28/9/22 2. Boolean Algebra & minimization of switching function

- Boolean Algebra - Postulates
- Boolean Algebra - laws & properties
- Digital logic gates
- SOP, POS - sum of products & ~~Poduct~~ product of sums
- Conversion between Canonical and Standard form
- K-map for three variable ~~and~~ & (Karnaugh map) 4 variable, with & without don't cares
- Logic design using logic gates

Boolean Algebra:-

* Boolean Algebra is a set of Rules and laws/theorems by which logical operations can be expressed mathematically
* It was introduced by "George Boole" (great mathematician)
* The most basic logical operator that are used in digital circuits are AND, OR, NOT operators
* In addition to these, there are certain derived operations like NAND, NOR, XOR, XNOR.

Boolean postulates:-

- Axium is anything which is assumed to be true but cannot be proved
- Axium (or) postulate (same)

"+" ⇒ OR operation

"•" ⇒ AND operation

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| S.NO | operation | |
|---|---|---|
| 1 | '+' operation '.' | $z = x+y$<br>$w = x \cdot y$ |
| 2 | Existence of Element '0' | $x+0 = x$ |
| | existence of Element '1' | $x \cdot 1 = x$ |
| 3 | Commutative operation | $x+y = y+x$<br>$x \cdot y = y \cdot x$ |
| 4 | Distributive operation | $x(y+z) = x \cdot y + x \cdot z$<br>$x+y \cdot z = (x+y) \cdot (x+z)$ |
| 5 | unary operation | $x+x = x$<br>$x \cdot x = x$ |
| 6 | Presence of two elements | Consider two elements $x, y \in m$ such that $x \neq$ |

Boolean laws and properties

- Boolean laws are used to simplify and manipulate boolean expression

- The various boolean laws are:-

1) AND laws
2) OR laws
3) Complementation laws
4) Commutative laws
5) Distributive laws
6) Idempotente laws
7) Associative laws

3) Demorgan's Laws/ Dual Laws

1) AND Laws:-

· The four basic AND laws are as follows:-

i) $A \cdot 0 = 0$

ii) $A \cdot 1 = A$

iii) $A \cdot \bar{A} = 0$

iv) $A \cdot A = A$

2) OR Laws

· The basic laws of OR are as follows:-

i) $A + 0 = A$

ii) $A + 1 = 1$

iii) $A + \bar{A} = 1$

iv) $A + A = A$

3) Complementation laws:-

· The basic Complementation laws are as follows

i) If $A = 0$ then $\bar{A} = 1$

ii) If $A = 1$ then $\bar{A} = 0$

$\bar{\bar{A}} = A$ (Double Complementation form)

4) Commutative laws

· The basic Commutative laws are as follows

i) $A + B = B + A$

ii) $A \cdot B = B \cdot A$

5) Distributive laws

The basic distributive laws are as follows

i) ~~A+A=A~~ $A(B+C) = A\cdot B + A\cdot C$

ii) ~~A.A=A~~ $A + B\cdot C = (A+B)(A+C)$

6) Idempotence laws

• The term Idempotence represents same value

i) $A + A = A$

ii) $A \cdot A = A$

7) Associative laws:-

• The basic Associative laws are as follows

i) $A+(B+C) = (A+B)+C$

ii) $A\cdot(B\cdot C) = (A\cdot B)\cdot C$

8) Demorgan's laws:-

• The basic Demorgan's laws are as follows :-

i) $\overline{A+B} = \overline{A}\cdot\overline{B}$

ii) $\overline{AB} = \overline{A}+\overline{B}$

Interchanging of '+' to '.' and '.' to '+' can be referred as duality

Logic gates:-

The logic gates are basic building blocks of a digital system

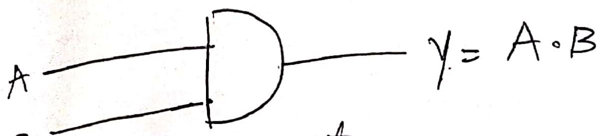
The basic logic gates are: AND, OR, NOT

→ In addition to this there are certain derived gates named as "NAND, NOR, X-OR, X-NOR".

→ NAND & NOR are called universal gates

## AND gate

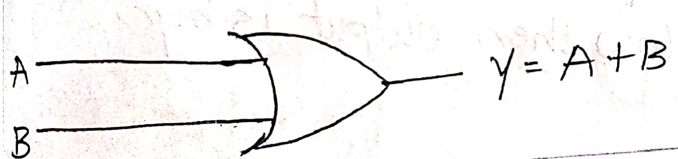
- An AND gate consists of two or more inputs and single output.
- If all the inputs are high then the output of AND gate is also high. If any of the inputs of AND gate is low then the output is also low.




AND gate

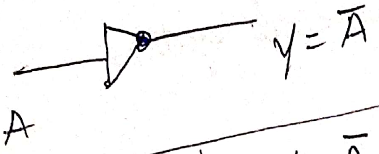
| A | B | $Y = A \cdot B$ |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 0 |
| 1 | 0 | 0 |
| 1 | 1 | 1 |

## OR gate:-




| A | B | $Y = A + B$ |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 1 |

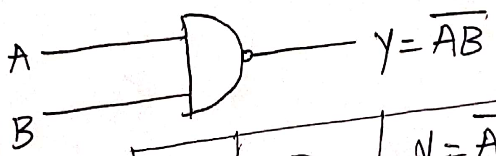
NOT gate.




| A | $Y = \bar{A}$ |
|---|---|
| 0 | 1 |
| 1 | 0 |

• If the input is high the output ~~high~~ low. If input is low the output is high.

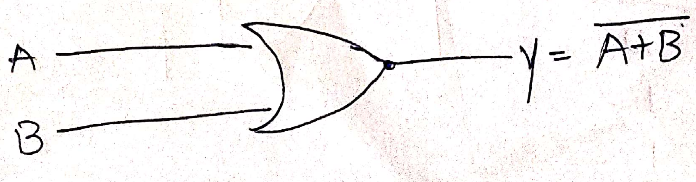
NAND gate. [AND + NOT]




| A | B | $Y = \overline{AB}$ |
|---|---|---|
| 0 | 0 | 1 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 0 |

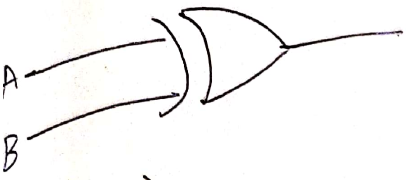
• If both the inputs are high then ~~the~~ output is low. If anyone of the input is low then output is high.

NOR gate [OR + NOT]



| A | B | $Y = A+B$ |
|---|---|---|
| 0 | 0 | 1 |
| 0 | 1 | 0 |
| 1 | 0 | 0 |
| 1 | 1 | 0 |

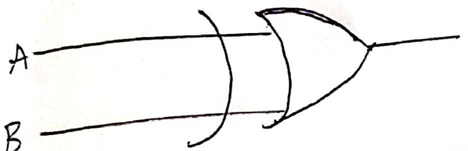
## XOR gate / EX-OR gate




$Y = A \oplus B$

$Y = \bar{A}B + A\bar{B}$

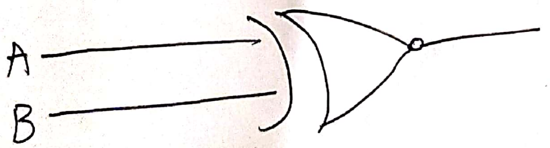
(or)




$Y = A \oplus B$

| A | B | $Y = A \oplus B$ |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 0 |

## X-NOR gate / EX-NOR gate [XOR + NOT]




$Y = A \odot B$

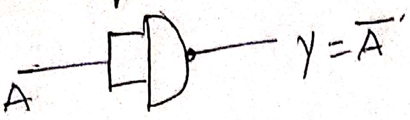
$Y = AB + \bar{A}\bar{B}$

| A | B | $Y = A \odot B$ |
|---|---|---|
| 0 | 0 | 1 |
| 0 | 1 | 0 |
| 1 | 0 | 0 |
| 1 | 1 | 1 |

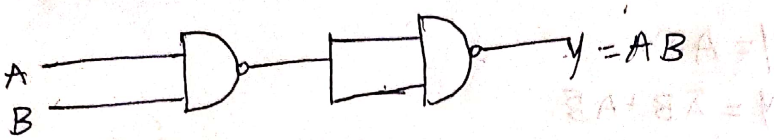
Note:- The gates NAND & NOR called universal gates

