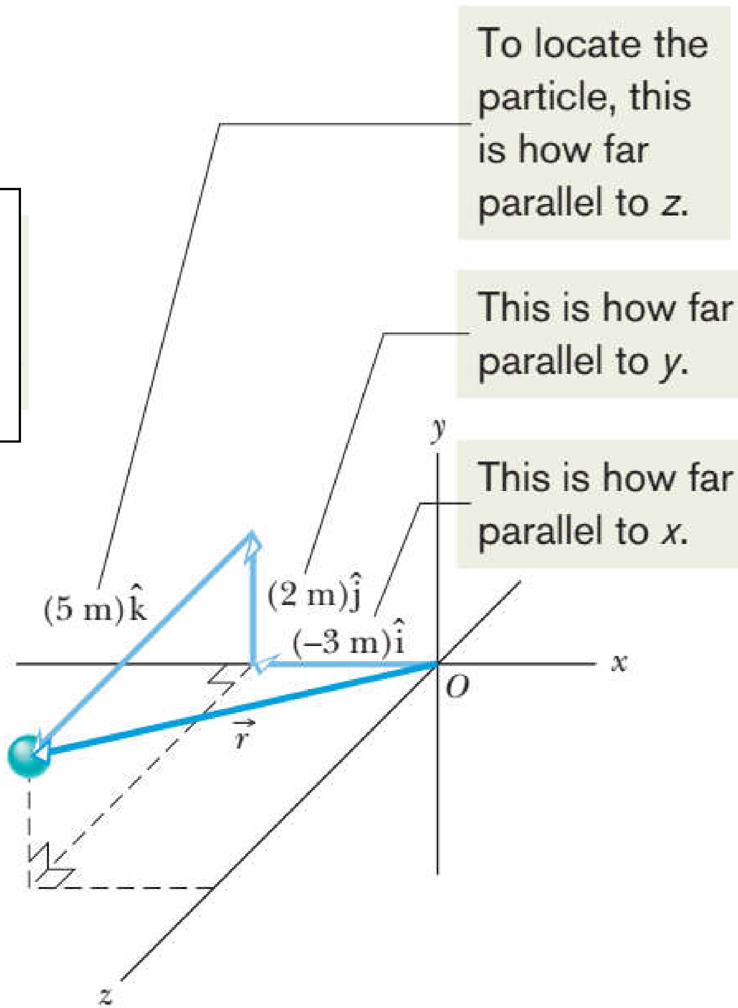
Velocity and Acceleration

Define a position first

$$\vec{r} = \boxed{} + \boxed{} + \boxed{}$$

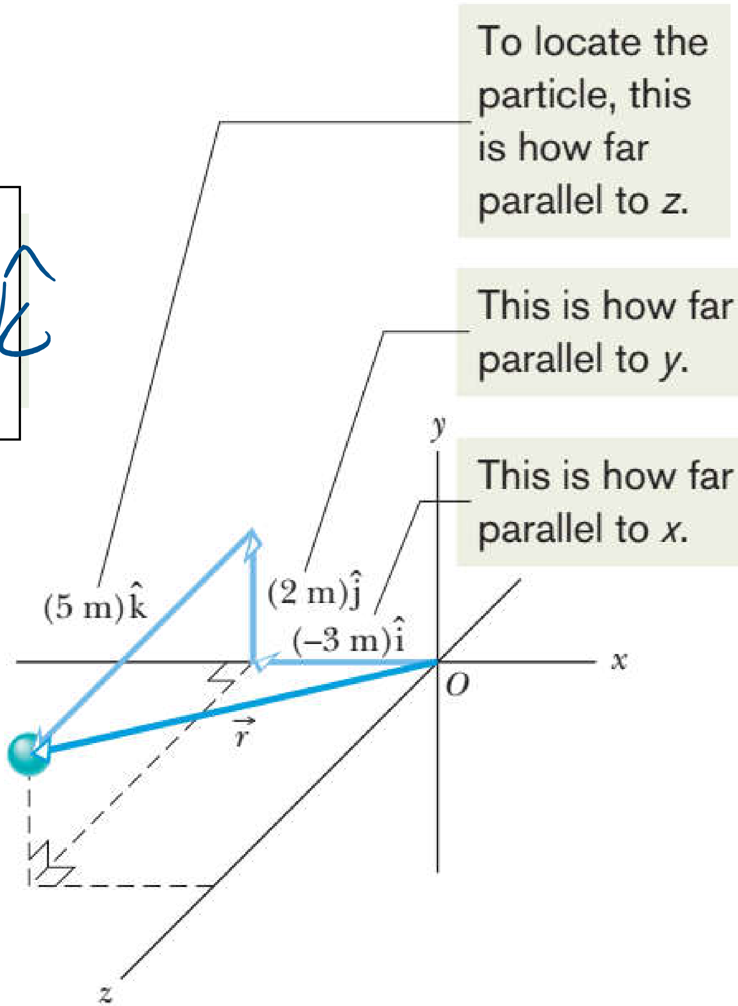


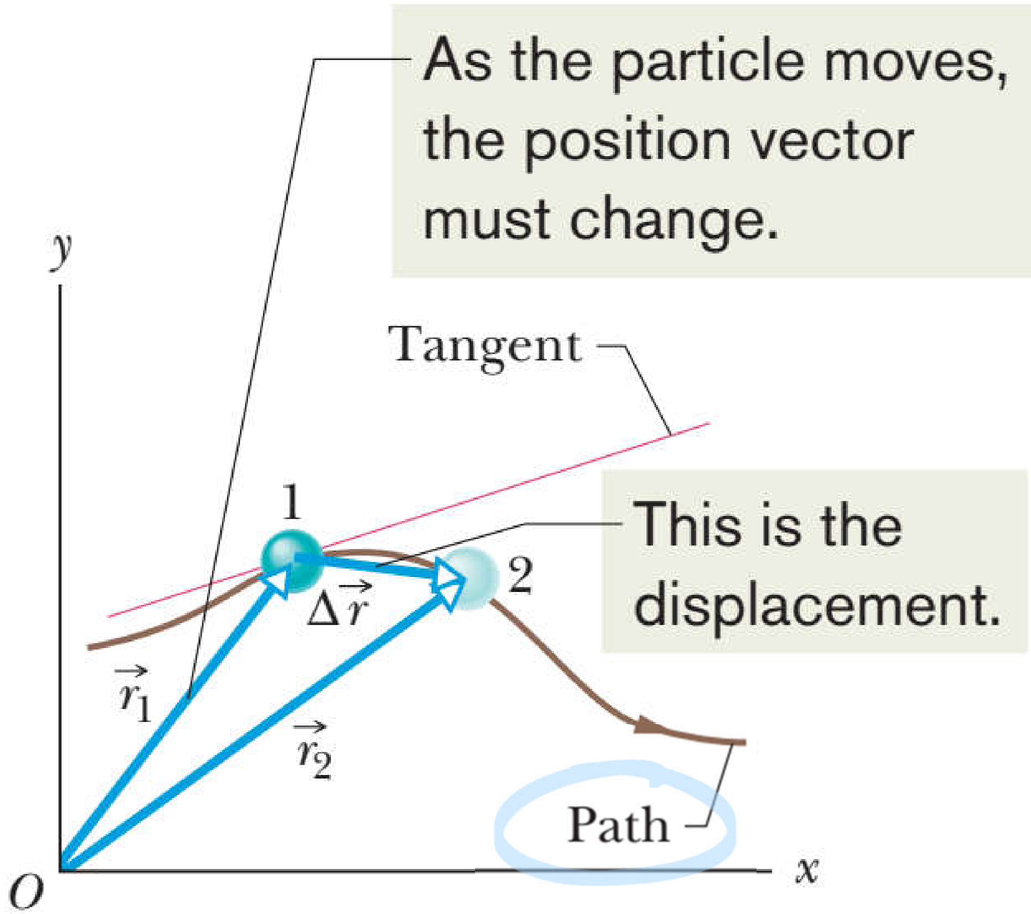
Define a position first

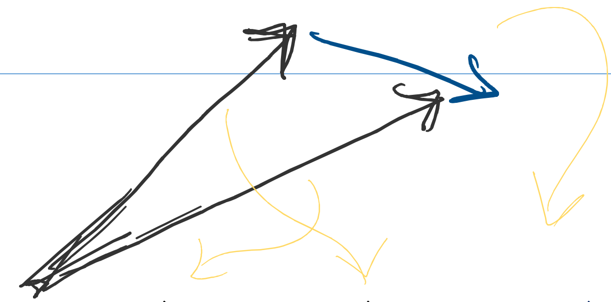
$$\vec{r} = \boxed{-3\,\hat{i}} + \boxed{2\,\hat{j}} + \boxed{5\,\hat{k}}$$

General position vector in 3D

$$\vec{r} = x\hat{\mathrm{i}} + y\hat{\mathrm{j}} + z\hat{\mathrm{k}}$$

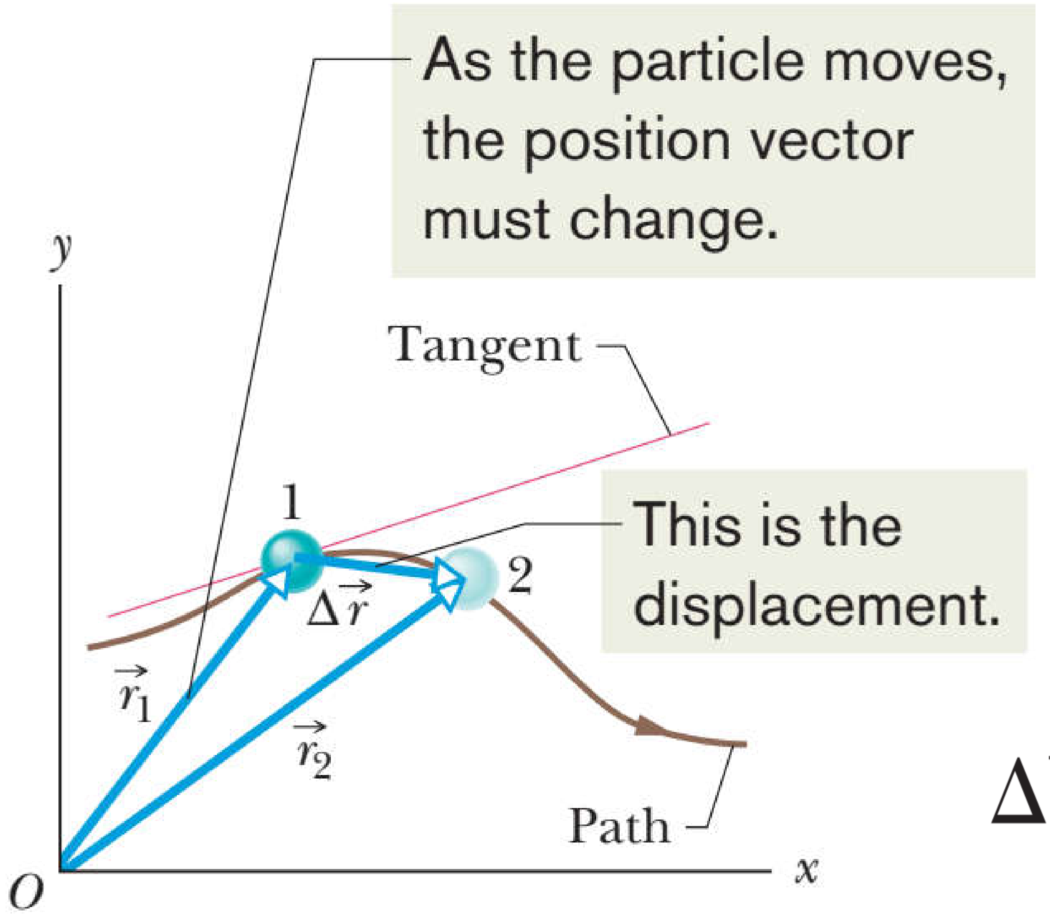






$$\overrightarrow{r_2} = \overrightarrow{r_1} + \Delta\vec{r}$$

$$\overrightarrow{\Delta r} = \overrightarrow{r_2} - \overrightarrow{r_1}$$
$$= (x_2\hat{i} + y_2\hat{j} + z\hat{k})$$
$$- (x_1\hat{i} + y\hat{j} + z\hat{k})$$



$$\vec{r_2} = \vec{r_1} + \Delta\vec{r}$$

$$\Delta\vec{r} = \vec{r_2} - \vec{r_1}$$

$$= (x_2\hat{i} + y_2\hat{j} + z\hat{k})$$

$$- (x_1\hat{i} + y\hat{j} + z\hat{k})$$

$$\Delta\vec{r} = \Delta x\hat{\mathrm{i}} + \Delta y\hat{\mathrm{j}} + \Delta z\hat{\mathrm{k}}$$

