + a_\perp^2}}$$

Acceleration in Two dimensions

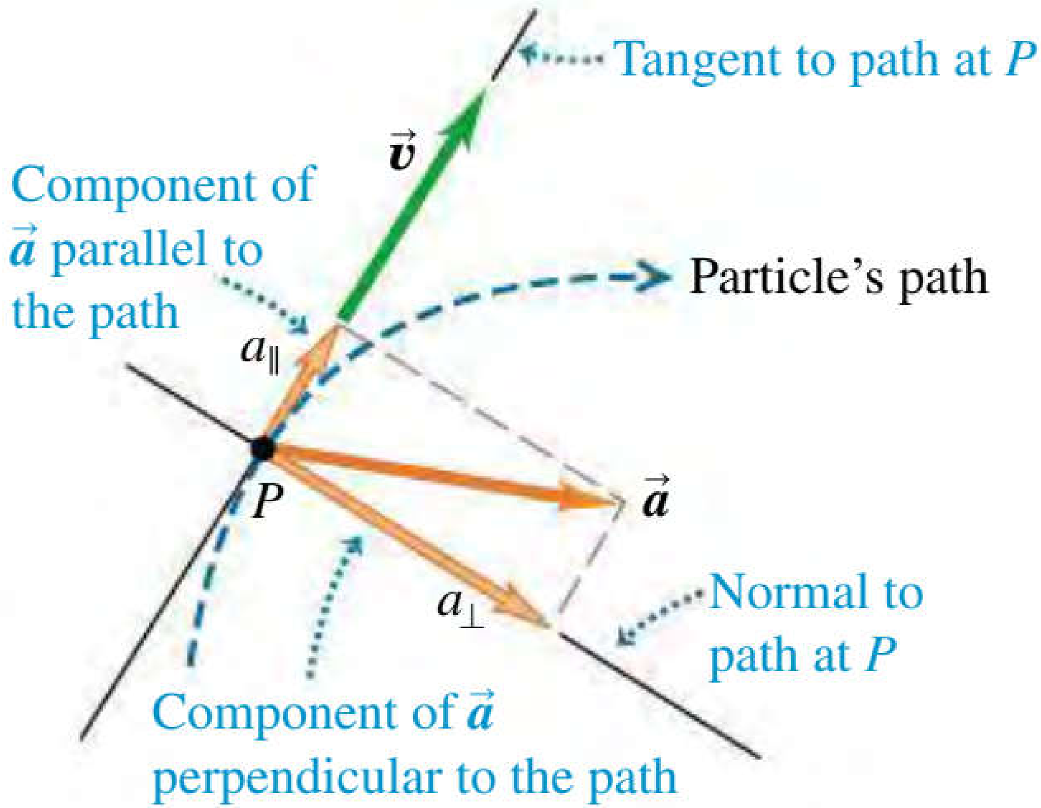


- The acceleration points inside the curvature of the path.
- Magnitude of a is inversely proportional to the radius of the curved path.

$a_{\perp} \longrightarrow a_{radial}$ because it points in the radial direction of the path curvature.

$a_{\parallel} \longrightarrow a_{tangential}$ because it points tangent to the path (curvature)

