Vector Multiplication

(Vector x Vector) = Vector

Cross Product

Rotational Information

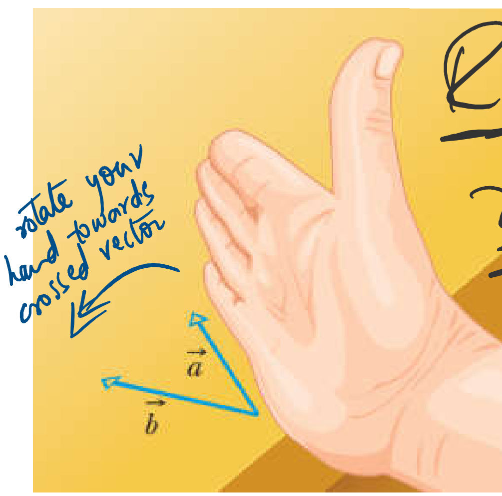
Important points

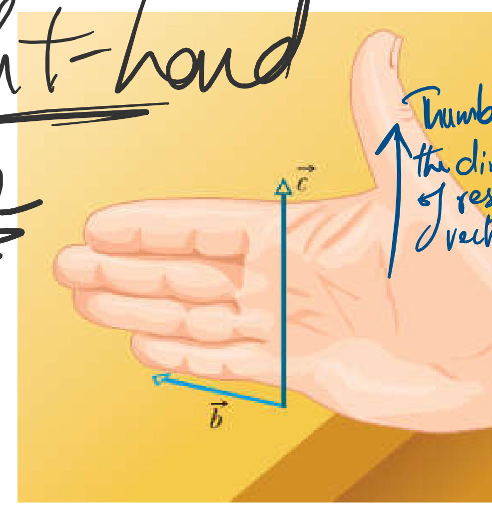
The system must be in three dimensions or more.

The resultant vector is always perpendicular to the two vectors multiplied.

