Date :

Calculus Project

Functions from data.

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Exercise # 1.

→ Which one is regression line?

Line # 2 is regression line in figure 4.

Line 1

points	residual	Sq of residuals	total or Sum of sq of residuals.
1	2−1 = 1	$(1)^2 = 1$	
2	1−2 = −1	$(-1)^2 = 1$	
3	3−3 = 0	0	
4	4−5 = −1	$(-1)^2 = 1$	9.
5	5−6 = −1	$(-1)^2 = 1$	
6	6−4 = 2	$(2)^2 = 4$	
7	6−7 = −1	$(-1)^2 = 1$	

Line 2.

points	residual	sq of residuals	Sum of sq of residuals
1	2−1.6 = 0.4	0.16	
2	1−2.3 = −1.3	1.69	
3	3−3.1 = −0.1	0.01	
4	6−3.8 = 2.2	4.84	7.15
5	4−4.6 = −0.6	0.36	
6	5−5.3 = −0.3	0.09	
7	6−6 = 0	0	

As, sum of sq of residual in line # 2 is minimum or less than line # 1, So, line # 2 is regression line.

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Q2. Find regression line through calculator.

Solution :-

x	1	1.5	2	2.5	3	3.5	4
y	1	2.5	6	9	10.5	14.5	15

By putting values of x and y in calculator we find, regression line.

$$y = mx + b.$$

$$\boxed{y = 5.035x - 4.23}$$

2nd method is :

(i) Mean of x and y values which is

$$\bar{x} = 2.5 \quad , \quad \bar{y} = 8.36$$

(ii) Standard deviations of x and y.

$$S_x = \sqrt{\frac{\sum(x_i - \bar{x})}{n-1}} \qquad \because n = \text{no of values.}$$

$$S_y = \sqrt{\frac{\sum(y_i - \bar{y})}{n-1}}$$

$$\boxed{S_x = 1.08} \quad , \quad \boxed{S_y = 5.49}$$

(iii) R : corelation coefficient

$$r = \frac{1}{n-1} \sum\left(\frac{x_i - \bar{x}}{S_x}\right)\left(\frac{y_i - \bar{y}}{S_y}\right)$$

$$\boxed{r = 0.99}$$

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(iv) Slope $m = r\left(\frac{S_y}{S_x}\right)$, $m = 5.08$