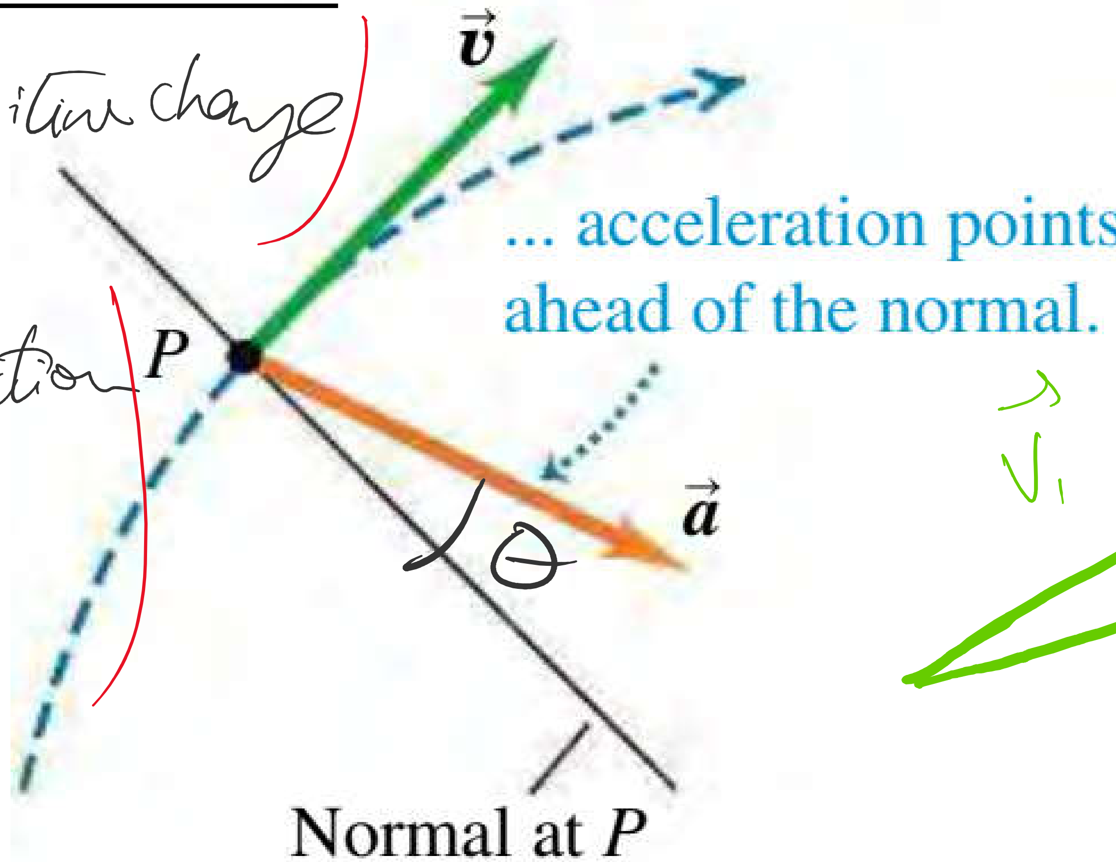
Acceleration in Two dimensions

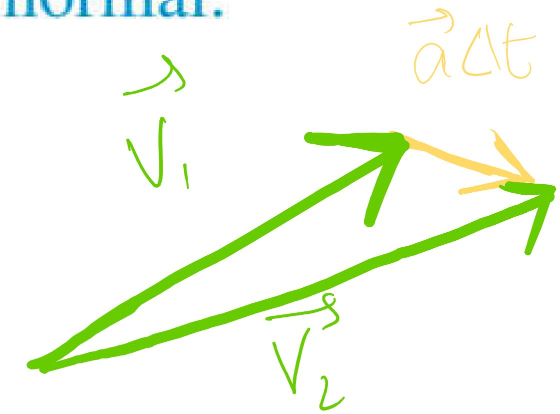


Acceleration in Two dimensions

$a_{\parallel}$ (produces positive change in velocity)

$a_{\perp}$ (changes direction towards the center of the curve)