Implementation of logic gates using NAND

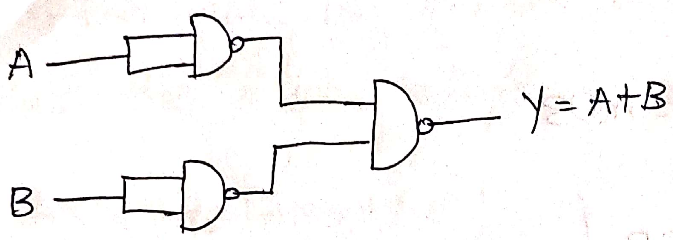
NOT gate:-



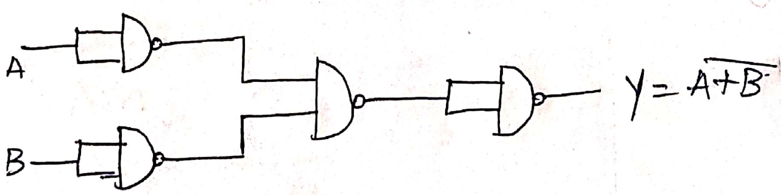

AND



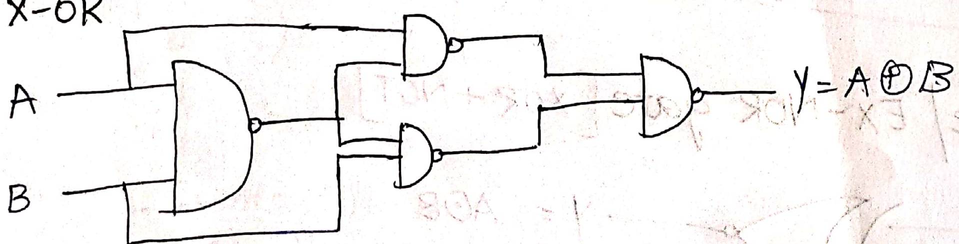

OR



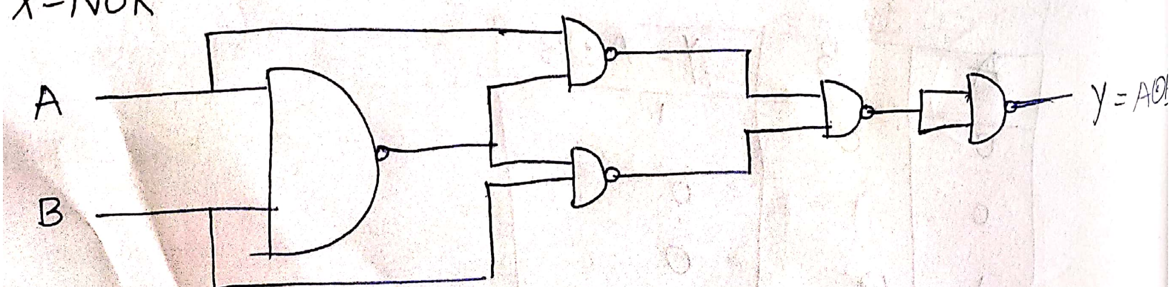

NOR




X-OR

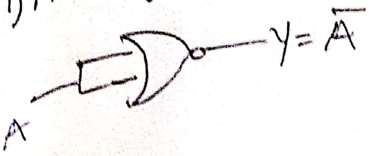



X-NOR

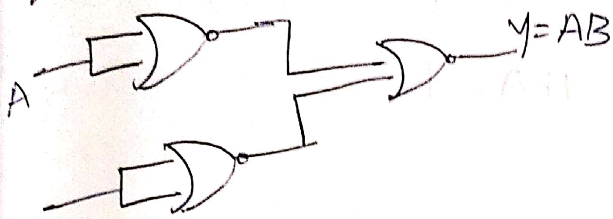


# Implementation of logic gates using NOR.

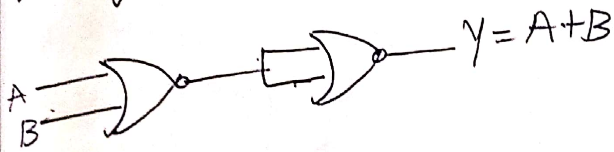
## 1) NOT gate



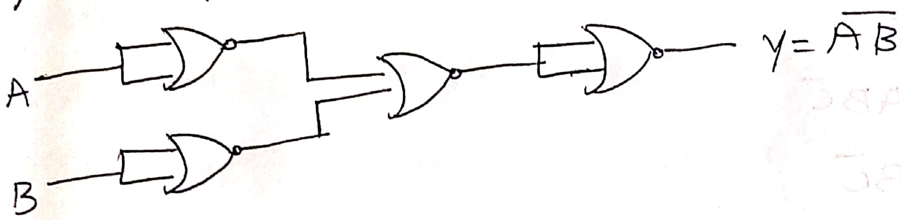

## 2) AND gate



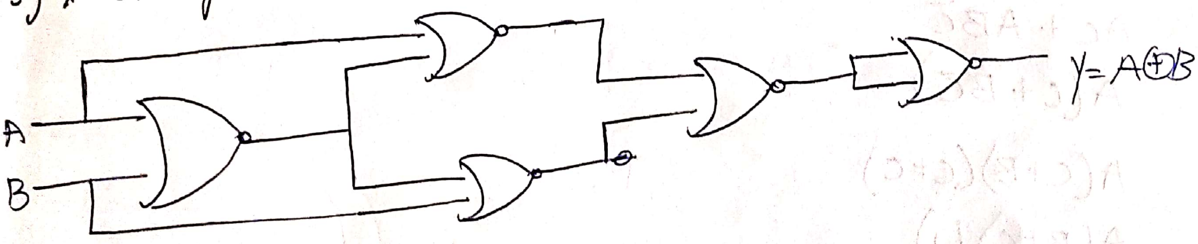

## 3) OR gate



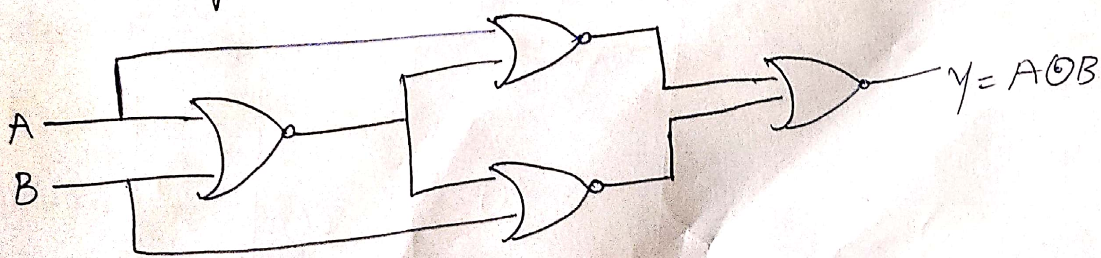

## 4) NAND gate




## 5) X-OR gate




## 6) X-NOR gate



Simplify the following using Boolean algebra

1) $(A+B)(A+\bar{B})$

2) $ABC+A\bar{B}C+AB\bar{C}$

3) $(A+B)(A+\bar{B})(\bar{A}+C)$

4) $AB+A(B+C)+B(B+C)$

5) $\bar{A}B(\bar{D}+\bar{C}D)+B(A+\bar{A}CD)$

$A \cdot A = A$

$A+A = A$

$A+\bar{A} = 1$

$A \cdot \bar{A} = 0$

$A+BC = (A+B)(A+C)$

$1+A = 1.$

1) $(A+B)(A+\bar{B})$

$(A \cdot A + A \cdot \bar{B} + B \cdot A + B \cdot \bar{B})$

$A + A(\bar{B}+B) + 0$

$A + A(1)$

$\Rightarrow A+A$

$= A$

2) $ABC+A\bar{B}C+AB\bar{C}$

$AC(B+\bar{B})+AB\bar{C}$

$AC(1)+AB\bar{C}$

$AC+AB\bar{C}$

$A(C+B\bar{C})$

$A(C+B)(C+\bar{C})$

$A(B+C)(1)$

$A(B+C)$

3) $(A+B)(A+\bar{B})(\bar{A}+C)$

$(A.A+\bar{A}.\bar{B}+B.A+B.\bar{B})(\bar{A}+C)$

$(A+A.\bar{B}+A.B+0)(\bar{A}+C)$

$(A+A(\bar{B}+B))(\bar{A}+C)$

$(A+A)(\bar{A}+C) \Rightarrow A(\bar{A}+C)$

$A.\bar{A}+A.C$

$0+AC \Rightarrow AC$

4) $AB+A(B+C)+B(B+C)$

$AB+AB+AC+B.B+BC$

$AB+AC+B+BC$

$AB+BC+AC+B$

$B(A+C)+AC+B$

(or)

$AB+AC+B(1+C)$

$AB+AC+B$

$B(A+1)AC$

$B+AC$

5) $\bar{A}B(\bar{D}+\bar{C}D)+B(A+\bar{A}CD)$

$=\bar{A}B\bar{D}+\bar{A}B\bar{C}D+AB+\bar{A}BCD$

$\bar{A}BD(\bar{C}+C)+\bar{A}B\bar{D}+AB$

$\bar{A}BD+\bar{A}B\bar{D}+AB$

$\bar{A}B(D+\bar{D})+AB$

$\bar{A}B+AB$

$B(\bar{A}+A)$

$B$

Reduce the following expressions

i) $F=A[B+\bar{C}(\overline{AB+AC})]$

ii) $F=A+B[AC+(B+\bar{C})\bar{D}]$

iii) $F=(\overline{A+B\bar{C}})[A\bar{B}+ABC]$

Show that $A\bar{B}C+B+B\bar{D}+AB\bar{D}+\bar{A}C=B+C$

i) $F = A[B+\overline{C}(\overline{AB+AC})]$

$= A(B+\overline{C}(\overline{AB}\cdot\overline{AC}))$

$= A[B+\overline{C}((\overline{A}+\overline{B})\cdot(\overline{A}+\overline{C}))$

