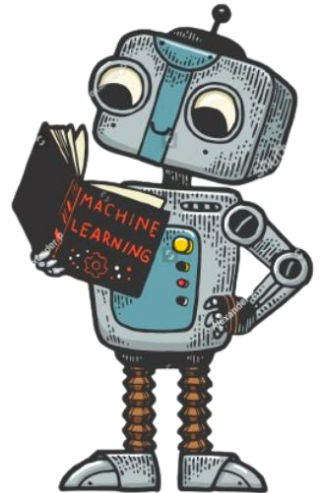


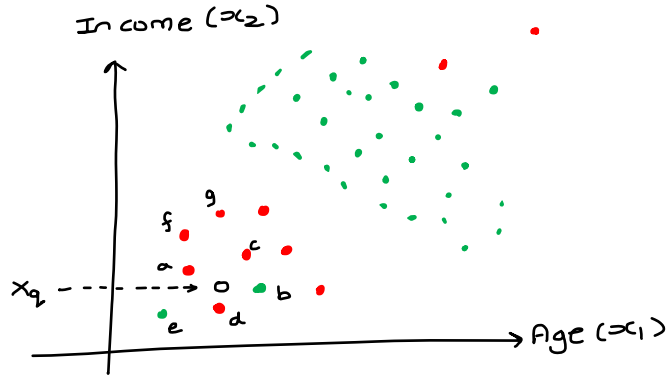
ML: K Nearest Neighbours

KNN-2

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Summary

KNN

old customers
existing customer

	x_1 Age	x_2 Income	y Default?
A			Y
B			N
C			Y
D			N
E			Y
...			

Target variable
(Dependent var)

Historical Data

New customer \rightarrow (x_q) $\left\{ \begin{array}{l} \text{Age} = 19 \\ \text{Income} = 10K \end{array} \right.$

Steps of KNN

- ① Compute distance from the query datapoint to all other points
- ② Sort the data points according to the corresponding distances
 $x_q \rightarrow [a, b, d, e, c, f, g, \dots]$

$$d(x_q, a) \rightarrow 1$$

$$d(x_q, b) \rightarrow 1.5$$

$$\boxed{k=4} \left\{ \begin{array}{l} Y=2 \\ N=2 \end{array} \right. \text{Split Brain situation}$$

- ③ choose k - nearest neighbour ($k=3$)

$$\text{④ voting} \quad \left. \begin{array}{l} a \rightarrow \text{Default} = Y \\ b \rightarrow \text{Default} = N \\ d \rightarrow \text{Default} = Y \end{array} \right\} \left\{ \begin{array}{l} Y=2 \\ N=1 \end{array} \right. \rightarrow [x_q \rightarrow \text{default} = \text{Yes}]$$

$$\boxed{\begin{array}{l} k = \text{default} = 5 \\ 3 \leq k \leq 11 \\ k = \text{odd value} \end{array}}$$

KNN - Non-parametric algo

m · Budget	Sales
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(parameters)

① Linear Regression

② Logistic Regression

Train Data
(x, y)

[Linear Reg.
Algo]

Best weights
 \bar{w}, w_0

prediction

(Learning the best
fit line)

line $\rightarrow \bar{w}x + w_0$

Sales = $w(m \cdot \text{Budget}) + w_0$
estimate sales $\uparrow 1000$

Train Data
(x, y)

[Logistic Reg.
Algo]

(Parameters)
Best weights
 \bar{w}, w_0

prediction

(Learning the best
separating hyperplane)
 $\bar{w}x + w_0$

KNN \rightarrow No parameter is being learnt

In the case of KNN to predict the outcome of any new data point, we need to iterate through the entire training data and here the algo doesn't learn any parameters

① parameters \rightarrow this is what the algo learns from the data (\bar{w}, w_0)

② Hyperparameter \rightarrow has to be provided by the user and the algo will behave differently for different values of Hyper-parameters

