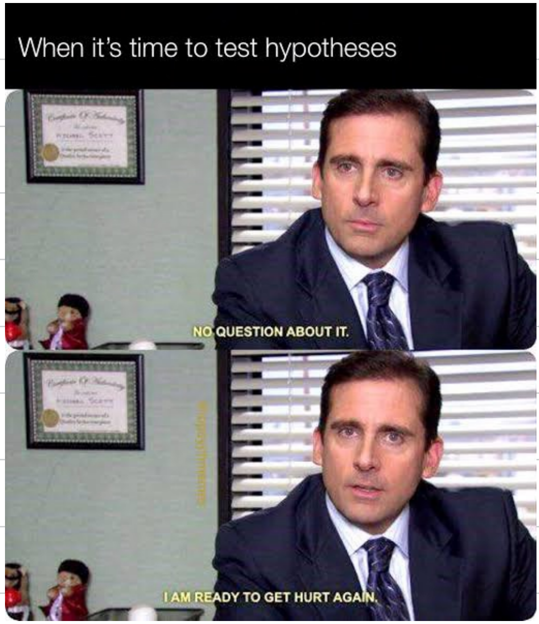
Day - 3 Miscellaneous

Jan 07 2024



Agenda

1. Confidence Interval – hypothesis testing
2. Power of test
3. 2 Sample – Z test

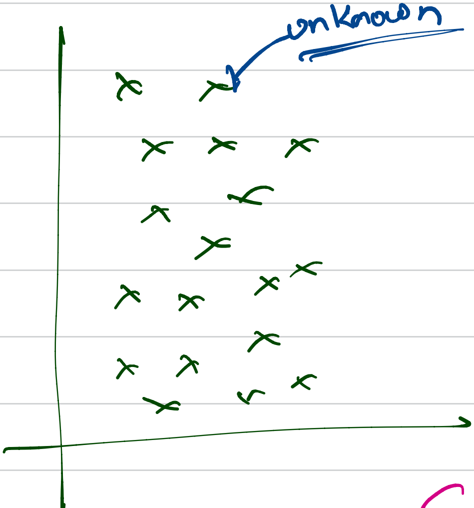
Interview Oriented

4. Z - test proportion

CONFIDENCE INTERVAL

→ Some analysis on heights of USA basket ball Player.

Take some random samples.



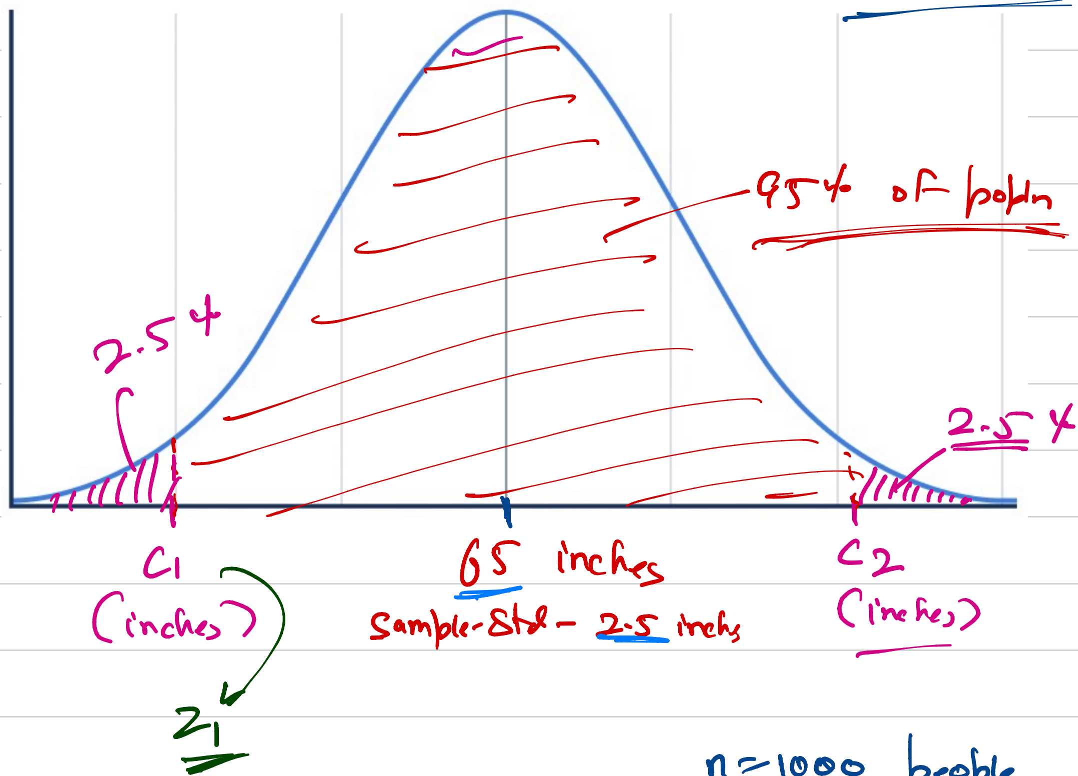
* CI → tells me with certain prob. that popln param (Means) Lies in what range

