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D.E Assignment#1.

1. $\frac{dy}{dx} = \frac{x}{y}$

Sol:- $y\,dy = x\,dx$

$\int y\,dy = \int x\,dx$

$\frac{y^2}{2} = \frac{x^2}{2} + \ln c$

Ans $y^2 = x^2 + c$.

2. $x\frac{dy}{dx} + y = x^2y^2$

$\div$ by 'x' on b/s.

$\frac{dy}{dx} + \frac{y}{x} = xy^2$

$// y' + P(x) = F(x)y^n //$.

$\div$ by y^2 we get,

$y^{-2}.y' + \frac{1}{xy} = x$. — (1)

$\rightarrow u = y^{1-n}$, $\frac{du}{dx} = \frac{d}{dx}(y^{-1})$.

$\frac{du}{dx} = -y^{-2}\,dy/dx$.

$\because dy/dx = -y^2\frac{du}{dx}$.

put values in (1)

$-\frac{y^2}{y^2}\frac{du}{dx} + \frac{1}{xy} = x \rightarrow -\frac{du}{dx} + \frac{1}{xy} = x$

$\rightarrow \frac{du}{dx} - \frac{1}{xy} = -x$

(2) $I.F = e^{\int P(x)dx}$

$I.F = e^{\int -\frac{1}{x}dx}$

~~Xing by~~ Soln of linear, $\because I.F = x^{-1}$

$u \times x^{-1} = \int -x \times x^{-1}\,dx + C$.

$\frac{u}{x} = -\int dx + C$. $\because u = \frac{1}{y}$

$\frac{u}{x} = -x + C \rightarrow \frac{1}{y^2} = -x^2 + cx$

$y^2 = \frac{1}{-x^2 + cx}$ or $\boxed{y = \frac{1}{-x^2 + C}}$ Ans.

(3) $(x^2+y^2)\,dx + xy\,dy = 0$

$$\frac{dy}{dx} = \frac{-(x^2+y^2)}{xy} \qquad \because y = vx$$

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

$$v + x\frac{dv}{dx} = \frac{-x^2-y^2}{xy}$$

$$x\,dv/dx = \frac{-x^2-y^2}{xy} - v$$

$$x\,dv/dx = \frac{-x^2-v^2x^2}{x^2v} - v$$

$$x\,dv/dx = \frac{-x^2-v^2x^2-x^2v^2}{x^2v}$$

$$x\,dv/dx = -x^2-2x^2v^2/x^2v$$

$$\text{"} = \frac{-1}{v} - 2v \rightarrow x\,dv/dx = -1-2v^2/v$$

$$x\,dv/dx = -(1+2v^2)/v$$

$$\left(v/1+2v^2\right)dv = -x/dx - dx/x$$

$\because$ integrate

$\because$ $t = 1+2v^2$

$$\int \frac{v}{1+2v^2}\,dv = \int -\frac{dx}{x}$$

$$\frac{1}{4}\int \frac{dt}{t} = -\ln|x| \qquad dt = 0 + 4v\,dv$$

$$\frac{1}{4}\ln|t| = -\ln|x| + \ln C$$

$$\frac{\ln}{4}|1+2v^2| = -\ln|x| + C$$

$$1+2v^2 = Cx^{-4}$$

$$\rightarrow 1 + 2y^2/x^2 = \frac{C}{x^4} \Rightarrow 2y^2 = \frac{C}{x^2} - x^2$$

$$\boxed{y^2 = \frac{Cx^{-2}}{2} - \frac{x^2}{2}}$$ Ans

4. $(x-y^2)\,dx + 2xy\,dy = 0$ non-exact eq form

$\frac{\delta M}{\delta y} = -2y \neq \frac{\delta N}{\delta x} = 2y$

case #1 $\frac{M_y - N_x}{N} = \frac{-2y-2y}{2xy} = \frac{-4y}{2xy}$

$P(x) = -2/x$

(ii) $I.F = e^{\int \frac{-2}{x} dx}$

$\boxed{I.F = x^{-2}}$

Xing by eq we get,

$\left(\frac{1}{x} - \frac{y^2}{x^2}\right)dx + \frac{2y}{x}\,dy = 0$

$\delta M/\delta y = -2y/x^2 = \delta N/\delta x$, now equal

So, exact eq

(iii) $F(x,y) = \delta F/\delta x = M(x,y)$

$F = \int M(x,y)\,dx + g(y)$

$F = \int\left(\frac{1}{x} - y^2/x^2\right)dx + g(y)$

$= \ln|x| + \frac{2y^3}{x^2} + g(y)$ — (A)