$= A[B+\overline{C}(\overline{A}\cdot\overline{A}+\overline{A}\cdot C+\overline{A}\,\overline{B}+\overline{B}\cdot C)]$

$= A[B+\overline{C}[\overline{A}+\overline{A}\cdot C+\overline{A}\,\overline{B}+\overline{B}C]]$

$= A[B+\overline{A}\overline{C}+\overline{A}\cdot\overline{C}\cdot C+\overline{A}\overline{B}\overline{C}+\overline{B}C\overline{C}]$

$= A[B+\overline{A}\overline{C}+0+\overline{A}\,\overline{B}\overline{C}+0]$

$= A[B+\overline{A}\overline{C}+\overline{A}\,\overline{B}\,\overline{C}]$

$= AB+A\cdot\overline{A}\overline{C}+A\cdot\overline{A}\,\overline{B}\,\overline{C}$

$= AB$

ii) $F = A+B[AC+(B+\overline{C})D]$

$= A+B[AC+BD+\overline{C}D]$

$= A+ABC+B\cdot BD+B\overline{C}D$

$= A+ABC+BD+B\overline{C}D$

$= A+ABC+BD(1+C)$

$= A+ABC+BD$

$= A(1+BC)+BD$

$= A+BD$

iii) $F = (\overline{A+B\overline{C}})(A\cdot\overline{B}+ABC)$

$= (\overline{A}\cdot\overline{B\overline{C}})(A\overline{B}+ABC)$

$= \overline{A}\cdot\overline{B}C(A\overline{B}+ABC)$

$= \overline{A}B\cdot C(A\overline{B}+ABC)$

$= \overline{A}\overline{B}C\cdot A\overline{B}+ABC\cdot\overline{A}\overline{B}\overline{C}$

$= \overline{A}\cdot A\overline{B}\,\overline{B}\cdot C+A\cdot\overline{A}\cdot B\cdot\overline{B}C\overline{C}$

$= 0+0$

$= 0$

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Duality ~~of~~

Duality of the expression is obtained simply by interchanging + to • and • to +

write the duality of the following expressions

1) $(x\bar{y} + y\bar{z})_d \Rightarrow (x+\bar{y})(y+\bar{z})$

2) $(AB+C)D+\bar{E} \Rightarrow [(A+B)C+D](\bar{E})$

3) $(A+B+C)D+\bar{E} \Rightarrow (ABC+D)\bar{E}$

4) $(x+\bar{y}+z)(x+\bar{z}) \Rightarrow \{x\bar{y}z + x\bar{z}$

Find the complements of the following expressions:

1) $(x\bar{y} + y\bar{z})$

2) $(AB+C)D+\bar{E}$

3) $(A+B+C)D+\bar{E}$

4) $(x+\bar{y}+z)(x+\bar{z})$

$\overline{A+B} = \bar{A}.\bar{B}$

$\overline{AB} = \bar{A}+\bar{B}$

$\bar{\bar{A}} = A$

1) $(x\bar{y}+y\bar{z})$

The complement of $(x\bar{y}+y\bar{z})$ is $(\overline{x\bar{y}+y\bar{z}})$

$(\overline{x\bar{y}+y\bar{z}}) \Rightarrow (\overline{x\bar{y}}).\overline{y\bar{z}}$

$\Rightarrow (\bar{x}+\bar{\bar{y}}).(\bar{y}+\bar{\bar{z}})$

$\Rightarrow (\bar{x}+y).(\bar{y}+z)$

$\overline{A+B} = \bar{A}.\bar{B}$

$\overline{AB} = \bar{A}+\bar{B}$

$\bar{\bar{A}} = A$

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2) $(AB+C)D+\overline{E}$

The Complement of $\overline{(AB+C)D+\overline{E}}$

$\overline{(AB+C)D+\overline{E}} \Rightarrow \overline{(AB+C)D} \cdot \overline{\overline{E}}$

$\Rightarrow \left[\left(\overline{AB+C}\right)+\overline{D}\right]E$

$\Rightarrow \left[\overline{AB}\cdot\overline{C}+\overline{D}\right]E$

$\Rightarrow \left[\left(\overline{A}+\overline{B}\right)\cdot\overline{C}+\overline{D}\right]E$

Boolean function representation.

A Boolean function contains n number of variable represented as $f(x_1, x_2, x_3, \ldots x_n)$. The various ways to represent boolean function are.

1) Sum of products form (SOP) (or) disjunctive form
2) product of sum form (POS) (or) Conjunctive form
3) Standard sum of products form. (or) Canonical sum of product form
(or)
Canonical sum of product form
(or)
minterm form
4) Standard products of sum for
(or)
Canonical sum of product form
(or)
max terms
5) Truth table form
6) venn diagram
7) Karnaugh map (or) Kmap

Examples

1) $AB + BC + CA$ (SOP)

2) $(A+B)(B+C)(C+A)$ (POS)

3) $f = AB + B\bar{C}$

$f = AB(C+\bar{C}) + B\bar{C}(A+\bar{A})$

$f = ABC + \underline{AB\bar{C} + AB\bar{C}} + \bar{A}B\bar{C}$.

$= 111 + 110 + 010$

$= m_7 + m_6 + m_2 \Rightarrow \sum m(2,6,7)$

4) $f = (A+B)(A+\bar{C})(B+\bar{C})$

$f = (A+B+C\cdot\bar{C})(A+\bar{C}+B\cdot\bar{B})(B+\bar{C}+A\cdot\bar{A})$

$f = (A+B+C)(A+B+\bar{C})(A+B+\bar{C})(A+\bar{B}+\bar{C})(A+B+\bar{C})(\bar{A}+B+\bar{C})$

$= (000)(001)(011)(101)$

$f = M_0 \cdot M_1 \cdot M_3 \cdot M_5 \Rightarrow \prod M(0,1,3,5)$.

Express the boolean function ~~of~~ $f = \bar{a}b + \bar{b}c$ in standard sum of products form.

$f = \bar{a}b + \bar{b}c$.

$f = \bar{a}b(c+\bar{c}) + \bar{b}c(a+\bar{a})$

$= \bar{a}bc + \bar{a}b\bar{c} + a\bar{b}c + \bar{a}\bar{b}c$

$= 011 + 010 + 101 + 001$

$= m_3 + m_2 + m_5 + m_1$

$\Rightarrow \sum m(1,2,3,5)$

$\bar{a} = 0$

$a = 1$.

Express the boolean expression $f=(\bar{A}+\bar{B})(B+C)$ into standard product of sum

It is a three variable expression. A, B, C are the var

$$f=(\bar{A}+\bar{B})(B+C)$$

$\boxed{a+bc=(a+b)(a+}$

$$=(\underbrace{\bar{A}+\bar{B}}_{a}+\underbrace{C.\bar{C}}_{bc})(B+C+A.\bar{A})$$

$$=(\bar{A}+\bar{B}+C)(\bar{A}+\bar{B}+\bar{C})(B+C+A)(B+C+\bar{A})$$

$a=0$

$\bar{a}=1$

$$=(\bar{A}+\bar{B}+C)(\bar{A}+\bar{B}+\bar{C})(A+B+C)(\bar{A}+B+C).$$

$$f=(\bar{A}+\bar{B}+C)(\bar{A}+\bar{B}+\bar{C})(A+B+C)(\bar{A}+B+C)$$

$$f=(110)(111)(000)(100)$$

$$=M_6 \cdot M_7 \cdot M_0 \cdot M_4$$

$$F=\pi M(0,4,6,7)$$

Express $f=A+\bar{B}C$ as a sum of min terms.

Express $f=xy+\bar{x}z$ as a product of maxterms.