Right-hand rule

rotate your hand towards crossed vector





Thumb gives the direction of resultant vector

Vector Multiplication

(Vector x Vector) = Vector

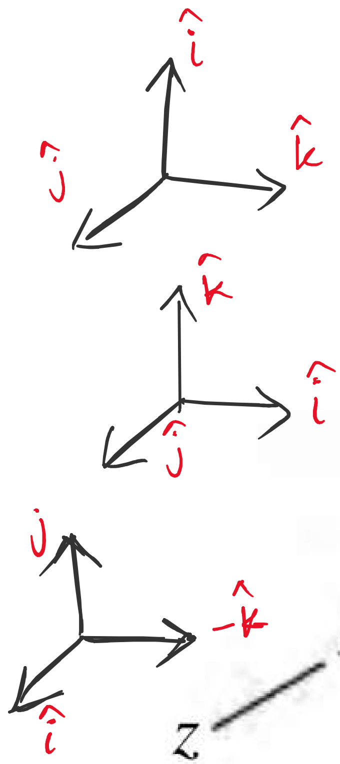
Cross Product

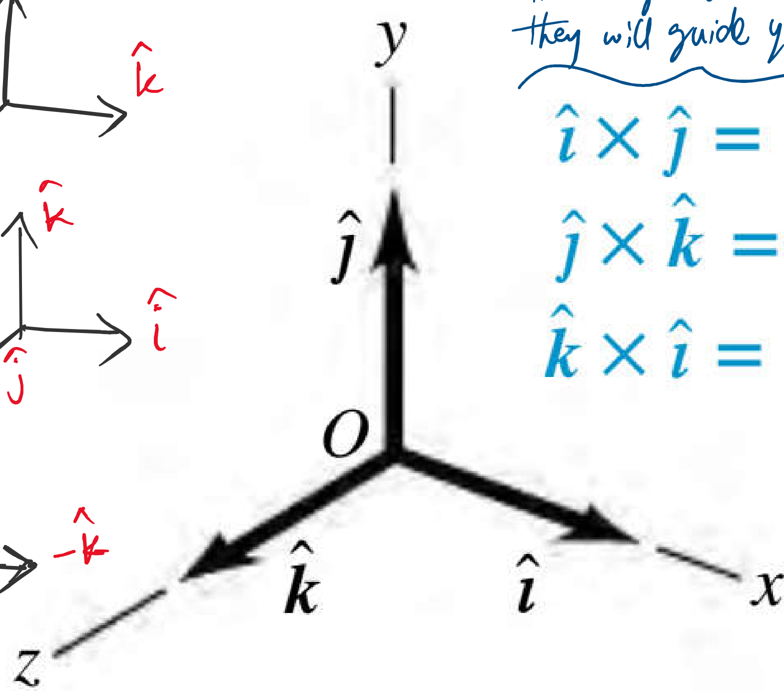
Rotational Information

$$\hat{\imath} \times \hat{\jmath} = -\hat{\jmath} \times \hat{\imath} = \hat{k}$$

$$\hat{\jmath} \times \hat{k} = -\hat{k} \times \hat{\jmath} = \hat{\imath}$$

$$\hat{k} \times \hat{\imath} = -\hat{\imath} \times \hat{k} = \hat{\jmath}$$





Hold tight to these
they will guide you

$$\hat{\imath} \times \hat{\jmath} = \hat{k}$$

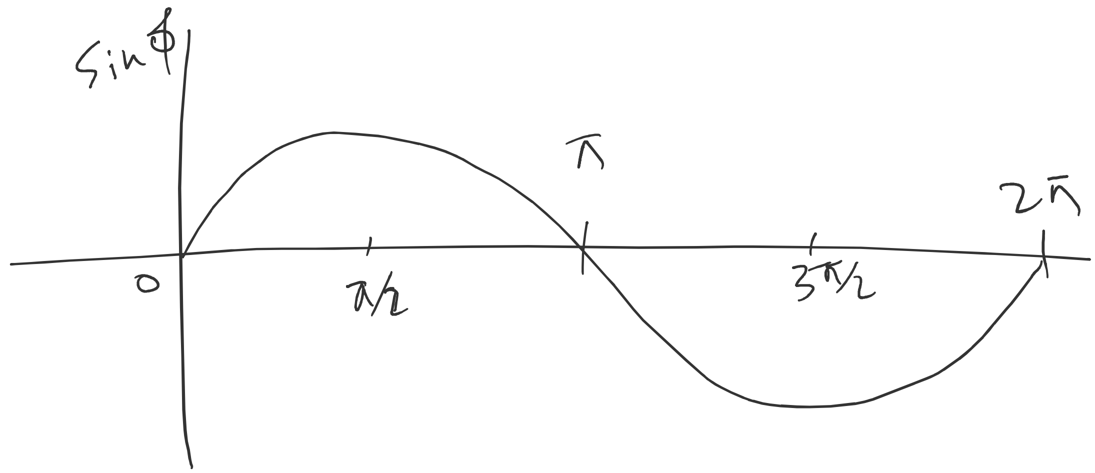
$$\hat{\jmath} \times \hat{k} = \hat{\imath}$$

$$\hat{k} \times \hat{\imath} = \hat{\jmath}$$

Vector Multiplication

(Vector x Vector) = Vector

Cross Product



Rotational Information

$$\vec{a} \times \vec{b} = |\vec{a}||\vec{b}| \sin \emptyset$$

If $\vec{a}$ and $\vec{b}$ are parallel or antiparallel, $\vec{a} \times \vec{b} = 0$. The magnitude of $\vec{a} \times \vec{b}$, which can be written as $|\vec{a} \times \vec{b}|$, is maximum when $\vec{a}$ and $\vec{b}$ are perpendicular to each other.

