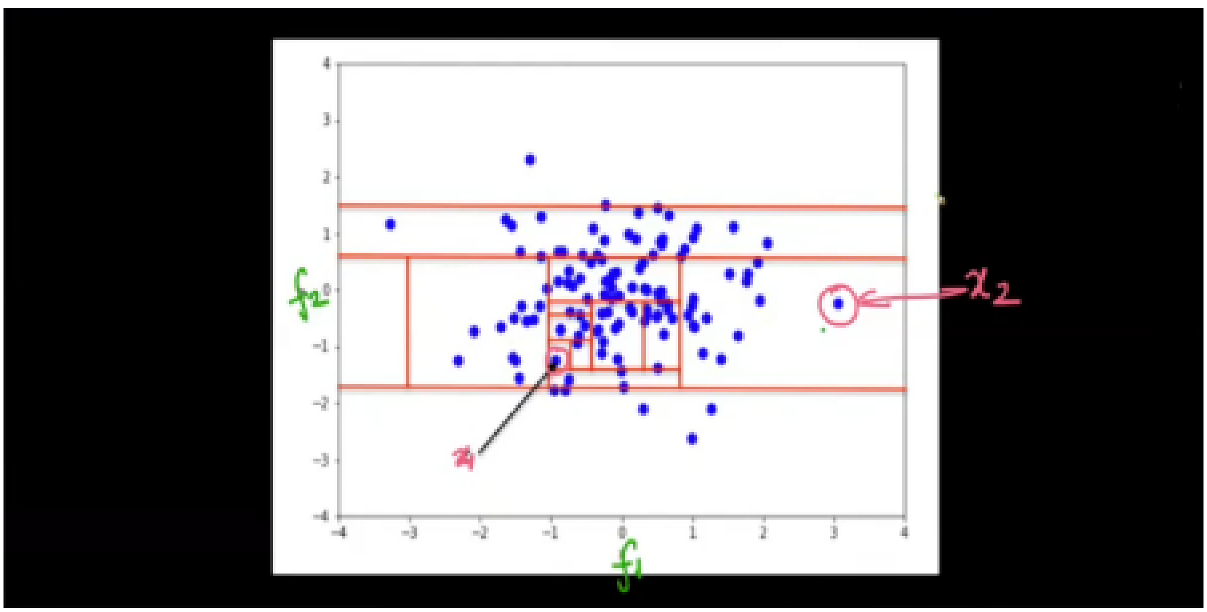
# Anomaly Detection - 2

## Isolation Forests (iForests)

- Consider a dataset D which contains data points $x_1, x_2, \ldots, x_n$. Just like Random forests, Isolation Forests build many trees.

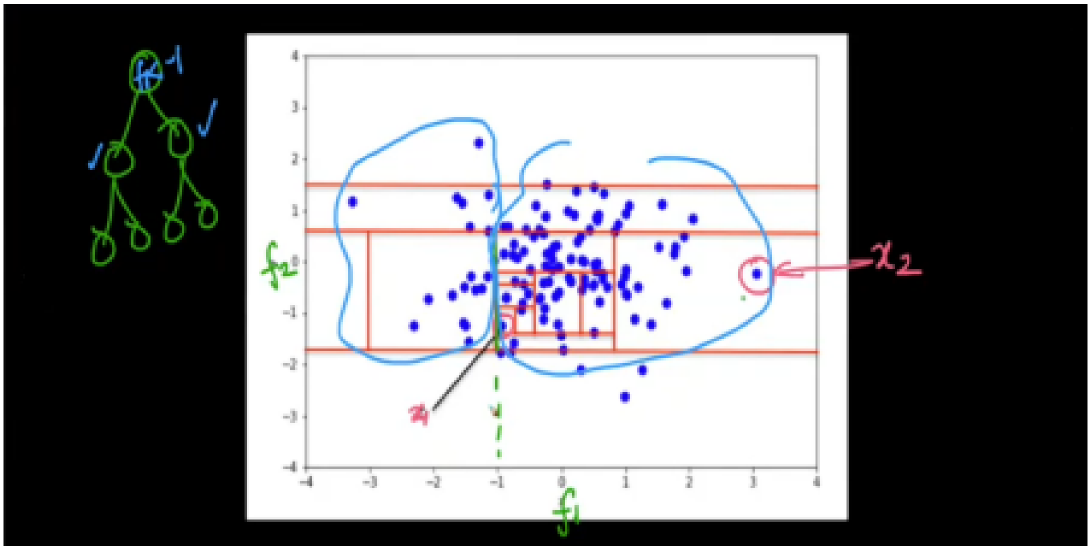
- Following are the steps involved in Isolation Forest:
  - Build many trees like random forests
  - For each tree:
    - Randomly pick a feature
    - Randomly threshold that features
    - Build each tree until the leaf consists of only one datapoint