Now, $\bar{y} = m\bar{x} + b$

$$8.36 = 5.08 \times 2.5 + b$$

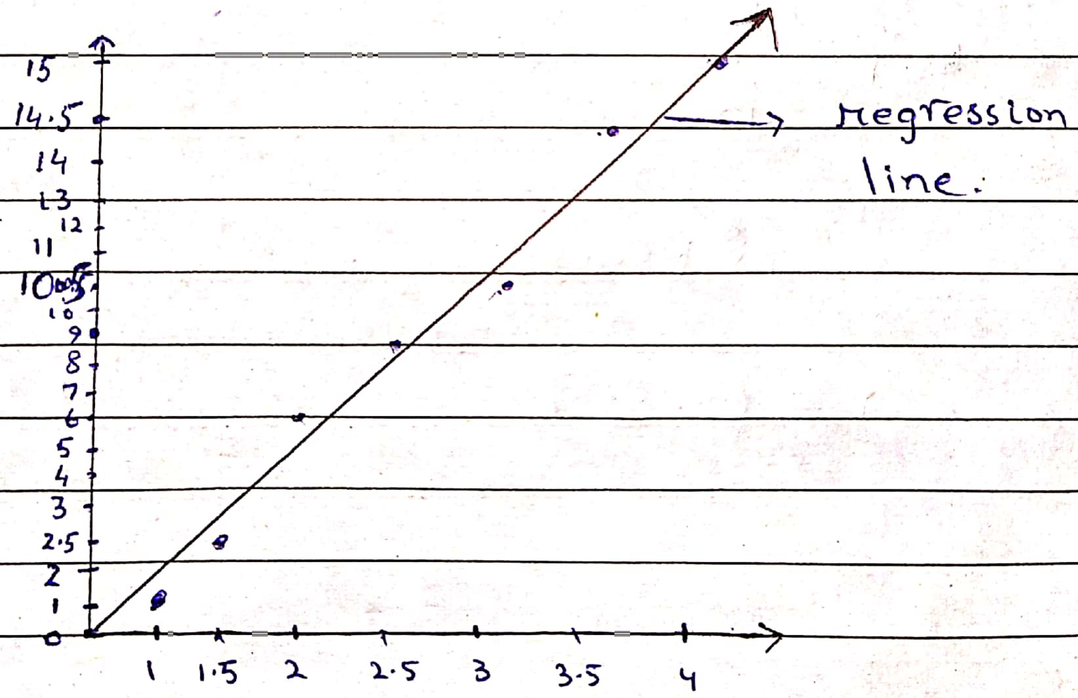
$$b = -4.23$$

putting values, in we get,

$$y = mx + b$$

$$\boxed{y = 5.08x - 4.23}$$

(b) Scatter plot of data



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Exercise 3

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casio fx 991Ez classwiz doesnot produce a co-relation coffcient when it calculates the regression line.

The regression line needs to be manually calculated from calculator by following methods

Mode + 6 (stats) + 2 (A + Bx)

Value of x ~~any~~ and y in table

Now the answer varies from table to table let suppose we took the values of Exercise 2 due to lack of table in Q3

x	y
1.0	1.0
1.5	2.5
2.0	6.0
2.5	9.0
3.0	10.5
3.5	14.5
4.0	15.0

The answer will be 0.990

Exercise #4

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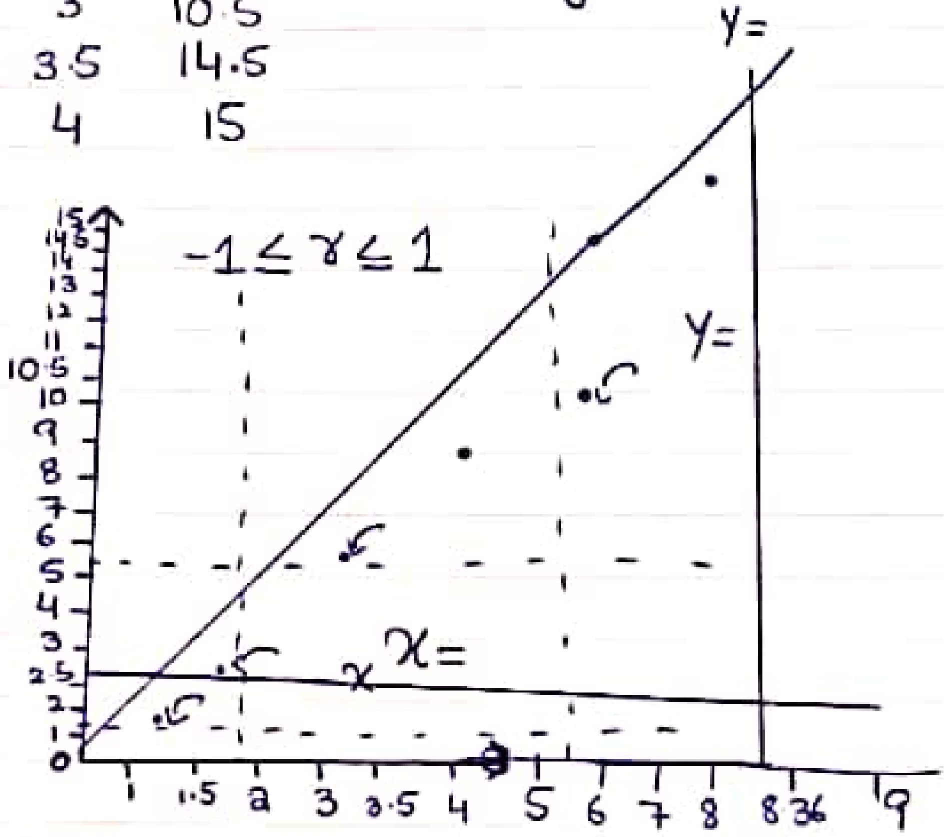
x	y
1	1
1.5	2.5
2	6
2.5	9
3	10.5
3.5	14.5
4	15

