EE112 Group 19

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1 Introduction

Our project was to design a circuit having a CE-CC cascading amplifier with a voltage gain of 20.

2 Assumptions

We assumed that the signal is of a very high frequency, hence capacitive impedances are presumed to vanish. We also considered the transistor's behavior up until the second-order approximation. Hence, $I_C = \beta I_B$ under all conditions, the CE terminal can support arbitrary voltages, and the BE terminal behaves as a regular silicon diode, approximated till the second-order.

We also took various frequencies to observe the behaviour of the cascading amplifiers at different frequencies.

3 The Circuit

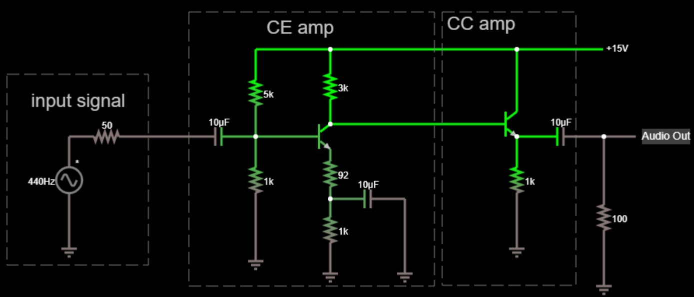


Figure 1: Main Circuit. Simulation

3.0.1 40 Hz

Below, we have the circuit diagram when the AC frequency is 40 Hz.

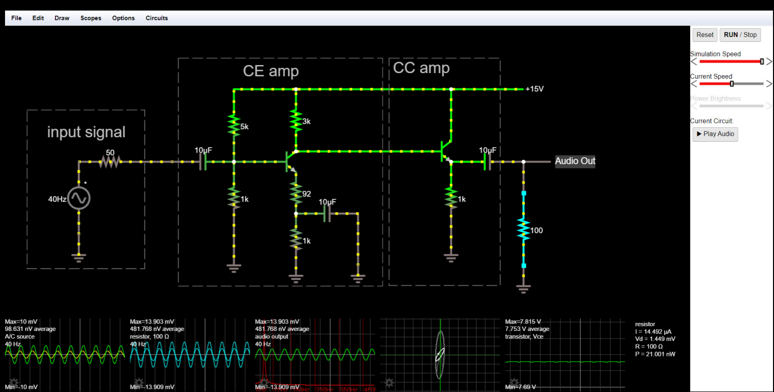


Figure 2: Circuit response at 40Hz. Observed amplification = 1.39

3.0.2 200 Hz

Below, we have the circuit diagram when the AC frequency is 200 Hz.

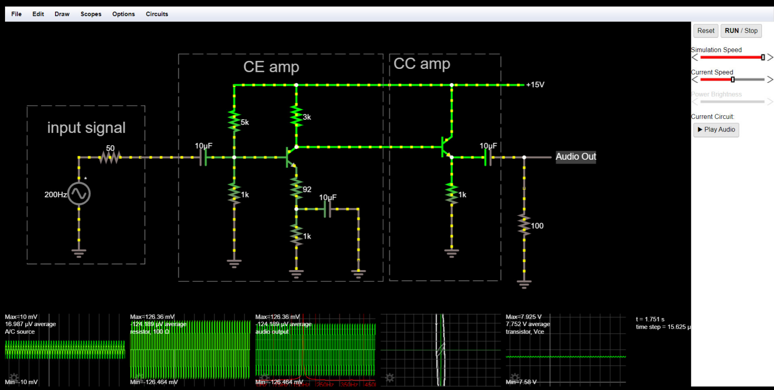


Figure 3: Circuit response at 200Hz (G3). Observed amplification = 12.64

3.0.3 440 Hz

Below, we have the circuit diagram when the AC frequency is 440 Hz.

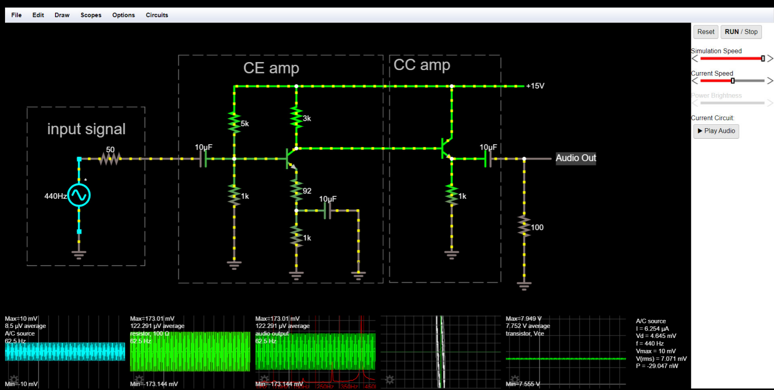


Figure 4: Circuit response at 400Hz (A4). Observed amplification = 17.3

3.0.4 40 kHz

Below, we have the circuit diagram when the AC frequency is 40 kHz.