- In isolation forests, we are building random trees. So if we pick feature $f_1$ and put a threshold there will be a vertical bar.

- Similarly, if we pick feature $f_2$ and put a threshold there will be a horizontal bar.

- For example, if we pick feature $f_1$ and select threshold as $f_1 < 1$, then our first root node will be based on this condition




- Based on the diagram above,
  - The node containing $x_1$ will be at more depth.

  - Observe that the point $x_1$ is in a dense region, and point $x_2$ is far away

  - That is because, to break point $x_1$ from all the other points, more and more splits will be required and that will increase the depth of the node containing point $x_1$.

- So, to sum it up, the idea behind Isolation Forest is:
  - On average outliers have lower depth in the random trees
  - On average, inliers have more depth in the random trees

## Evaluation of Isolation Forest

- Imagine, we have to build random trees.
  - For each point $x_i$ in the dataset, we can get an average depth.

- We use this average depth to convert it into a metric.

- The basic intuition is that the lesser the average depth, the higher the likelihood is there that it is an outlier

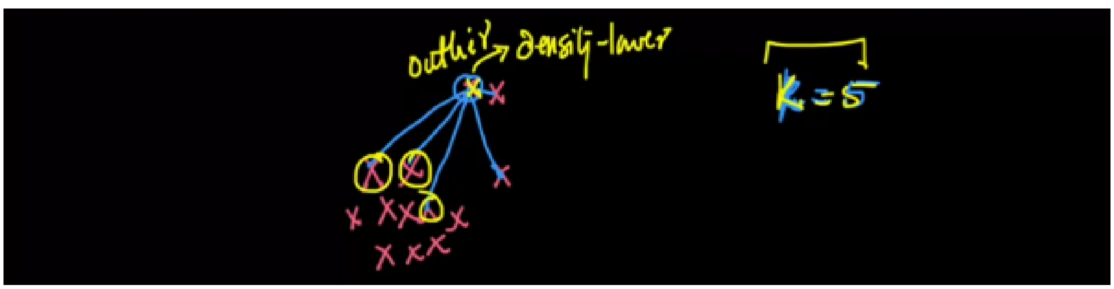
## Disadvantages

- They are biased towards axis parallel splits.
  - Because of this, the boundary will not be smoothened.
  - Because the model is biased towards the axis, it will classify the point as an inlier and as an outlier

## Local Outlier Factor (LOF)

- Core idea: to compare the density of a point with its neighbors' density
- If the density of a point is less than the density of its neighbors, we flag that point as an outlier

- Imagine a bunch of datapoints as shown below




- We compute the density of a point based on average distance.
- If the average distance between a point and its **K** nearest neighbors is large, it is more likely that the point will be an outlier
- Also, the larger the value of **K**, the more confident are the results.