The Rabbit Problem

$$x = -0.31t^2 + 7.2t + 28$$
$$y = 0.22t^2 - 9.1t + 30.$$

$$\vec{r}(t) = x(t)\hat{\mathrm{i}} + y(t)\hat{\mathrm{j}}$$

at $t = 15$ s

$x(t) = -0.31(15^2) + 7.2(15) + 28$

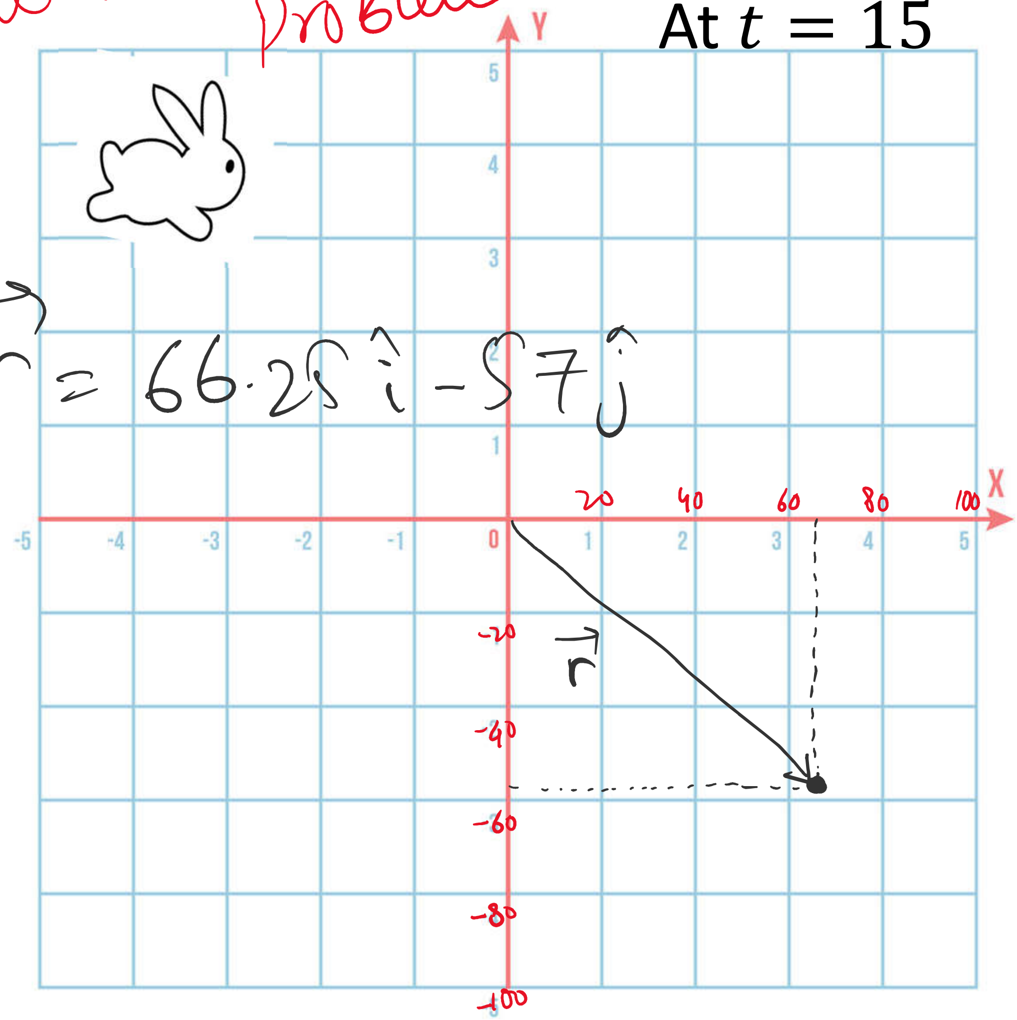
$= 66.25$ m

$y(t) = 0.22(15^2) - 9.1(15) + 30$

$= -57$ m

At $t = 15$

$\vec{r} = 66.25\,\hat{i} - 57\,\hat{j}$



The Rabbit Problem

$$x = -0.31t^2 + 7.2t + 28$$
$$y = 0.22t^2 - 9.1t + 30.$$

$$\vec{r}(t) = x(t)\hat{\mathrm{i}} + y(t)\hat{\mathrm{j}}$$

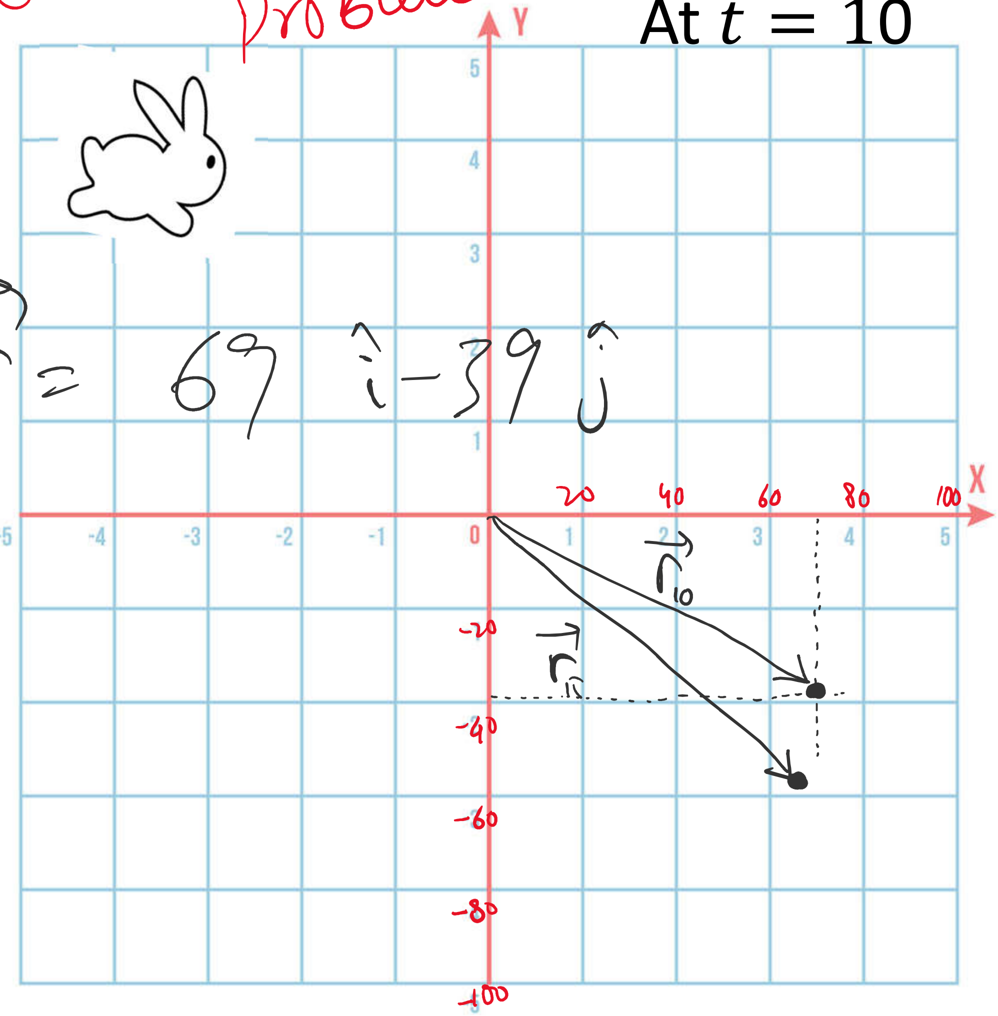
At $t = 10$

$$\vec{r} = 69\,\hat{i} - 39\,\hat{j}$$

at $t = 10\ s$

$$x(t) = -0.31(10^2) + 7.2(10) + 28$$
$$= 69 \text{ m}$$

$$y(t) = 0.22(10^2) - 9.1(10) + 30$$
$$= -39 \text{ m}$$



The Rabbit Problem

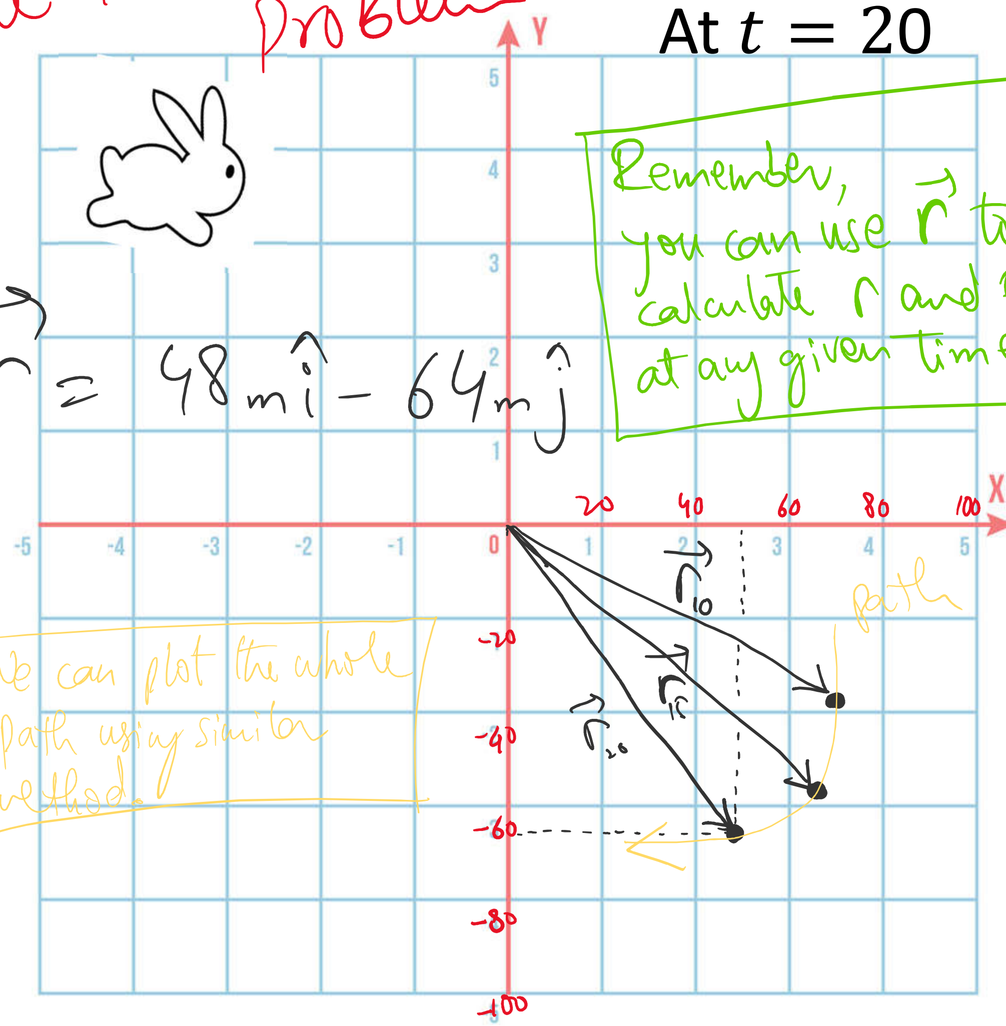
$$x = -0.31t^2 + 7.2t + 28$$
$$y = 0.22t^2 - 9.1t + 30.$$

$$\vec{r}(t) = x(t)\hat{\mathrm{i}} + y(t)\hat{\mathrm{j}}$$

At $t = 20$

$\vec{r} = 48\,\mathrm{m}\,\hat{i} - 64\,\mathrm{m}\,\hat{j}$

Remember, you can use $\vec{r}$ to calculate r and θ at any given time



we can plot the whole path using similar method.