1) $f=A+\bar{B}C$

$$f=A(B+\bar{B})(C+\bar{C})+\bar{B}C(A+\bar{A})$$

$$=(AB+A\bar{B})(C+\bar{C})+\bar{B}CA+\bar{B}C\bar{A}$$

$$=ABC+ABC̄+\underline{A\bar{B}C}+A\bar{B}\bar{C}+\underline{A\bar{B}C}+\bar{A}\bar{B}C.$$

$$=111+110+\underline{101}+100+\cancel{101}+001$$

$$=m_7+m_6+m_5+m_4+m_1$$

$$=\sum m(1,4,5,6,7)$$

2) $f=\underbrace{xy}_{A}+\underbrace{\bar{x}z}_{BC}$ $\quad A+BC=(A+B)(A+C)$

$$f=(xy+\bar{x})(xy+z)$$

$$f=(x+\bar{x})(\bar{x}+y)(x+z)(y+z)$$

$$f=(\bar{x}+y)(x+z)(y+z).$$

$$f=(\bar{x}+y+z.\bar{z})(x+z+y.\bar{y})(y+z+x.\bar{x})$$

$$f=(\underline{\bar{x}+y+z})(\bar{x}+y+\bar{z})(x+y+z)(x+\bar{y}+z)(x+y+z)(\underline{\bar{x}+y+z})$$

$$f=(\bar{x}+y+z)(\bar{x}+y+z)(x+y+z)(x+\bar{y}+z)(x+y+z).$$

$f = (100)(101)(000)(010)$
$f = M_4 \cdot M_5 \cdot M_0 \cdot M_2 \Rightarrow \pi M(0,2,4,5)$

Conversion between canonical forms

The conversion between canonical form can be obtained simply by complementing

$\overline{SOP} = POS$

$\overline{POS} = SOP$

Note:- The minterms complement is equivalent to max terms i.e $(\overline{m_j} = M_j)$

Reduce the Expression. $f = \sum m(0,2,4,6,7)$ in Product of max terms.

given $f = \sum m(0,2,4,6,7)$ it is a 3 variable expression (A,B,C)

the minterms given are $m_0, m_2, m_4, m_6, m_7$

$\neq \pi M(1,3,5)$

$m_0 = 000 = \bar{A}\bar{B}\bar{C}$

$m_2 = 010 = \bar{A}B\bar{C}$

$m_4 = 100 = A\bar{B}\bar{C}$

$m_6 = 110 = AB\bar{C}$

$m_7 = 111 = ABC$

As per the rule $\overline{m_j} = M_j$

$f = \sum(0,2,4,6,7) = \pi M(1,3,5)$

$M_1 = 001 \Rightarrow AB\bar{C}$

$M_3 = 011 \Rightarrow A\bar{B}\bar{C}$

$M_5 = 101 \Rightarrow \bar{A}B\bar{C}$

Express $\overline{A}+\overline{B}$ into ~~mean~~

$f = \overline{A}+\overline{B}$

It is in Sop form (2 Variables)

minterms.

$f = \overline{A}(B+\overline{B}) + \overline{B}(A+\overline{A})$

$f = \overline{A}B + \overline{A}\overline{B} + A\overline{B} + \overline{A}\overline{B}$

$f = \overline{A}B + \overline{A}\overline{B} + A\overline{B}$

$f = 01 + 00 + 10$

$f = m_1 + m_0 + m_2$

$f = \sum m(0,1,2)$

maxterms. $= \prod M(3)$

Express the expression $f = (A+B)(A+\overline{C})(\overline{B}+\overline{C})$ into max terms and minterms.

$f = (A+B)(A+\overline{C})(\overline{B}+\overline{C})$

maxterms

$f = (A+B+C\cdot\overline{C})(A+\overline{C}+B\cdot\overline{B})(\overline{B}+\overline{C}+A\cdot\overline{A})$

$f = (A+B+C)(A+B+\overline{C})(A+B+\overline{C})(A+\overline{B}+\overline{C})(A+\overline{B}+\overline{C})(\overline{A}+\overline{B}+\overline{C})$

$f = (A+B+C)(A+B+\overline{C})(A+\overline{B}+\overline{C})(\overline{A}+\overline{B}+\overline{C})$

$= (000)(001)(011)(111)$

$f = M_0 \cdot M_1 \cdot M_3 \cdot M_7$

$f = \prod M(0,1,3,7)$

minterms

$f = \sum m(2,4,5,6)$

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Express the following in minterm and maxterms

i) $A+\bar{B}C$

ii) $A+\bar{B}C+AB\bar{D}+ABCD$

iii) $(A+\bar{B})(\bar{B}+\bar{C}+A)(A+\bar{C})$

iv) $A\bar{B}+B\bar{C}$

v) $A(\bar{B}+A)B$

i) $A+\bar{B}C$

minterms.

$f = A(B+\bar{B})(C+\bar{C}) + \bar{B}C(A+\bar{A})$

$f = (AB+A\bar{B})(C+\bar{C}) + A\bar{B}C + \bar{A}\bar{B}C$

$= ABC + AB\bar{C} + A\bar{B}C + \bar{A}\bar{B}C + A\bar{B}C + A\bar{B}\bar{C}$

$= ABC + AB\bar{C} + A\bar{B}C + A\bar{B}\bar{C} + A\bar{B}C + \bar{A}\bar{B}C$

$= 111 + 110 + 101 + 100 + 001$

$= ABC + AB\bar{C} + A\bar{B}C + \bar{A}BC + A\bar{B}\bar{C} + A\bar{B}C$

$= ABC + AB\bar{C} + A\bar{B}C + A\bar{B}\bar{C} + \bar{A}\bar{B}C$

$= 111 + 110 + 101 + 100 + 001$

$= m_7 + m_6 + m_5 + m_4 + m_1$

$f = \sum m(1,4,5,6,7)$

maxterms $= f\prod M(0,2,3)$

Implementation of Boolean expression using unive[rsal]
gates

i Implementation of boolean expression using NAND gate.

Step-1 :- ~~Draw~~ Write the expression in the Sop form

Step-2 :- Draw AND OR logic diagram

Step-3 :- Place a bubble at the output of AND gate an[d] inputs of OR gate

Step-4 :- Add an inverter (NOT) where the bubble is place[d]

Step-5 :- Remove the ~~Consea~~ Consecutive NOT gates

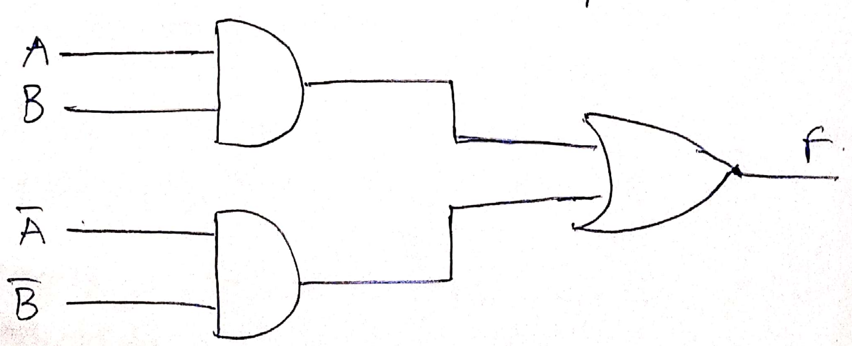
Step-6 :- Replace bubbled OR gate with NAND.

Step-7 :- Represent the Complement input in NAND

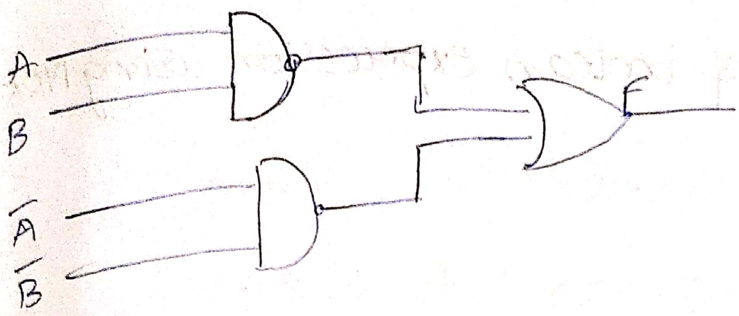
Implement the following boolean expression $f = AB + \bar{A}\bar{B}$ using ~~Nand~~ NAND gate.

Step-1 :- Sop form $f = AB + \bar{A}\bar{B}$

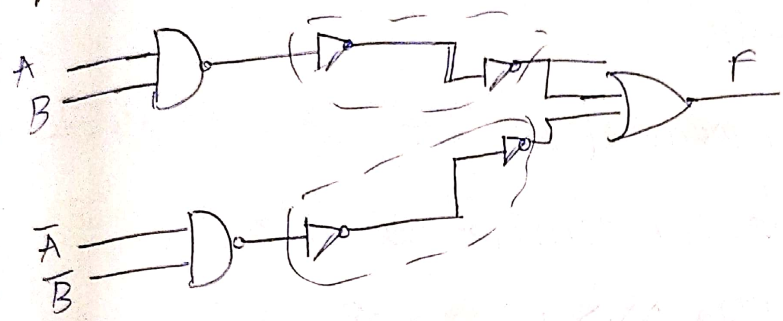
Step-2 :- Draw AND-OR logic diagram



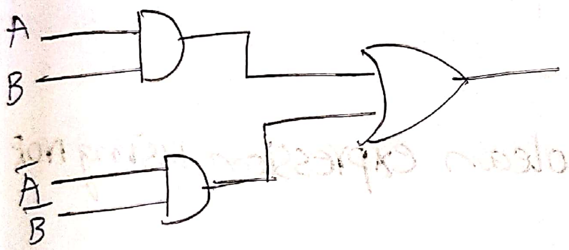
step:-3 :- place a bubble at the output of AND gate and inputs of OR



