Q1: Let

$$f(x) = \begin{cases} 5a + bx^2, & x < -2, \\ 5, & x = -2, \\ ax - 3b, & x > -2. \end{cases}$$

Determine the values of a and b so that f is continuous at $x = -2$. What can be said of the differentiability of f at this point?

A $a = -4$, $b = 1$, not differentiable at $x = -2$

B $a = 5$, $b = -5$, not differentiable at $x = -2$

C $a = -4$, $b = 1$, differentiable at $x = -2$

Q2: Discuss the continuity and differentiability of the function f at $x = 0$ given

$$f(x) = \begin{cases} -9x - 6 & \text{if } x < 0, \\ x^2 - 9x - 6 & \text{if } x \geq 0. \end{cases}$$

A The function is not continuous, so it is not differentiable at $x = 0$.

B The function is continuous and differentiable at $x = 0$.

C The function is not continuous but differentiable at $x = 0$.

D The function is continuous but not differentiable at $x = 0$.

Q3: Discuss the differentiability of a function f at $x = -4$ given

$$f(x) = \begin{cases} 8x + 7 & \text{if } x < -4, \\ 2x + 5 & \text{if } x \geq -4. \end{cases}$$

A $f(x)$ is not differentiable at $x = -4$ because $f'(-4^+) \neq f'(-4^-)$.

B $f(x)$ is differentiable at $x = -4$ because f is continuous at $x = -4$.

C $f(x)$ is not differentiable at $x = -4$ because $f(-4)$ is undefined.

D $f(x)$ is differentiable at $x = -4$ because $f'(-4^+) = f'(-4^-)$.

Q4 : Let

$$f(x) = \begin{cases} -4c + mx, & x < 1, \\ cx^2 - 4m, & x \geq 1. \end{cases}$$

If $f(1) = 12$ and f is continuous at $x = 1$, determine the values of m and c. What can be said of the differentiability of f at this point?

A $m = -12, c = -6$, differentiable at $x = 1$

B $m = -12, c = -6$, not differentiable at $x = 1$

C $m = -4, c = -4$, not differentiable at $x = 1$

D $m = -4, c = -4$, differentiable at $x = 1$

Q 5: Suppose

$$f(x) = \begin{cases} -6x - 4, & x \leq -1, \\ 3x^2, & x > -1. \end{cases}$$

What can be said of the differentiability of f at $x = -1$?

A The function $f(x)$ is differentiable at $x = -1$ as $\lim_{x \to -1^-} f(x) \neq \lim_{x \to -1^+} f(x)$ but is not continuous.

B The function $f(x)$ is not differentiable at $x = -1$ because $f(-1)$ is undefined.

C The function $f(x)$ is not differentiable at $x = -1$.

D The function $f(x)$ is not differentiable at $x = -1$ because $f(x)$ is continuous at $f(-1)$.

E The function $f(x)$ is continuous but not differentiable at $x = -1$ because $f'(-1^-) \neq f'(-1^+)$.

Q6 : Suppose

$$f(x) = \begin{cases} x^2 - 15, & x \leq 1, \\ 2x - 16, & x > 1. \end{cases}$$

What can be said of the differentiability of f at $x = 1$?

A The function is not continuous, so it is not differentiable at $x = 1$.

B The function is not continuous but differentiable at $x = 1$ because $\lim_{x \to 1^-} f(x) = \lim_{x \to 1^+} f(x) = f(1)$.

C The function is continuous and differentiable at $x = 1$ because $f'(1^-) = f'(1^+)$.

D The function is continuous but not differentiable at $x = 1$ because $\lim_{x \to 1^-} f(x) = \lim_{x \to 1^+} f(x) = f(1)$.