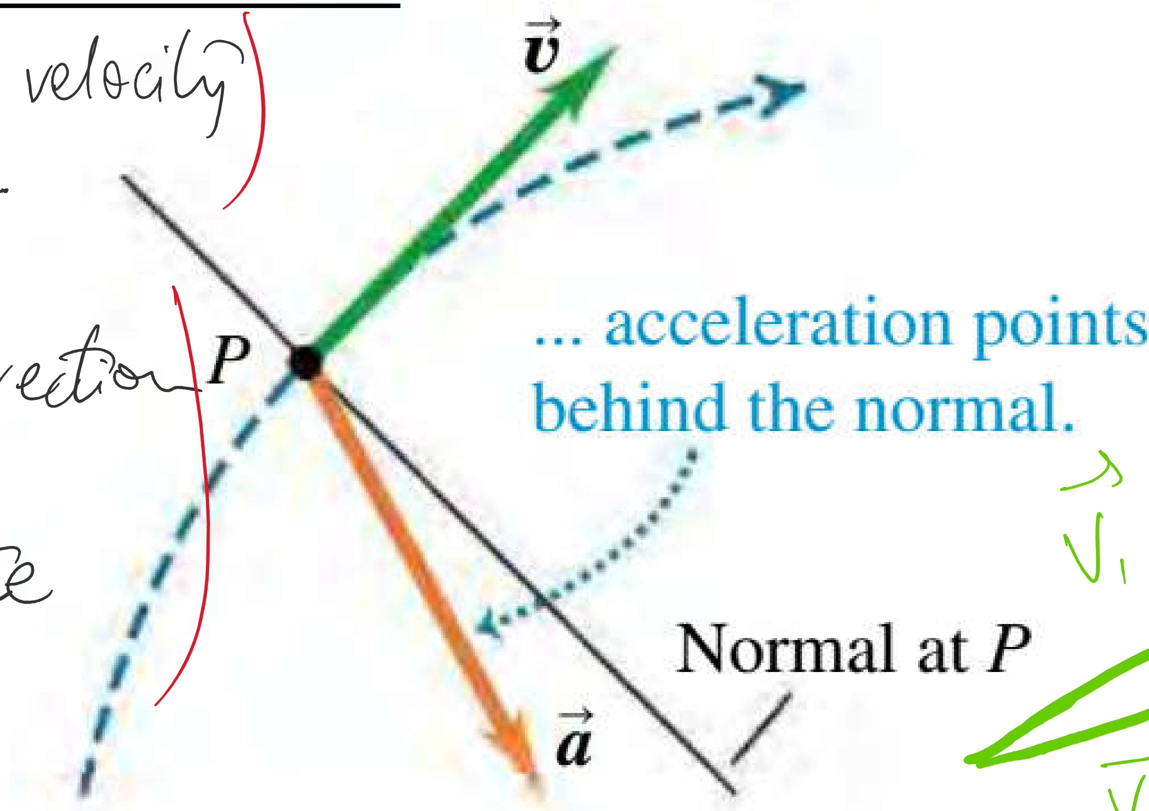


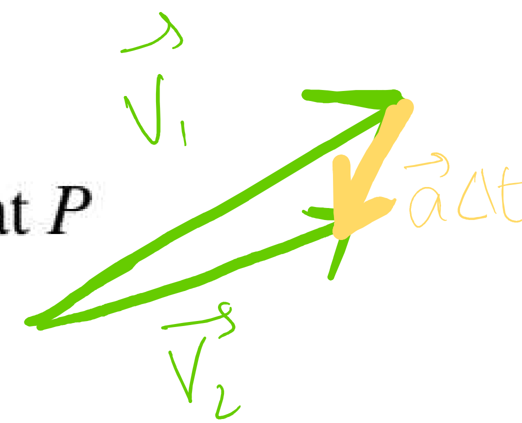


Acceleration in Two dimensions

$a_{\parallel}$ (reduces the velocity magnitude)

$a_{\perp}$ (changes direction towards the center of the curve)

