Vectors and Vector Operations

- **Points**
- Space
- Coordinate system(s)
- Vectors

dimensionless

That which has no part. -Euclid

Represented by ordered pair

$$\underbrace{(a,b,c)}_{\text{point}}$$

$$\overbrace{(a,b,c,d,e,f,g,h)}^{\text{point}}$$

space of fruits
space of students
space of cars

This must be where Transformers lived

- Points
- **Space**
- Coordinate system(s)
- Vectors

Set with some added ~~structure~~

- made out of points

- all the points are unique

- space can be described for any set of objects

number of classes of these objects will tell you the number of dimensions of space

- Points
- Space
- **Coordinate system(s)**
- Vectors

type of
– space with defined properties (geometric)

Cartesian
polar
cylindrical
spherical

- Points
- Space
- Coordinate system(s)
- **Vectors**

Mathematicians

objects that follow certain rules.

Physicists

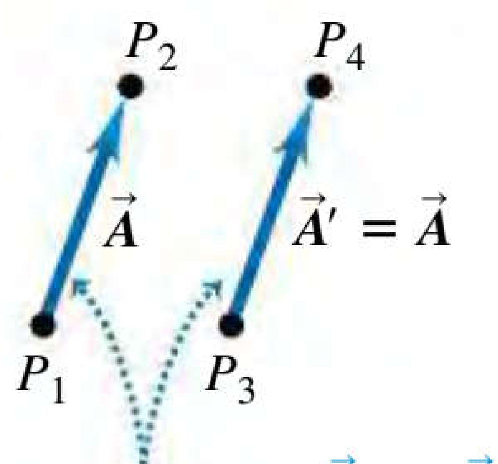
tools to represent quatities specific in direction.

Computer scientists

- array of object
- points in dataset

- Points
- Space
- Coordinate system(s)
- **Vectors**

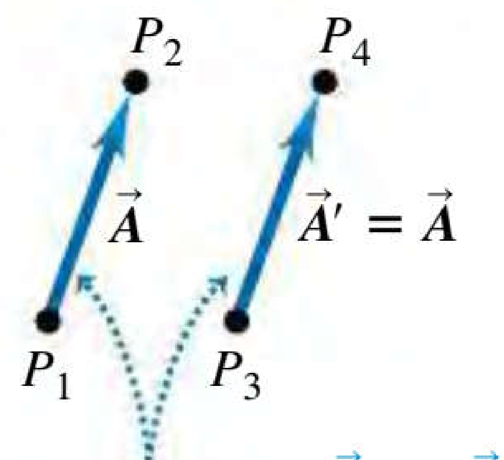
Vectors can exist between any two points



Displacements $\vec{A}$ and $\vec{A}'$ are equal because they have the same length and direction.

Vectors may also contain information of the evolution

Vectors can exist between any two points



Displacements $\vec{A}$ and $\vec{A}'$ are equal because they have the same length and direction.

Vectors may also contain information of the evolution

How many vectors are here?

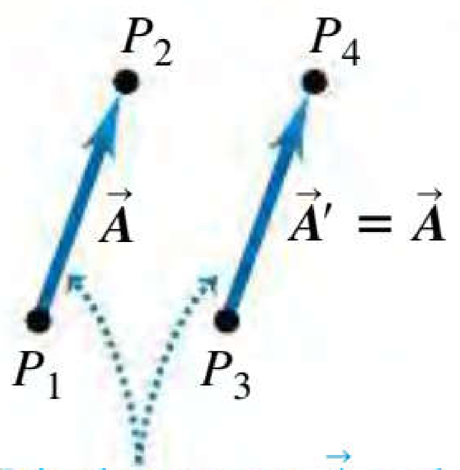
$\vec{P}_{12}$ $\vec{P}_{14}$ $\vec{P}_{13}$ $\vec{P}_{23}$ $\vec{P}_{24}$ $\vec{P}_{34}$

$\vec{P}_{21}$ $\vec{P}_{41}$ $\vec{P}_{31}$ $\vec{P}_{32}$ $\vec{P}_{42}$ $\vec{P}_{43}$

12 possible

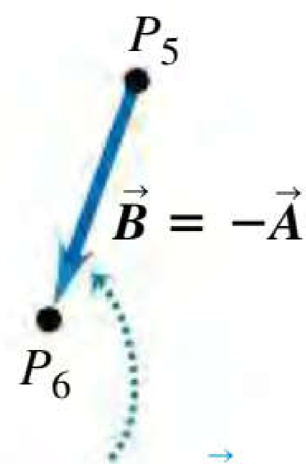
negative vectors

Vectors can exist between any two points



Displacements $\vec{A}$ and $\vec{A}'$ are equal because they have the same length and direction.

Vectors may also contain information of the evolution



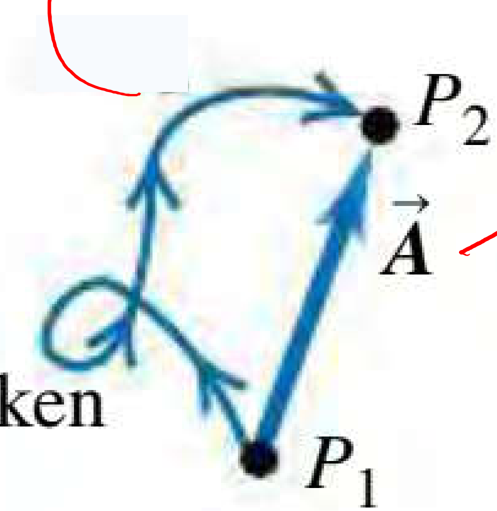
Displacement $\vec{B}$ has the same magnitude as $\vec{A}$ but opposite direction; $\vec{B}$ is the negative of $\vec{A}$.

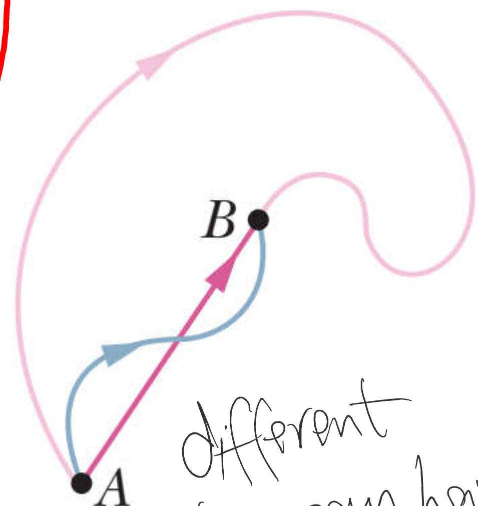
(This is not vector)

Distance and Displacement

(This is a vector)