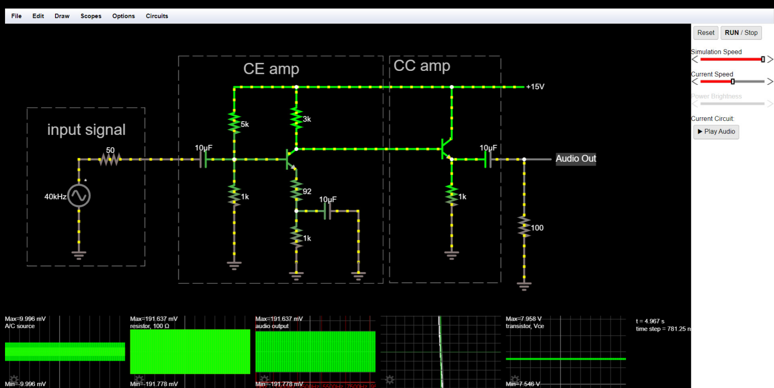


Figure 5: Circuit response at 40kHz. Observed amplification = 19.16

4 Analysis

4.1 Component values

Our Analysis presumes a voltage source providing 15 V DC. All coupling capacitors are marked 10 µF and the driven load is assumed to be 100 Ω

4.1.1 CE biasing

Voltage Divider We took $R_1 = 5\,\text{k}\Omega, R_2 = 1\,\text{k}\Omega$, providing the the CE stage transistor Q_1 with a base voltage of $15\,\text{V} \cdot \frac{1\,\text{k}\Omega}{5\,\text{k}\Omega + 1\,\text{k}\Omega} = 2.5\,\text{V}$.

Q-point Parameters We took $R_C = 3\,\text{k}\Omega, R_E = 1\,\text{k}\Omega, r_e = 92\,\Omega$. With a $V_B = 2.5\,\text{V}, V_E = V_B - 0.7\,\text{V} = 1.8\,\text{V}$, resulting in quiescent current of $I_C \approx I_E = \frac{V_E}{r_e + R_E} = 1.65\,\text{mA}$. This also creates a DC voltage at the collector of 10 V, which biases the next stage.

AC resistances In the AC load line, an additional diode resistance emerges. It's value is $r'_e = \frac{25\,\text{mV}}{I_E} = 15.2\,\Omega$

4.1.2 CC Biasing

Q-point Parameters We took $R_E = 1\,\text{k}\Omega$. With a $V_B = 10\,\text{V}$ supplied from the last stage, $V_E = V_B - 0.7\,\text{V} = 9.3\,\text{V}$, this results in quiescent current of $I_C \approx I_E = \frac{V_E}{R_E} = 9.3\,\text{mA}$.

AC resistances In the AC load line, an additional diode resistance emerges. It's value is $r'_e = \frac{25\,\text{mV}}{I_E} = 2.69\,\Omega$

4.2 AC load line

Here we present the components and their arrangement in the Ac load line.

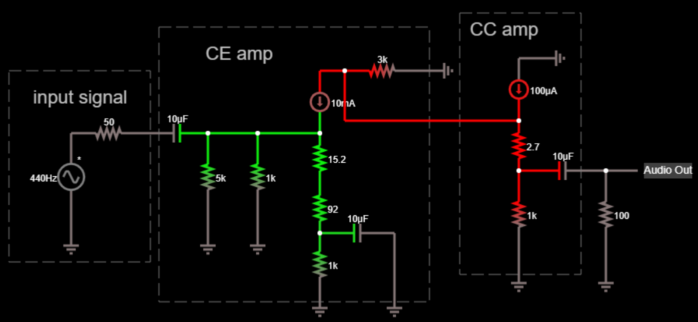


Figure 6: AC load line. Representation (Inaccurate numerically)

4.2.1 CE stage

$z_{\mathbf{in(base)}}$ We can observe that the resistances visible from the emitter are $r'_e + r_e + (R_E \parallel Z_C) = r'_e + r_e + \frac{1}{\frac{1}{R_E} + \jmath 2\pi f C}$. Hence, $z_{\text{in(base)}} = \beta \left(r'_e + r_e + \frac{R_E}{1 + \jmath 2\pi R_E C f} \right)$