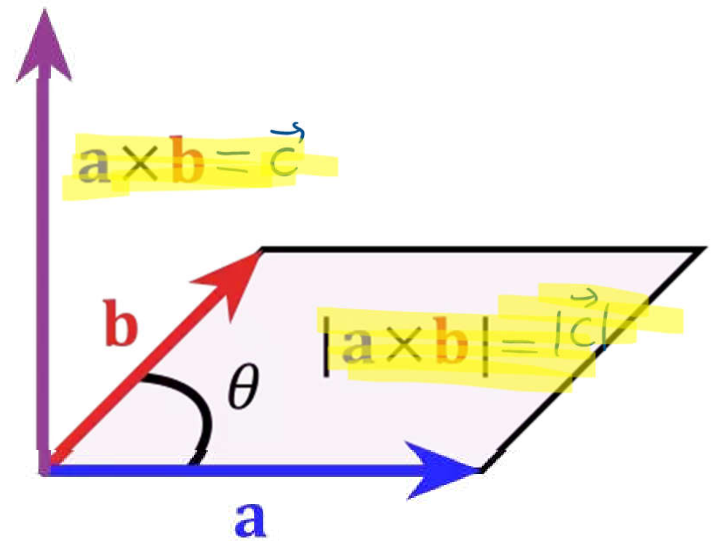
Vector Multiplication

(Vector x Vector) = Vector

Cross Product

Determinant (because determinants show how area is stretched and rotated)

$$\vec{a} \times \vec{b} = det\left(\begin{bmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{bmatrix}\right)$$



$$\vec{c} = \hat{\imath}(a_y b_z - a_z b_y) - \hat{\jmath}(a_x b_z - a_z b_x) + \hat{k}(a_x b_y - a_y b_x)$$

$$\vec{c} = c_x \hat{\imath} + c_y \hat{\jmath} + c_z \hat{k}$$

Vector Multiplication

(Vector x Vector) = Vector

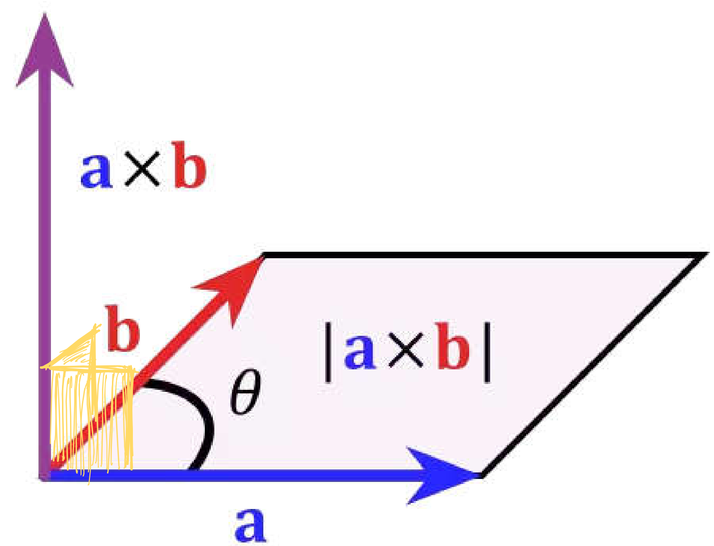
Cross Product

for parallel and antiparallel vectors, the area of parallelogram will remain zero.