$\bar{x} = 2.5$

$S_x = 1.08$

$\bar{y} = 8.36$

$S_y = 5.49$



Solution

For x

$$\frac{1+1.5+2+2.5+3+3.5+4}{7} = \frac{17.5}{7}$$

$$\frac{\sqrt{(1-2.5)^2+(1.5-2.5)^2+(2-2.5)^2+\cancel{(2.5-2.5)^2}+(3-2.5)^2+(3.5-2.5)^2+(4-2.5)^2}}{6}$$

$$\frac{2.25+1+0.25+0.25+1+2.25}{6}$$

$$= 1.1$$

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1.1 is approx to value of 1.08

$\boxed{Sx = 1.08}$

For y

$$\frac{1+2.5+6+9+10.5+14.5+15}{7} = \frac{58.5}{7}$$

$$\sqrt{\frac{(1-8.36)^2+(2.5-8.36)^2+(6-8.36)^2+(9-8.36)^2+(10.5-8.36)^2+(14.5-8.36)^2+(15-8.36)^2}{6}}$$

$$\frac{54.16+34.33+5.56+0.40+4.57+37.6+44.08}{6}$$

$(1-8.36)^2+(2.5-8.36)^2$

~~54~~ $54.16+34.33+5.56+0.40+4.57+$
$37.6+44.08$

$$\frac{180.7}{6} = \sqrt{30.11} = 5.48$$

5.48 is also approx to 5.49

$\boxed{Sy = 5.49}$

iNotes

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Correlant coeffient

$$r = \frac{1}{n-1} \Sigma \underbrace{\left(\frac{x_1 - \bar{x}}{Sx}\right)}_{Zx_1} \underbrace{\left(\frac{y_1 - \bar{y}}{Sy}\right)}_{Zy_1}$$

$$\frac{1}{6}\left(\frac{1-2.5}{1.08}\right)\left(\frac{1-8.36}{5.49}\right)$$

$+$

$$\frac{1}{6}\left(\frac{1.5-2.5}{1.08}\right)\left(\frac{2.5-8.36}{5.49}\right)$$

$+$

$$\frac{1}{6}\left(\frac{2-2.5}{1.08}\right)\left(\frac{6-8.36}{5.49}\right)$$

$+$

$$\cancel{\frac{1}{6}\left(\frac{2.5-2.5}{1.08}\right)\left(\frac{9-8.36}{5.49}\right)}$$

$+$

$$\frac{1}{6}\left(\frac{3-2.5}{1.08}\right)\left(\frac{10.5-8.36}{5.49}\right)$$

$+$

$$\frac{1}{6}\left(\frac{3.5-2.5}{1.08}\right)\left(\frac{14.5-8.36}{5.49}\right)$$

$+$

$$\frac{1}{6}\left(\frac{4-2.5}{1.08}\right)\left(\frac{15-8.36}{5.49}\right)$$

$$R = \frac{1}{6}\left(\frac{11.04}{1.08 \times 5.49}\right.$$

$$+ \frac{5.84}{1.08 \times 5.49}$$

$$+ \frac{1.18}{1.08 \times 5.49}$$

$$+ \frac{1.07}{1.08 \times 5.49}$$

$$+ \frac{6.14}{1.08 \times 5.49}$$

$$+ \left.\frac{9.96}{1.08 \times 5.49}\right)$$

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$$R = \frac{1}{6}\left(\frac{35.23}{1.08 \times 5.49}\right)$$

$$\boxed{R = 0.990}$$

iNotes

Exercise #5

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(a) To find co-relation co-efficient, using the formula.

$$r = \frac{n \cdot \Sigma XY - \Sigma X \cdot \Sigma Y}{\sqrt{[n\Sigma X^2 - (\Sigma X)^2] \cdot [n\Sigma Y^2 - (\Sigma Y)^2}} \quad \text{--- (A)}$$

The mean of x and y values are,

$\Sigma X = 153$

$\Sigma Y = 4083.331$

Sum of squared x values, y values,

$\Sigma X^2 = 1785$ $\quad\quad$ $\Sigma Y^2 = 926552.7289$

Sum of product of paired values.