$z_{\mathbf{in(stage)}}$ Here the divider resistors combine with $z_{\text{in(base)}}$ to give us the total input impedance of our system.

$$
\begin{aligned}
z_{\text{in(stage)}} =& Z_C + \left(R_1 \parallel R_2 \parallel z_{\text{in(base)}} \right) \\
=& \frac{1}{\jmath 2\pi f C} + \frac{1}{R_1^{-1} + R_2^{-1} + \frac{1}{\beta \left(r'_e + r_e + \frac{R_E}{1 + \jmath 2\pi R_E C f} \right)}} \\
=& \frac{1}{\jmath 2\pi f C} + \frac{(r'_e + r_e)(1 + \jmath 2\pi R_E C f) + R_E}{\left(\frac{(r'_e + r_e)(R_1 + R_2)}{R_1 R_2} + \frac{1}{\beta} \right)(1 + \jmath 2\pi R_E C f) + \frac{R_E}{R_1} + \frac{R_E}{R_2}}
\end{aligned}
$$

$A_{\mathbf{unloaded}}$ First we shall consider the voltage divider formed between $z_{\text{in(base)}}$ and the rest of impedances. We only however need to consider division until after the first capacitor, as the rest of the components are in parallel and share the voltage.

$$
\begin{aligned}
A_{\text{divider}} =& \frac{\left(R_1 \parallel R_2 \parallel z_{\text{in(base)}}\right)}{Z_C + \left(R_1 \parallel R_2 \parallel z_{\text{in(base)}}\right)} \\
=& 1 - \frac{\frac{1}{\jmath 2\pi f C}}{\frac{1}{\jmath 2\pi f C} + \frac{(r_e'+r_e)(1+\jmath 2\pi R_E C f)+R_E}{\left(\frac{(r_e'+r_e)(R_1+R_2)}{R_1R_2}+\frac{1}{\beta}\right)(1+\jmath 2\pi R_E C f)+\frac{R_E}{R_1}+\frac{R_E}{R_2}}} \\
=& 1 - \frac{1}{1 + \frac{((r_e'+r_e)(1+\jmath 2\pi R_E C f)+R_E)(\jmath 2\pi f C)}{\left(\frac{(r_e'+r_e)(R_1+R_2)}{R_1R_2}+\frac{1}{\beta}\right)(1+\jmath 2\pi R_E C f)+\frac{R_E}{R_1}+\frac{R_E}{R_2}}} \\
=& \frac{1}{1 + \frac{\left(\frac{(r_e'+r_e)(R_1+R_2)}{R_1R_2}+\frac{1}{\beta}\right)(1+\jmath 2\pi R_E C f)+\frac{R_E}{R_1}+\frac{R_E}{R_2}}{((r_e'+r_e)(1+\jmath 2\pi R_E C f)+R_E)(\jmath 2\pi f C)}} \\
=& \frac{\left(\left(r_e'+r_e\right)\left(1+\jmath 2\pi R_E C f\right)+R_E\right)\left(\jmath 2\pi f C\right)}{\left(\left(r_e'+r_e\right)\left(1+\jmath 2\pi R_E C f\right)+R_E\right)\left(\jmath 2\pi f C\right)+\left(\frac{(r_e'+r_e)(R_1+R_2)}{R_1R_2}+\frac{1}{\beta}\right)\left(1+\jmath 2\pi R_E C f\right)+\frac{R_E}{R_1}+\frac{R_E}{R_2}}
\end{aligned}
$$

Let us consider the following,

$$
\begin{aligned}
i_e =& \frac{V_{base}-0}{z_{\text{emitter}}} = \frac{0-V_collector}{z_{\text{collector}}} \\
\implies A_{\text{transistor}} =& -\frac{z_{\text{collector}}}{z_{\text{emitter}}} \\
=& -\frac{R_C}{r_e'+r_e+\frac{R_E}{1+\jmath 2\pi R_E C f}} && \text{(When unloaded,} z_{\text{collector}} = R_C\text{)} \\
\implies A_{\text{unloaded}} =& A_{\text{divider}} A_{\text{transistor}} \\
=& \frac{\left(\left(r_e'+r_e\right)\left(1+\jmath 2\pi R_E C f\right)+R_E\right)\left(\jmath 2\pi f C\right)}{\left(\left(r_e'+r_e\right)\left(1+\jmath 2\pi R_E C f\right)+R_E\right)\left(\jmath 2\pi f C\right)+\left(\frac{(r_e'+r_e)(R_1+R_2)}{R_1R_2}+\frac{1}{\beta}\right)\left(1+\jmath 2\pi R_E C f\right)+\frac{R_E}{R_1}+\frac{R_E}{R_2}} \\
& \cdot -\frac{R_C}{r_e'+r_e+\frac{R_E}{1+\jmath 2\pi R_E C f}} \\
=& -\frac{\left(1+\jmath 2\pi R_E C f\right)\left(\jmath 2\pi R_C C f\right)}{\left(\left(r_e'+r_e\right)\left(1+\jmath 2\pi R_E C f\right)+R_E\right)\left(\jmath 2\pi f C\right)+\left(\frac{(r_e'+r_e)(R_1+R_2)}{R_1R_2}+\frac{1}{\beta}\right)\left(1+\jmath 2\pi R_E C f\right)+\frac{R_E}{R_1}+\frac{R_E}{R_2}}
\end{aligned}
$$