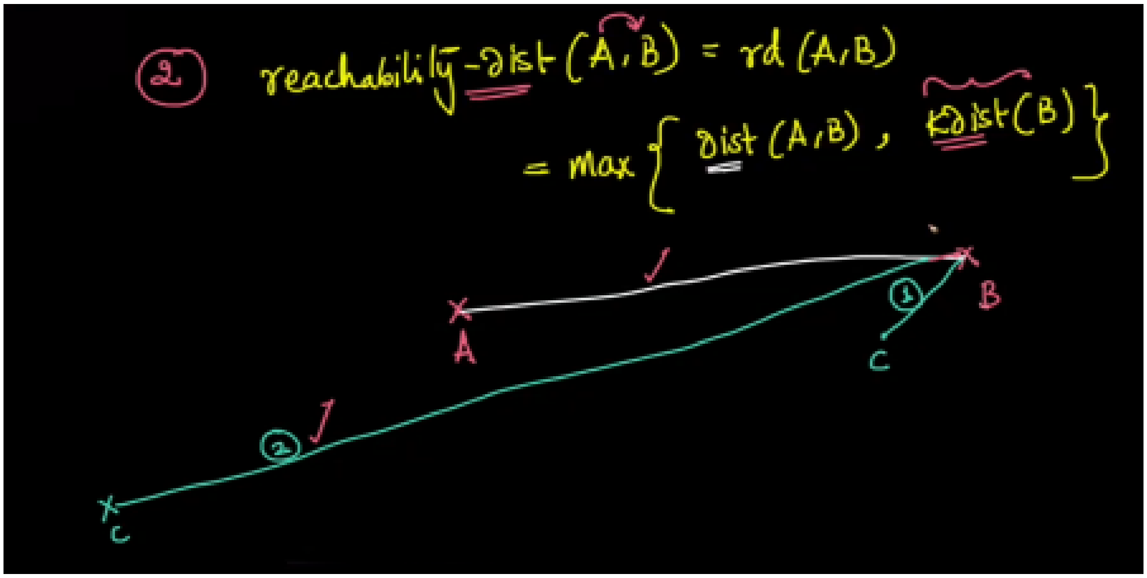
### 1(a) K-distance

- We define the K-distance of point **A** as the distance of point **A** to its **K**$^{th}$ nearest neighbor
- In general, the larger the value of k-distance is, the farther away the point is from other data points

**1(b) Set: Nk (A)**: It is a set of k-nearest neighbors of point **A**.

### 2. Reachability distance

- From point **A** to point **B**, we define reachability distance as
  - a maximum of the distance from point **A** to point **B** and the maximum k-distance of point **B**
- Consider point **B** with some **k** nearest neighbors shown in the diagram below.




- There is a possibility that some neighbors might be close(condition 1) and some neighbors might be very far away(condition 2)
- In this case, there is a neighbor of point **B** whose k-distance is greater than the distance between point **A** and **B**, and hence, it is considered as its reachability distance.

### 3. Local Reachability Density

- It is often represented as **$lrd_k$(A)**, which tells the local reachability density of a point **A.**
- It is defined as the average reachability distance between point **A** and **k** neighbors

$$\text{So, } lrd_k(A) = \frac{\sum_{B\in N_k(A)} rd_k(A,B)\$}{N_k(A)}$$

- The summation in the above equation represents the sum of reachability distances from a point **A** and set of neighbors **B** as **B** Є $\mathbf{N_k(A)}$

- We define Local Outlier Factor of point as follows:

$$LOF_k(A) = \frac{\sum_{B \in N_k(A)} lrd_k(B)}{|N_k(A)|.ldr_k(A)}$$

  - $\mathbf{lrd_k(A)}$ is the density of point **A**

- The expression $\frac{\sum_{B \in N_k(A)} lrd_k(B)}{|N_k(A)|}$ is the average neighborhood density

- So, LOF of point **A** is nothing but the average neighborhood density(lrd) of point **A** divided by the density of **A**

## Interpretation of LOF

- If **LOF(A) = 1**, then we can say that the point has the same density(lrd) as its **k** nearest neighbors

- If **LOF(A) > 1**, then the **k** neighbors of point **A** have a higher density than point **A**.
  - That does not mean point **A** is an outlier. It may or may not be.
  - But if **LOF(A) >>> 1**, then the point is an outlier.

- If **LOF(A) < 1**, then the point has more density than its nearest neighbors.

## Disadvantages of LOF

- Finding optimal K

- Finding threshold.
  - If LOF(A) >> 1, what is the threshold??

- Cannot handle high dimensional data efficiently

- High Time Complexity