541 → Patient

[2, 3, 10, 11, 14 - - - - .]

541 Patients

95% Probab



n = 1000 people

For what z_1, do we have 2.5% to the left

z_1 = norm.ppf(0.025) = −1.96

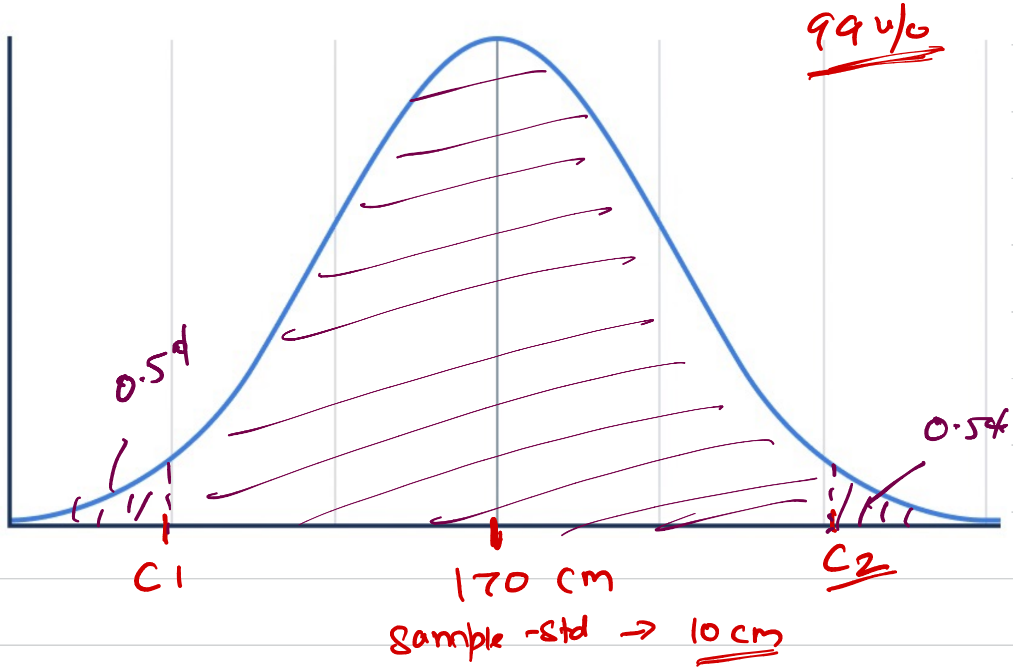
z_2 = norm.ppf(0.975) = +1.96

$c_1 \Rightarrow z_1$ std left of mean

$\Rightarrow 65 - 1.96 \times 2.5$

$\Rightarrow$ 60.10 inches, $c_2 = 69.9$

Sample std dev ≈ std-error



Z_1 = norm.ppf(0.005) => −2.57

Z_2 = norm.ppf(0.995) = 2.57

$C_1 = 170 - 2.57 \times 10$

$C_2 = 170 + 2.57 \times 10$

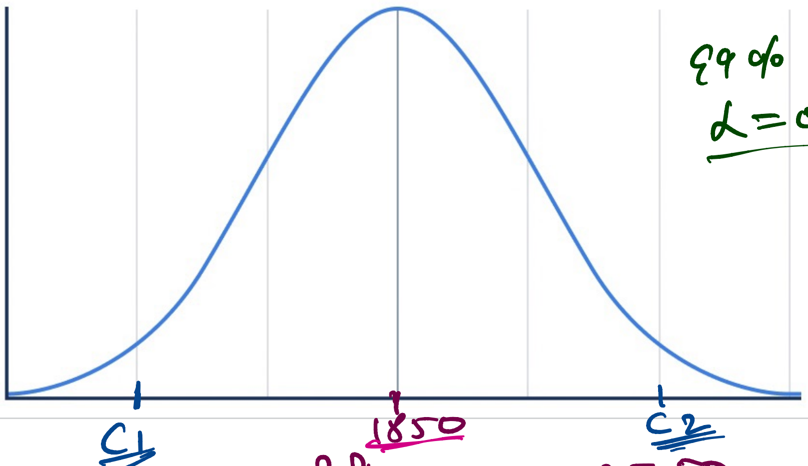
Nike → 1800 shoes
100 — std - dev(Popln)

Marketing team A → 50 stores