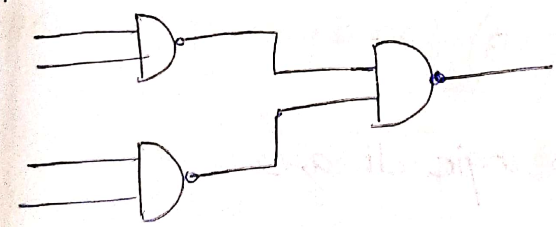

step-4:-



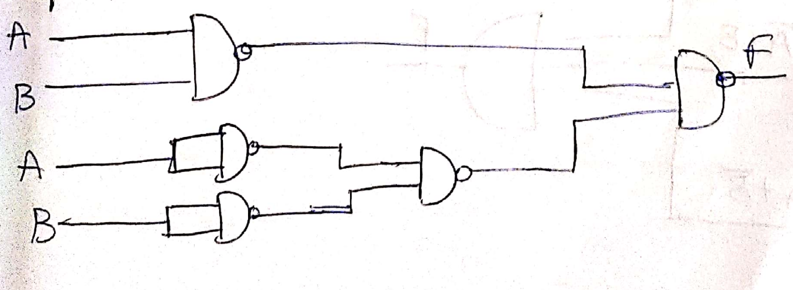

step-5:-




step:-6:-



step:-7



Implement the expression ... using
universal gates (AND)

Implementation of boolean expression using NOR

Step 1:- Write the expression in POS form

Step 2:- Draw the OR-AND logic diagram

Step 3:- Place the bubble at the O/P of OR gate and inputs of AND gate.

Step 4:- Add an inverter (NOT gate) after the bubble

Step 5:- Remove the Consecutive NOT gate.

Step 6:- Replace bubble AND with NOR gate.

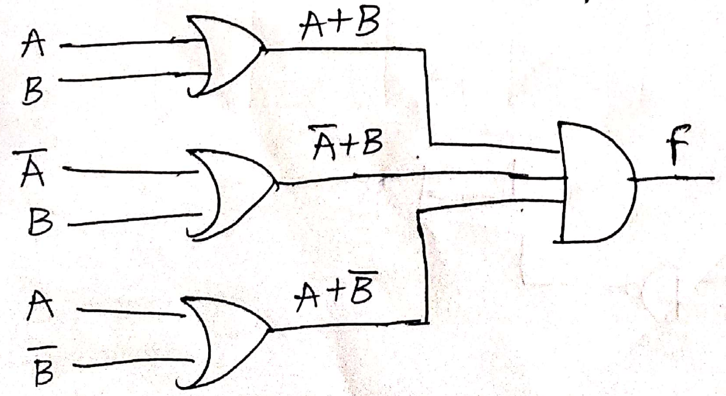
Step 7:- Represent the Complement input with single NOR gate.

Express the following boolean expression using NOR gate.

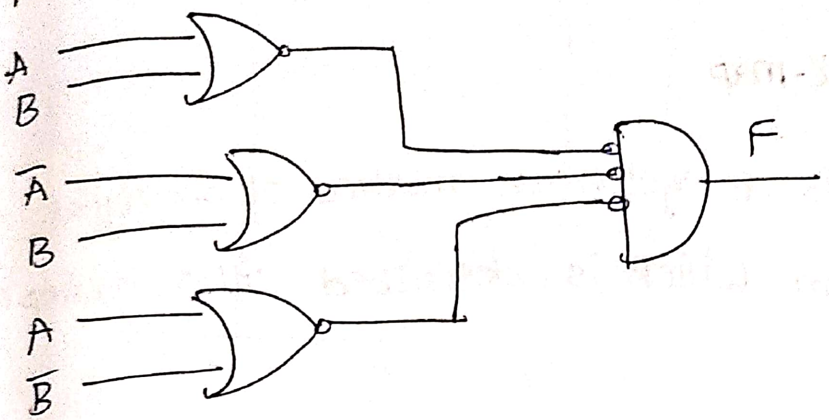
$f = (A+B)(\bar{A}+B)(A+\bar{B})$

Step-1:- POS form $(A+B)\ (\bar{A}+B)\ (A+\bar{B})$

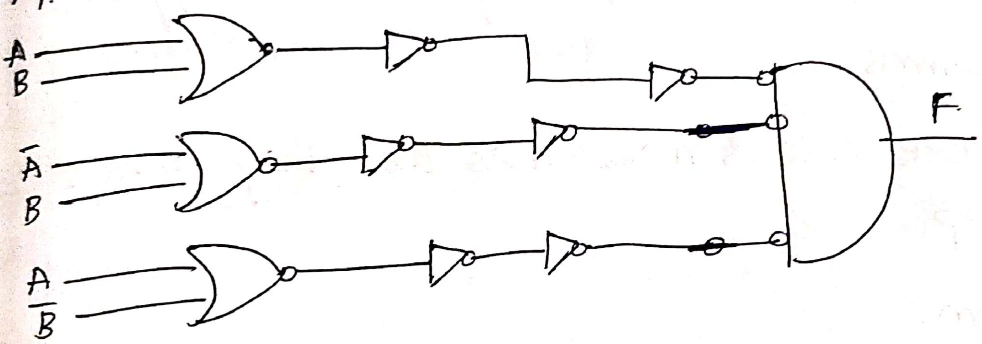
Step-2:- Draw the OR-AND logic diagram.



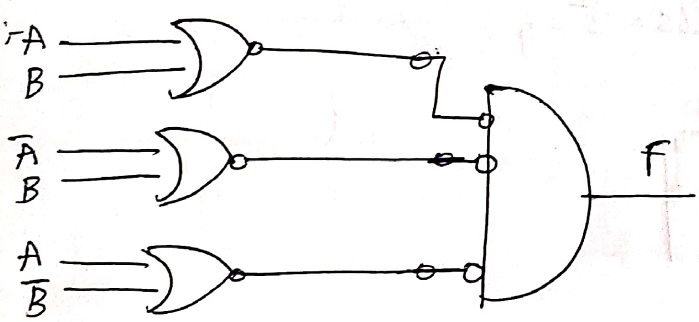
step-3:- Place a bubble at the output of OR gate and inputs of AND gate.



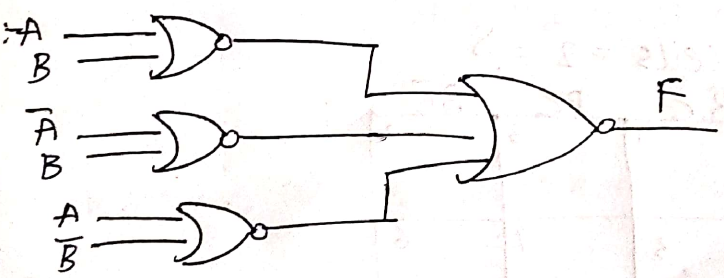

step:-4.



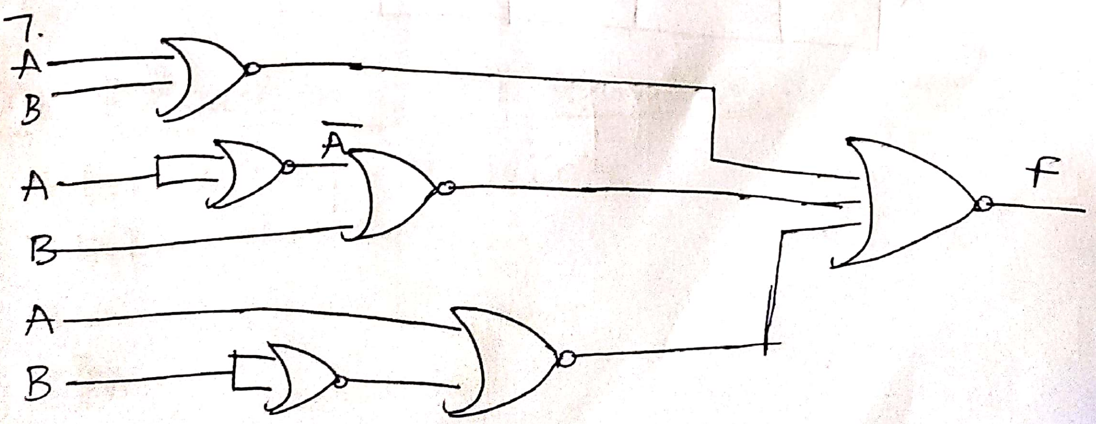

step-5:-




step 6 :-




step:- 7.



Implement the expression $f = (A+B)(A+\bar{B}+C)$ u

NOR

, Karnaugh map / K-map

Karnaugh map is a systematic method of simplified boolean expression which is described either in Sop or Pos forms.

K-map makes use of gray code to specify the rows and columns.

If there are n variables then they will be $2^n$ cells in K-map

Sop form

$n = 2$ ; No. of cells $= 2^2 = 4$.