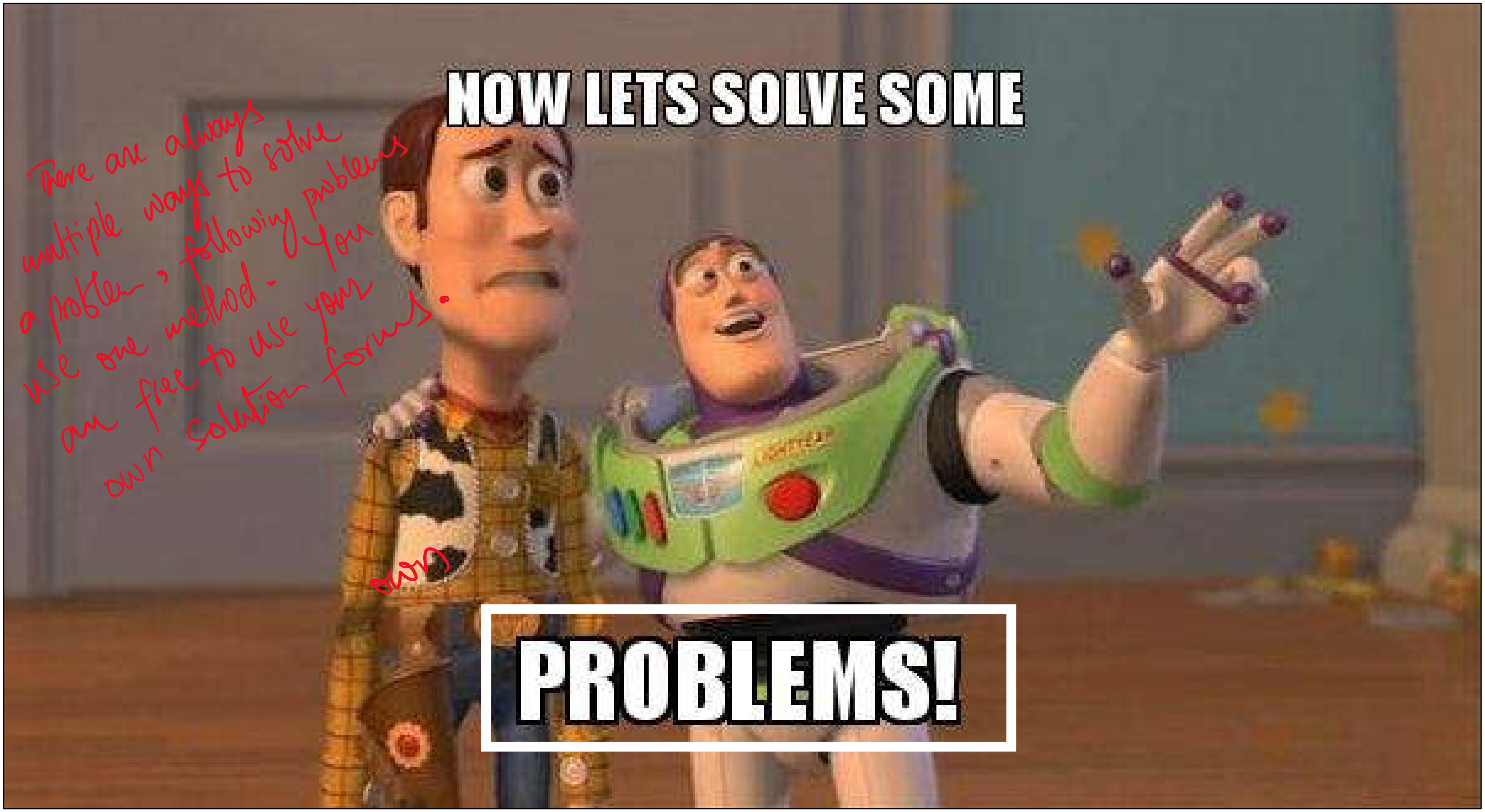
Determinant (because determinants show how area is stretched and rotated)

- Length of $\vec{a} \times \vec{b}$ is the same as *area of parallelogram*.

- $\vec{a} \times \vec{b}$ is perpendicular to the $\vec{a}$ and $\vec{b}$



Lecture 2



There are always multiple ways to solve a problem, following problems use one method. You are free to use your own solution forms.

own

If $\vec{a} = 3\hat{i} - 4\hat{j}$ and $\vec{b} = -2\hat{i} + 3\hat{k}$, what is $\vec{c} = \vec{a} \times \vec{b}$?

$$\vec{c} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -4 & 0 \\ -2 & 0 & 3 \end{vmatrix} = \hat{i}(-12) - \hat{j}(9+0) + \hat{k}(0-8)$$