Now, P.d w.r.t y. (iv)

$\delta F/\delta y = 0 + 2y/x + g'(y)$

$\because \delta F/\delta y = N(x,y)$

$\cancel{\frac{2y}{x}} = \cancel{2y/x} + g'(y)$

$g'(y) = 0$

(v) int w.r.t (y) $\int g'(y)\,dy = \int 0\,dy$

$\boxed{g(y) = c}$. put in (A)

$F(x,y)$: $\ln|x| + \frac{y^2}{x^2} = c.$

Ans

5. $e^{y}\left(\frac{dy}{dx}-1\right)=e^{x}$

$\frac{dy}{dx}-1=e^{x-y}$

$x-y=t$.

$1-\frac{dt}{dx}=\frac{dt}{dx}$

$-\frac{dt}{dx}=e^{t}$

$-dx=\frac{dt}{e^{t}} \rightarrow -dx=e^{-t}dt$.

$-\int dx=\int e^{-t}dt \Rightarrow -x=-e^{-t}+C$.

$e^{-t}=x+C \Rightarrow 1=e^{t}(x+c)$.

$1=e^{x-y}(x+c) \Rightarrow \underline{e^{x-y}(x+c)-1=0}$

Ans.

6. $\sin y \dfrac{dy}{dx} = \cos x(2\cos y - \sin x)$

$(2\cos x\cos y - \cancel{2}\cos x\sin x)dx - \sin y\, dy = 0$

$\Rightarrow \dfrac{\partial M}{\partial y} = -2\cos x\sin y \neq 0 = \dfrac{\partial N}{\partial x}$

So, case #1 $\dfrac{M_y - N_x}{N} = \dfrac{-2\cos x\sin y}{-\sin y}$

$= 2\cos x$, $I.F = e^{\int 2\cos x\,dx} = e^{2\sin x}$

$\Rightarrow (e^{2\sin x}\cos x\cos y - e^{2\sin x}\cos x\sin x)dx$
$- e^{2\sin x}\sin y\, dy = 0$

$\dfrac{\partial M}{\partial y} = -2e^{2\sin x}\cos x\sin y = \dfrac{\partial N}{\partial x}$

$F(x,y) = \dfrac{\partial F}{\partial x} = M(x,y)$, $\dfrac{\partial F}{\partial y} = N(x,y)$.

$F = \int M(x,y)dx + g(y)$ X
$F = \int(2e^{2\sin x}\cos x\cos y)\,\cancel{dx} + g(y)$
X $\quad - e^{2\sin x}\cos x\sin x)dx$

$F = \int N(x,y)\,dy + h(x) \rightarrow$ (A)
$F = \int -[e^{2\sin x}\sin y]dx + h(x)$.

$\Rightarrow F = +e^{2\sin x}\cos y + h(x)$. now P.d w.r.t x

$\dfrac{\partial F}{\partial x} = 2e^{2\sin x}\cos x\cos y + h'(x) \because \dfrac{\partial F}{\partial x} = M(x,y)$

$2e^{2\sin x}\cancel{\cos x\cos y} - \cancel{2}e^{2\sin x}\cos x\sin x = \cancel{2e^{2\sin x}\cos x\cos y} + h'(x)$

int w.r.t 'x'

$\int h'(x)dx = \int(e^{2\sin x}\cos x\sin x)dx$

$h(x) = e^{2\sin x}(2\sin x - 1) + C.$

put in (A)

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$$y = -\cos^{-1}\left(-c - \frac{2e^{2\sin x}\sin x + e^{2\sin y}}{4e^{2\sin x}}\right)$$

Ans.

7. $x(3x+2y^2)dx + 2y(1+x^2)dy = 0$

$\frac{\partial M}{\partial y} = 4xy = \frac{\partial N}{\partial x}$ exact eq.

$\frac{\partial F}{\partial x} = F(x,y) = M(x,y)$.

$F = \int (3x^2+2xy^2)dx + g(y)$. (A)

$F = x^3 + x^2y^2 + g(y)$. (2) p.d w.r.t y.

$\frac{\partial F}{\partial y} = N(x,y) = \cancel{2yx^2} \mp 2y \mp \cancel{2yx^2} + g'(y))$

So, $g'(y) = +2y \Rightarrow$ int w.r.t y.