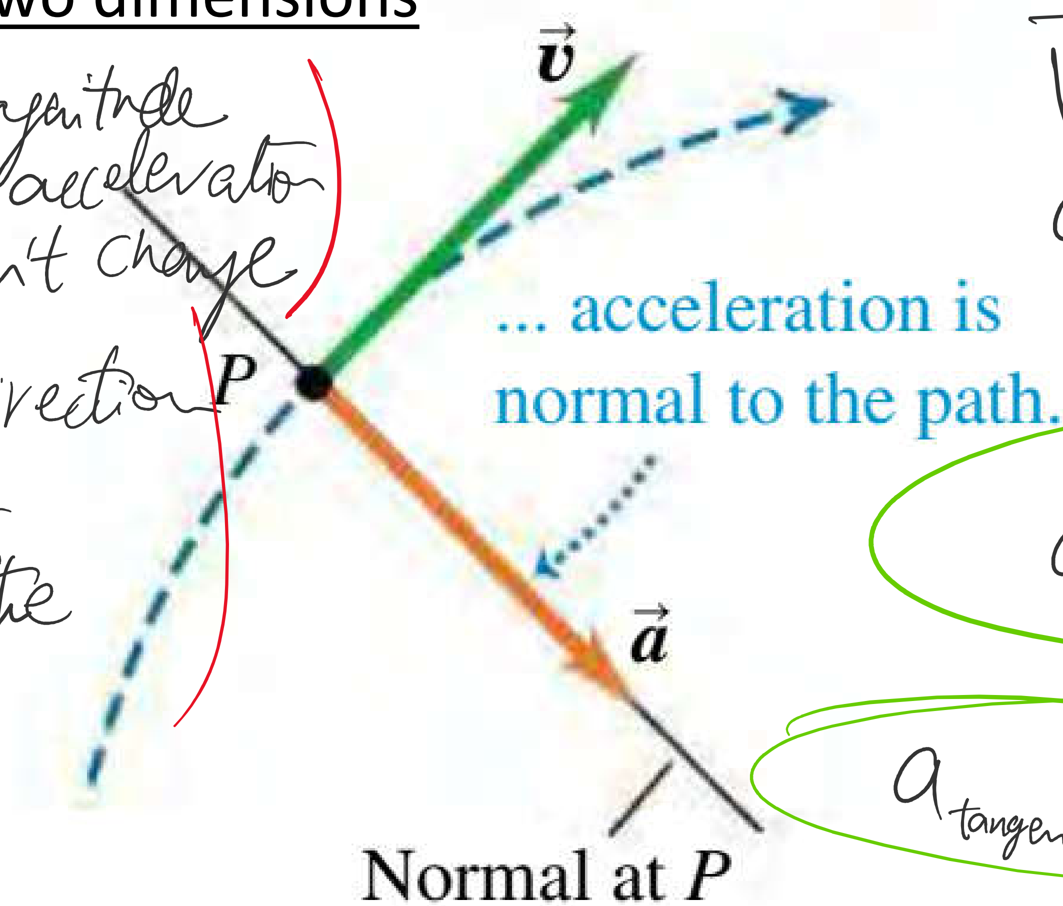




Acceleration in Two dimensions

$a_{\parallel} = 0$ (the magnitude of the acceleration doesn't change)

$a_{\perp}$ (changes direction towards the center of the curve)