$\mathbf{z_{out(stage)}}$ Using Thevenin's theorem, we can clearly see that, using the fact that a current source isolates voltaicly, that $z_{\text{out(stage)}} = R_C$.

4.3 CC stage

We can clearly see that once we replace the CC stage with an appropriate voltage source and resistance, we get a simple voltage division until the load.

$\mathbf{z_{in(stage)}}$ The resistances visible from the emitter are $r_e' + (R_E \parallel (Z_C + R)) = r_e' + \frac{1}{\frac{1}{R_E}+\frac{1}{\frac{1}{\jmath 2\pi f C}+R_L}}$.

Hence, $z_{\text{in(stage)}} = \beta\left(r_e' + \frac{R_E(1+\jmath 2\pi R_L C f)}{1+\jmath 2\pi R_L C f+\jmath 2\pi R_E C f}\right)$

$\mathbf{A_{in}}$ The AC voltage component fed into the base of CC transistor Q_2 is given by a voltage division,

$$
\begin{aligned}
A_{\text{in}} =& \frac{z_{\text{in(stage), CC}}}{z_{\text{out(stage), CE}} + z_{\text{in(stage), CC}}} \\
=& \frac{r_e' + \frac{R_E(1+\jmath 2\pi R_L C f)}{1+\jmath 2\pi R_L C f+\jmath 2\pi R_E C f}}{\frac{R_C}{\beta} + r_e' + \frac{R_E(1+\jmath 2\pi R_L C f)}{1+\jmath 2\pi R_L C f+\jmath 2\pi R_E C f}} \\
=& \frac{r_e'\left(1+\jmath 2\pi R_L C f+\jmath 2\pi R_E C f\right)+R_E\left(1+\jmath 2\pi R_L C f\right)}{\left(\frac{R_C}{\beta}+r_e'\right)\left(1+\jmath 2\pi R_L C f+\jmath 2\pi R_E C f\right)+R_E\left(1+\jmath 2\pi R_L C f\right)}
\end{aligned}
$$

$A_{\textbf{load}}$ Dividing further on the emitter side, the Thevenin voltage presented to the load is

$$\begin{aligned}
A_{\text{load}} =& \frac{(R_E \parallel (Z_C + R))}{r_e' + (R_E \parallel (Z_C + R))} \\
=& \frac{\frac{R_E(1+\jmath 2\pi R_L C f)}{1+\jmath 2\pi R_L C f + \jmath 2\pi R_E C f}}{r_e' + \frac{R_E(1+\jmath 2\pi R_L C f)}{1+\jmath 2\pi R_L C f + \jmath 2\pi R_E C f}} \\
=& \frac{(1+\jmath 2\pi R_L C f)}{\frac{r_e'}{R_E}(1+\jmath 2\pi R_L C f + \jmath 2\pi R_E C f) + (1+\jmath 2\pi R_L C f)}
\end{aligned}$$

$A_{\textbf{out}}$ The final amplification seen is

$$\begin{aligned}
A_{\text{out}} = A_{\text{in}} A_{\text{load}} =& \frac{r_e'(1+\jmath 2\pi R_L C f + \jmath 2\pi R_E C f) + R_E(1+\jmath 2\pi R_L C f)}{\left(\frac{R_C}{\beta} + r_e'\right)(1+\jmath 2\pi R_L C f + \jmath 2\pi R_E C f) + R_E(1+\jmath 2\pi R_L C f)} \\
& \cdot \frac{(1+\jmath 2\pi R_L C f)}{\frac{r_e'}{R_E}(1+\jmath 2\pi R_L C f + \jmath 2\pi R_E C f) + (1+\jmath 2\pi R_L C f)} \\
=& \frac{R_E(1+\jmath 2\pi R_L C f)}{\left(\frac{R_C}{\beta} + r_e'\right)(1+\jmath 2\pi R_L C f + \jmath 2\pi R_E C f) + R_E(1+\jmath 2\pi R_L C f)}
\end{aligned}$$