Parametric

non-parametric

Hyperparameter

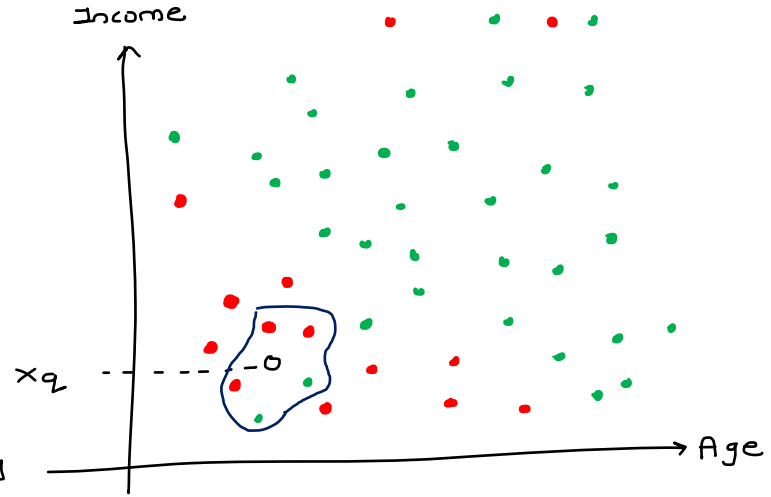
K impact on Bias & Variance

$K=1$ (we are dependent on few neighbours)
overfitting



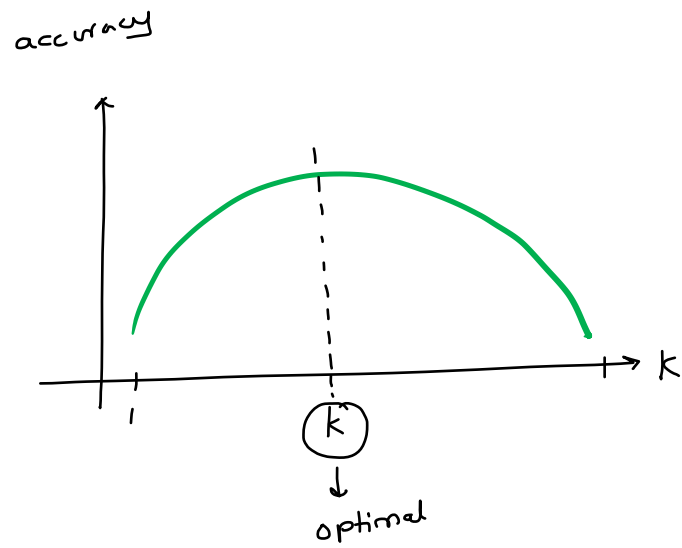
$$\begin{aligned} 2\beta &= \text{modi} \\ 3 &= R \cdot G \end{aligned} \quad \rangle$$

In case k is very small, we are being dependent on few neighbours and if any of the neighbour change, the outcome may get impacted \rightarrow overfitting [High variance, Low Bias]



$K=100$ In this case when k is very high, we are being dependent on so many neighbours and the change in one or two neighbours will not impact my final outcome but the results are over generalised and hence very high error is expected

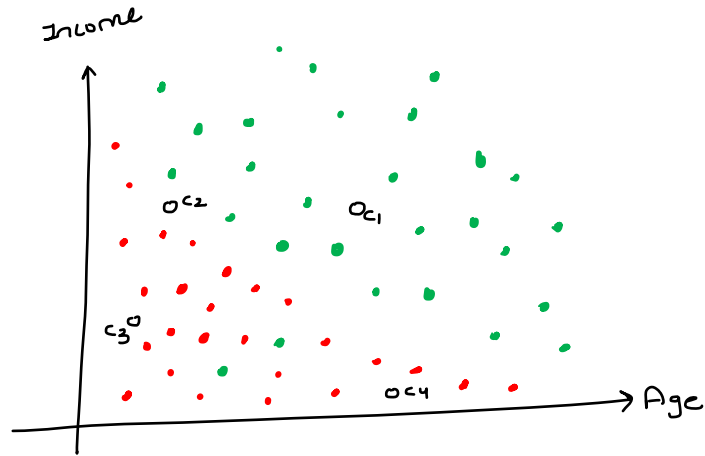
underfitting [Low variance, High Bias]



Hyperparameter Tuning

	Age	Income	Y
c1			
c2			
c3			
...			
c100			

Training Data



	Age	Income	Y
c1			
c2			
c3			
...			
c10			

Testing

for each data point in test data
we will run the entire KNN
steps

c1 → KNN (Training data)
c2 → KNN (" ")
c3 →
...

Time complexity of KNN

x_1	x_2	x_3	\dots	x_d

n

$$x_q \rightarrow [x_{1q}, x_{2q}, \dots, x_{dq}]$$

$$D = |x_1 - x_{1q}| + |x_2 - x_{2q}| + \dots + |x_d - x_{dq}|$$

n

- ① Distance calculation $\rightarrow O(nd)$
 - ② Sort distance ascending $\rightarrow O(n \log n)$
 - ③ pick k -values $\rightarrow O(k)$
 - ④ voting \rightarrow very less
- } very low

$$\text{Time complexity} = O(nd + n \log n)$$

problem with KNN

- ① As n increase KNN will go slow
- ② As d increase KNN will go slow

Advantages

- ① KNN can generate non-linear boundary
- ② Simple to understand
- ③ very powerful on smaller data
- ④ very easily applied in multi-class classification.