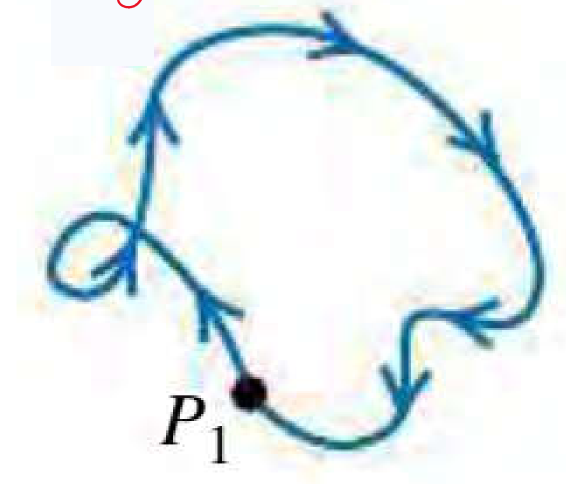
Path taken





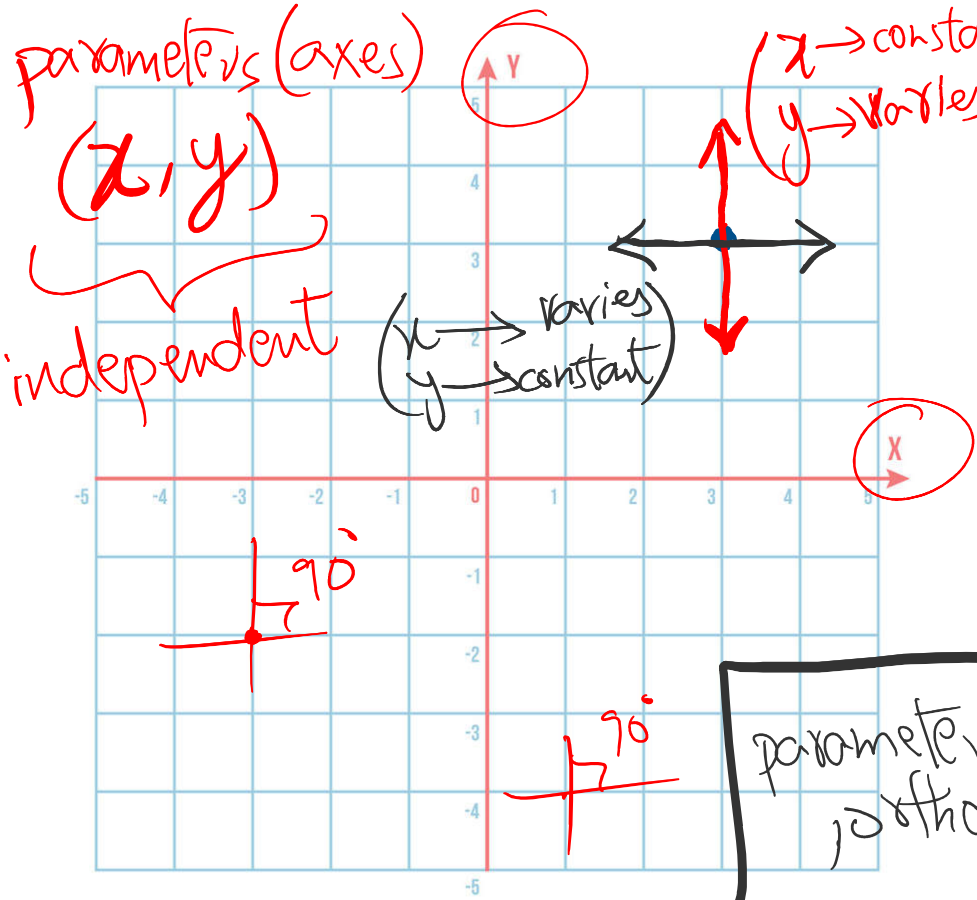
different paths may have different distance values