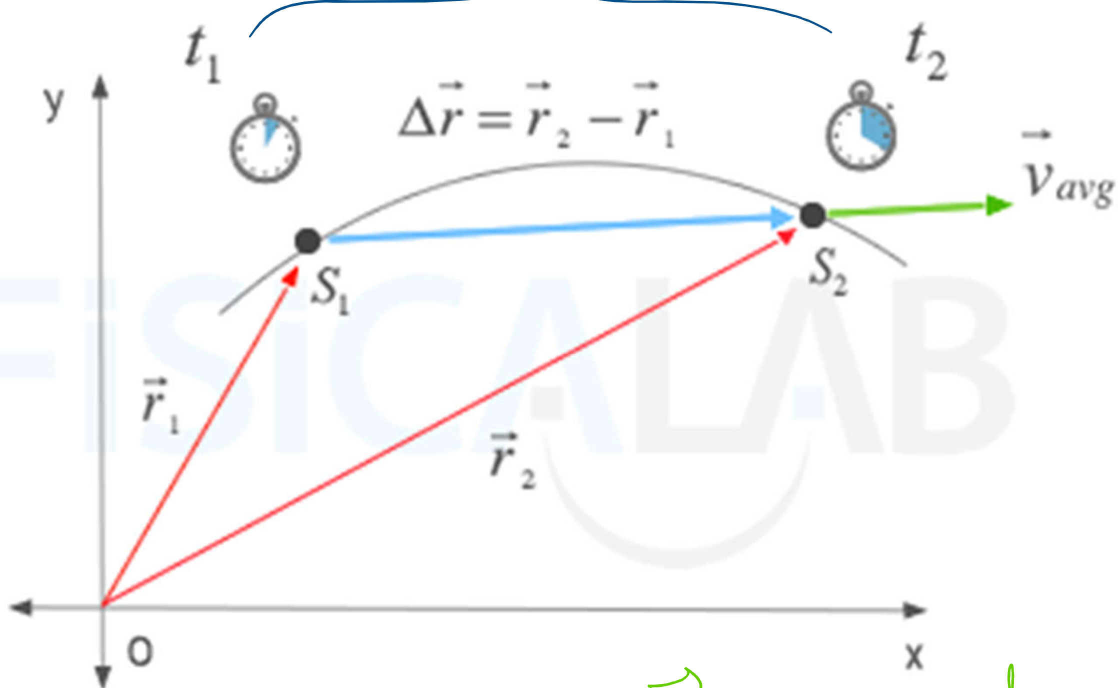
at $t = 20$ s

$x(t) = -0.31(20^2) + 7.2(20) + 28$
$= 48$ m

$y(t) = 0.22(20^2) - 9.1(20) + 30$
$= -64$ m

Velocity

$\Delta t = t_2 - t_1$



Average Velocity

$$\vec{v}_{\text{avg}} = \frac{\Delta\vec{r}}{\Delta t}.$$

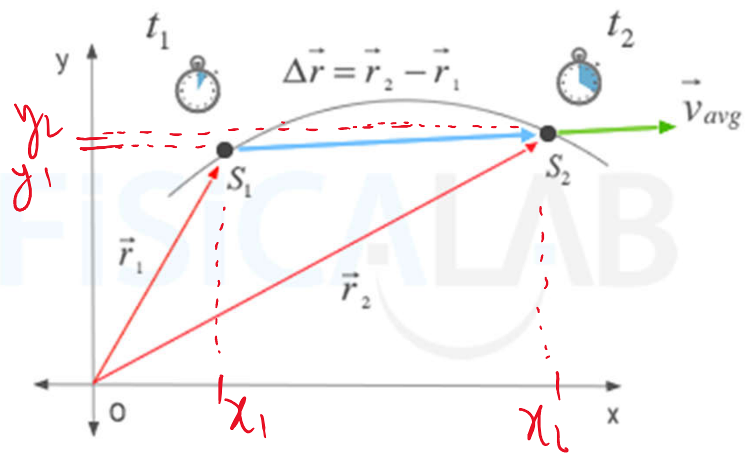
Also note that two points of position generates single point of velocity.

Position of $\vec{v}$ vector is at point S_2 because this is the final position of the particle in our calculation.

Velocity

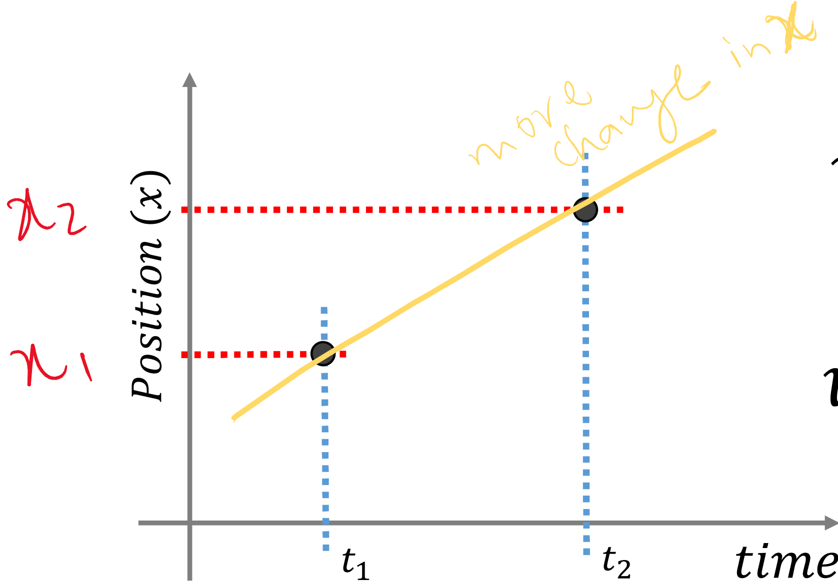
Average Velocity

$$\vec{v}_{\text{avg}} = \frac{\Delta \vec{r}}{\Delta t}.$$



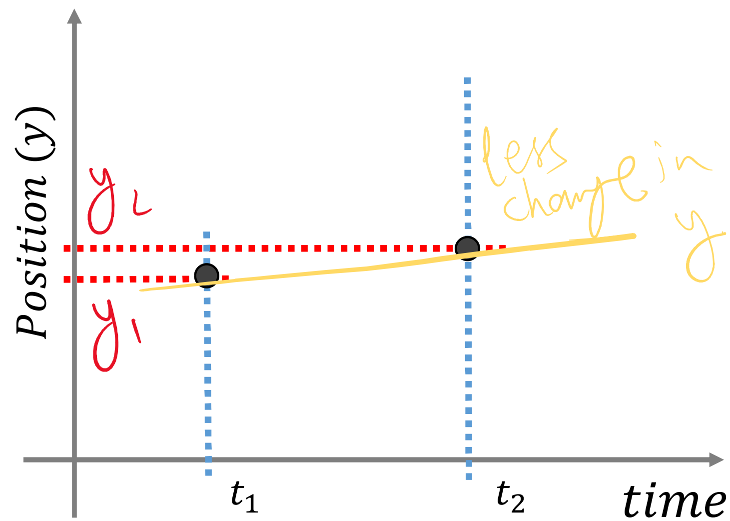
direction of $\vec{v}_{avg}$ is the same as direction of $\Delta\vec{r}$

$$\vec{v}_{avg} = \frac{\Delta\vec{r}}{\Delta t}$$



$$v_x = \frac{\Delta x}{\Delta t}$$

$$v_y = \frac{\Delta y}{\Delta t}$$

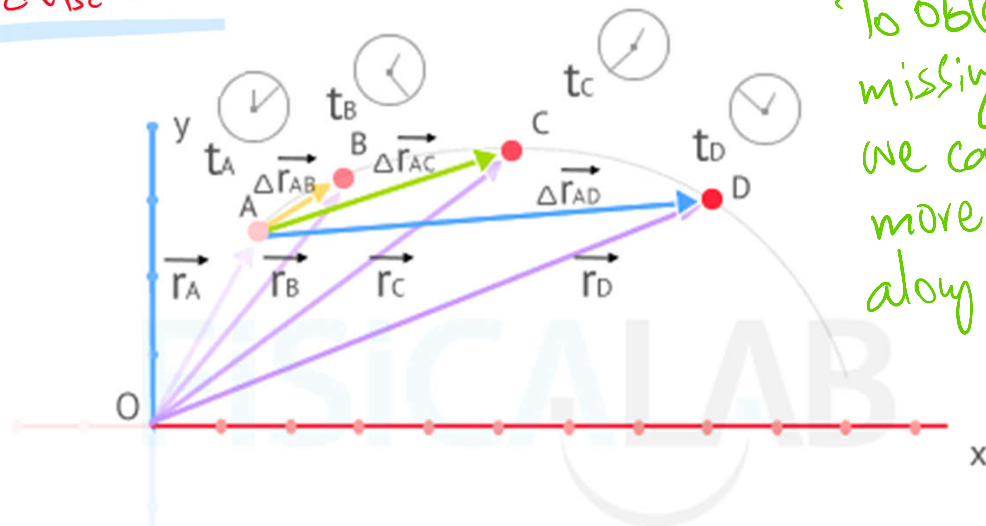


Note:
$\Delta t_{AB} = \Delta t_{BC} = \Delta t_{CD}$

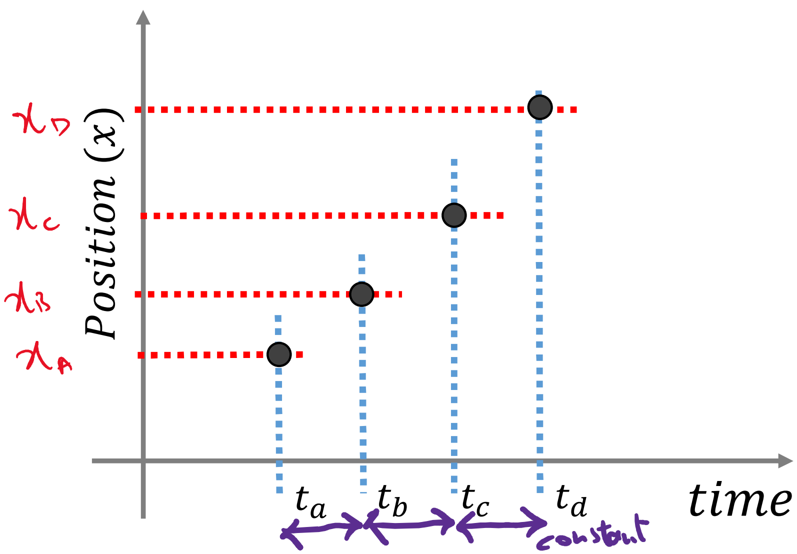
Velocity

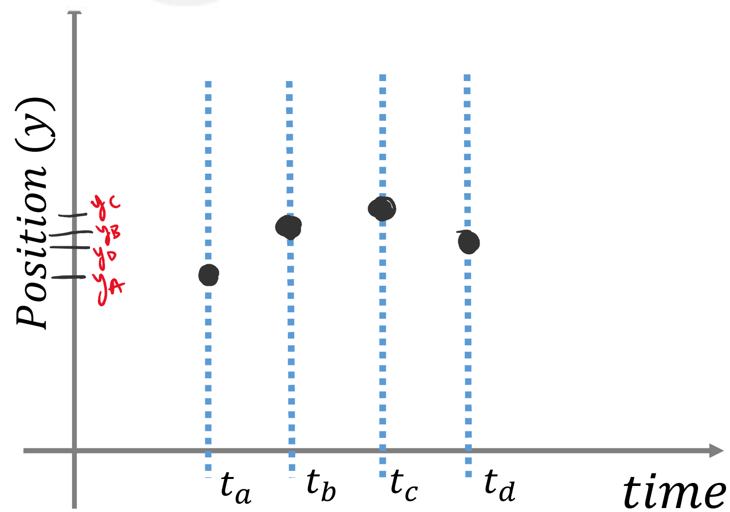
Average Velocity

$$\vec{v}_{avg} = \frac{\Delta \vec{r}}{\Delta t}.$$



To obtain the missing information we can use more points along the path.