1850 shoes / week

" " B → 5 stores
1900 shoes / week

1. → Z-score
2. → Critical value
3. → Confidence Interval

A



99 % confidence
$\alpha = 0.01$

Popln → 100 n = 50

$\alpha = 0.01$

confidence level $= 1 - \alpha$

$= 0.99$ / 99 %

Formula

Ho: They've no effect ($\mu = 1800$)

Ha: They made an effect ($\mu \neq 1800$)

$$CI = \text{Sample mean} \pm |Z_{crit}| \times \text{sample std}$$

$$= 1850 \pm |2.57| \times \frac{100}{\sqrt{50}}$$

$Z_{critical} = 2.57$

$\Rightarrow$ 1813 $\rightarrow$ 1886

1800

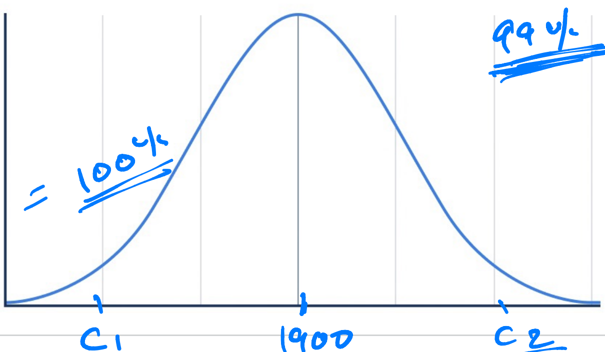
* If popln parameter doesn't lie in CI $\rightarrow$ Reject Ho