Various Distance consideration KNN

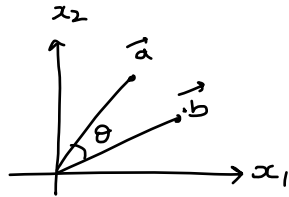
① Minkowski Distance

$$d = \left[\sum_d |x_{ad} - x_{bd}|^p \right]^{1/p} \longrightarrow \text{for custom distance metric}$$

② $p=1$ [L1] $d = \sum_d |x_{ad} - x_{bd}|$

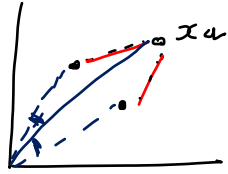
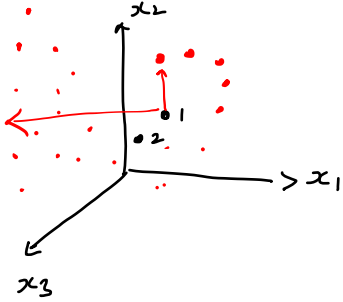
③ $p=2$ [L2] $d = \left(\sum_d |x_{ad} - x_{bd}|^2 \right)^{1/2} \longrightarrow \text{Data is less}$

④ Cosine Similarity $\cos(\theta) = \frac{\vec{a} \cdot \vec{b}}{\|\vec{a}\| \|\vec{b}\|} \longrightarrow \text{In the case of highly dimensional data}$

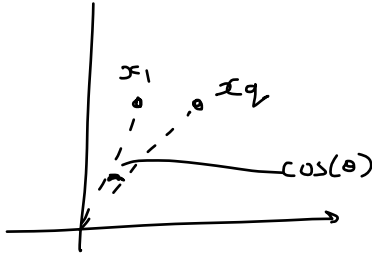


$$\mathcal{E} \cdot d = \left[(x_q - x_1)^2 + \underline{(x_q - x_2)^2} + \dots + (x_q - x_{100})^2 \right]^{\frac{1}{2}}$$

$\mathcal{E}d(1) \stackrel{\sim}{=} \mathcal{E}d(2)$ Because of higher dimensions



cosine similarity in higher dimension
the angles $\cos(\theta)$ appears very
different and easy for algo to differentiate.
=



Weighted KNN

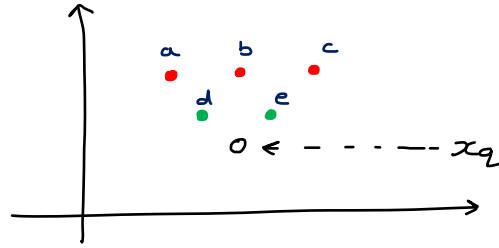
$$d(x_q, a) = 0.5$$

$$d(x_q, b) = 0.5$$

$$d(x_q, c) = 0.5$$

$$d(x_q, d) = 0.2$$

$$d(x_q, e) = 0.2$$



$$K = 5$$

$$x_q \rightarrow \text{Red}$$

Idea \rightarrow Data point close to the query data point should be given more weight in the voting

$$\text{weight} \propto \frac{1}{\text{distance}}$$

$$w(i) = \frac{1}{d(i)}$$

$$x_q \begin{cases} \text{Red} = [2+2+2] = 6 \\ \text{Green} [5+5] = 10 \end{cases}$$

\downarrow
green

$$\left. \begin{array}{l} w_1 = \frac{1}{0.5} = 2 \\ w_2 = \frac{1}{0.5} = 2 \\ w_3 = \frac{1}{0.5} = 2 \end{array} \right\} \text{Red}$$
$$\left. \begin{array}{l} w_4 = \frac{1}{0.2} = 5 \\ w_5 = \frac{1}{0.2} = 5 \end{array} \right\} \text{Green}$$

KNN- Imputation (for missing value imputation)

	Age	Income	Tenure	Distance
A	21	30	2	4.24
B	22	34	5	2.24
C	24	38	1	5.00
D	25	32	2	1.41
E	24	31	4	2.00
q	24	33	NA	

The missing data point can be predicted by taking avg of closest neighbors

$$|K|=3$$

$$Tenure(q) = \frac{Tenure(B) + Tenure(E) + Tenure(D)}{3}$$

$$= \frac{5 + 2 + 4}{3} = 3.6$$