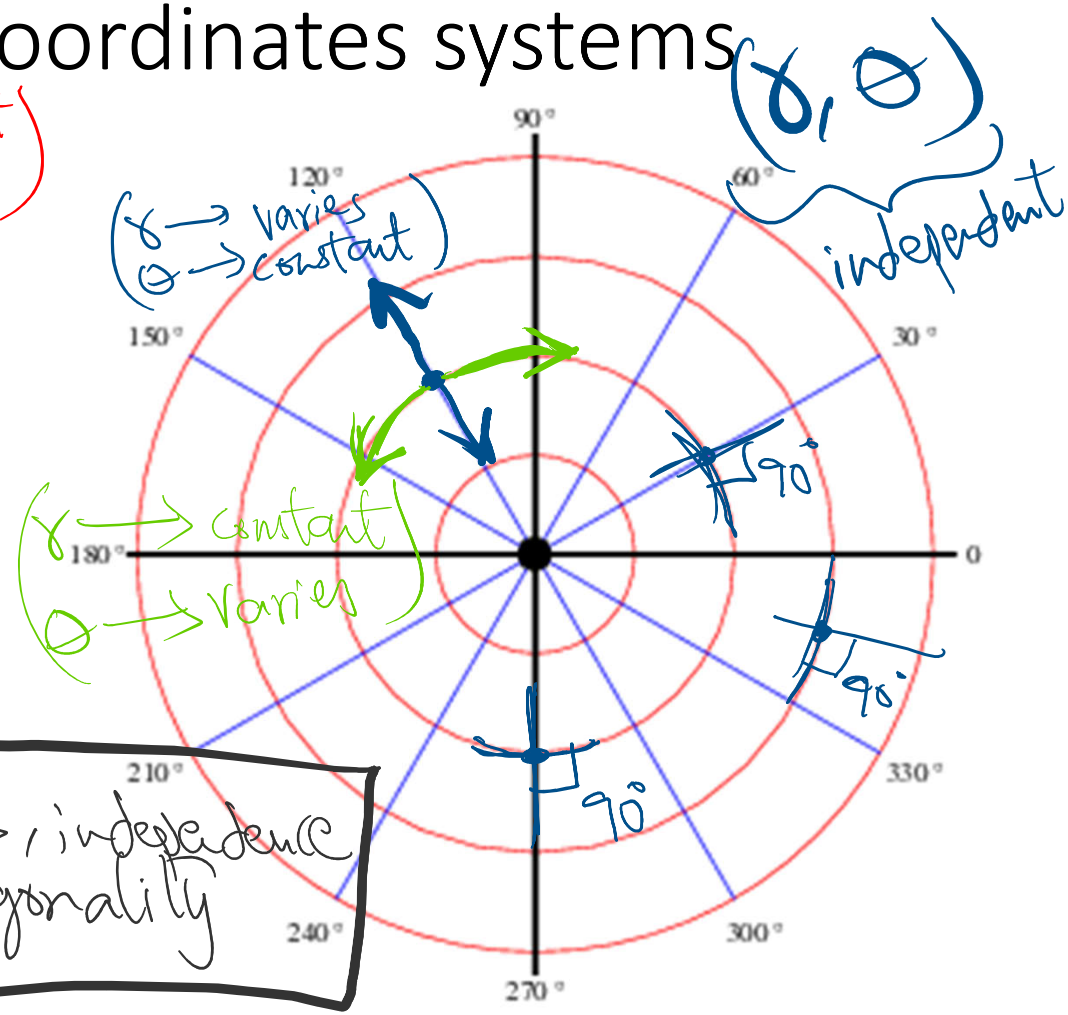
non-zero distance



zero displacement

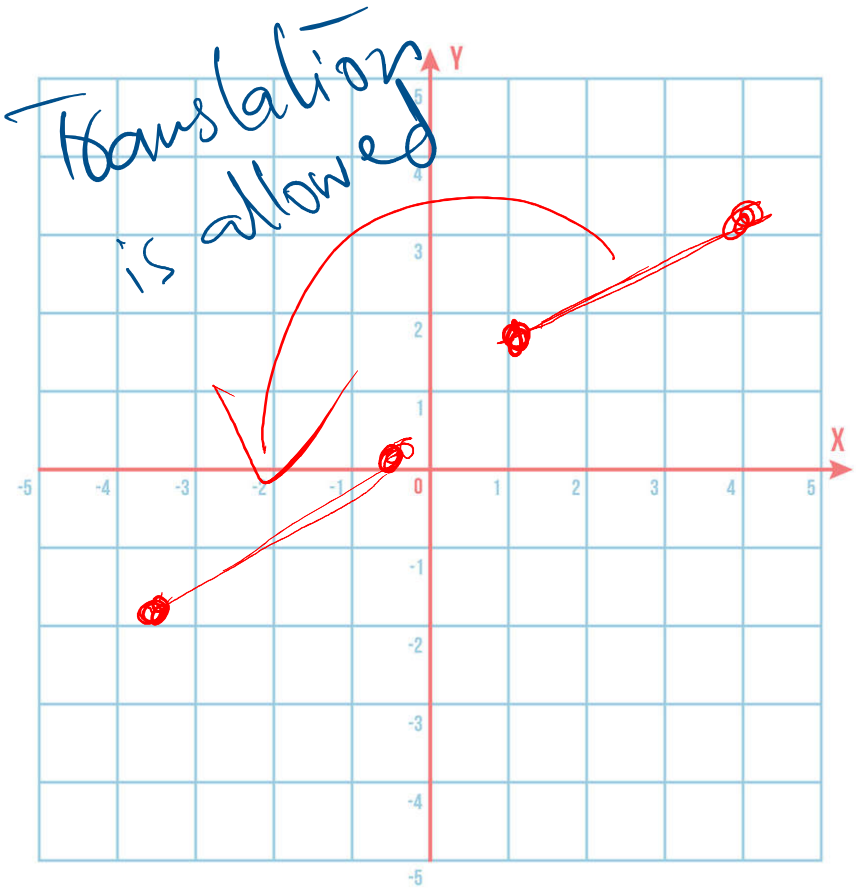
Vectors are drawn in coordinates systems