$\int g'(y)dy = +y^2 + c = g(y)$.

$F = x^3 + x^2y^2 + y^2 + c$ put in (A)

or $F(x,y) : x^3 + x^2y^2 + y^2 = c$.

$x^3 + y^2(1+x^2) = c \rightarrow \boxed{\frac{c - x^3}{(1+x^2)} = y^2}$

Ans

8. $e^{-y} \sec^2 y \, dy = dx + x\,dy$

$e^{-y} \sec^2 y = \frac{dx}{dy} + x$

$\sec^2 y = e^y \, dx/dy + x e^y$

$\int \sec^2 y = \int d(xe^y) + C$

$\tan y = xe^y + C$ Ans

9. $(x^2 + y^2)dx + (x^2 - xy)\,dy = 0$

degree is same, homo eq,

$y = vx$, $dy = v\,dx + x\,dv$

$(x^2 + v^2x^2)\,dx + (x^2 - vx^2)\,dy = 0$

$(x^2 + v^2x^2)dx + (x^2 - vx^2)(v\,dx + x\,dv) = 0$

$\cancel{x^2 + v^2x^2}$

$x^2 dx + \cancel{v^2x^2\,dx} + x^2v\,dx + x^3 dx - \cancel{v^2x^2\,dx}$

$- x^3 v\,dv = 0$

$x^2 dx(1+v) + x^3 dv(1-v) = 0$

$\int \frac{1-v}{1+v} dv = -\int \frac{dx}{x}$

$\cancel{v - 2\ln(v+1)}$

$v - 2\ln(v+1) = -\ln(x) + C$

back substitute, we get,

$\frac{y}{x} - 2\ln\left(\frac{y}{x} + 1\right) = -\ln(x) + C$

Ans

$\because \frac{1-v+1-1}{v+1}$

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10. $y - x\dfrac{dy}{dx} = a\left(y^2 + \dfrac{dy}{dx}\right).$

Sol:- $y - x\dfrac{dy}{dx} = ay^2 + a\dfrac{dy}{dx}$

$y - ay^2 = x\dfrac{dy}{dx} + a\dfrac{dy}{dx}.$

$\cancel{y(1-a)}\; y - ay^2 = \dfrac{dy}{dx}(x+a)$

$\int \dfrac{dx}{x+a} = \int \dfrac{dy}{y-ay^2}$

$\ln|x+a| = \int \frac{1}{y}dy - \int \frac{a}{1-ay}dy.$

$C + \ln|x+a| = \ln y - \ln|1-ay|$

$\ln|x+a| = \ln\left|\dfrac{y}{1-ay}\right| + C.$

$$\boxed{C = \frac{(x+a)(1-ay)}{y}}$$

Ans

11. $(x+1)y' + 1 = 2e^{-y}$

Sol:-

$(x+1)\dfrac{dy}{dx} = 2e^{-y} - 1$

$\dfrac{dx}{x+1} = \dfrac{dy}{2e^{-y}-1}$

integrating,

$\int \dfrac{dx}{x+1} = \int \dfrac{e^{y}}{2-e^{y}}dy \Rightarrow \ln|x+1| + c = \int \dfrac{e^{y}}{2-e^{y}}dy.$

$\because z = 2\cancel{e}e^{y} \rightarrow dz = -e^{y}dy \rightarrow -dz = e^{y}dy.$

Substitute, $-\int \dfrac{dz}{z} = \ln|x+1| + C.$

$-\ln|z| = \ln|x+1| + C \Rightarrow -\ln|2-e^{y}| = \ln|x+1| + C$

$\rightarrow \ln|2-e^{y}| + \ln|x+1| = C.$

$\rightarrow C = \dfrac{(2-e^{y})(x+1)}{\cancel{x+1}} \Rightarrow C = \left(\dfrac{2}{e^{y}} - 1\right)(x+1)$

Ans

12. $x^2\frac{dy}{dx} + y(x+y) = 0$

$\frac{dy}{dx} = \frac{-(yx+y^2)}{x^2}$

homogeneous eq. So

$y = vx$, $\frac{dy}{dx} = v + x\frac{dv}{dx}$

$v + x\frac{dv}{dx} = \frac{-(vx^2+v^2x^2)}{x^2}$

$// = -(v+v^2)$

$x\frac{dv}{dx} = -2v - v^2$

$\int \frac{dv}{+(2v+v^2)} + \int \frac{dx}{x} = 0$

$\int \frac{1}{v}dv + \int \frac{1}{(v+2)}dv = -\ln x + C$

OR.