$$\boxed{\vec{c} = -12\hat{i} - 9\hat{j} - 8\hat{k}}$$

a — 2D

b — 2D

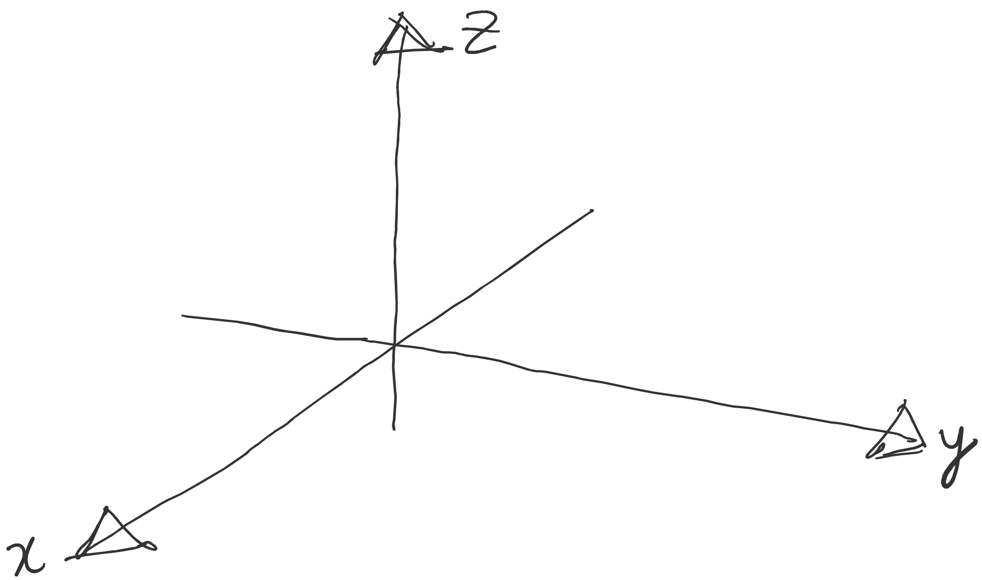
c — 3D

Warning: 3D vectors will require two angles (θ, ϕ) in spherical polar coordinates

Magnitude

$$|\vec{c}| = \sqrt{(-12)^2 + (-9)^2 + (-8)^2}$$
$$= (144 + 81 + 64)^{1/2}$$
$$= \sqrt{289}$$
$$\boxed{|\vec{c}| = 17}$$

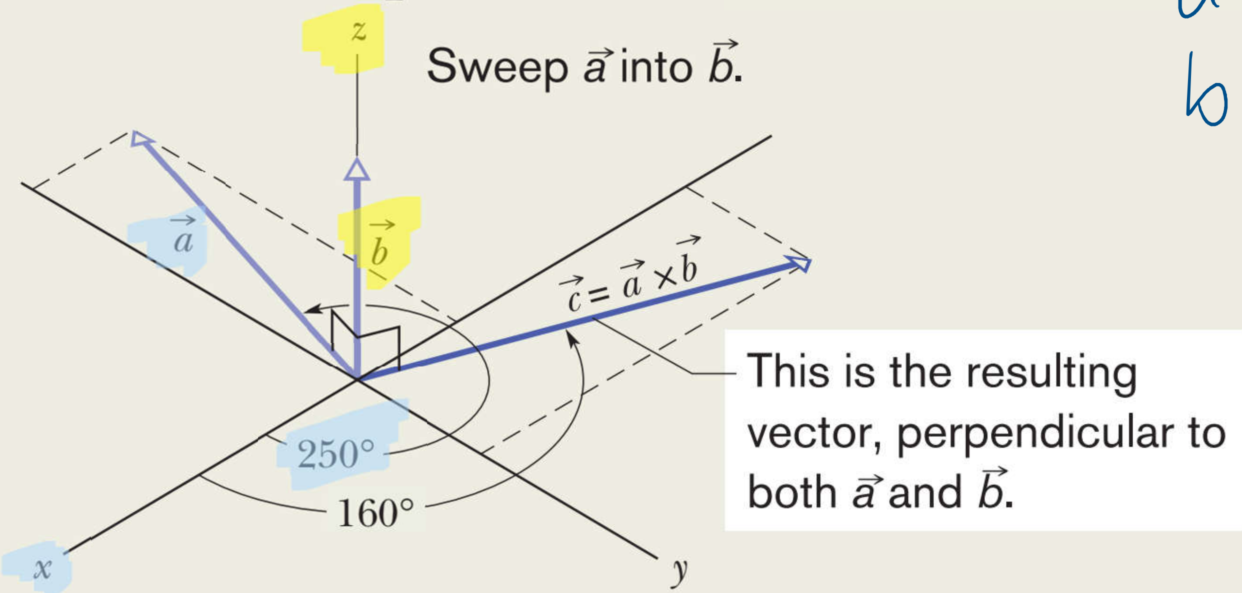
vector $\vec{a}$ lies in the xy plane, has a magnitude of 18 units, and points in a direction 250° from the positive direction of the x axis. Also, vector $\vec{b}$ has a magnitude of 12 units and points in the positive direction of the z axis. What is the vector product $\vec{c} = \vec{a} \times \vec{b}$?