0.5%

\+ 99%

\+ 0.5%

= 100%



100 - Pop

n=5

$\boxed{\alpha = 0.01}$

Sample-mean $\pm$ Z_{crit} $\times$ Sample-std

$$1900 \pm |2.57| \times \frac{100}{\sqrt{5}}$$

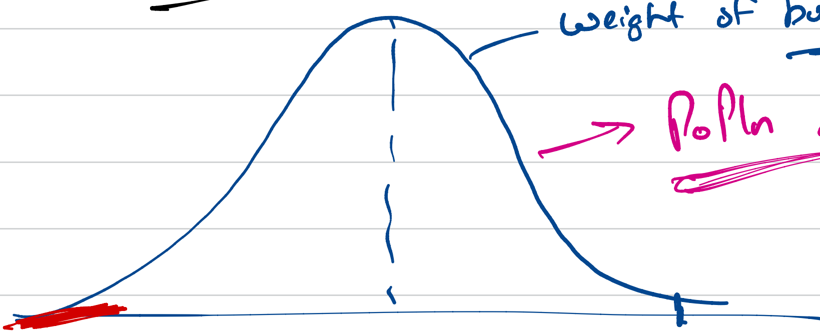
$$[1795 \;—\; 2004]$$

1800 in this range?? → Yes

Team B sucks!! Can't reject H_0

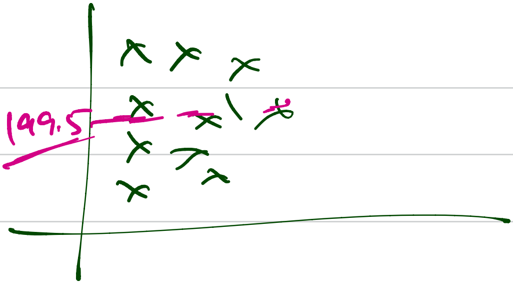
POWER OF TEST

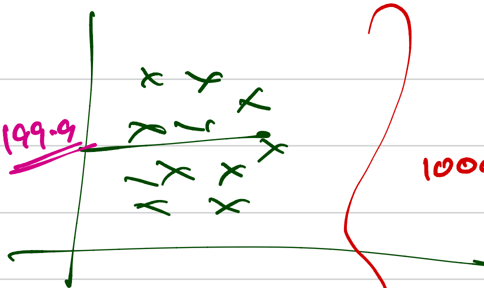
weight of burger



→ Popln dist

200 gm

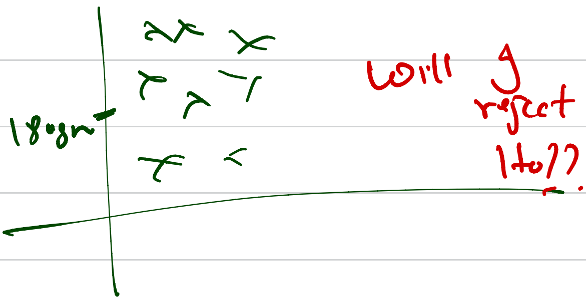




1000s of tr..

Ho: $\omega = 200$

Ha: $\omega \neq 200$

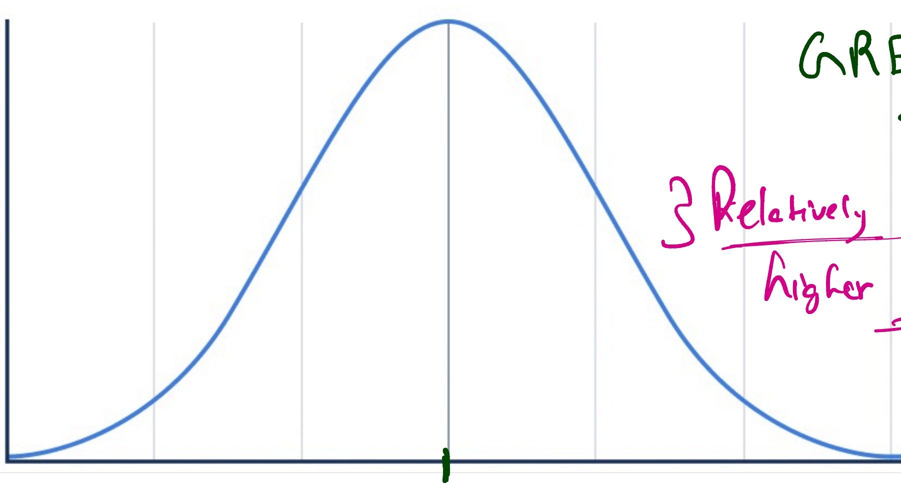


will I reject Ho??