$\Sigma X \cdot Y = 34695.733$

N = number of pair of values.

putting values in (A) we get,

$$r = \frac{18(34695.733) - (153)(4083.331)}{\sqrt{[18(1785) - 153^2] \cdot [18 \cdot 926552.7289 - 4083.33^2]}}$$

$r \simeq 0.367$

regression line: $y = -0.03x + 227.07$

(B) As r is close to +1, the more tightly the data points hug regression line, and more appropriate the regression line is a model.

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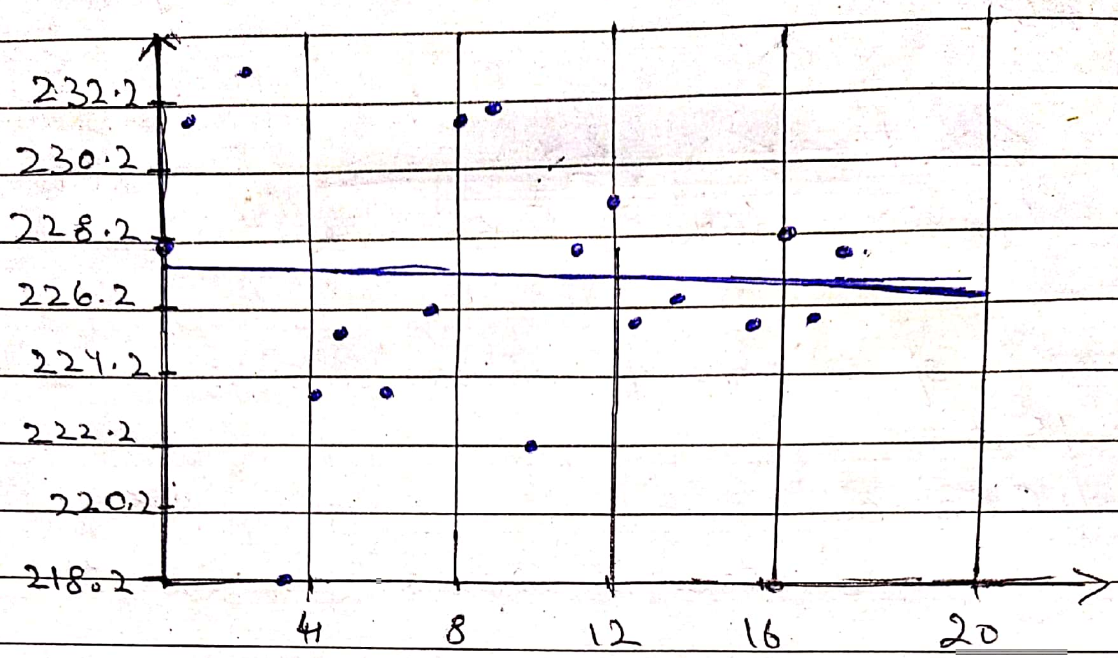

– Regression line ($y = -0.03X + 227.07$)

(c) For year 2012, x is 18.

$Y = -0.03x + 227.07$
put x in equation, we get

$Y = 226.53 \, mh^{-1}$

(d) (i) Time taken is Zero.
(ii) Co-relation exists
(iii) ~~As~~ ~~v~~alue of r is close to +1
thus, more ~~closely~~ data points lie on regression line.