(A, B)

| A \ B | $\bar{B}$ | B |
|---|---|---|
| $\bar{A}$ | 0 | 1 |
| A | 2 | 3 |

$n = 2$ ; no. of cells $= 2^3 = 8$

(A, B, C)

| A \ BC | $\bar{B}\bar{C}$ | $\bar{B}C$ | BC | $B\bar{C}$ |
|---|---|---|---|---|
| $\bar{A}$ | 0 | 1 | 3 | 2 |
| A | 4 | 5 | 7 | 6 |

$n=4$; No. of cells :- $2^4 = 16$ cells
$(A,B,C,D)$

| $AB \backslash CD$ | $\bar C\bar D$ | $\bar C D$ | $CD$ | $C\bar D$ |
|---|---|---|---|---|
| $\bar A\bar B$ | 0 | 1 | 3 | 2 |
| $\bar A B$ | 4 | 5 | 7 | 6 |
| $AB$ | 12 | 13 | 15 | 14 |
| $A\bar B$ | 8 | 9 | 11 | 10 |

$n=5$; No. of cells :- $2^5 = 32$ cells.
$(A,B,C,D,E)$

$\bar E$

| $AB \backslash CD$ | $\bar C\bar D$ | $\bar C D$ | $CD$ | $C\bar D$ |
|---|---|---|---|---|
| $\bar A\bar B$ | 0 | 1 | 3 | 2 |
| $\bar A B$ | 4 | 5 | 7 | 6 |
| $AB$ | 12 | 13 | 15 | 14 |
| $A\bar B$ | 8 | 9 | 11 | 10 |

$E$

| $AB \backslash CD$ | $\bar C\bar D$ | $\bar C D$ | $CD$ | $C\bar D$ |
|---|---|---|---|---|
| $\bar A\bar B$ | 16 | 17 | 19 | 18 |
| $\bar A B$ | 20 | 21 | 23 | 22 |
| $AB$ | 28 | 29 | 31 | 30 |
| $A\bar B$ | 24 | 25 | 27 | 26 |

Note:- 1) The minterms (or) SOP terms are denoted by 1 in K-map

2) The maxterms (or) POS terms are denoted by 0 in K-map

## K-map construction for POS terms (or) maxterms

$n=2$; no. of cells $= 2^2 = 4$ cells

| $A \backslash B$ | $B$ | $\bar B$ |
|---|---|---|
| $A$ | | |
| $\bar A$ | | |

$n = 3$, no. of cells $= 2^3 = 8$ cells.

A, B, C

| $A \backslash BC$ | $B+C$ | $B+\bar{C}$ | $\bar{B}+\bar{C}$ | $\bar{B}+C$ |
|---|---|---|---|---|
| $A$ | 0 | 1 | 3 | 2 |
| $\bar{A}$ | 4 | 5 | 7 | 6 |

$n = 4$; no. of cells $= 2^4 = 16$ cells

A, B, C, D

| $AB \backslash CD$ | $C+D$ | $C+\bar{D}$ | $\bar{C}+\bar{D}$ | $\bar{C}+D$ |
|---|---|---|---|---|
| $A+B$ | 0 | 1 | 3 | 2 |
| $A+\bar{B}$ | 4 | 5 | 7 | 6 |
| $\bar{A}+\bar{B}$ | 12 | 13 | 15 | 14 |
| $\bar{A}+B$ | 8 | 9 | 11 | 10 |

- The grouping of terms in a K-map can be done in the following ways

1) Octet
2) Quad
3) Pair
4) Single

Reduce or simplify the following Boolean expression using K-MAP

1) $f(A,B) = \bar{A}\bar{B} + \bar{A}B + AB$

Given expression is $f(A,B) = \bar{A}\bar{B} + \bar{A}B + AB$

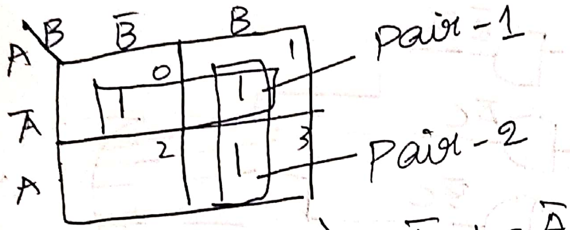
It is in Sop form.

minterms $= \bar{A}\bar{B} + \bar{A}B + AB$

$= 00 + 01 + 11$

$= \sum m(0, 1, 3)$

$f(A,B) = \sum m(0,1,3)$




Pair-1 $= \bar{A}(B+\bar{B}) = \bar{A}\cdot 1 = \bar{A}$

Pair-2 $= B(A+\bar{A}) = B \cdot 1 = B$

$f = \bar{A} + B$

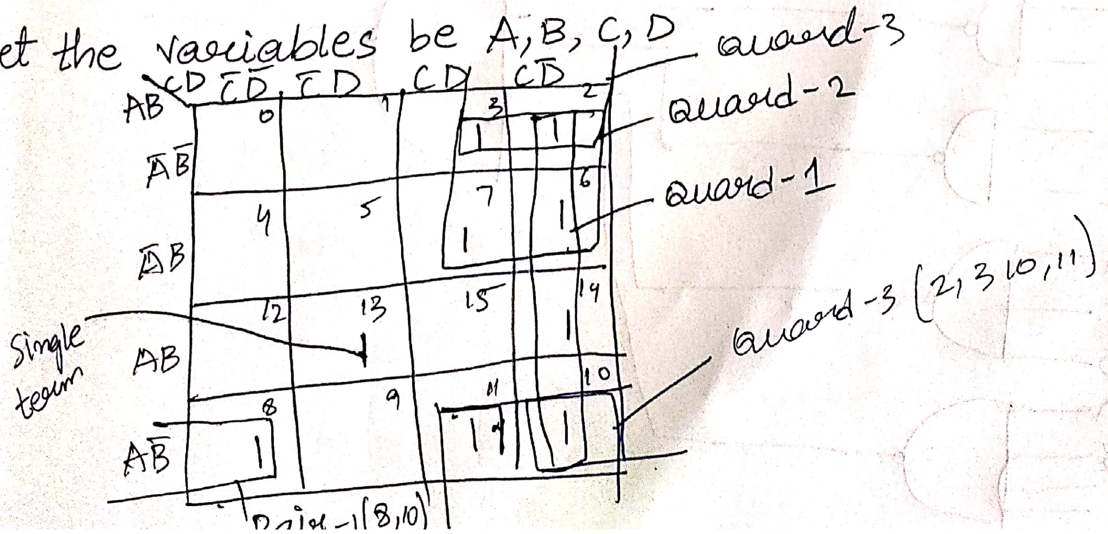
2) $F = \sum m(2, 3, 6, 7, 8, 10, 11, 13, 14)$ using K-map

Given boolean Expression $f = \sum m(2, 3, 6, 7, 8, 10, 11, 13, 14)$

It is in minterms ~~of~~ form

The above expression contain 4 variable.