- First, check axis label
- Second, try to draw the vectors $\vec{a}$ and $\vec{b}$ on these axes.

Lecture 2

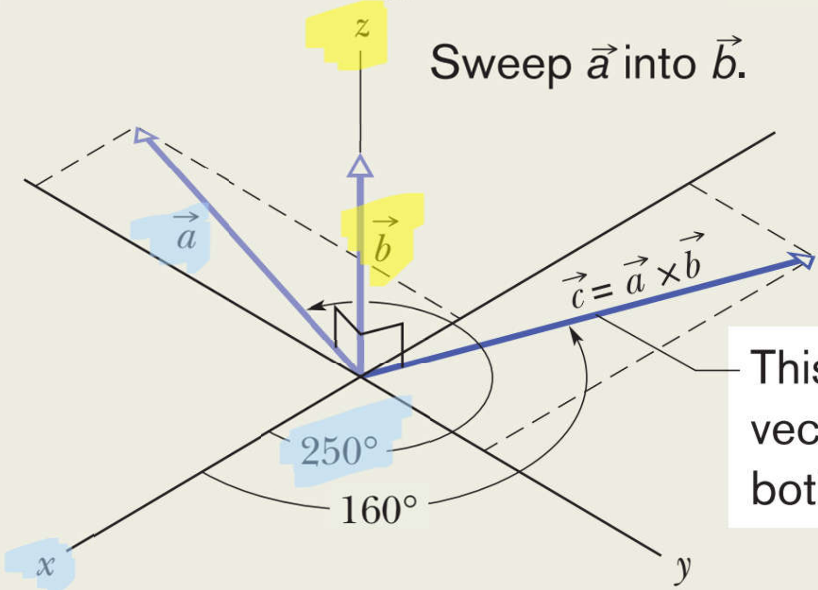
vector $\vec{a}$ lies in the xy plane, has a magnitude of 18 units, and points in a direction 250° from the positive direction of the x axis. Also, vector $\vec{b}$ has a magnitude of 12 units and points in the positive direction of the z axis. What is the vector product $\vec{c} = \vec{a} \times \vec{b}$?



$$\left.\begin{array}{l} a \text{ — } xy \text{ plane} \\ b \text{ — } z \text{ axis} \end{array}\right\} \phi = 90^\circ$$

$$|\vec{c}| = |a||b| \sin(90^\circ)$$
$$= (18)(12)(1)$$
$$\boxed{|\vec{c}| = 216 \text{ units}}$$

vector $\vec{a}$ lies in the xy plane, has a magnitude of 18 units, and points in a direction 250° from the positive direction of the x axis. Also, vector $\vec{b}$ has a magnitude of 12 units and points in the positive direction of the z axis. What is the vector product $\vec{c} = \vec{a} \times \vec{b}$?



This is the resulting vector, perpendicular to both $\vec{a}$ and $\vec{b}$.

a — xy plane
b — z axis

$\phi = 90°$

since $\vec{c}$ must be orthogonal to both $\vec{a}$ and $\vec{b}$

$\theta_c = 250° - 90°$ (from $+x$-axis)

$\boxed{\theta_c = 160°}$ (from $+x$-axis)

vector $\vec{a}$ lies in the xy plane, has a magnitude of 18 units, and points in a direction 250° from the positive direction of the x axis. Also, vector $\vec{b}$ has a magnitude of 12 units and points in the positive direction of the z axis. What is the vector product $\vec{c} = \vec{a} \times \vec{b}$?