→ No

Incorrectly reject Ho: } Type 1 error

False Positive



GRE → MS

340

} Relatively Higher Power

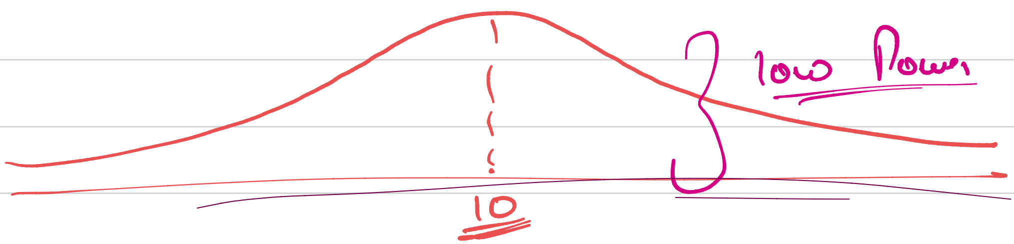
Ho: marks = 320
Ha: marks ≠ 320

out of 1000s of sample → 1-2 sample

I'll incorrectly reject Null hypothesis

Prop. that in a dist. I will correctly reject Ho → POWER

correctly not reject Ho



$H_0: \mu = 10$
$H_a: \mu \neq 10$

V. often incorrectly reject Ho } Type-1 error

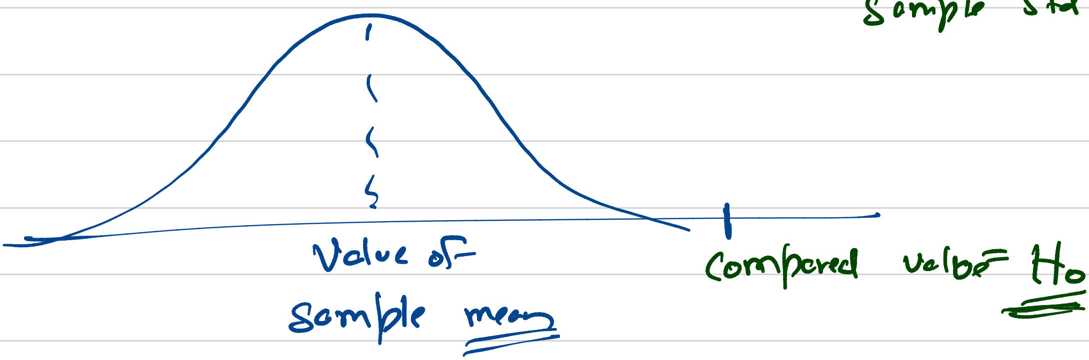
False Positive

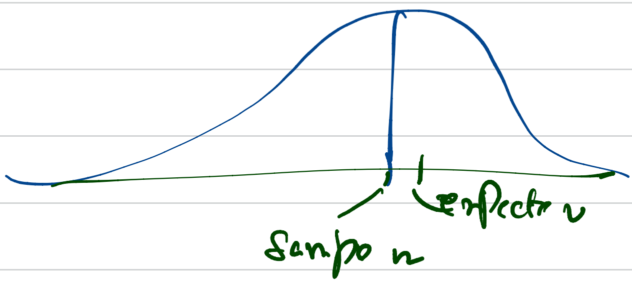
Prob → making Type 1 error

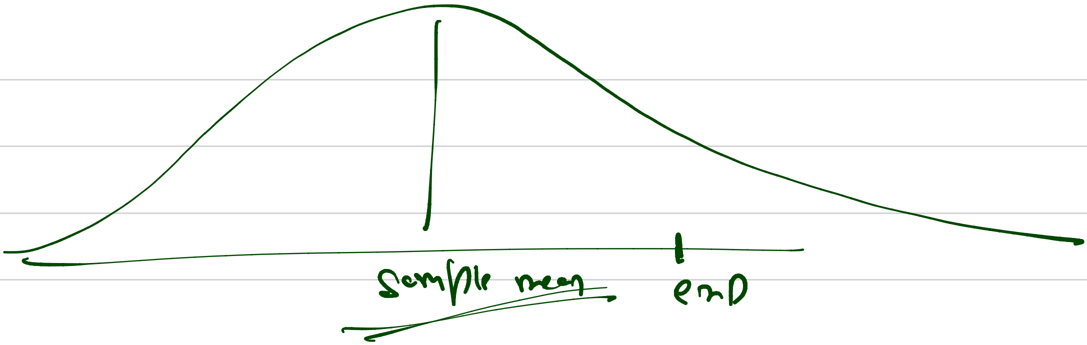
What param affect power?!

1. Sample Size
2. Variance
3. Confidence Interval
4. Effect Size → $\frac{\text{Diff b/w mean}}{\text{sample std.}}$

obs mean − expected mean







→ Power → how confident you are about the result

Type-1 error → False positive
→ Reject Ho → actually true

Type-2 error → Fail to reject Ho, actually False
→ False Negative

Use Power to Focus on type-2 error

2 - Sample T-test

{ } - S_1

{ } - S_2

If they are statistically diff

huge no. of samples in $\underbrace{S_1}_{200}$ & $\underbrace{S_2}_{200}$

OR

I know popln std-dev for dist of S_1 & S_2

ttest-ind

2 - Sample - Z test

Sample-mean → S_1/x_1

Sample-mean S_2/x_2

$$Z = \frac{x_1 - x_2}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

PoPln std-dev For x_1

Popln std-dev for x_2

B.coz Sample size high

$\sigma_1 \approx S_1$ → Sample std-dev

Primary Cond: Big sample size (at least 100)

or

Atleast 50-size (But PoPln Std-dev is known)

