

Assignment # 2
Differential Equations (MT-224)
Date of Submission: 5th May, 2021
Total marks: 6 (CLO-3)

Find the general solution of the following:

[marks: 16; weightage: 4]

1. $y'' + 4y' + 3y = 0$

Answer: $y = c_1 e^{-x} + c_2 e^{-3x}$

2. $y''' - y'' + y' - y = 0$

Answer: $y = c_1 e^x + c_2 \sin x + c_3 \cos x$

3. $2x^2 y'' + 3xy' - 15y = 0$,

Answer: $y = C_1 x^{\frac{5}{2}} + C_2 x^{-3}$

4. $y'' - 3y' + 2y = x^2 e^x$

Answer: $y = c_1 e^x + c_2 e^{2x} - \frac{1}{3} x^3 e^x - x^2 e^x$

5. $y'' + 4y = x e^x + x \sin 2x$

Answer: $y = c_1 \sin 2x + c_2 \cos 2x + \frac{1}{5} x e^x - \frac{2}{25} e^x + \frac{1}{16} x \sin 2x - \frac{1}{8} x^2 \cos 2x$

6. $y'' - 2y' + y = x e^x \ln x$

Answer: $y = c_1 e^x + c_2 x e^x + e^x \left(\frac{-5}{36} x^3 + \frac{x^3}{6} \ln x \right)$

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7. $x^2 y'' - x(x+2)y' + (x+2)y = x^3$

Answer: $y = c_1 x + c_2 x e^x - x - x^2$

8. $y'' - 4y' - 12y = 2t^3 - t + 3$

Answer: $y = C_1 e^{-2t} + C_2 e^{6t} - \frac{1}{6} t^3 + \frac{1}{6} t^2 - \frac{1}{9} t - \frac{5}{27}$

9. $y'' + 5y' + 6y = 2x$

Answer: $y = A e^{-2x} + B e^{-3x} + \frac{x}{3} - \frac{5}{18}$

10. $y'' + 5y' - 9y = e^{-2x} + 2 - x$

Answer: $y = A e^{1.40x} + B e^{-6.40x} - \frac{1}{15} e^{-2x} + \frac{1}{9} x - \frac{13}{81}$

11. $y'' - 100y = 9t^2 e^{10t} + \cos t - t \sin t$

Answer: $y = C_1 e^{10t} + C_2 e^{-10t} + \frac{t \sin t}{101} + \frac{3t^3}{20} e^{10t} - \frac{9t^2}{400} e^{10t} + \frac{9t}{4000} e^{10t} - \frac{99 \cos t}{10201}$

12. $y'' - 2y' + 2y = e^x \tan x$

Answer: $y = e^x (C_1 \cos x + C_2 \sin x) - e^x \cos x \ln(\sec x + \tan x)$

13. $x^2 y'' - 4xy' + 6y = 2x^4 + x^2$

Answer: $y = C_1 x^2 + C_2 x^3 + x^4 = x^2 \ln x$

14. $x^2 y'' + 10xy' + 8y = x^2$

Answer: $y = C_1 x^{-1} + C_2 x^{-8} + \frac{1}{50} x^2$

15. $x^2 y'' - 3xy' + 13y = 4 + 3x$

Answer: $y = x^2 [C_1 \cos(3 \ln x) + C_2 \sin(3 \ln x)] + \frac{4}{13} + \frac{3}{10} x$

16. $x^3 y''' - 3x^2 y'' + 6xy' - 6y = 3 + \ln x^3$

Answer: $y = C_1 x + C_2 x^2 + C_3 x^3 - \frac{\ln x}{2} - \frac{17}{12}$

Solve the following initial value problems

[marks: 4; weightage: 2]

17. $y'' - 2y' + y = \frac{1}{x}e^x,$

$y(1) = 0, y'(1) = 1$

Answer: $y = e^{x-1}(e-1)(1-x) + xe^x \ln x$

18. $y'' + 4y = \sin^2 2x$

$y(1) = 0, y'(0) = 0$

Answer: $y = -\frac{1}{6}\cos 2x + \frac{1}{6}\cos^2 2x + \frac{1}{12}\sin^2 2x$

19. $y'' - 6y' - 7y = -9e^{-2x}$

$y(0) = -2, y'(0) = -13$

Answer: $y = e^{-x} - 2e^{7x} - e^{-2x}$

20. $y'' - 4y' + 4y = 2e^{2x} - 12\cos 3x - 5\sin 3x$

$y(0) = -2, y'(0) = 4$

Answer: $y = -2e^{2x} + 5xe^{2x} + x^2 e^{2x} + \sin 3x$