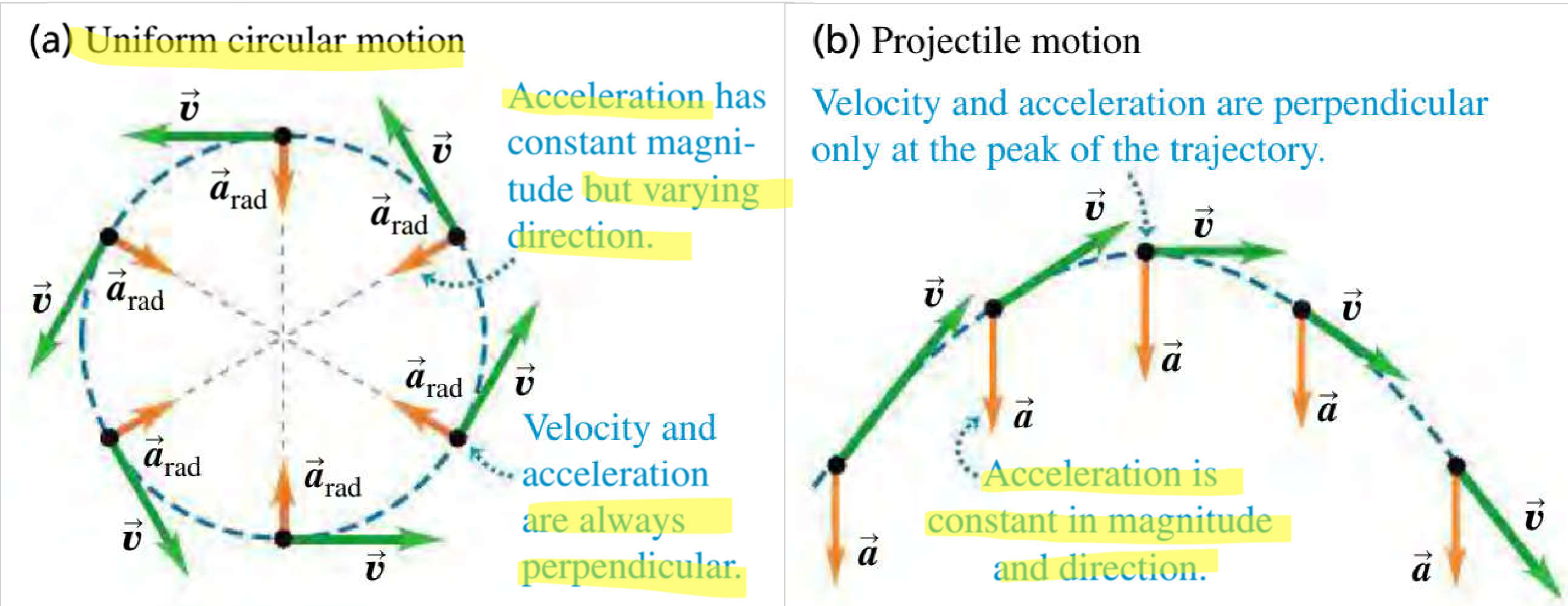


$\vec{V}$ and $\vec{a}$ are perpendicular

$\vec{a} = \vec{a}_{radial}$

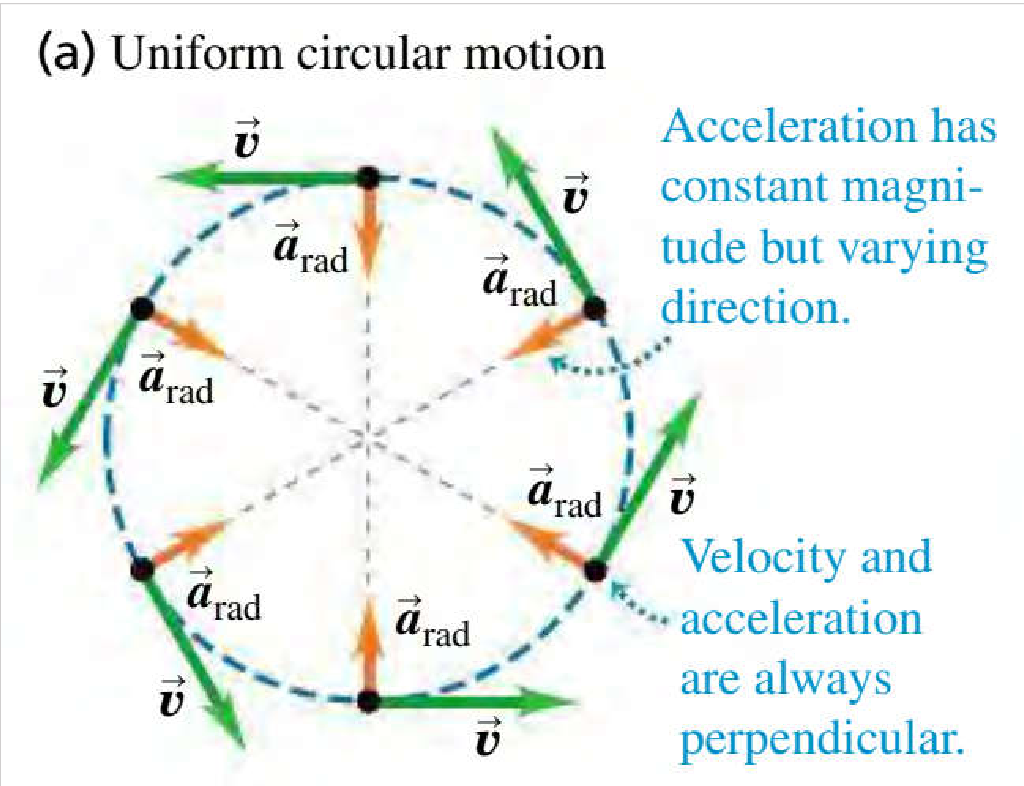
$a_{tangential} = 0$

Acceleration in Two dimensions



Acceleration in Two dimensions

$$\boxed{\vec{a}_r = -\frac{v^2}{r}\hat{r}}$$



$$a = \frac{v^2}{r} \qquad \text{(centripetal acceleration)}$$

$v = 2\pi r/T$

$a_r = 4\pi^2 r/T^2$

tangential Acceleration

$a_t = \frac{dv}{dt}$ or $\frac{\Delta v}{\Delta t}$

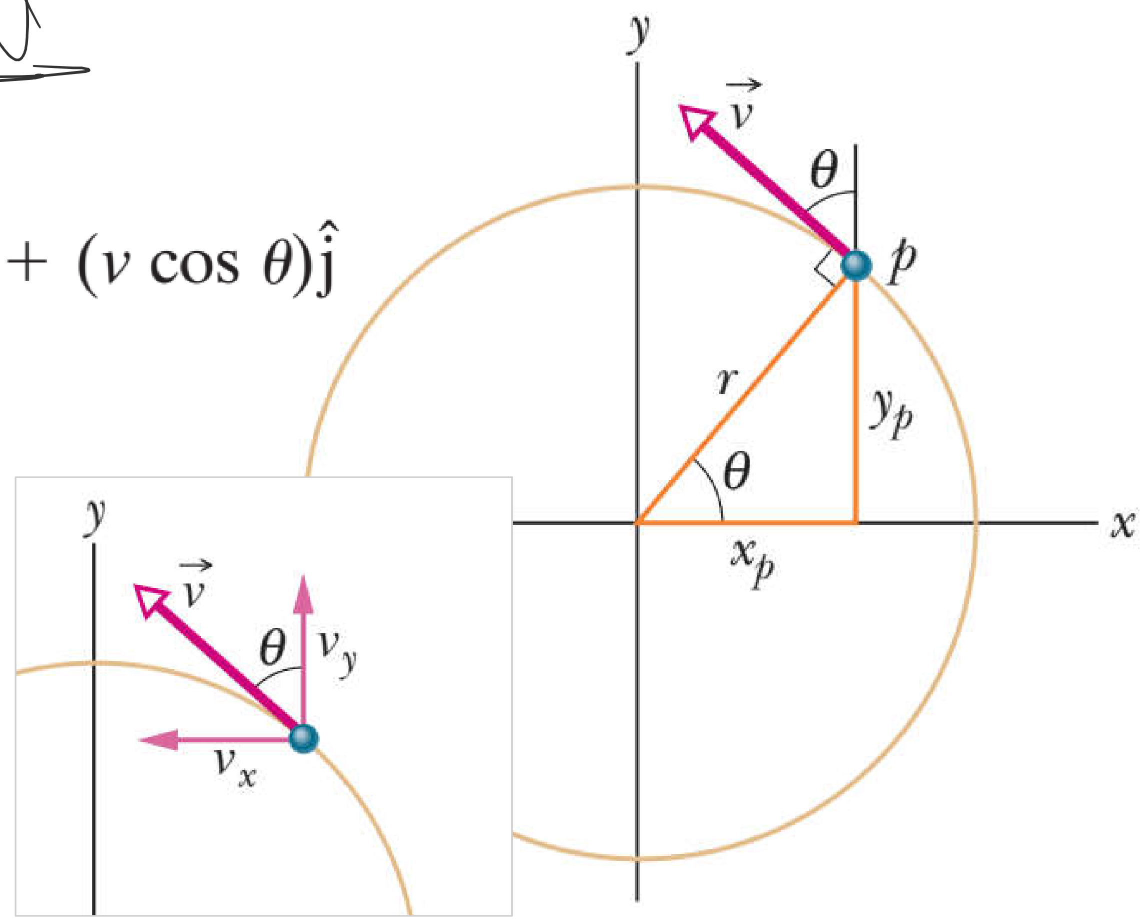
$a_t = r\frac{\Delta\omega}{\Delta t}$