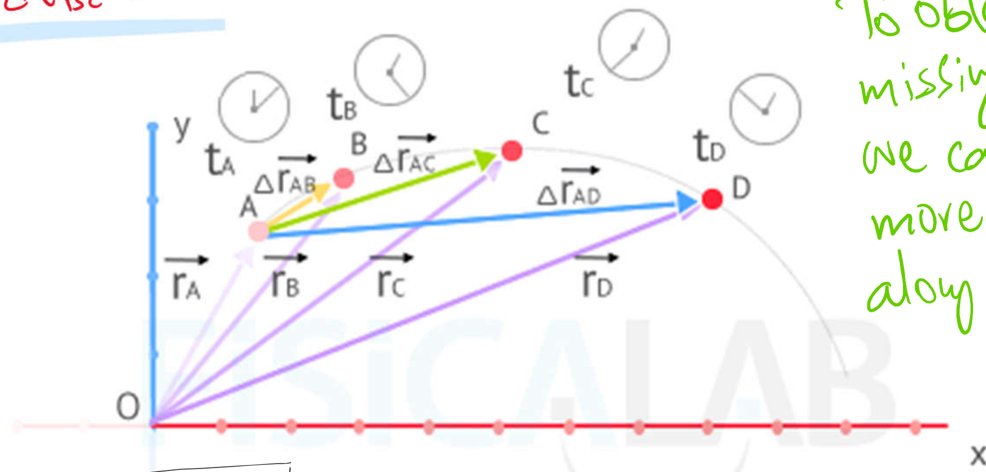


Note:
$\Delta t_{AB} = \Delta t_{BC} = \Delta t_{CD}$

Velocity

Average Velocity

$$\vec{v}_{avg} = \frac{\Delta \vec{r}}{\Delta t}.$$



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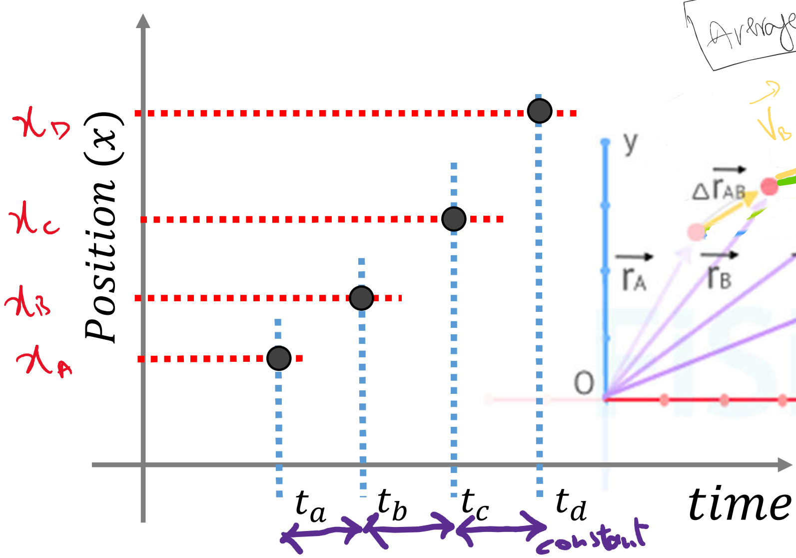
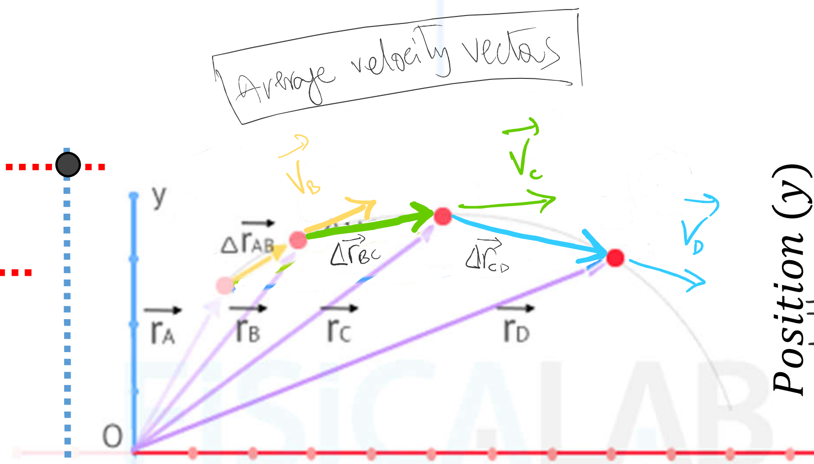
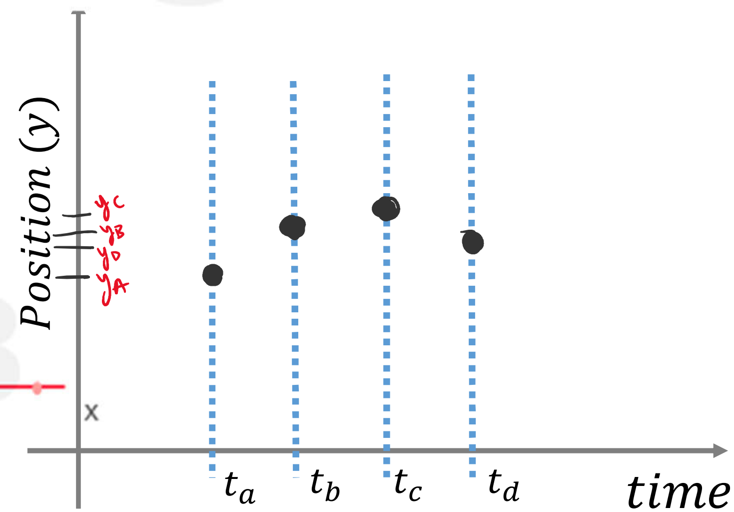


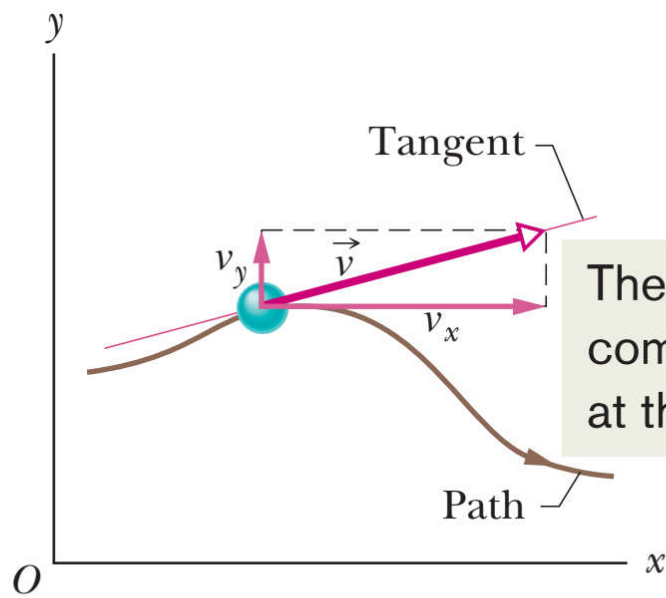




Velocity

Reducing Δt will cause points to converge on a single point and $\Delta\vec{r}$ will becomes the same as tangent on that point.

Therefore $\longrightarrow$ The velocity vector is always tangent to the path.



These are the x and y components of the vector at this instant.

Instantaneous Velocity

$$\Delta t \to 0$$

Approaches to zero, NOT EQUAL TO ZERO

$$\vec{v} = \frac{d\vec{r}}{dt}.$$

$\Delta \longrightarrow d$

Lecture 3

$$x = -0.31t^2 + 7.2t + 28$$
$$y = 0.22t^2 - 9.1t + 30.$$

Bring back the rabbit!