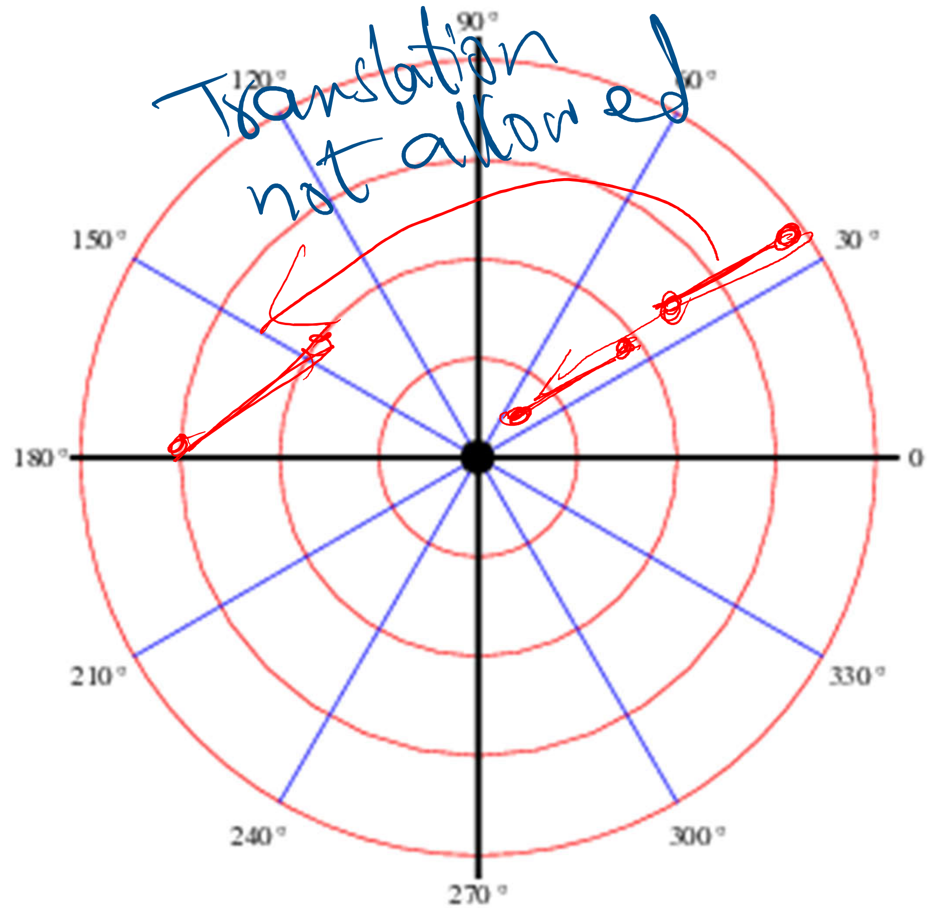




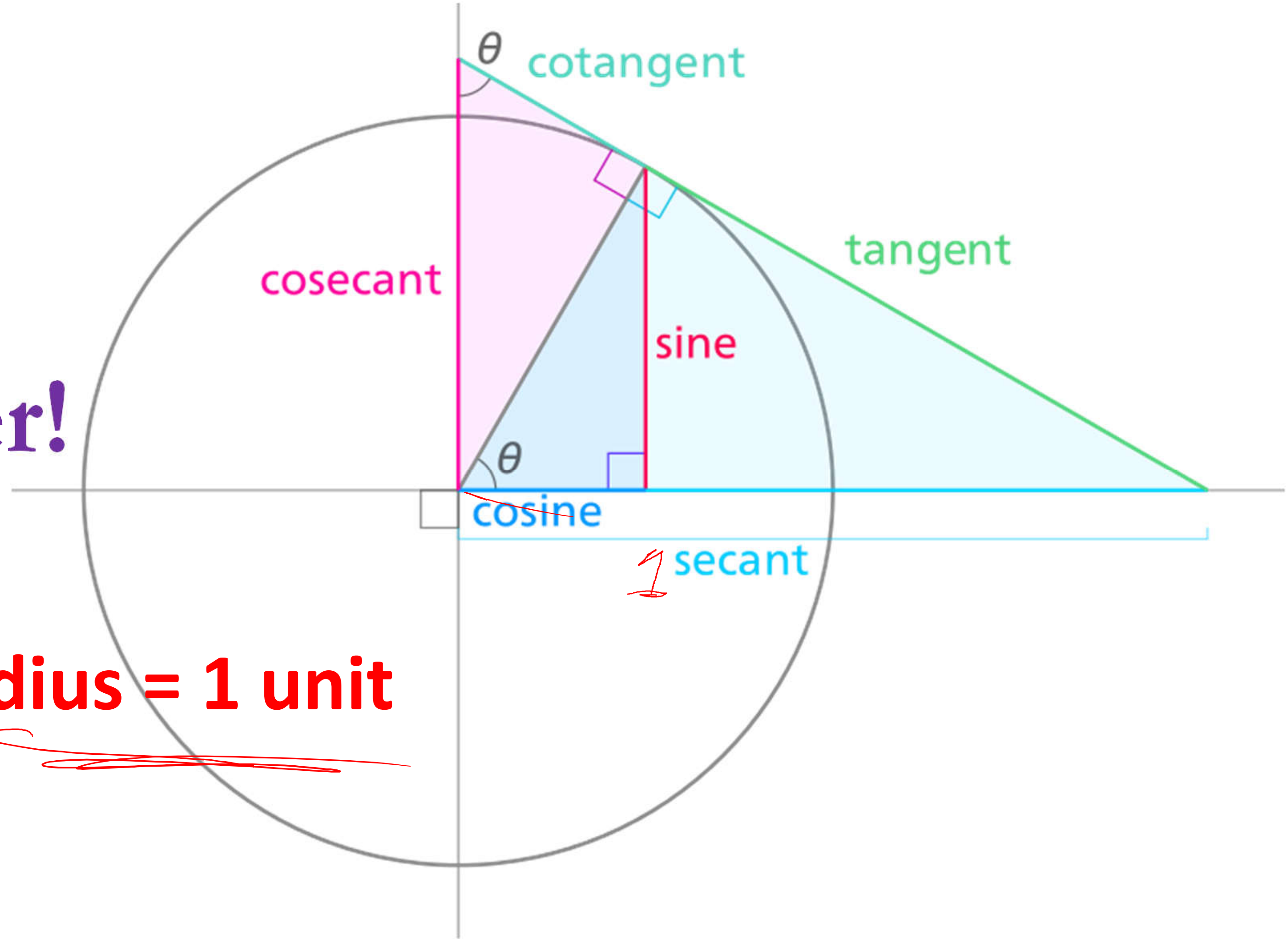
$\boxed{\text{parameters, independence, orthogonality}}$

Vectors are drawn in coordinates systems





A gentle Reminder!



Unit circle; **radius = 1 unit**

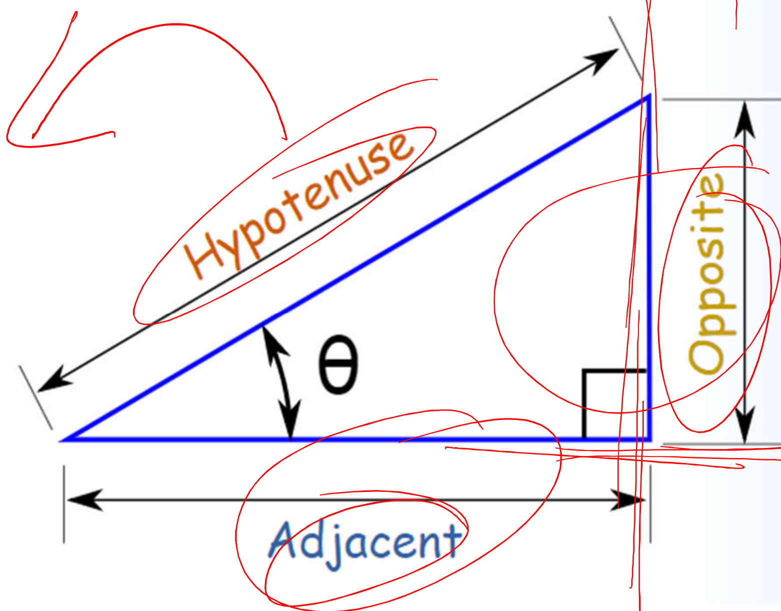
still just a reminder!

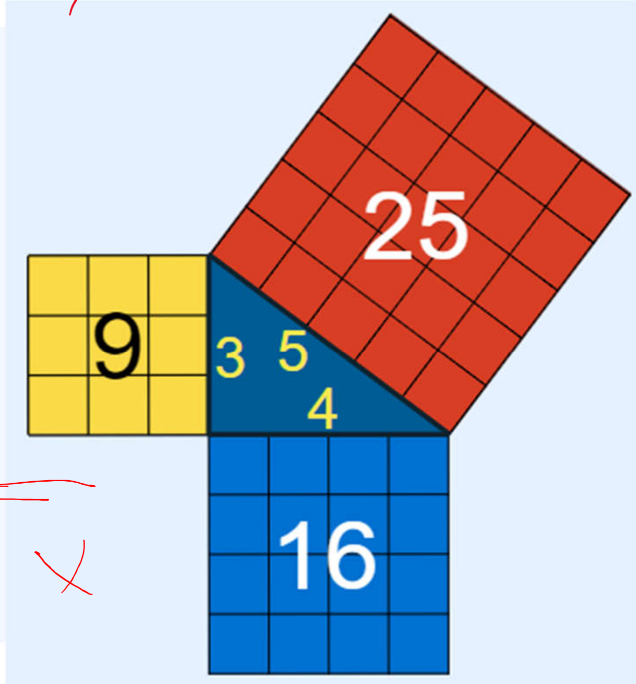
Pythagoras theorem

$\sin\theta = \dfrac{\text{Opposite}}{\text{Hypotenuse}}$

$\cos\theta = \dfrac{\text{Adjacent}}{\text{Hypotenuse}}$

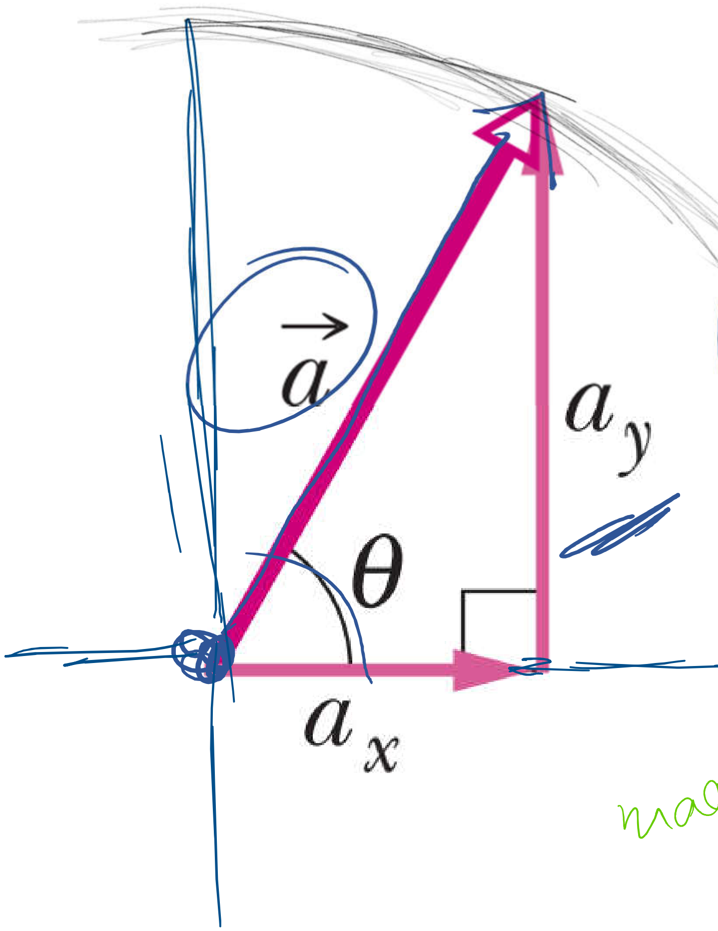
$\tan\theta = \dfrac{\text{Opposite}}{\text{Adjacent}}$