Let the variables be A, B, C, D



Quard-1 :- $C\bar{D}$

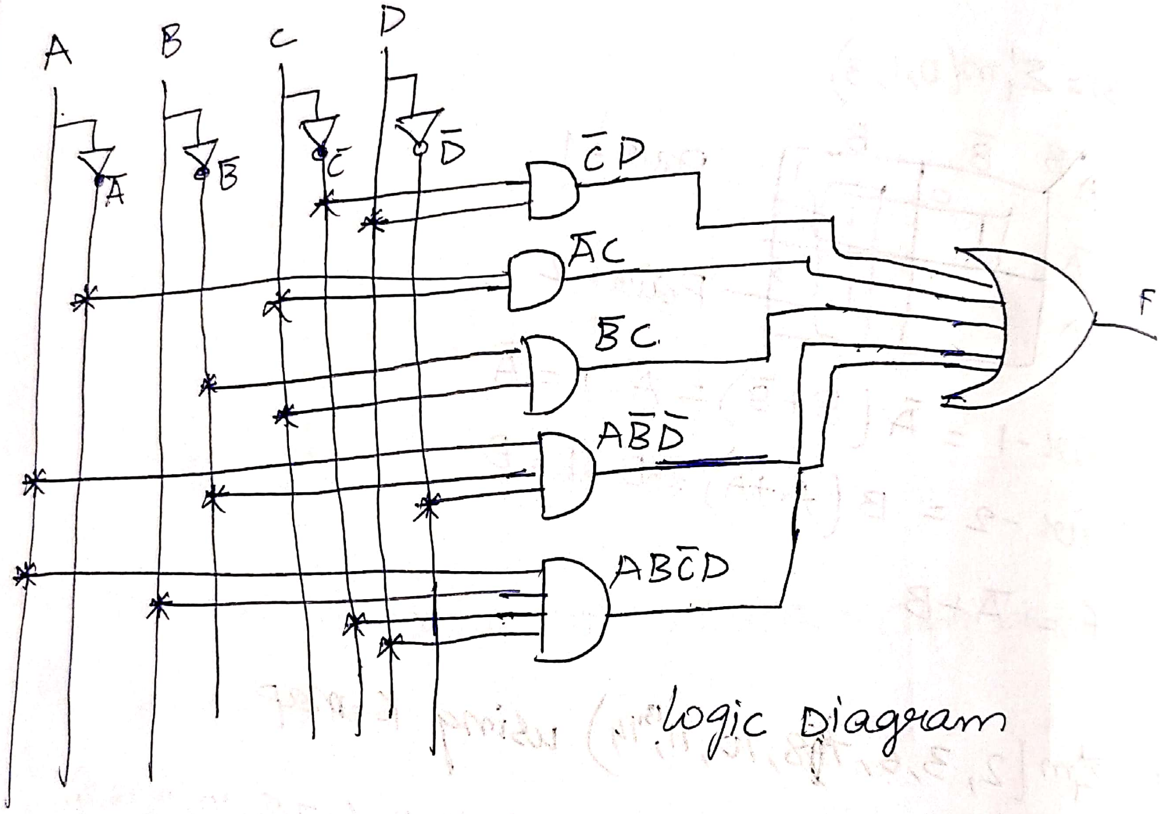
Quard-2 :- $\bar{A}C$

Quard-3 :- $\bar{B}C$

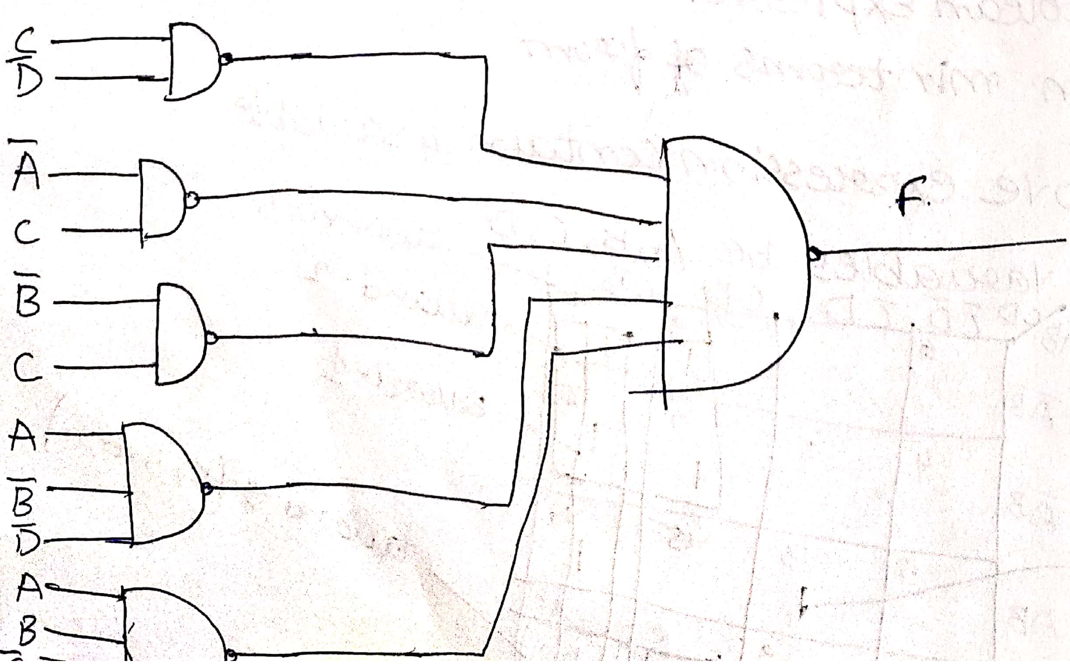
Pair 1 :- $A\bar{B}\bar{D}$

single :- $AB\bar{C}D$

$\therefore F = \bar{C}D + \bar{A}C + \bar{B}C + A\bar{B}\bar{D} + AB\bar{C}D$

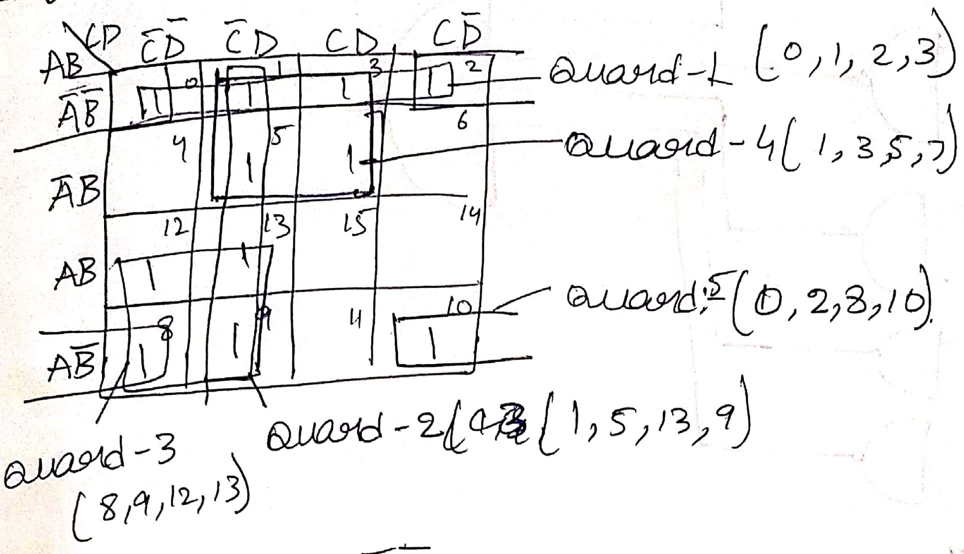



logic Diagram



Simplify the following boolean expression using K-map

1) $f = \Sigma m(0,1,2,3,5,7,8,9,10,12,13)$




quard 1 :- $\bar{A}\bar{B}(1) = \bar{A}\bar{B}$

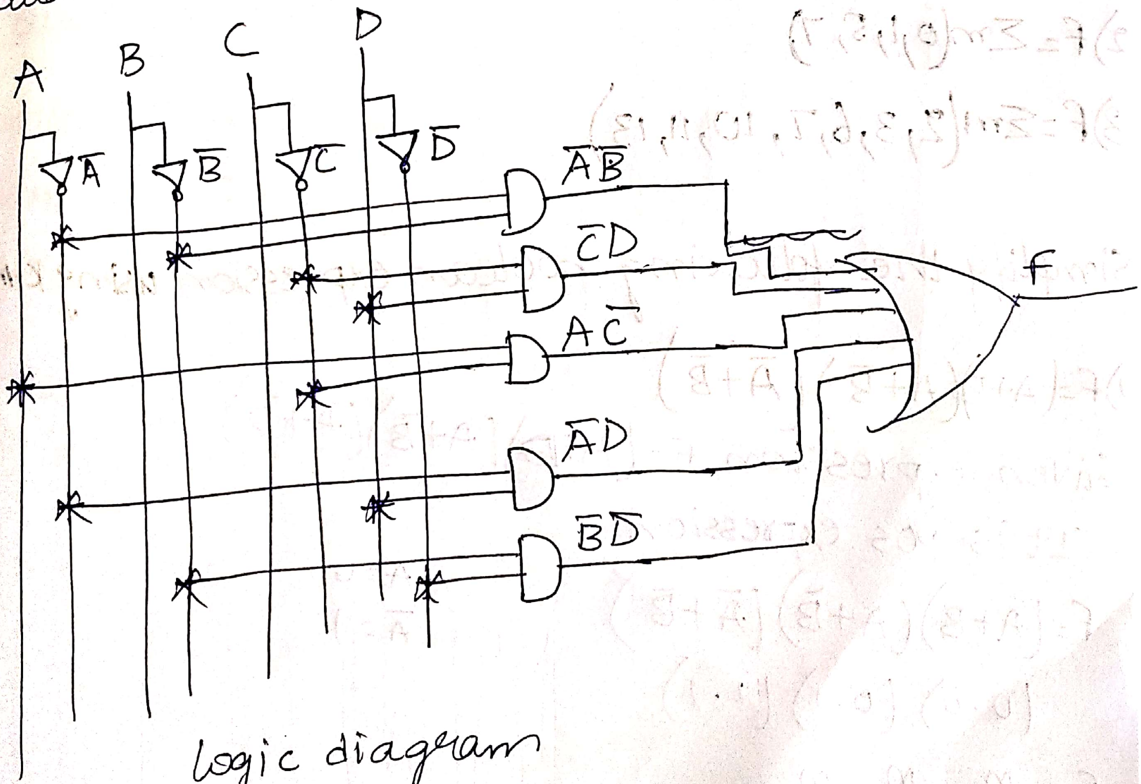
quard 2 :- $\bar{C}D$

quard 3 :- $A\bar{C}$

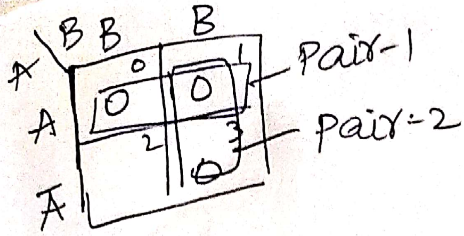
quard 4 :- $\bar{A}D$

quard 5 :- $\bar{B}\bar{D}$

$f = \bar{A}\bar{B} + \bar{C}D + A\bar{C} + \bar{A}D + \bar{B}\bar{D}$




logic diagram

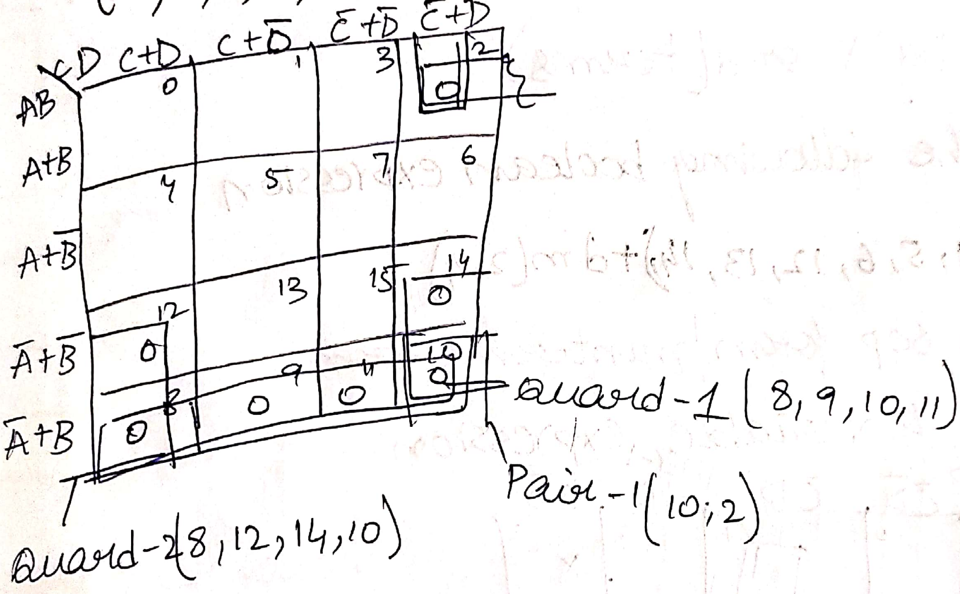



pair 1 = $A$

pair 2 = $\overline{B}$

$F = A \cdot \overline{B}$

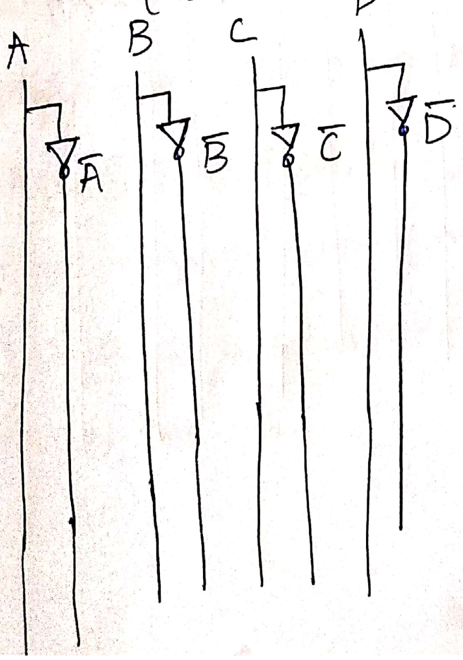
2) $f = \pi M(2, 8, 9, 10, 11, 12, 14)$




quard-1 = $\overline{A} + B$

quard-2 = $\overline{A} + D$

Pair-1 $(B + \overline{C} + D)$



2) $F = \pi M(0, 2, 8, 9, 10, 11, 15)$

3) $F = \pi M(0, 3, 7, 8, 9, 10, 11, 15)$