Z- Proportion test

① 1- proportions

→ Dev → web applications

70 % → Satisfied → $\frac{\#\ \text{of satisfied}}{\text{total}\ \#\ \text{of Users}}$

↓

Increase → new Feature → P %

If $\hat{P}$ % is statistically diff than 70 %

Ho: No diff

Ha: $P \neq 70\%$

$$Z\text{-test} = \frac{\hat{P} - P}{\sqrt{\frac{P(1-P)}{n}}}$$

observe

Sample of Satisfied Customer

Ho

70

normally distributed

2 - Sample - Z-test

$$Z = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}(1-\hat{p})\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}}$$

Web-page A → 1000 Visitors → 50 of them buy subscribl

" B → 500 Visitor → 30 - buy subscription.

$P_1 = \frac{50}{1000}$ / $P_2 = \frac{30}{500}$

$$\hat{p} = \frac{x_1 + x_2}{n_1 + n_2} \Rightarrow \frac{50+30}{1000+500}$$

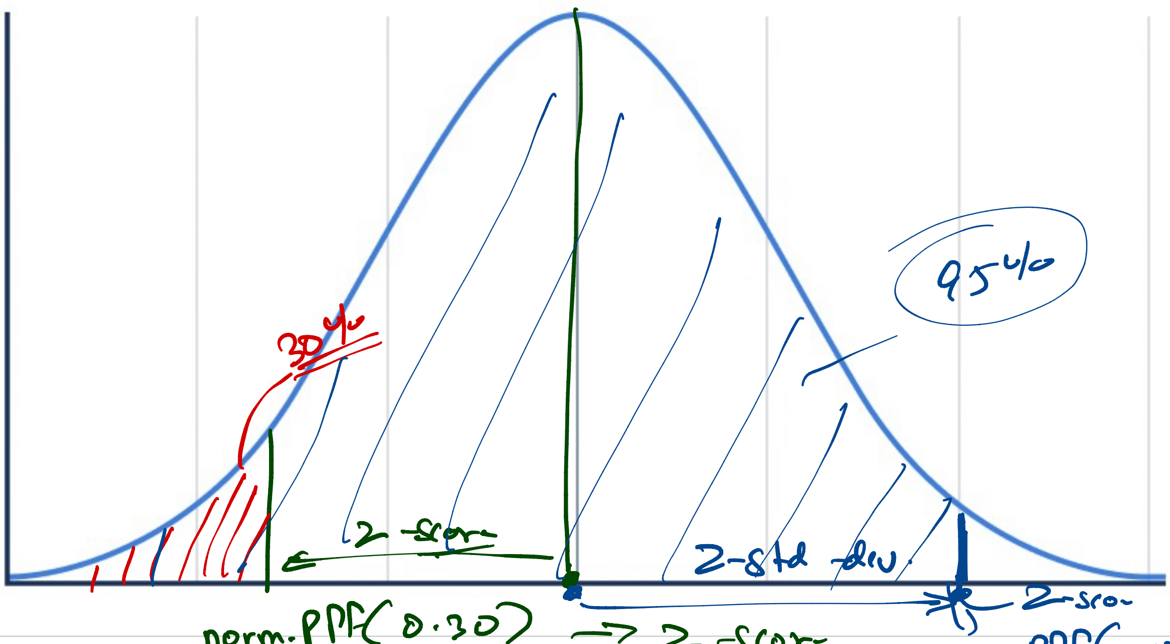
$$\Rightarrow \frac{80}{1500}$$

$$\frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}(1-\hat{p})\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}} = \frac{\frac{50}{1000} - \frac{30}{500}}{\sqrt{\frac{80}{1500}\left(1-\frac{80}{1500}\right)\left(\frac{1}{1000}+\frac{1}{500}\right)}}$$