$$v_x = \frac{dx}{dt}, \quad v_y = \frac{dy}{dt}$$

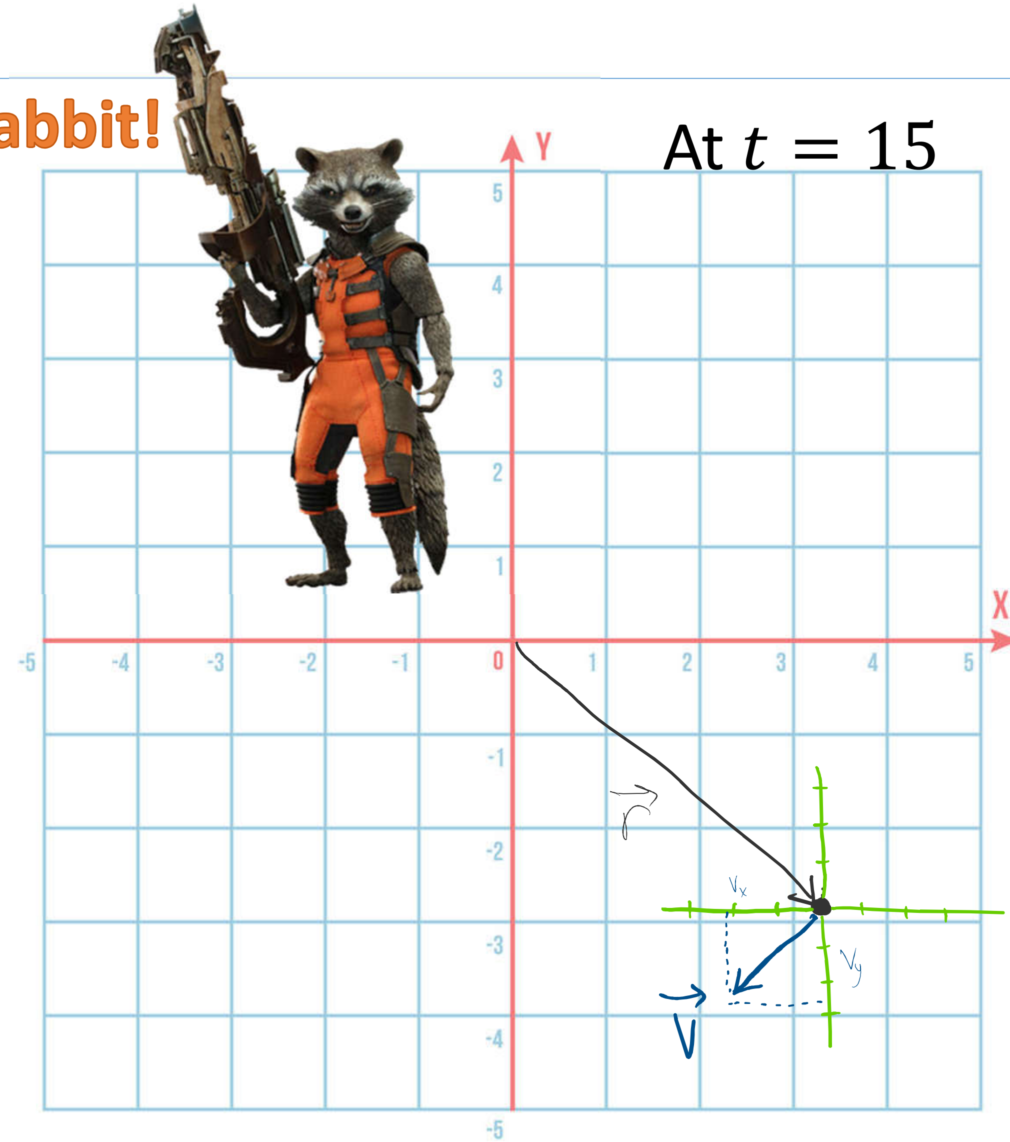
$$V_x(t) = -0.62t + 7.2$$

$$V_y(t) = 0.44t - 9.1$$

at $t = 15$ sec

$$\vec{V} = -2.1\frac{m}{s}\hat{i} - 2.5\frac{m}{s}\hat{j}$$

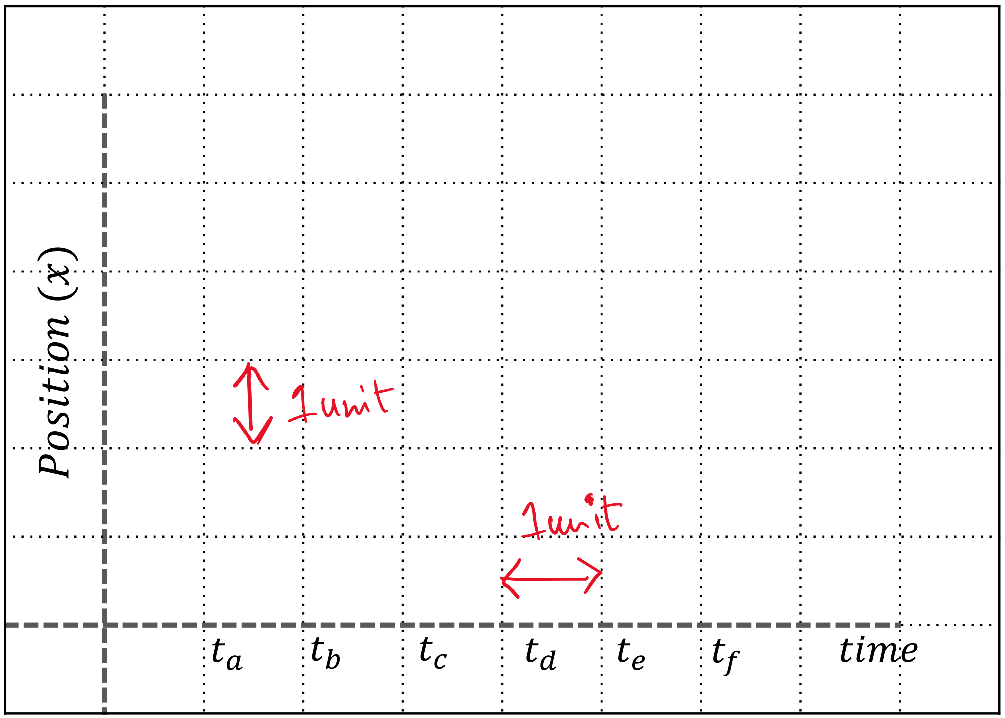
At $t = 15$

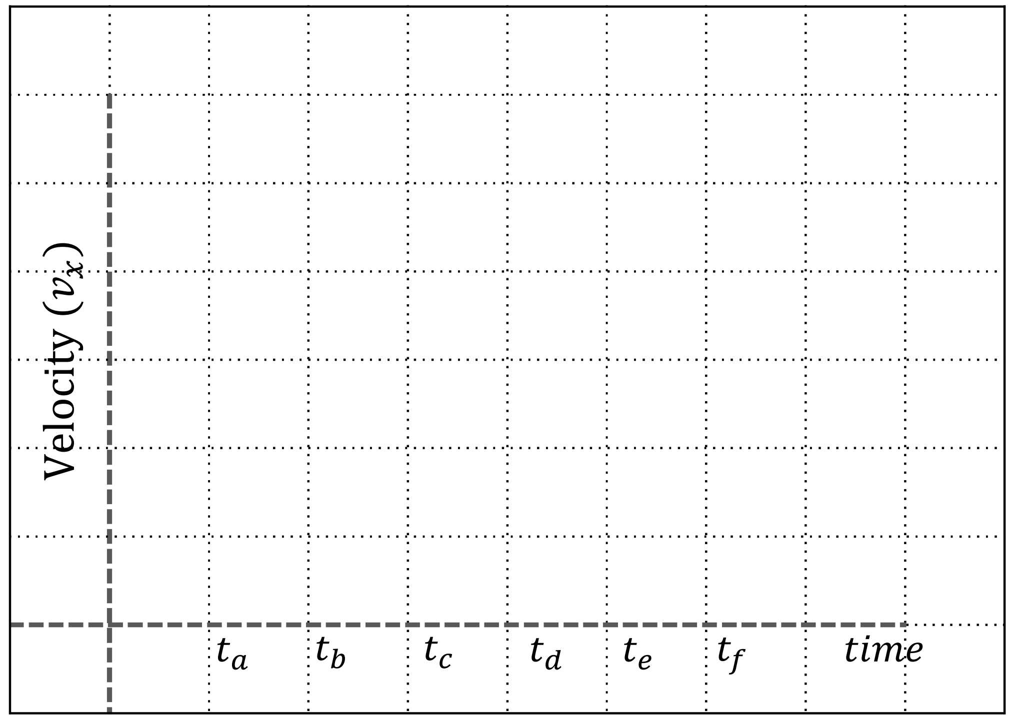


Constant Velocities – graphical analysis

$$\Delta v_{12} = \Delta v_{23} = \Delta v_{34} = \Delta v_{45} = \cdots$$

One dimension

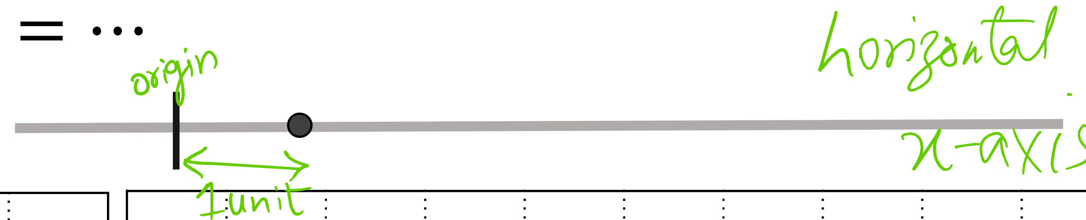


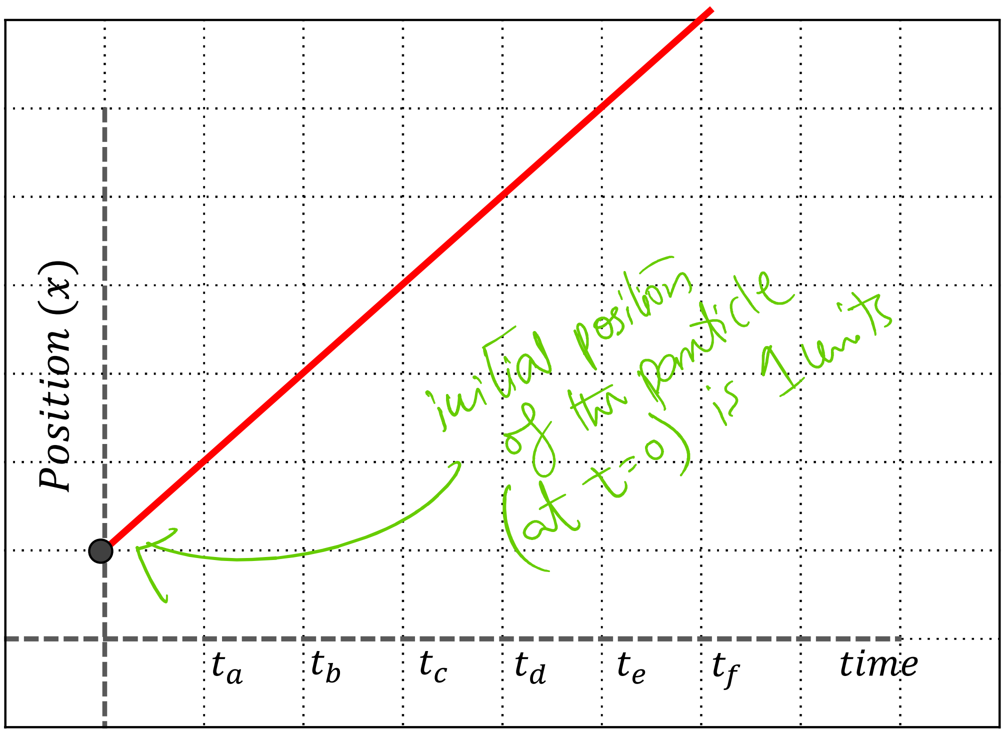


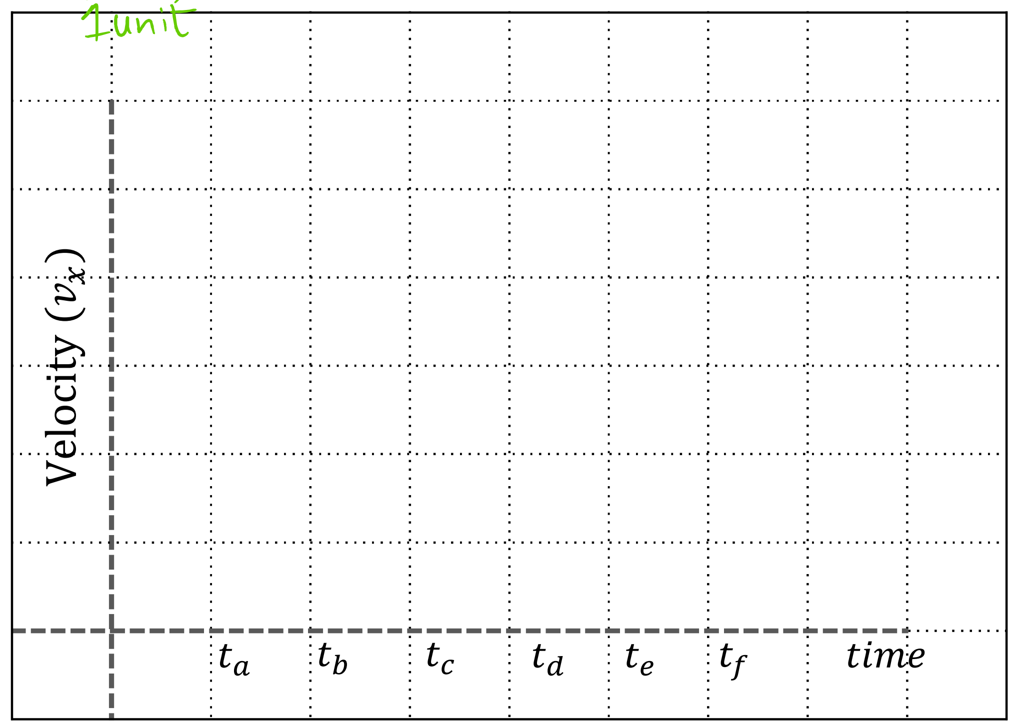
Constant Velocities – graphical analysis

$$\Delta v_{12} = \Delta v_{23} = \Delta v_{34} = \Delta v_{45} = \cdots$$

Tracing the object position







Constant Velocities – graphical analysis

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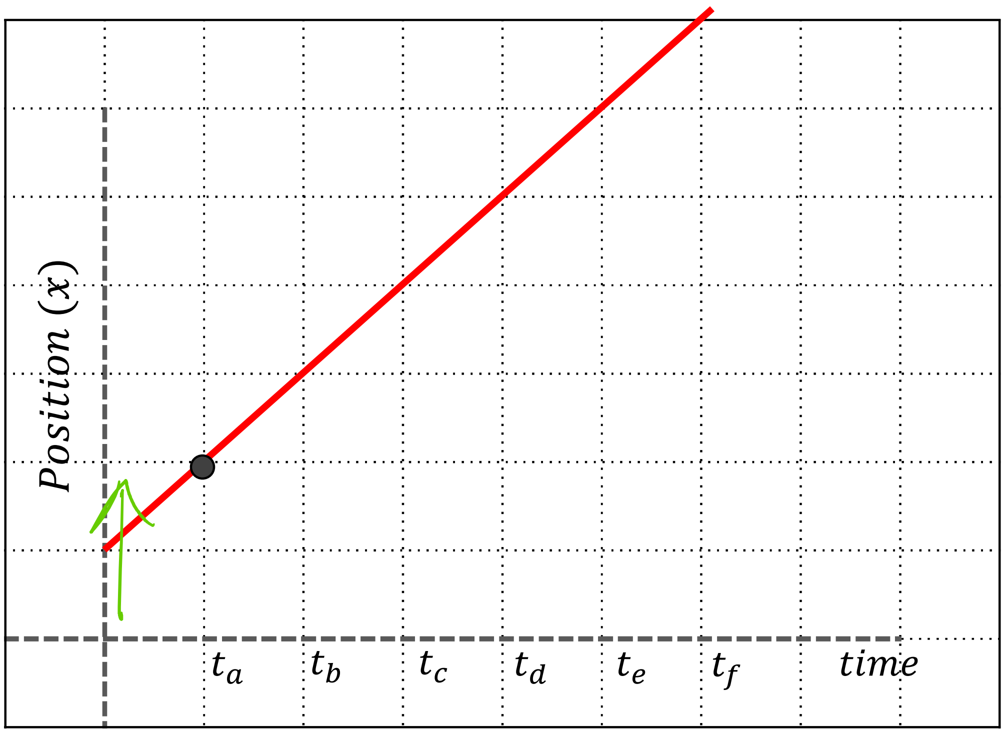
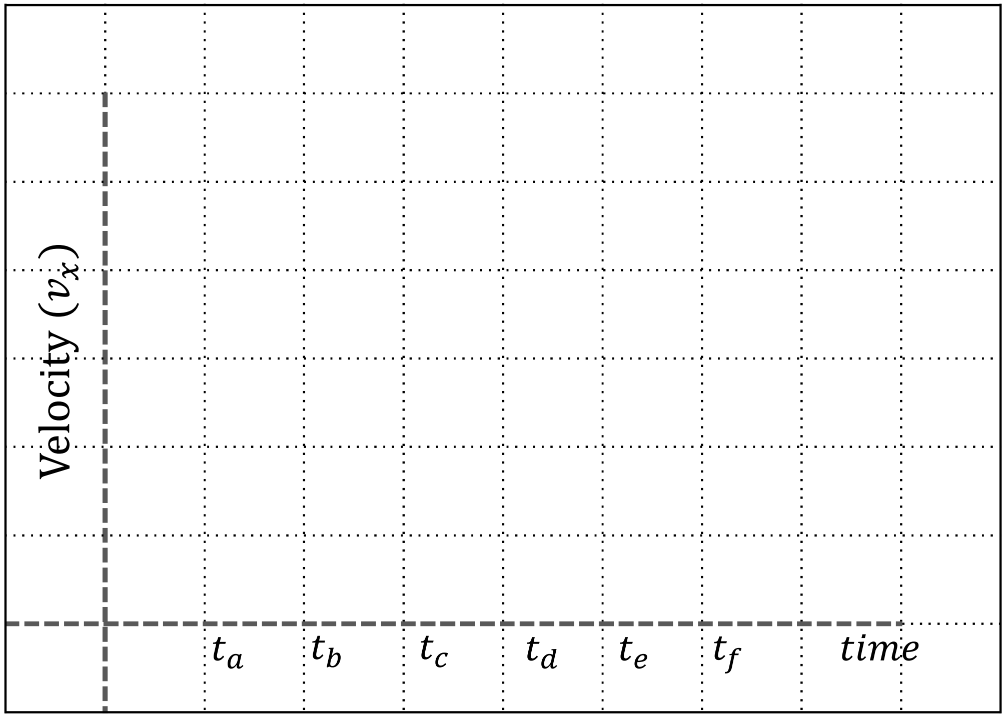
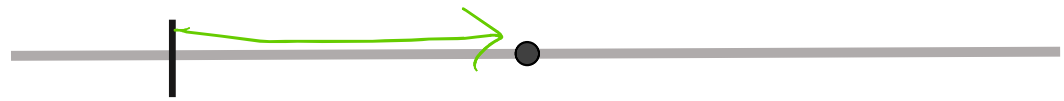