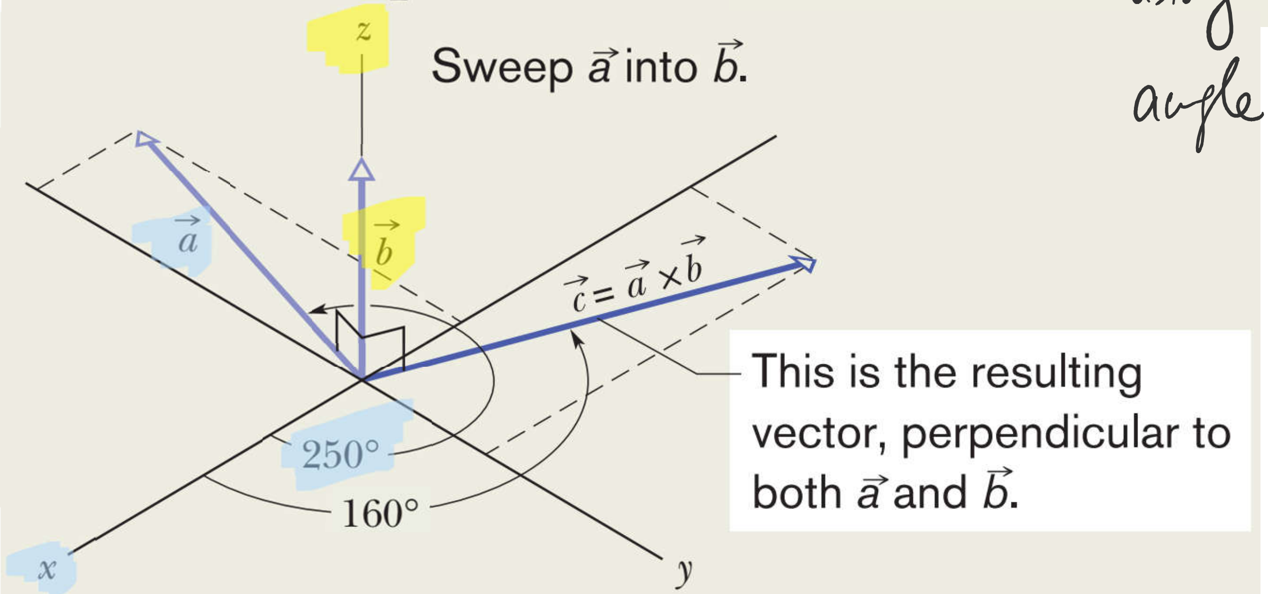
Using the magnitude and the angle of resultant $\vec{c}$ we can find

$$\vec{c} = c_x \hat{i} + c_y \hat{j}$$

$c_x = |c| \cos(160)$
$= 216(-0.939)$
$\cong -203$

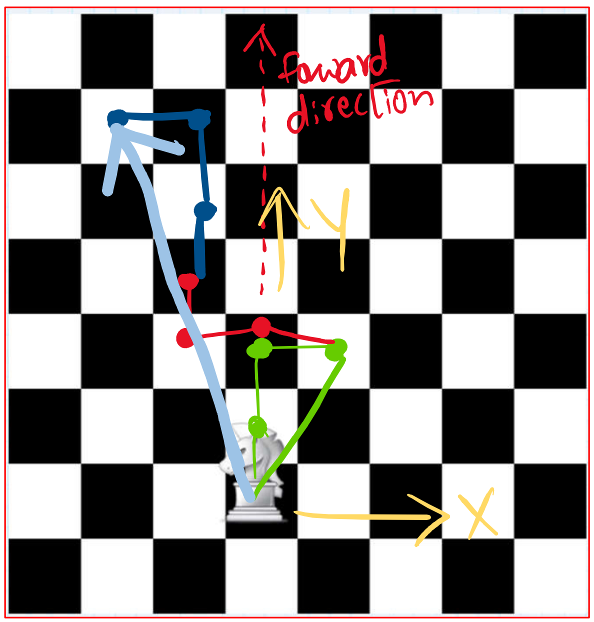
$c_y = |c| \sin(160)$
$= 216(0.342)$
$\cong 74$

$$\boxed{\vec{c} = -203\hat{i} + 74\hat{j}}$$



•19 In a game of lawn chess, where pieces are moved between the centers of squares that are each 1.00 m on edge, a knight is moved in the following way: (1) two squares forward, one square rightward; (2) two squares leftward, one square forward; (3) two squares forward, one square leftward. What are (a) the magnitude and (b) the angle (relative to "forward") of the knight's overall displacement for the series of three moves?