$\alpha = \frac{\Delta\omega}{\Delta t}$

find derivation in **Fundamentals of Physics**, Chapter **4** section **5** (Uniform Circular Motion), page 77 optional

$$\vec{v} = v_x\hat{\mathrm{i}} + v_y\hat{\mathrm{j}} = (-v\sin\theta)\hat{\mathrm{i}} + (v\cos\theta)\hat{\mathrm{j}}$$

$$\vec{v} = \left(-\frac{v y_p}{r}\right)\hat{\mathrm{i}} + \left(\frac{v x_p}{r}\right)\hat{\mathrm{j}}$$



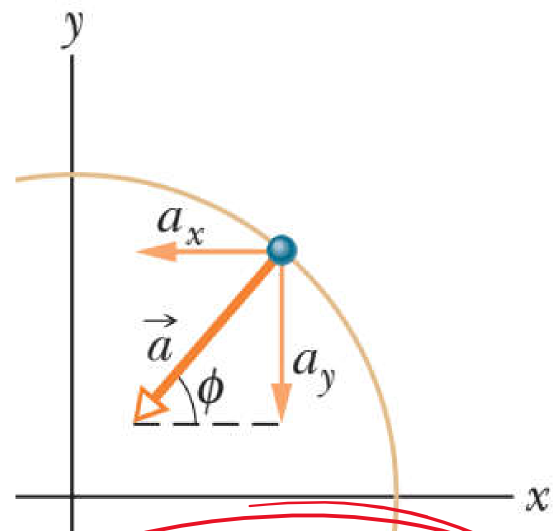
Optional

$$\vec{v} = v_x\hat{\mathrm{i}} + v_y\hat{\mathrm{j}} = (-v \sin\theta)\hat{\mathrm{i}} + (v\cos\theta)\hat{\mathrm{j}}$$