$\int \frac{dv}{v(v+2)+1-1} = -\ln(x) + C$

$\int \frac{dv}{(v+1)^2-(1)^2} = -\ln(x) + C$

$\frac{1}{2}\ln\left|\frac{v}{v+2}\right| = -\ln(x) + C$

$\frac{1}{2}\ln\left(\frac{y/x}{y/x+2}\right) = -\ln(x) + C$

$\ln\left(\frac{y}{y+2x}\right) = 2\ln\left|\frac{c}{x}\right|$

$\boxed{c^2 = \frac{x^2 y}{y+2x}}$ Ans.

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$(\sec x \tan x \tan y - e^x)dx + \sec x \sec^2 y\, dy = 0$

Sol:-

$\Rightarrow \sec x \tan x \tan y\, dx - e^x dx + \sec x \sec^2 y\, dy$

$\sec x \tan x \tan y\, dx + \sec x \sec^2 y\, dy = e^x dx$

$d[\sec x \tan y] = e^x dx$

$\int d[\sec x \tan y] = \int e^x dx$

$\sec x \tan y = e^x + C$

$\tan y = \frac{e^x + C}{\sec x}$

$y = \tan^{-1}\left(\frac{e^x + c}{\sec x}\right)$

Ans.

Q4. $x\cos x \frac{dy}{dx} + y(x\sin x + \cos x) = 1$,

$x\cos x \frac{dy}{dx} + xy\sin x + y\cos x = 1$

÷ by $x\cos x$

$\frac{dy}{dx} + y\tan x + \frac{y}{x} = \frac{1}{x\cos x}$

$y' + y\left(\tan x + \frac{1}{x}\right) = \frac{1}{x\cos x}$

$y' + P(x)\cdot y = Q(x)$

$I.F = e^{\int\left(\tan x + \frac{1}{x}\right)dx}$

$IF = e^{\ln|\sec x| + \ln x}$

$I.F = x\sec x$

$y \times x\sec x = \int \frac{1}{x\cos x} \times x\sec x\, dx + C$

$y \cdot x\sec x = \int \sec^2 x\, dx + C$

$xy\sec x = \tan x + C$

$xy = \sin x + C\cos x$

$\boxed{y = \frac{\sin x}{x} + C\frac{\cos x}{x}}$ OR

$\boxed{y = \frac{\tan x}{x\sec x} + \frac{C}{x\sec x}}$ Ans

15. $x \ln x \dfrac{dy}{dx} + y = 2 \ln x$.

$\dfrac{dy}{dx} + \dfrac{y}{x \ln x} = \dfrac{2}{x}$.

$y' + P(x) y = Q(x)$,

$I.F = e^{\int \frac{1/x}{\ln(x)} dx}$

$= e^{\ln(\ln(x))}$

$y \times \ln(x) = \int \dfrac{2}{x} \times \ln x \, dx + C$. $= \boxed{\ln(x)}$

$y = 2\int F(x) dx \cdot F'(x) + c$, So,

$y = \dfrac{\cancel{2}(\ln(x))^{\cancel{2}}}{\cancel{2} \cdot \times \ln(\cancel{x})} + \dfrac{C}{\ln(x)}$

$y = \ln(x) + C/\ln(x) \longrightarrow$ Ans.

16. $y' + \dfrac{4}{x} y = x^3 y^2$. Sol:-

$1-2 = -1$

$y^{-2} y' + \dfrac{4}{xy} = x^3$

$u = y^{-1}$

$\dfrac{du}{dx} = -y^{-2} \dfrac{dy}{dx}$

$-du/dx + \dfrac{4u}{x} = x^3$

$u' - \dfrac{4u}{x} = -x^3$.

$I.F = e^{\int -\frac{4}{x} dx}$

$\boxed{I.F = x^{-4}}$

$\dfrac{uy}{x^4} = \int \dfrac{-x^3}{x^4} dx + C$.

$u/x^4 = -\ln x + \ln C$.

$\frac{1}{y} = -\ln(x)x^4 + Cx^4$

$y = \frac{1}{x^4(-\ln(x)+C)}$. Ans.