$z_{\textbf{out(stage)}}$ The resistances visible from the base is R_C. Hence,

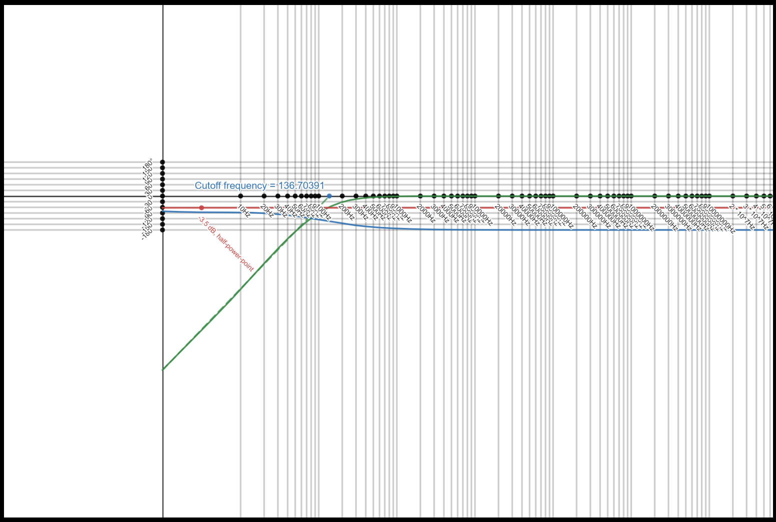
$$\begin{aligned}
z_{\text{out(stage)}} =& Z_C + \left(R_E \parallel \left(r_e' + \frac{R_C}{\beta}\right)\right) \\
=& \frac{1}{\jmath 2\pi C f} + \frac{1}{R_E^{-1} + \frac{1}{r_e' + \frac{R_C}{\beta}}} \\
=& \frac{R_E + r_e' + \frac{R_C}{\beta} + \left(r_e' + \frac{R_C}{\beta}\right)\jmath 2\pi R_E C f}{j 2\pi C f \left(R_E + r_e' + \frac{R_C}{\beta}\right)}
\end{aligned}$$

Notice that when we increase the frequencies i.e. eliminate the frequency factors, we get left with some pretty simple expressions.

$$\begin{aligned}
As&f \to \infty, \\
A_{\text{unloaded, CE}} \approx& -\frac{R_C}{r'_{\text{e, CE}} + r_e} \\
A_{\text{out, CC}} \approx& \frac{R_L R_E}{\left(r'_{\text{e, CC}} + \frac{R_C}{\beta}\right)(R_L + R_E) + R_L R_E} \\
A_v = A_{\text{unloaded, CE}} \cdot A_{\text{out, CC}} \approx& -\frac{R_C R_L R_E}{\left(r'_{\text{e, CE}} + r_e\right)\left(\left(r'_{\text{e, CC}} + \frac{R_C}{\beta}\right)(R_L + R_E) + R_L R_E\right)}
\end{aligned}$$

When we input all the values using an equation notebook, we find that the approximate amplification is roughly -20.59.

When we input the above equations and values for the full AC analysis in an equation notebook and graph them, we obtain that the amplifier functions roughly like a high-pass filter, with -10dB/Decade slope in the stop band and cut-off frequency of about 136.7 Hz. The amplification at infinity is roughly 21.05.



Desmos graph of the frequency response.Link

Interestingly, lowering the load further results in band-pass like behaviour in the aforementioned stop band.

5 Bad Points

Frequency Dependence The voltage gain of the cascading amplifier depends on the frequency of the signal. In particular, using the given values, it acts like a high-pass filter. This isn't useful for phone calls with deeply voiced people as this will distort lower frequencies.

Power Usage It has a poor efficiency and only a fraction of power drawn is used to drive the AC signal. The DC power drawn is itself 183 mW.

Capacitor Size In order to decrease the cut-off frequency, we need to increase the capacitances of our AC couplers. But this lengthens the time taken by the capacitors to charge up to the RC DC component, leading over upto a minute of down time in higher cases. One may proportionally decreases the magnitudes of the resistors, but that results in a increase in DC power consumption.‘

6 Conclusion

Hence we have met our design goals of a CE-CC cascade amplifier with a voltage gain of 20.