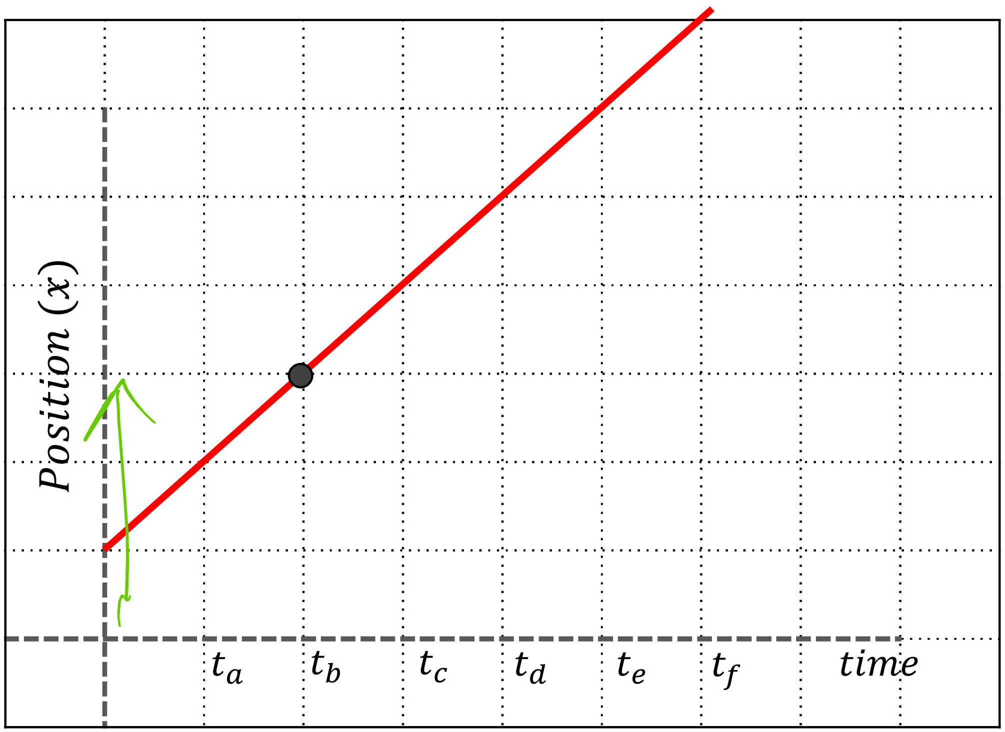


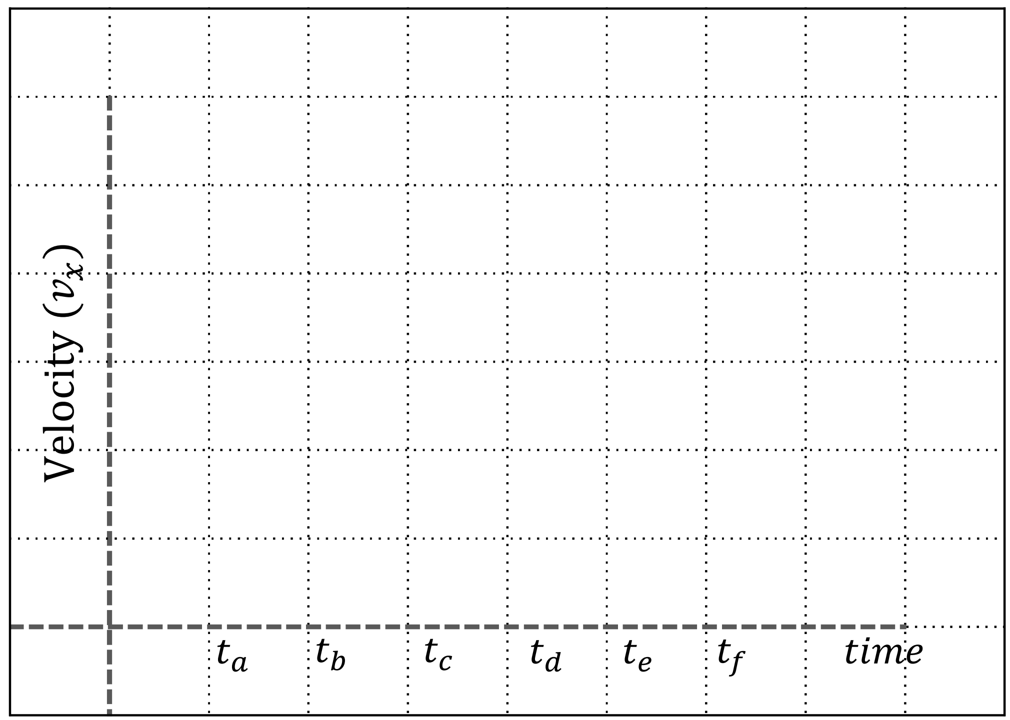
Constant Velocities – graphical analysis

$$\Delta v_{12} = \Delta v_{23} = \Delta v_{34} = \Delta v_{45} = \cdots$$

Tracing the object position



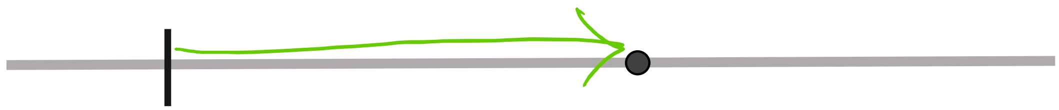


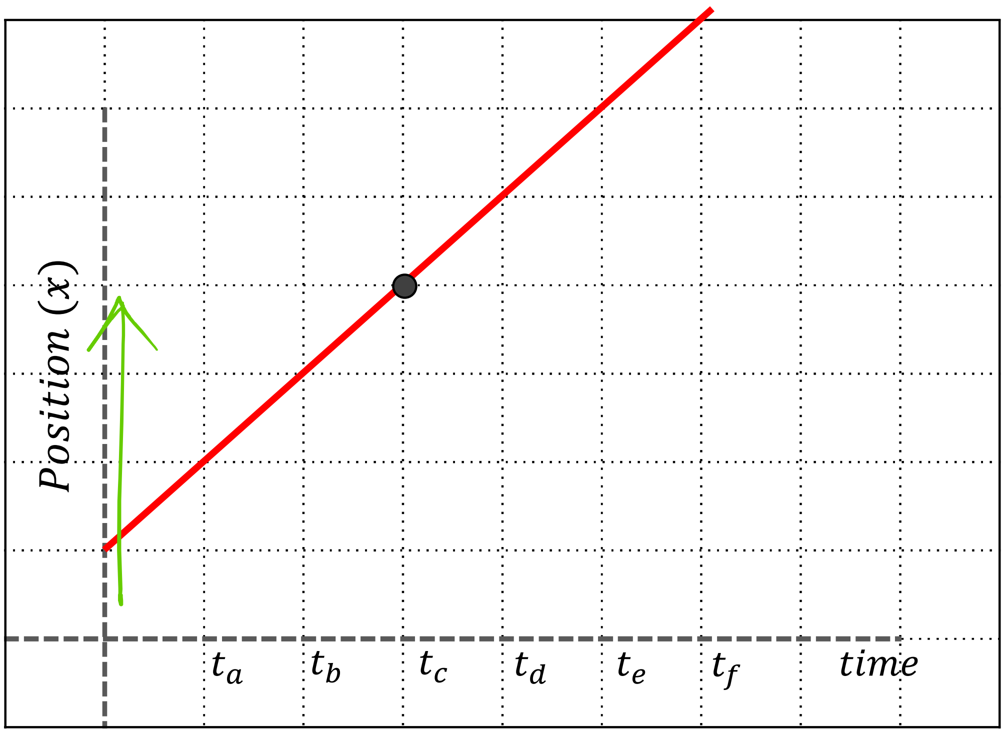


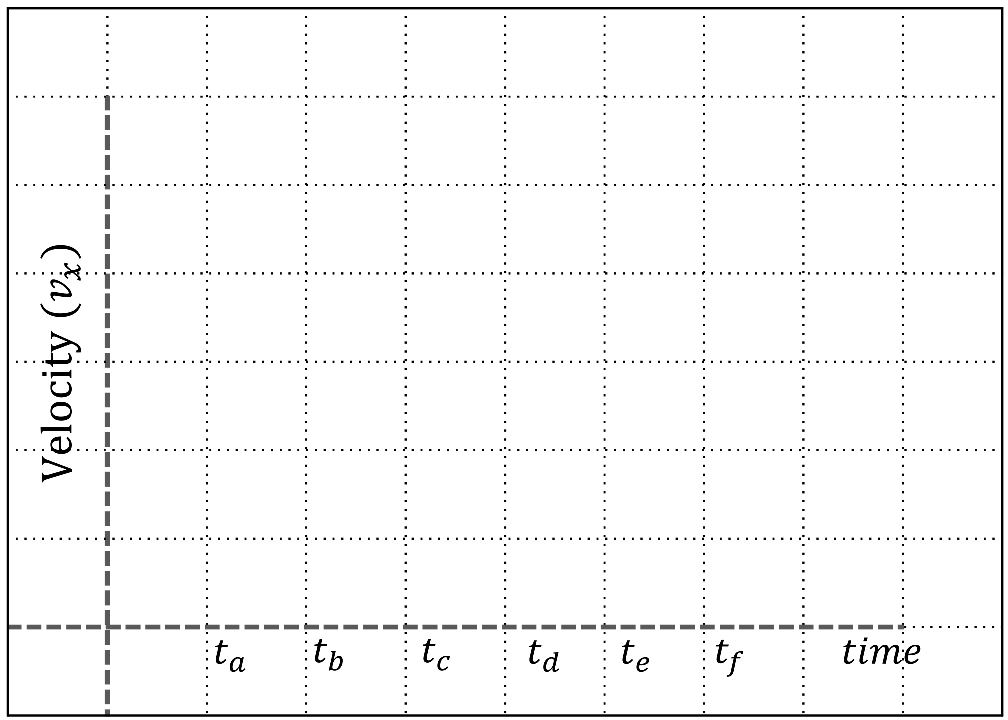
Constant Velocities – graphical analysis