Problem-Solving Tactics, book pg#45

Combined form of a vector using Right-Triangle



cartesian   polar

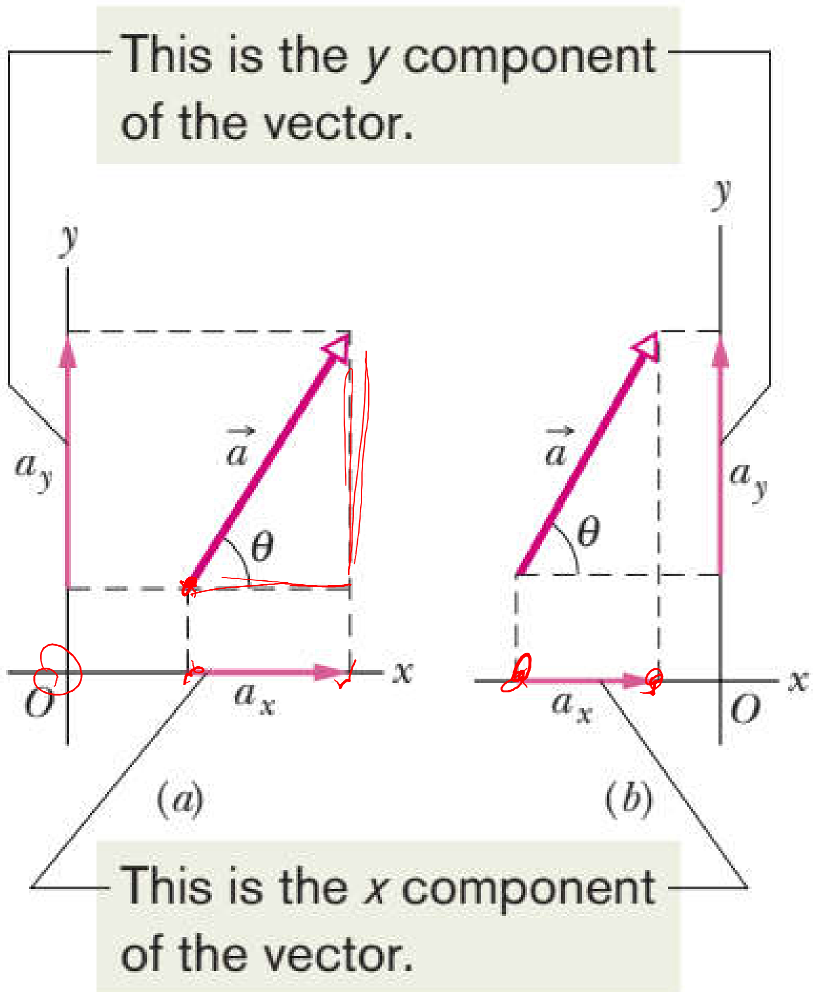
$$a_y = a \sin\theta$$

$$a_x = a \cos\theta$$

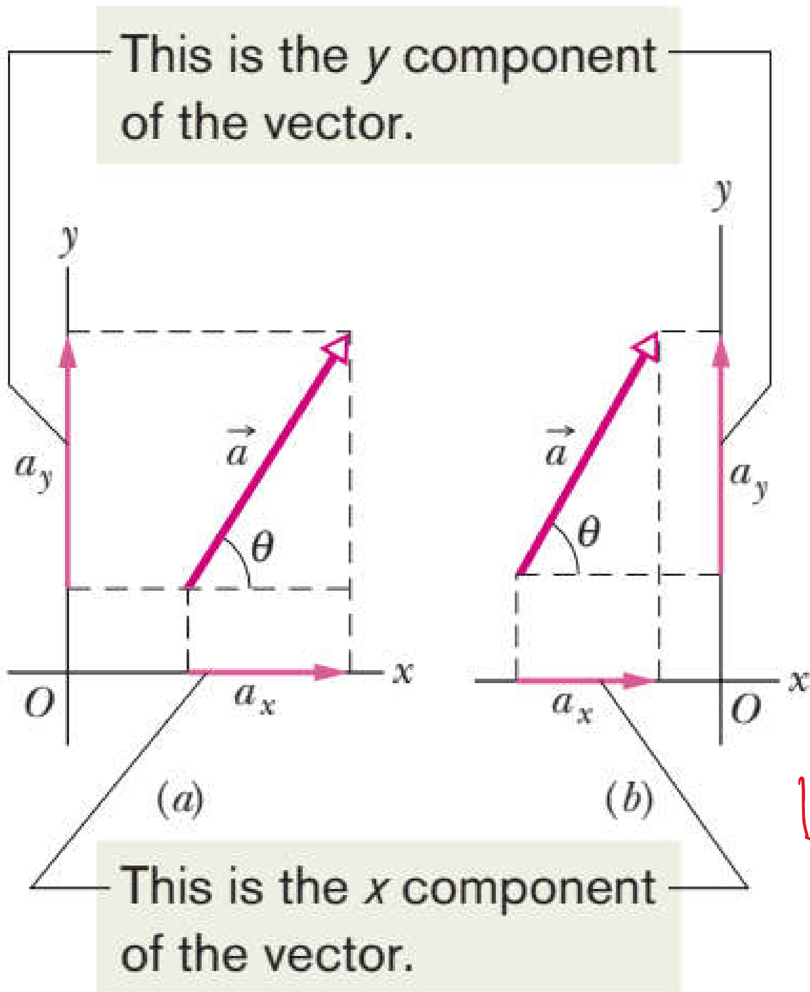
Transformation equations (from polar to cartesian)

$$a = \sqrt{a_x^2 + a_y^2}$$

magnitude



Vectors are essentially independent of the position



Vectors are essentially independent of the position

angle from horizontal

$$a_x = a\cos\theta \quad \text{and} \quad a_y = a\sin\theta,$$

$$\underbrace{a = \sqrt{a_x^2 + a_y^2}}_{\text{length}} \quad \underbrace{\tan\theta = \frac{a_y}{a_x}}_{\text{direction}}$$

tan repeats some angles.