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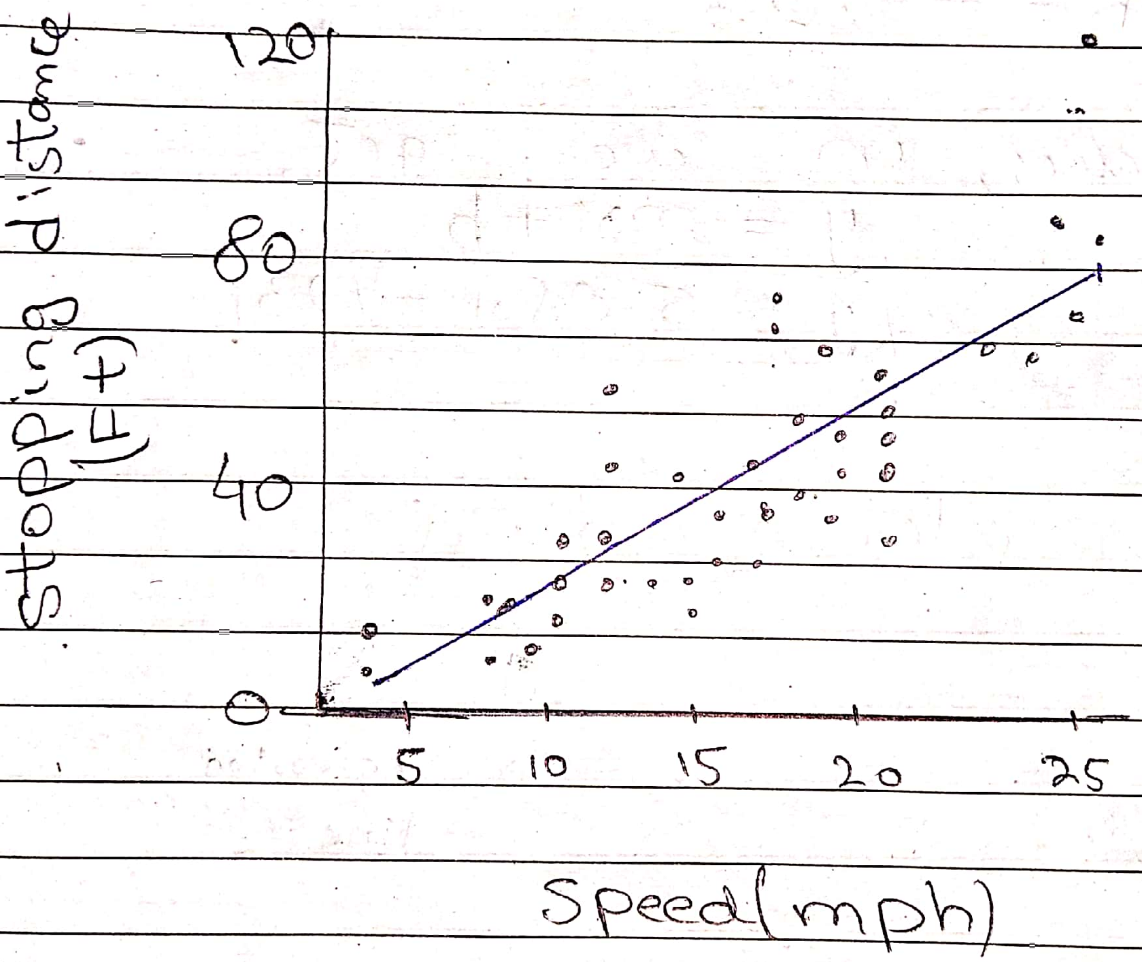


r = 0.807

Exercise 6 Page 1

(A) :: Exponential Models ::