$$\Delta v_{12} = \Delta v_{23} = \Delta v_{34} = \Delta v_{45} = \cdots$$

Tracing the object position



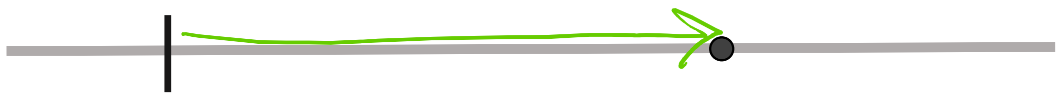


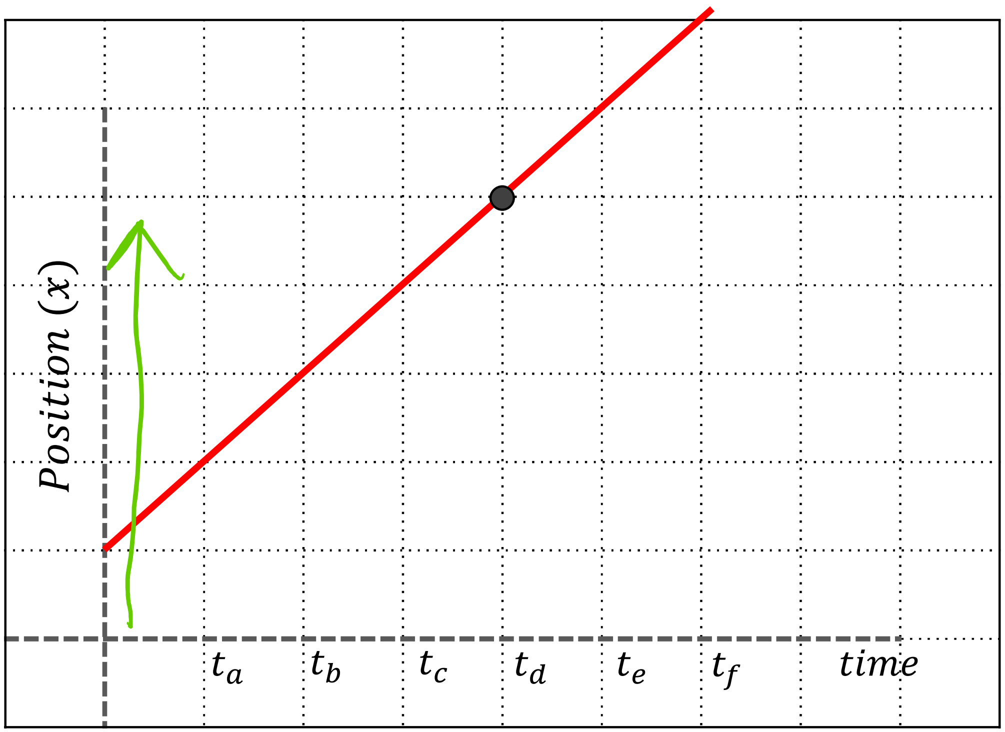


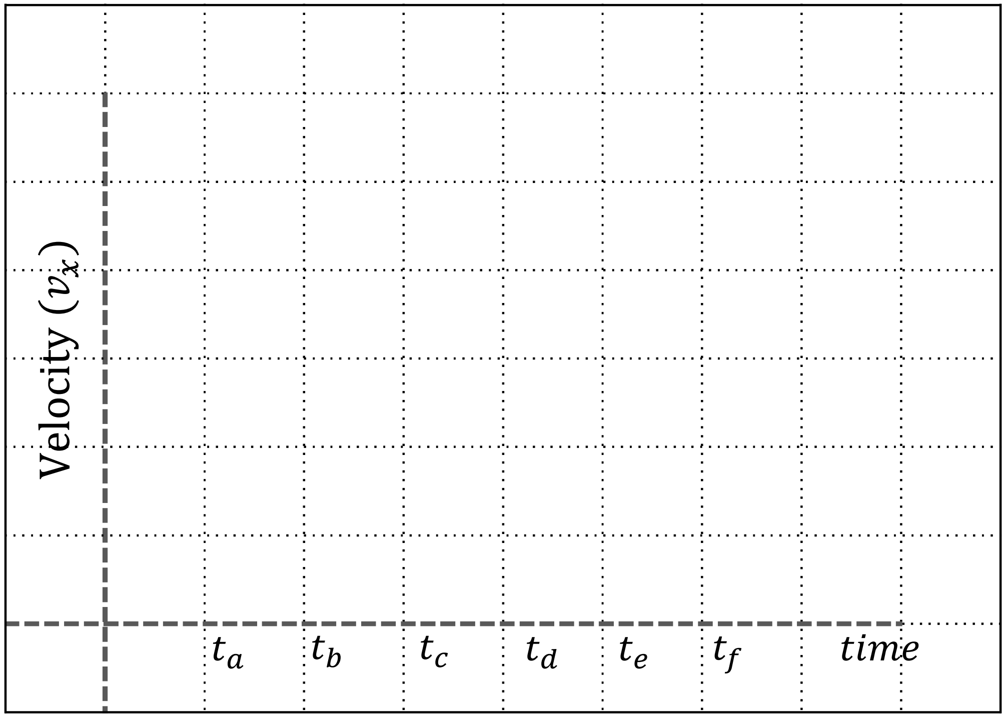
Constant Velocities – graphical analysis