Question 2 Mathematical Modeling.

i) Population, how long it will take to triple?

Sol:-

D.E for growth is,

$\frac{dP}{dt} = kP \rightarrow \frac{dP}{P} = k\,dt$. integrating

$P = e^{kt} \cdot e^{c} \rightarrow P = Ce^{kt}$

I.C:- $P(0) = P_0$. $P_0 = C$

$P = P_0 e^{kt} \rightarrow$ (i) double

$2P_0 = P_0 e^{kt} \rightarrow 2\cancel{P_0} = \cancel{P_0} e^{5k}$

$2 = e^{5k} \rightarrow 5k = \ln(2)$.

$\boxed{k = 0.13863.} \rightarrow P = P_0 e^{0.1383t}$

(ii) triple

$3P_0 = P_0 e^{kt}$

$3 = e^{0.13863t} \rightarrow \ln(3) = \ln e^{0.13863t}$.

$\ln(3) = 0.13863t$. $\because \ln(e) = 1$

$t \simeq 7.9$ years to triple

iii) quadraple. $4\cancel{P_0} = \cancel{P_0} e^{0.13863t}$

$\ln(4) = 0.1386t \Rightarrow t = 10$

10 years to quadraple. Ans

(ii) Radioactive isotope ...

Sol:-

$$\frac{dA}{dt} = kA \rightarrow A = ke^{kt}$$

$$A(0) = A_0,$$

$$A_0 = C \quad A = A_0 e^{kt}$$

$$\frac{1}{2}\cancel{A_0} = \cancel{A_0} e^{k(3.3)}$$

$$3.3k = \ln(1/2)$$

$$k = -0.21004$$

$$A = A_0 e^{-0.21004t}$$

$A_0 = 1$ gram and 90% decay

$$A = (1 - 90/100) A_0 \quad \text{find } A$$

$$A = 0.1 \text{ gram}$$

$$0.1\cancel{A_0} = \cancel{A_0} e^{-0.21004t}$$

$$0.1 = e^{-0.21004t}$$

$$t \approx 10.9 \text{ hours}$$

iii) Thermometer reading 70°F

How hot is oven?

Sol:- D.E for newton law of cooling,

$$\frac{dT}{dt} = k(T - T_m) \rightarrow T = T_m + Ce^{kT}$$

I.C $T(0) = 70°F$

$70 = T_m + C \Rightarrow \boxed{C = T_0 - T_m}$

$T = T_m + (T_0 - T_m)e^{kt}$

$110 = T_m + (70 - T_m)e^{0.5k}$ after ½ min,

$110 - T_m = (70 - T_m)e^{0.5k} \rightarrow (1)$

$145 = T_m + (70 - T_m)e^{0.5k}$ after 1min,

$145 - T_m = (70 - T_m)e^{0.5k} \rightarrow (2)$

÷ (1) and (2), we get,

$e^{0.5k} = \frac{145 - T_m}{110 - T_m}$ (3) and we have $e^{0.5k} = \frac{110 - T_m}{70 - T_m}$

$$\frac{145 - T_m}{110 - T_m} = \frac{110 - T_m}{70 - T_m}$$

$10150 - 215T_m = 12100 - 220T_m$

$5T_m = 1950$ $\boxed{T_m = 390°F}$

Temperature of oven is/was

390°F.

(ii) 30 volt eMF is applied to LR series circuit.

D.E for LR series circuit is,

$$L\frac{di}{dt} + Ri = E(t)$$

$$0.1\frac{di}{dt} + 50i = 30 \rightarrow \frac{di}{dt} + 500i = 300$$

$$I.F = e^{\int P(t)dt} \longrightarrow e^{\int 500dt} = e^{500t}$$

$$i \times e^{500t} = \int 300 \times e^{500t} dt + c$$

$$e^{500t} i = \frac{3e^{500t}}{5} + c.$$

$$\boxed{i = \frac{3}{5} + ce^{-500t}}$$

$$i(0) = 0$$

$$0 = \frac{3}{5} + c \rightarrow \boxed{c = \frac{-3}{5}}$$

$$i(t) = \frac{3}{5} - \frac{3}{5}e^{-500t}$$

$$t \rightarrow \infty$$

$$e \rightarrow 0.$$

$$\boxed{i(t) = \frac{3}{5}A}$$

or

$$\boxed{\lim_{t \to \infty} i(t) = \frac{3}{5}A.}$$ Ans.

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