$$y = ae^{bx}$$

$$Y = \ln y$$

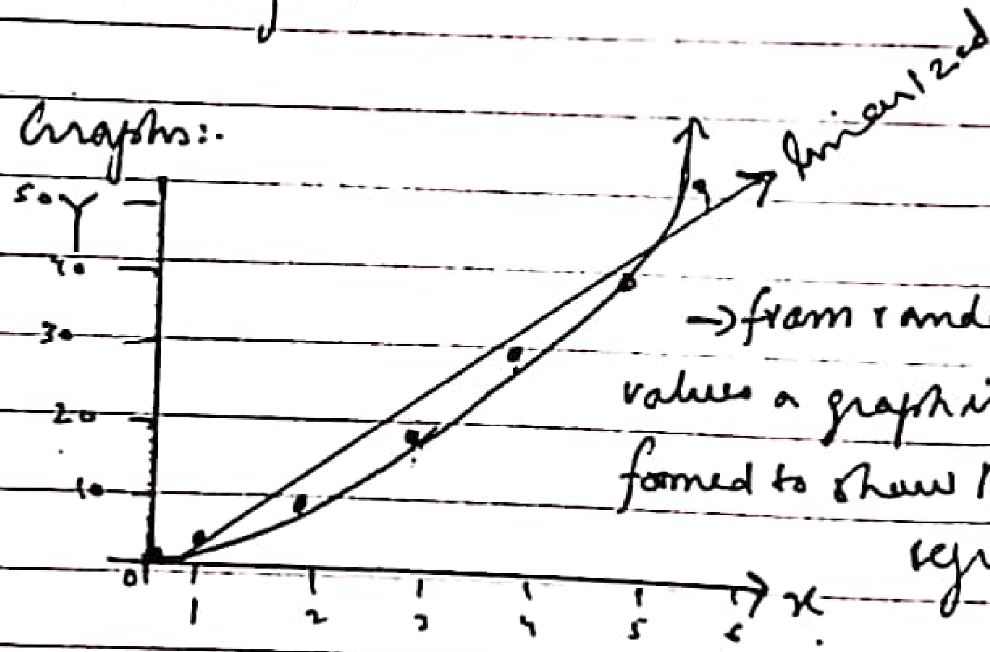
$$Y = \ln(ae^{bx})$$

$$Y = \ln(a) + \ln(e^{bx})$$

$$Y = \ln(a) + bx \ln e$$

$$Y = \underbrace{\ln(a)}_{\text{Intercept}} + \underbrace{b}_{\text{slope}}x \quad \because \ln e = 1$$

Generic Graphs ::



→ from random values a graph is formed to show the regression

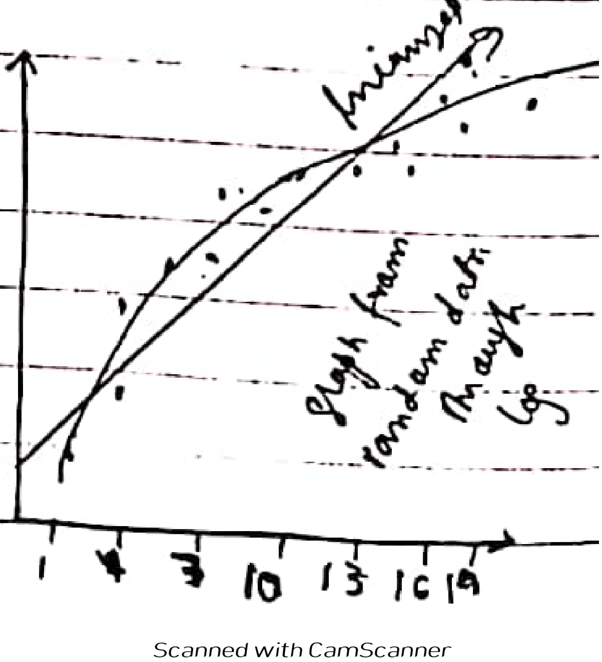
(B) :: Logarithmic Models ::

$$X = \ln x$$

$$Y = a + b \ln x$$

$$Y = a + bX \ln e$$

$$Y = \underbrace{a}_{\text{Intercept}} + \underbrace{b}_{\text{slope}} X$$



Project Exercise 6.2

Page 2

(C):. Power function Model:.

We have to convert the equation always to the linearized model equation by taking ln on both sides.

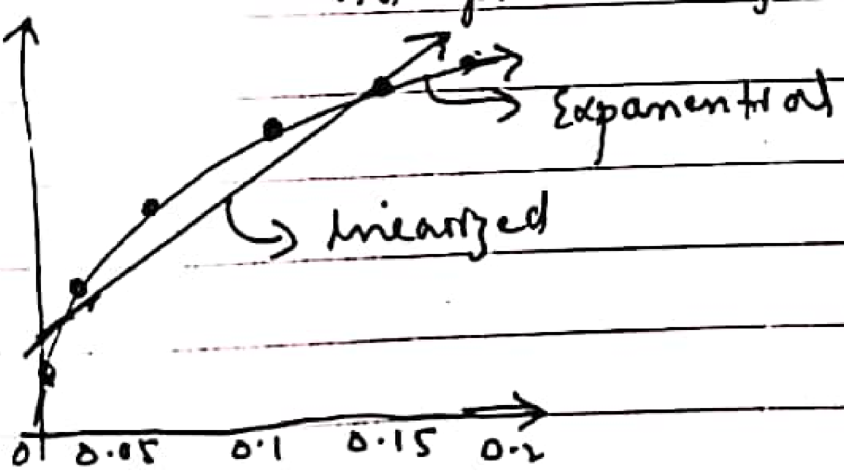
$Y = \ln y \qquad X = \ln x$

$y = a x^b$

$\ln y = \ln a + \ln x^b$

$Y = \ln a + b X$ as $X = \ln x$

intercept ($\ln a$), slope (b)



(D):. If we have taken log rather than ln then
∴ take $Y = \log y$

$\Rightarrow Y = \log y$

$\Rightarrow Y = \log(a e^{(bx)})$

$\Rightarrow Y = \log a + \log^e(bx)$ → It is same as the natural log (ln).