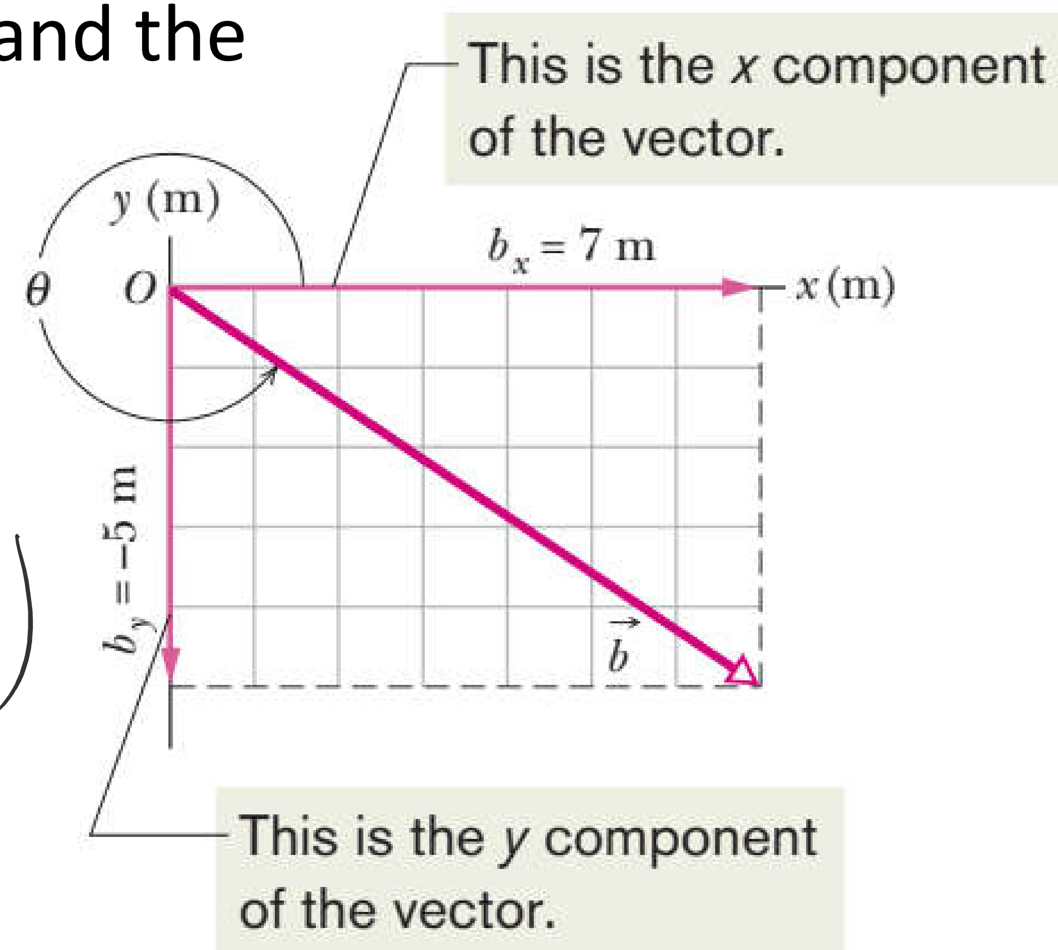
Find length of the vector $\vec{b}$ and the unknown angle θ.

$$b = \sqrt{b_x^2 + b_y^2}$$

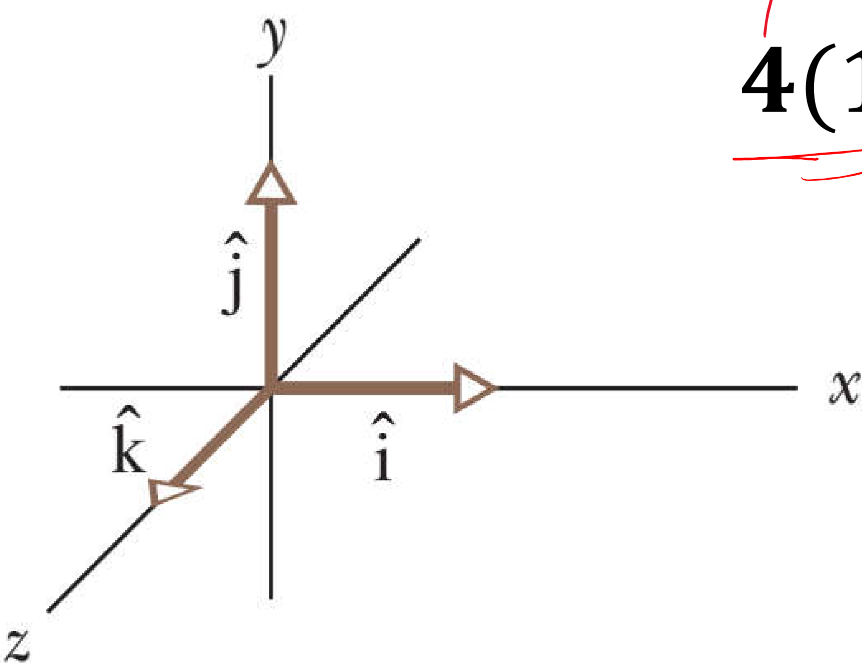
$$= \qquad m$$

$$\tan\theta = \frac{b_y}{b_x} \Rightarrow \theta = \tan^{-1}\left(\frac{b_y}{b_x}\right)$$