Don't forget to be creative and use your own solution forms



Motion in X

$= +1 -2 -1$

$= -2$

Motion in Y

$= +2 +1 +2$

$= 5$

Magnitude of motion vector

$= \sqrt{(5)^2 + (-2)^2}$

$\boxed{= \sqrt{29}}$ units

Angle from forward direction

$\theta = \tan^{-1}\left(\frac{5}{-2}\right)$

$= -68.19°$

This is the angle from -x-axis

$\theta = 180 - 68.19$

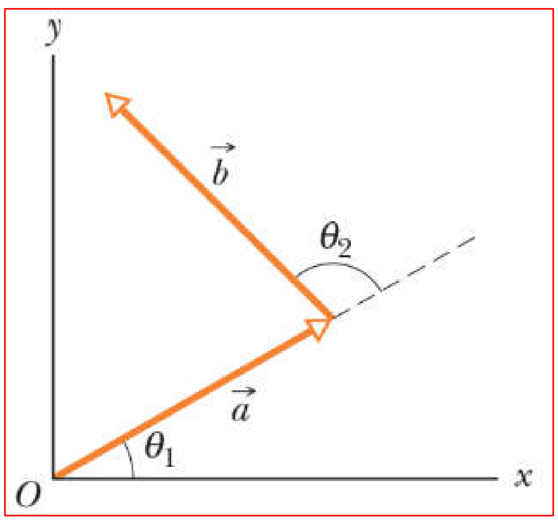
$\boxed{\theta = 111°}$ from +x axis

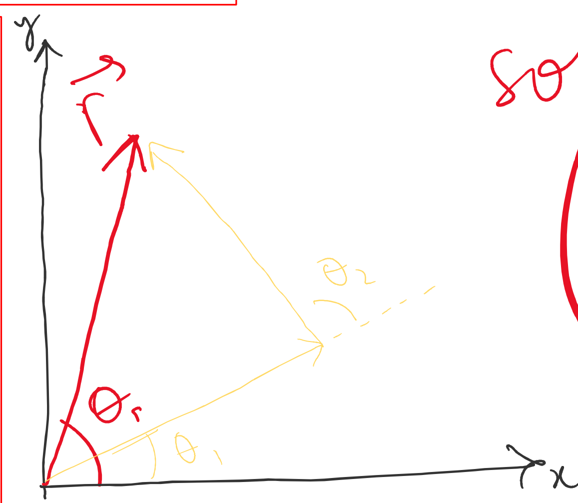
$\theta = 111 - 90$

$\boxed{\theta = 21.8°}$

from the forward direction.

•15 SSM ILW WWW The two vectors $\vec{a}$ and $\vec{b}$ in Fig. 3-28 have equal magnitudes of 10.0 m and the angles are $\theta_1 = 30°$ and $\theta_2 = 105°$. Find the (a) x and (b) y components of their vector sum $\vec{r}$, (c) the magnitude of $\vec{r}$, and (d) the angle $\vec{r}$ makes with the positive direction of the x axis.





Try to solve it yourself now. If you need you can always check like the solution

We need to find the resultant

$\vec{r} = \vec{r}_x + \vec{r}_y$

$\vec{r}_x = \vec{a}_x + \vec{b}_x$

$= |a| \cos\theta_a + |b| \cos\theta_b$

$= 10 \cos(30°) + 10 \cos(105°)$

$= 10(0.866 + (-0.707))$

$= 10(0.159)$

$\boxed{\vec{r}_x = 1.59 \text{ m}}$

$\vec{r}_y = |a| \sin\theta_a + |b| \sin\theta_b$

$= 10 \sin(30°) + 10 \sin(105°)$

$= 10(0.5 + 0.707)$

$\boxed{= 12.07 \text{ m}}$

$|\vec{r}| = \sqrt{|\vec{r}_x|^2 + |\vec{r}_y|^2}$

$= \sqrt{1.59^2 + 12.07^2}$

$|\vec{r}| = 12.17 \text{ m}$

$\theta = \tan^{-1}\left(\frac{r_y}{r_x}\right) = \tan^{-1}\left(\frac{12.07}{1.59}\right)$ $\boxed{\theta = 82.5°}$

Practice problems:

Problems from **Fundamentals of Physics**
-Jearl Walker

Chapter 3 : Vectors

Page#57

2, 5, 13, 20, 26