Exercise 7

Page 1

Ⓐ:

$y = ae^{bx}$ Taking the assumed data from the question.

taking log on both side natural (ln)

$\ln y = \ln ae^{bx}$

$\ln y = \ln a + \ln e^{bx}$

$\ln y = \ln a + bx \ln e$

$\ln y = \ln a + bx$ $\because \ln e = 1$

Now take $x = w$ and $z = \ln y$

$$\boxed{z = \ln a + bw}$$

w	0	1	2	3	4	5	6	7
z	1.36	1.66	1.974	2.262	2.284	2.833	3.135	3.434

Another table for value of $z \to \ln y$.

for finding the required values, we must remember the formula

$$\boxed{a_1 = \frac{N\sum w_i z_i - \sum w_i \cdot \sum z_i}{N\sum w_i^2 - (\sum w_i)^2}, \quad a_0 = z_{avg} - a_1 \overline{w}, \quad z = a_0 + a_1 w}$$

Exercise 7

[Page 2]

finding the values to substitute in the equations to get the required values.

$[N = 8]$

$\sum w_i = 0+1+2+3+4+5+6+7 = 28$

$\sum z_i = 1.86 + 1.66 + 1.974 + 2.262 + 2.284$
$+ 2.833 + 3.135 + 3.434 = 18.942$

$\sum w_i^2 = 0^2 + 1^2 + 2^2 + 3^2 + \dots + 7^2 = 140$

$\sum w_i . z_i = 0 + 1.66 + 3.948 + 6.786 + 11.36$
$+ 14.16 + 18.81 + 24.038 = 80.762$

Now formulas

$$a_1 = \frac{N \sum w_i z_i - \sum w_i \sum z_i}{N \sum w_i^2 - (\sum w_i)^2}$$

$$a_1 = \frac{8 \times 80.762 - 28 \times 18.942}{8 \times 140 - 784} = \frac{646.096 - 530.376}{336}$$

$\boxed{a_1 = 0.3444}$

$\bar{Z} = \frac{Z}{8} = 2.3677$

$\bar{W} = \frac{W}{8} = 3.5$

$a_0 = \bar{Z} - a_1 \bar{w}$

$a_0 = 2.3677 - (0.3444 \times 3.5)$

$\boxed{a_0 = 1.1623}$

Exercise 7

Page 3

⇒ Now equating two equations:

$$Z = a_0 + a_1 w$$

$$Z = 1.1623 + 0.344 w$$

$$Z = \ln(a) + bx$$

↓ from here we get

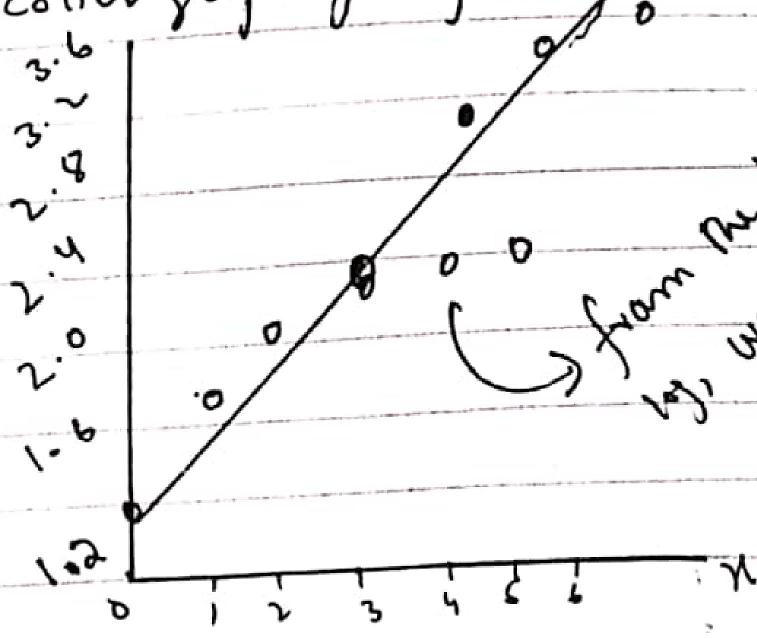
$\ln a = 1.1623 \rightarrow$ which $\rightarrow a = \exp(1.1622)$