$$\Delta v_{12} = \Delta v_{23} = \Delta v_{34} = \Delta v_{45} = \cdots$$

Tracing the object position







Constant Velocities – graphical analysis

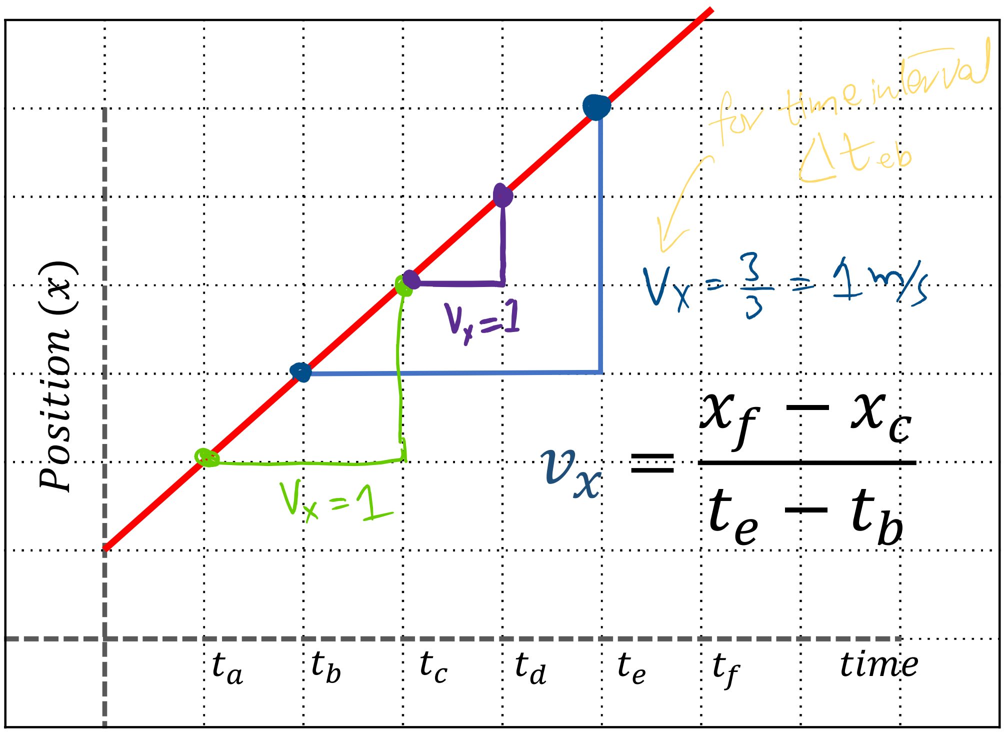
Slope of straight line is constant

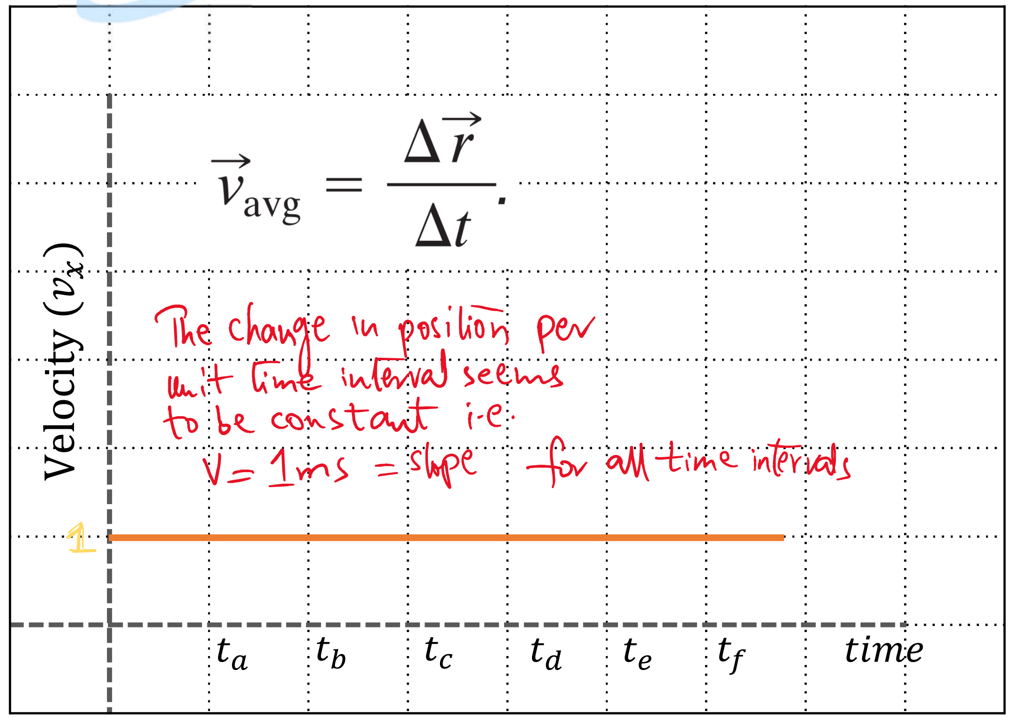
$\Updownarrow$

Velocity of equidistant motion is constant

$$\Delta v_{12} = \Delta v_{23} = \Delta v_{34} = \Delta v_{45} = \cdots$$

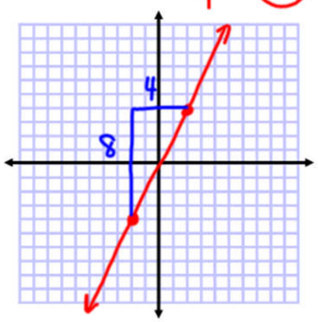
Velocity is "**slope**" in xt graph



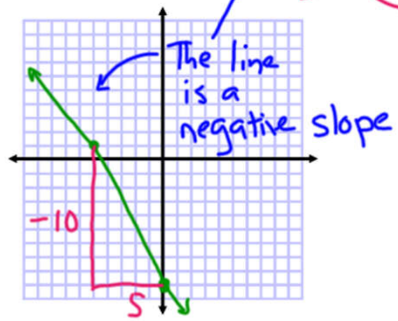


Find the Slope

(2, 4) (-2, -4) $\frac{8}{4} = 2$



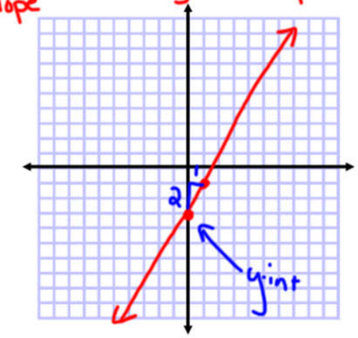
(-5, 1) (0, -9) $\frac{-10}{5} = -2$



Find the Slope and Graph

y = $\boxed{2}$x $\boxed{-\ 3}$

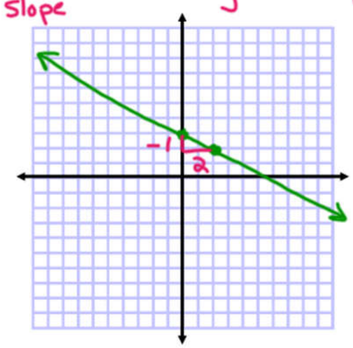
slope y-intercept



Slope = $\frac{2}{1}$ y-int = -3

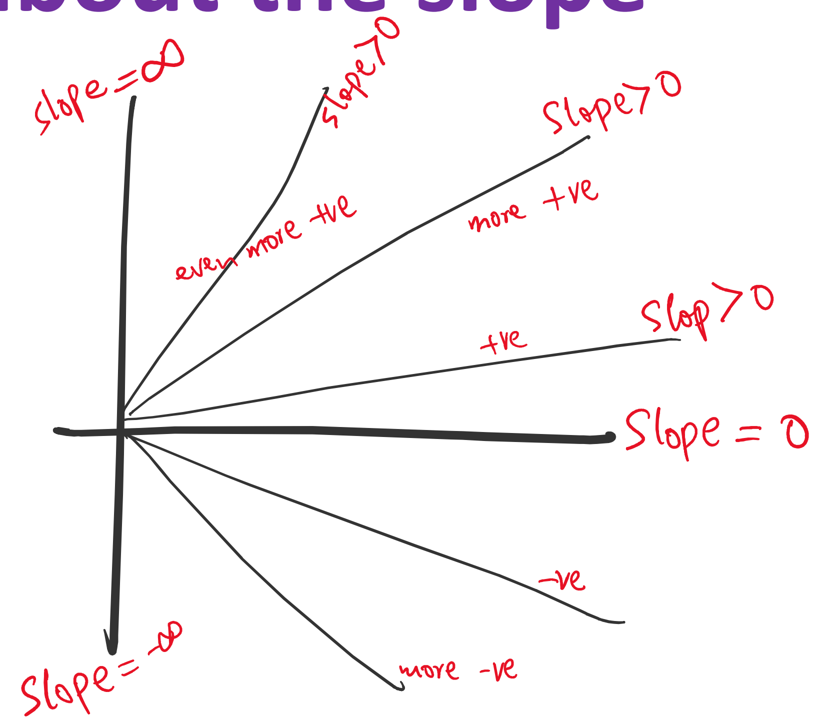
y = $\boxed{-½}$x $\boxed{+\ 3}$

slope y-intercept

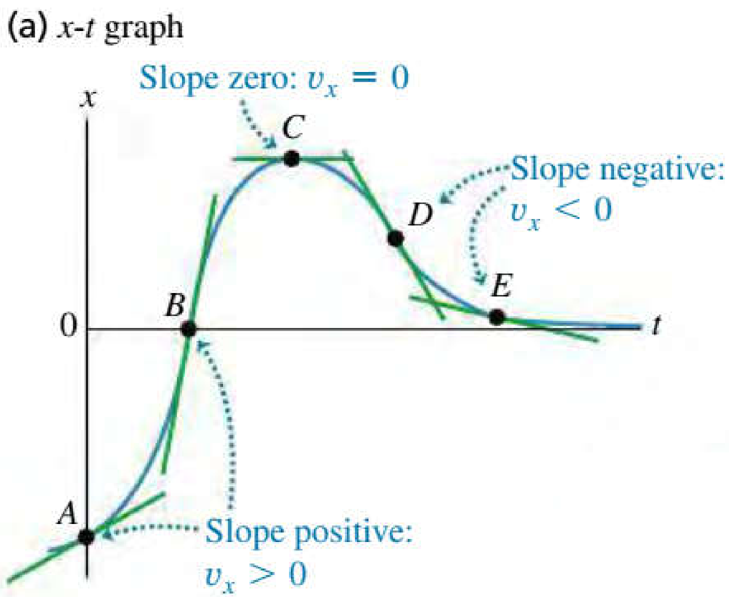


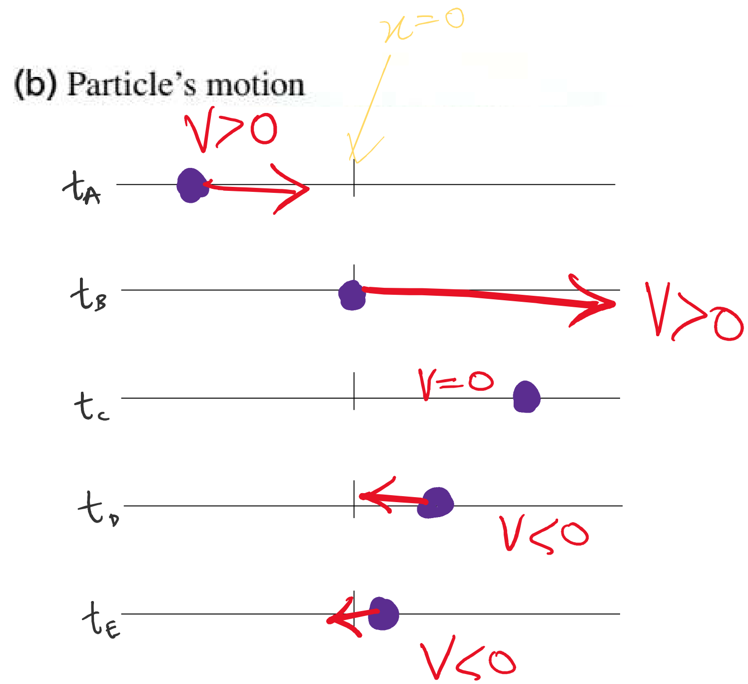
Slope = $-\frac{1}{2}$ y-int = 3

Reminder!
about the slope

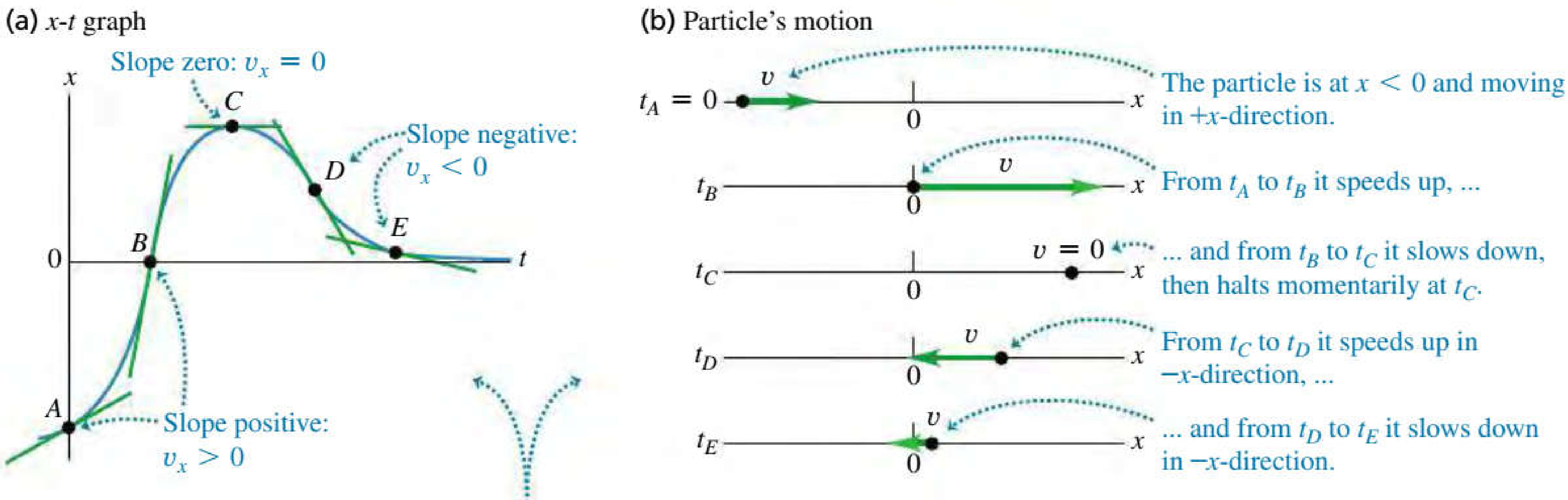


Slope analysis





Slope analysis

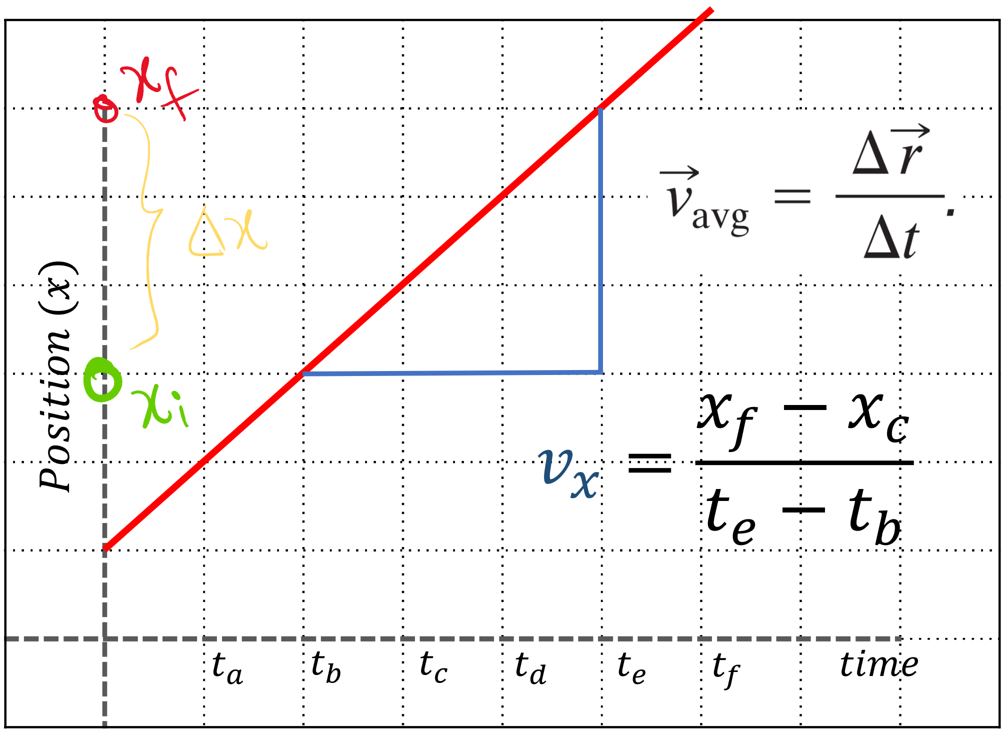


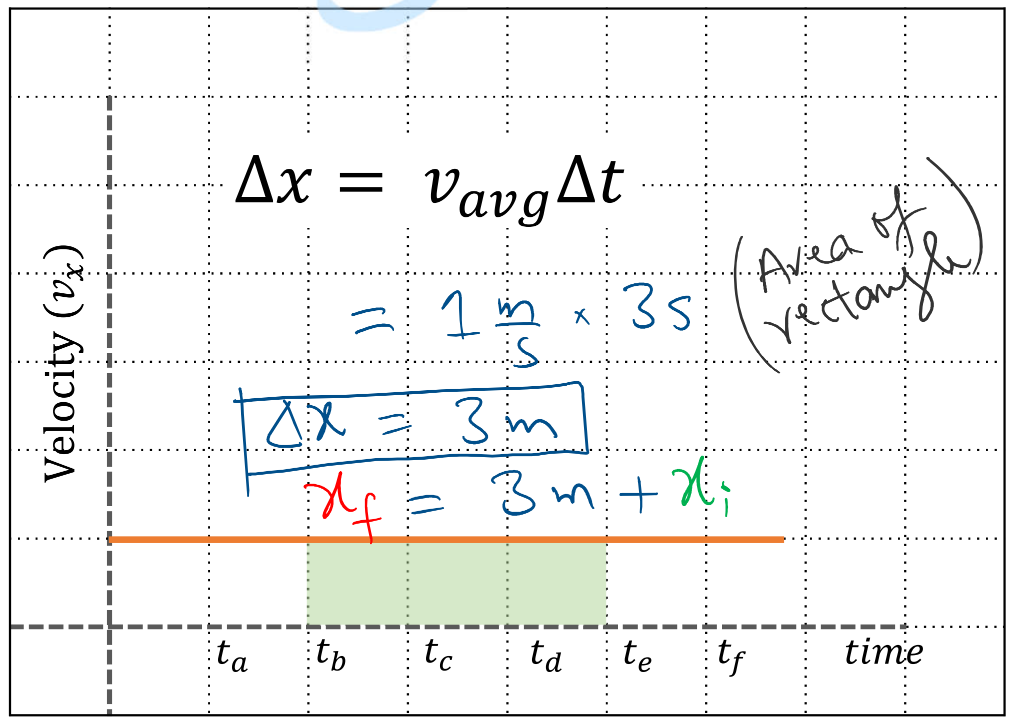
- On an x-t graph, the slope of the tangent at any point equals the particle's velocity at that point.
- The steeper the slope (positive or negative), the greater the particle's speed in the positive or negative x-direction.

Constant Velocities – graphical analysis

$$\Delta v_{12} = \Delta v_{23} = \Delta v_{34} = \Delta v_{45} = \cdots$$

Position is "***area under the curve***" in vt graph





Simulation can help you learn by real-time interactions

click to visit website

1. Set a=0 and play around with velocity and position values

2. Calculate distance (area under the curve) from vt graph

click to visit website