$$\theta =$$



Concept of unit vectors

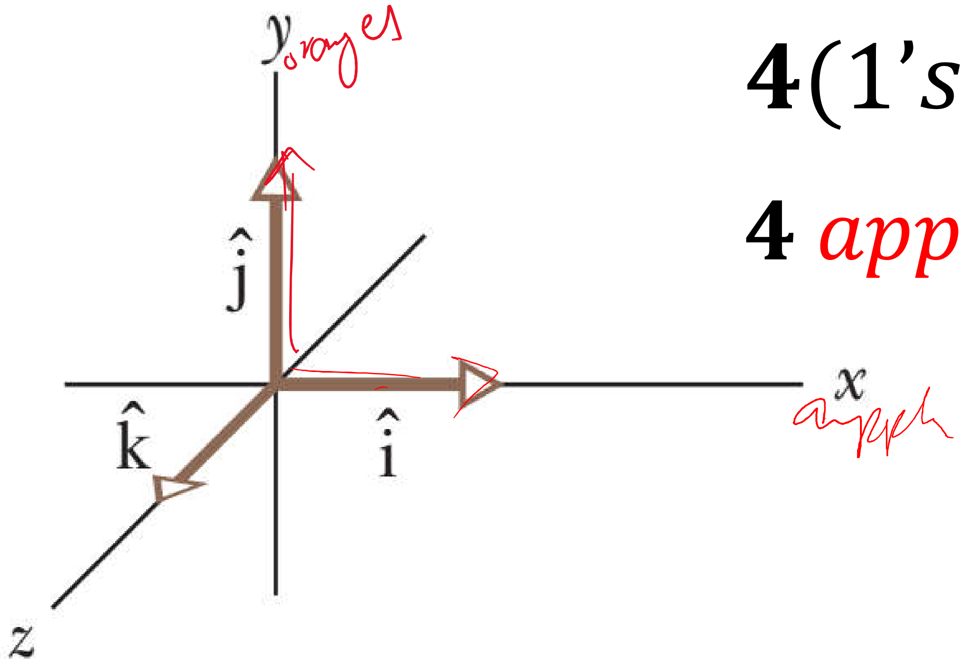


quantity

$$\mathbf{4}(1's) + \mathbf{3}(1's) = \mathbf{7}(1's)$$

unit objects

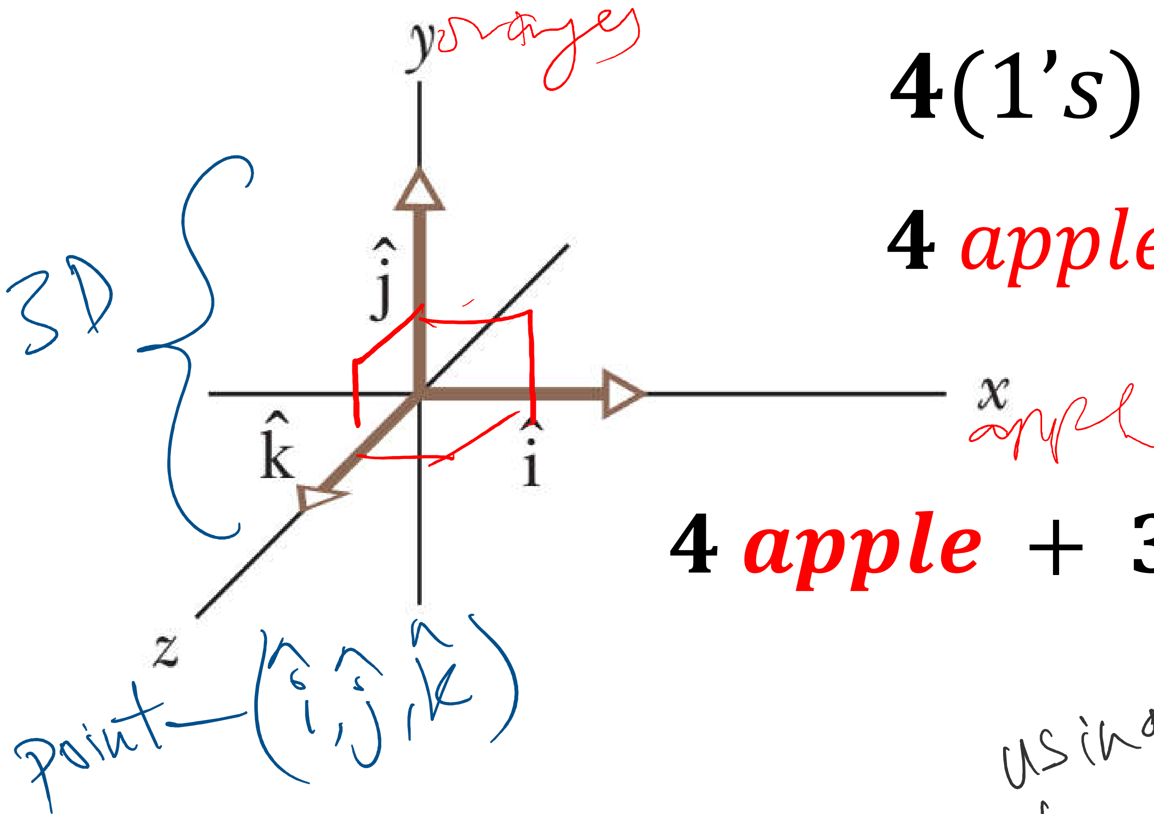
Concept of unit vectors



$$\mathbf{4}(1's) + \mathbf{3}(1's) = \mathbf{7}(1's)$$

$$\mathbf{4}\,\color{red}{apple} - \mathbf{3}\,\color{red}{apple} = \mathbf{1}\,\color{red}{apple}$$

Concept of unit vectors



$$\mathbf{4}(1's) + \mathbf{3}(1's) = \mathbf{7}(1's)$$

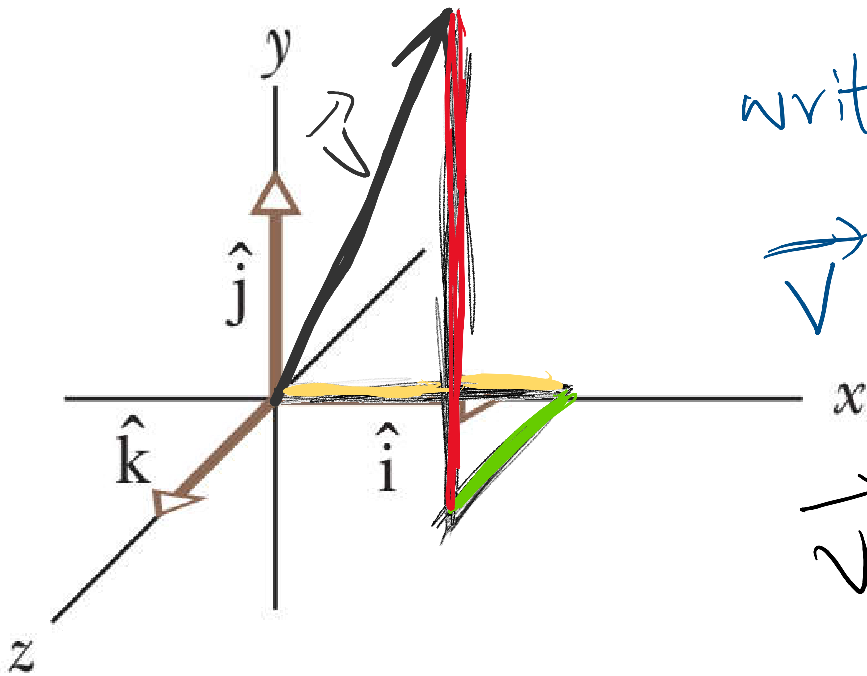
$$\mathbf{4}\ apple - \mathbf{3}\ apple = \mathbf{1}\ apple$$