$$\boxed{a = 3.2}$$

for b, we can get $\boxed{b = 0.344}$

Answers $\rightarrow$ $\boxed{\begin{matrix} a = 3.2 \\ b = 0.344 \end{matrix}}$

(B) :: Scatter graph plotting.



from the technique of ln, natural log, we got the non, linear model plotted linearly into the graph.

The End

Exercise : 8

(a) power functional model

(b) solution:

Sum of d = 5.6835
Sum of T = 8.5249

mean of d = 0.6315
mean of Y = T = 0.9472

Sum of Squares (SS) = 4.4879
Sum of Products (SP) = 6.7303

Regression equation = $Y = bd + a$

$b = \frac{SP}{SS} = \frac{6.74}{4.49} = 1.499$

$a = M(Y) - bM(d) = 0.95 - (1.5 \times 0.63) = 0.00017$

so, $Y = 1.499d + 0.00017$

(C) gradient(m) = (2.4-0) / (1.6-0) = 1.5
Using Pluto (1.6, 2.4) and (0,0)

Equation ⇒ $Y - y1 = m(x - x1)$ ⇒ $Y - 0 = 3/2 \times (x - 0)$;
$y = \log T$, $x = \log d$

$\log T = 3/2 \times \log d$

$\log T = \log d$^(3/2)
T^2 $= (d^{3/2})^2$ ⇒ $T^2 = d^3$

(d) Kepler's third law of planetary motion

The square of a planet's orbital period is proportional to the cube of length of the semi-major axis of its orbit. The law implies that the period for a planet to orbit the Sun increases rapidly with the radius of its orbit.

$$\frac{(a^3)}{(T^2)} = \frac{(G(M+m))}{4\pi^2} = \frac{(GM)}{4\pi^2} = 7.496 \times 10^{-6} \left(\frac{AU^3}{days^2}\right)$$ is Constant.

Exercise 9:

(a) $T - T_a = (T_0 - T_a)e^{(-kt)}$

When $t = 4$, $T = 75$ and $T_a = 27$

So:

$$75 - 27 = (T_0 - 27)e^{(-4k)}$$
$$48 = (T_0 - 27)e^{(-4k)} - 1 \rightarrow (1)$$

When $t = 1$, $t = 82.2$, $T_a = 27$

So;

$$82.2 - 27 = (T_0 - 27)e^{-k} - 2$$
$$55.2 = (T_0 - 27)e^{-k} \rightarrow (2)$$

Divide eq (1) by (2)

$$\frac{48}{55.2} = \frac{(T_0 - 27)e^{-4k}}{(T_0 - 27)e^{-k}}$$

$$0.87 = e^{-3k}$$
$$\ln 0.87 = -3k$$
$$\therefore k = \left(-\frac{1}{3}\right)(\ln 0.87)$$

giving

$$T = 27 + (T_0 - 27)e^{(\frac{1}{3})(\ln 0.87)t)}$$

(b) When $t = 4$, $T = 75$

$$75 = 27 + (T_0 - 27)e^{((1/3)(\ln 0.87)(4))}$$

$$48 = (T_0 - 27)(0.831)$$

$$T_0 = 27 + 57.8$$

$$T_0 = 84.8°C$$

This is the initial temperature of Coffee

(c) $T = 32$ $t = ?$

$$32 - 27 = (84.8 - 27) \times e^{((1/3)(\ln 0.87)t)}$$

$$0.0865 = e^{((1/3)(\ln 0.87)t)}$$

$$\frac{(\ln(0.0865) \times 3)}{(\ln 0.87)} = t$$

$$t = 52.726 \text{ min}$$

The ~~first~~ time at which the temperature would be 5 degree from the Room temperature at that point T is 32 as given above.