Pop - mean → Sample mean

→ mean of sample means

PPF = Percentile



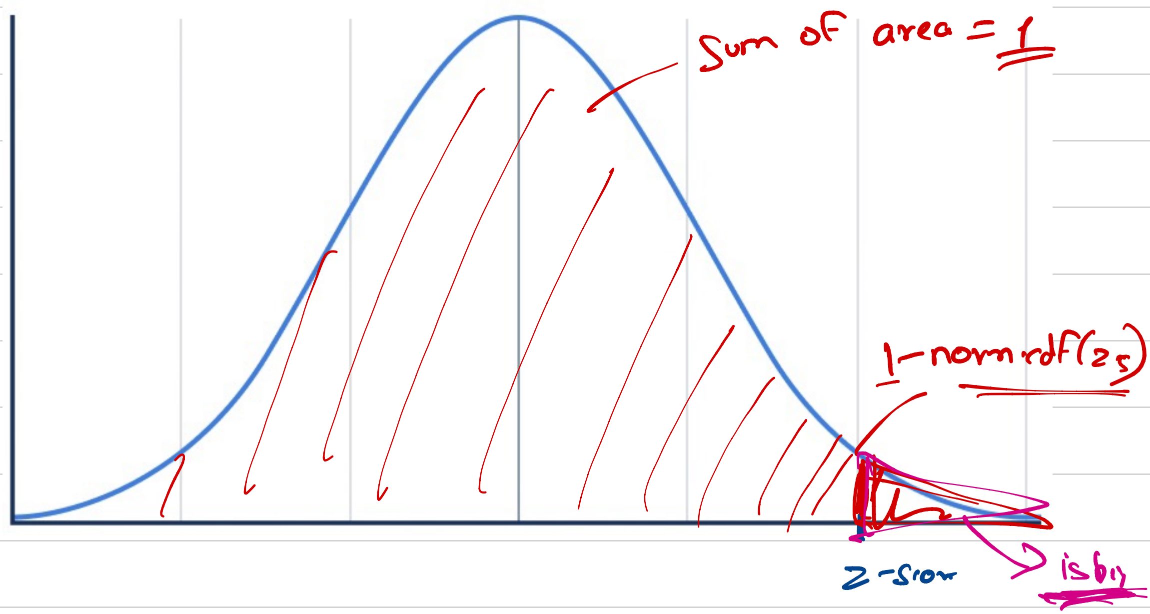
norm.ppf(0.30) → z-score

norm.ppf(0.95)

np.array(Range [1 — 100])

20% → 20 for sur

np. Percentile(array 20) → 20



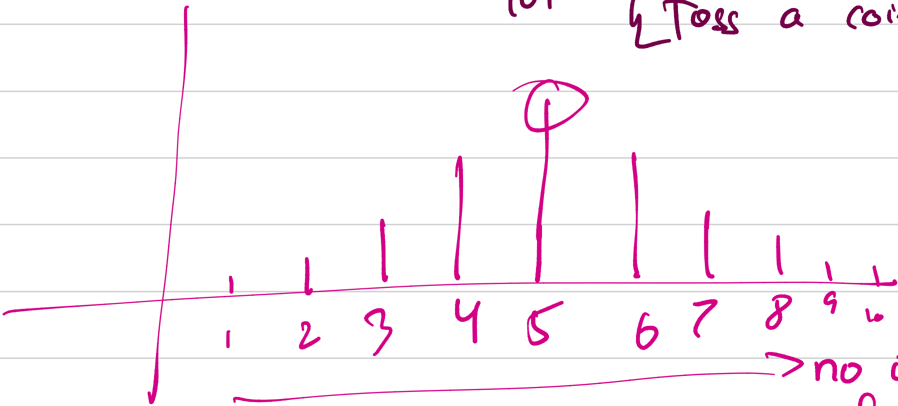
norm.cdf(z-score) → to the left of Z-score

area is big → P-value is big

→ Can't reject Ho

PMF → discrete distribution

1012 tim { Toss a coin 10 tim



$\boxed{\text{binom.pmf}(5, 10, 0.5)}$ →

pmf(6, 10, 0.5)

accused

Prob of observing data assuming H_0 is true

$P(\text{data} / H_0 \text{ is true (innocent)}) = \underline{0.03}$

Possible → <u>highly unlikely</u> → he's still <u>innocent</u>

If he's innocent

Judge → guilty → Type 1 error

False <u>Positive</u>

P-value of seeing this data

and assuming Innocent

→ 0.3

He was guilty → Judge say innocent

Type-2 error.

False negative

We fail to reject Ho, its actually False