$$\mathbf{4}\ \boldsymbol{apple} + \mathbf{3}\ \boldsymbol{oranges} =$$

5 appanges

using Pythagoras theorem

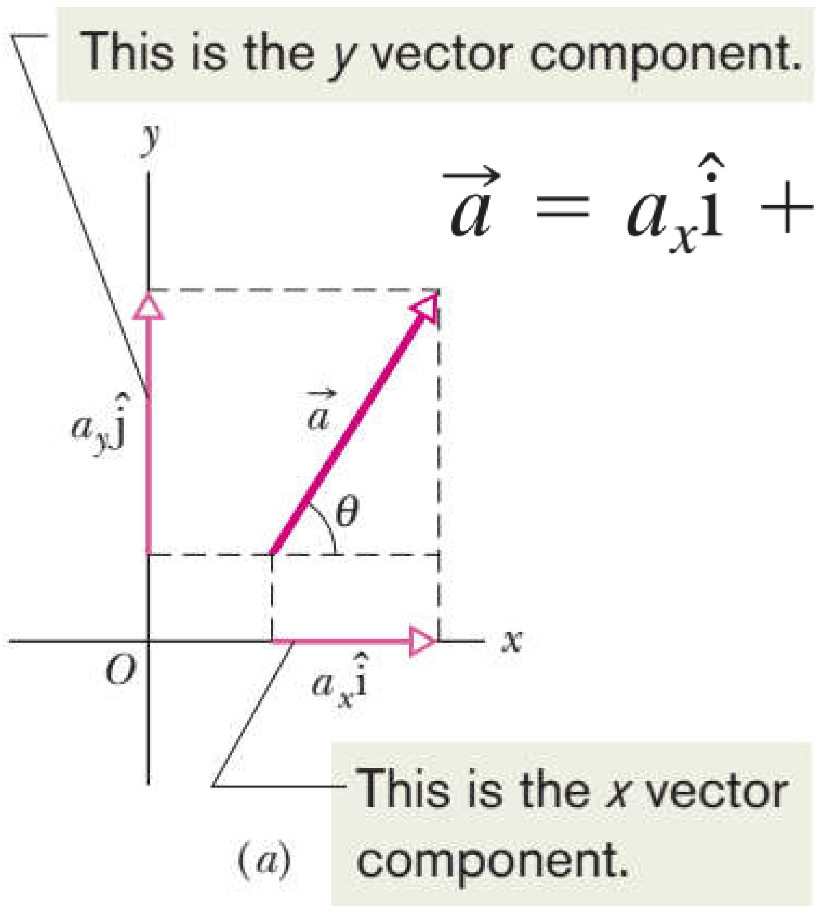
Concept of unit vectors



writting a vector in 3D

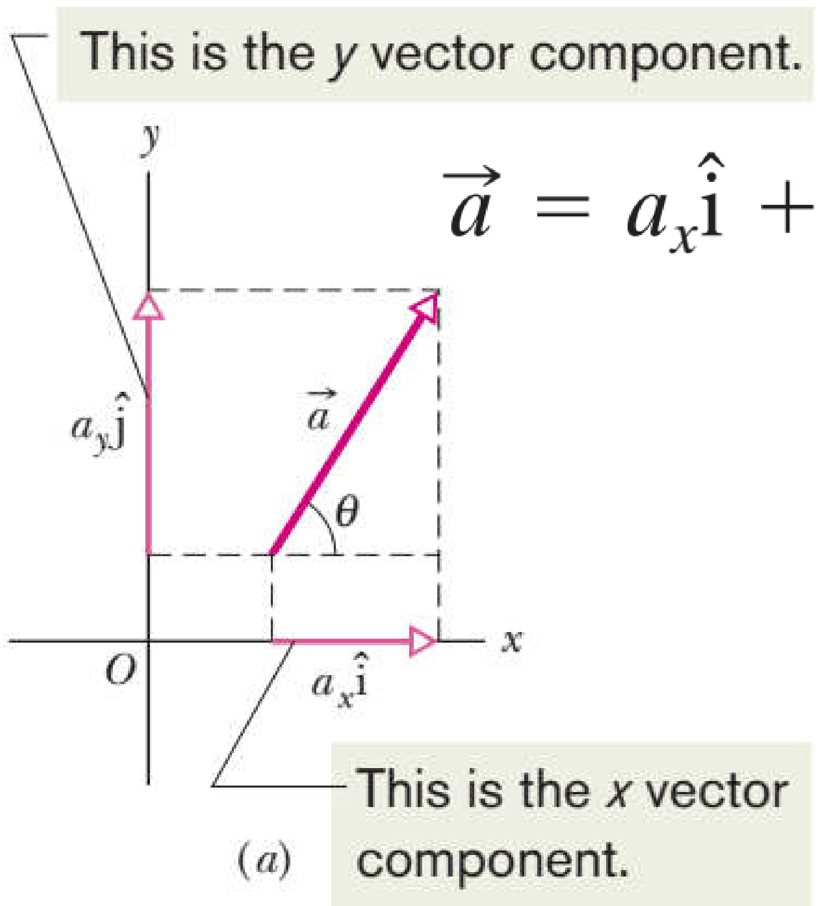
$$\vec{V} = V_x \hat{i} + V_y \hat{j} + V_z \hat{k}$$

$$\vec{V} = V_y \hat{j} + V_z \hat{k} + V_x \hat{i}$$

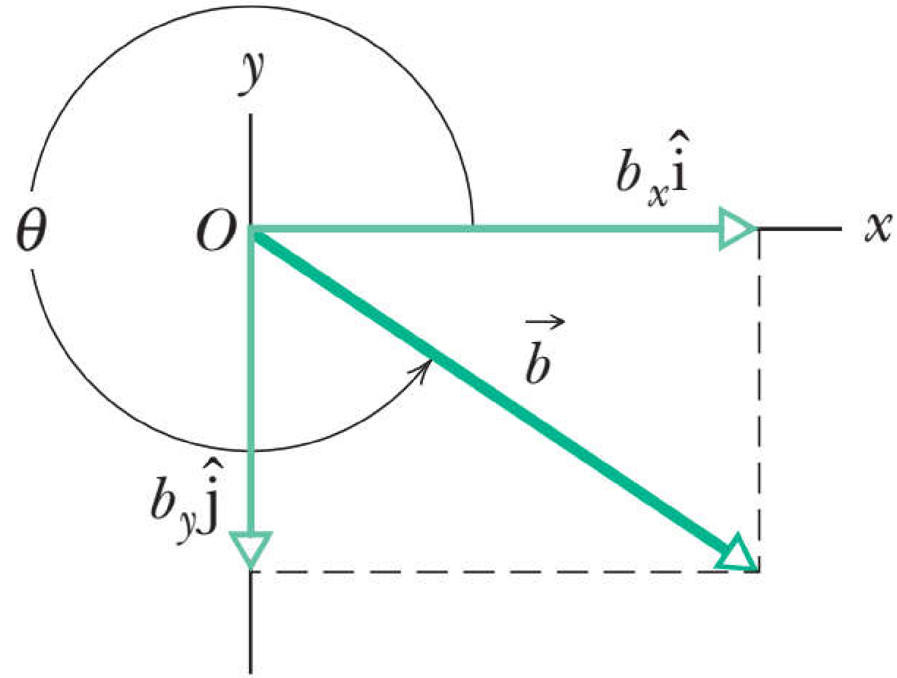


(a)

$$\vec{a} = a_x\hat{i} + a_y\hat{j}$$



$$\vec{a} = a_x\hat{\mathrm{i}} + a_y\hat{\mathrm{j}}$$



$$\vec{b} = b_x\hat{i} + b_y\hat{j}$$

$$\vec{a} = (4.2\text{ m})\hat{i} - (1.5\text{ m})\hat{j},$$
$$\vec{b} = (-1.6\text{ m})\hat{i} + (2.9\text{ m})\hat{j},$$
$$\vec{c} = (-3.7\text{ m})\hat{j}.$$

$$\vec{R} = \overrightarrow{R_x} + \overrightarrow{R_y}$$

=

$$\theta = \tan^{-1}\left(\frac{R_y}{R_x}\right)$$

=