$$\vec{v} = \left(-\frac{vy_p}{r}\right)\hat{\mathrm{i}} + \left(\frac{vx_p}{r}\right)\hat{\mathrm{j}}$$

$$\vec{a} = \frac{d\vec{v}}{dt} = \left(-\frac{v}{r}\frac{dy_p}{dt}\right)\hat{\mathrm{i}} + \left(\frac{v}{r}\frac{dx_p}{dt}\right)\hat{\mathrm{j}}$$

$$\vec{a} = \left(-\frac{v^2}{r}\cos\theta\right)\hat{\mathrm{i}} + \left(-\frac{v^2}{r}\sin\theta\right)\hat{\mathrm{j}}.$$

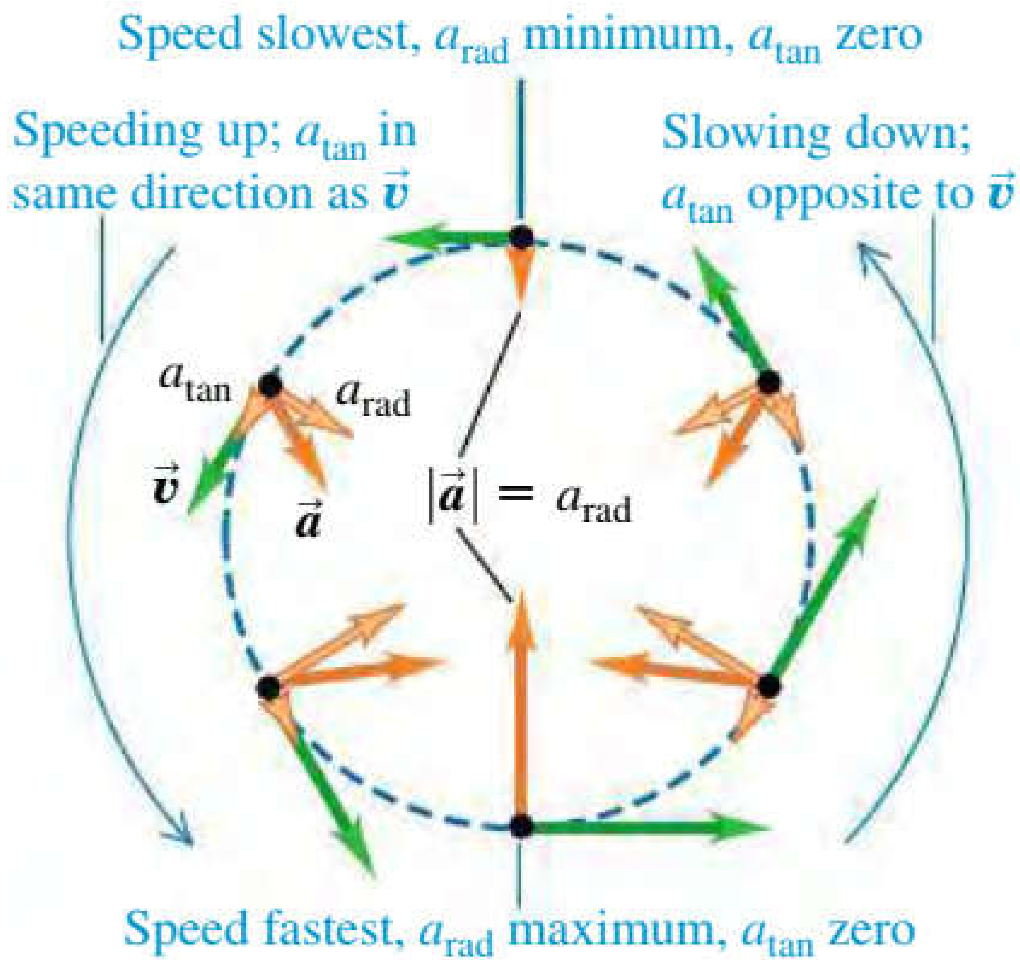


$$\boxed{a = \frac{v^2}{r}\sqrt{1} = \frac{v^2}{r}}$$

Non-uniform Circular Motion

Not every circular motion is uniform circular motion.

Non-uniform Circular Motion



Non-uniform Circular Motion