4) $F = \pi M(2, 7, 9, 10, 11, 12, 14, 15)$

Don't care Condition :- The Combinations for which the values can't be specified are called Don't care Condition (or) optional ~~Conditiad~~ Condition. It is denoted by 'X' or d(terms)

Simplify the following boolean expression

1) $f = \Sigma m(1, 5, 6, 12, 13, 14) + d\,m(2, 4)$.

It is in Sop form / minterms form.

It is a 4 variable expression.

| AB \ CD | $\bar{C}\bar{D}$ | $\bar{C}D$ | $CD$ | $C\bar{D}$ |
|---|---|---|---|---|
| $\bar{A}\bar{B}$ | (0) | 1 (1) | (3) | X (2) |
| $\bar{A}B$ | X (4) | 1 (5) | (7) | 1 (6) |
| $AB$ | 1 (12) | 1 (13) | (15) | 1 (14) |
| $A\bar{B}$ | (8) | (9) | (11) | (10) |

pair-1

Quard-1

quard-2 (4, 12, 6, 14)

Quard-1 = $B\bar{C}$

Quard-2 = $B\bar{D}$

Pair 1 = $\bar{A}\bar{C}D$

$f = B\bar{C} + B\bar{D} + \bar{A}\bar{C}D$

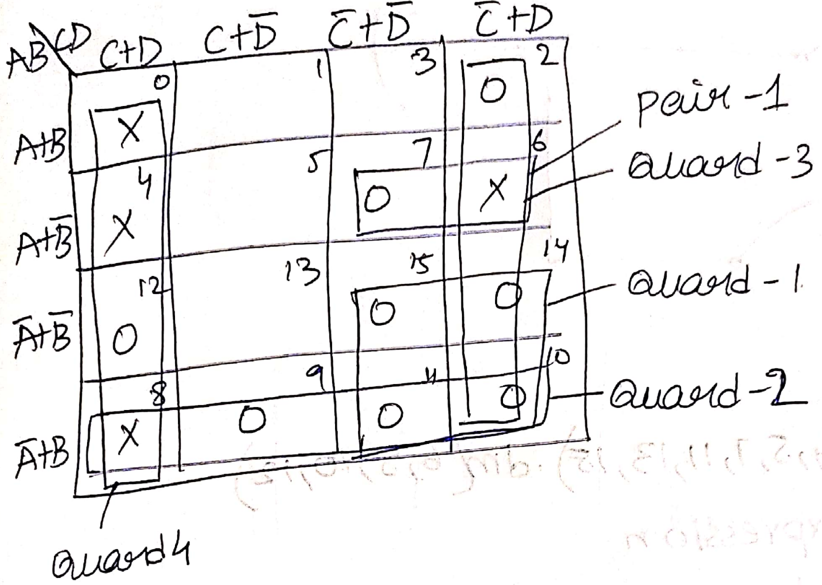
simplify following expressions

1) $f = \pi M(2,7,9,10,11,12,14,15) \cdot dm(0,4,6,8)$

given expression.

$f = \pi M(2,7,9,10,11,12,14,15)\, dm(0,4,6,8)$

The above expression contains 4 variables (A,B,C,D)




quard 1 :- $\bar{A}+\bar{C}$

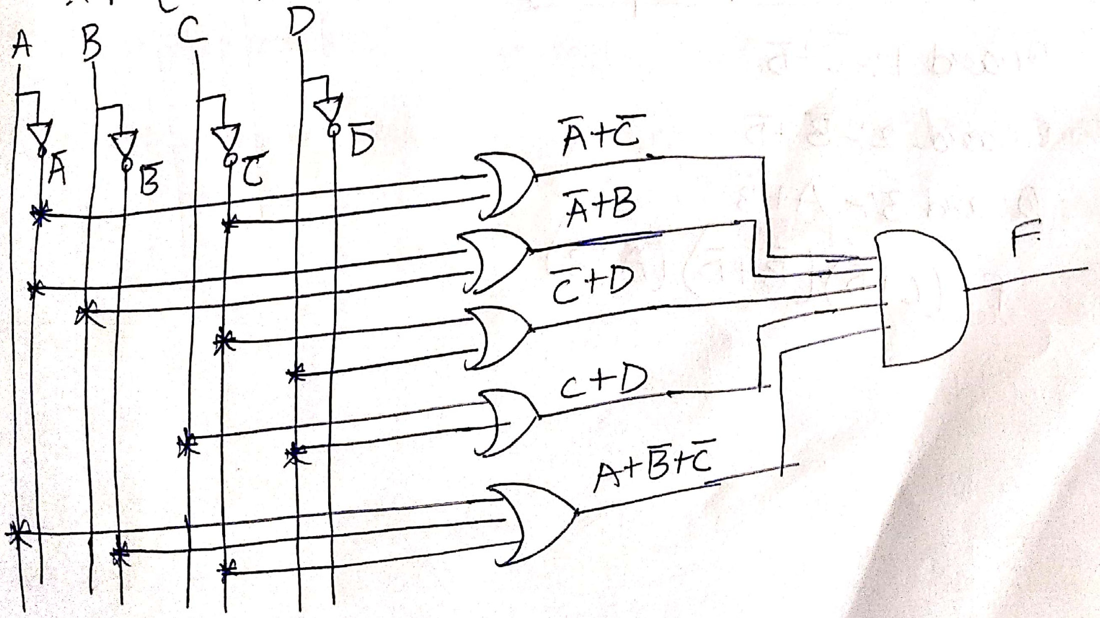
quard 2 :- $\bar{A}+B$

quard 3 :- $\bar{C}+D$

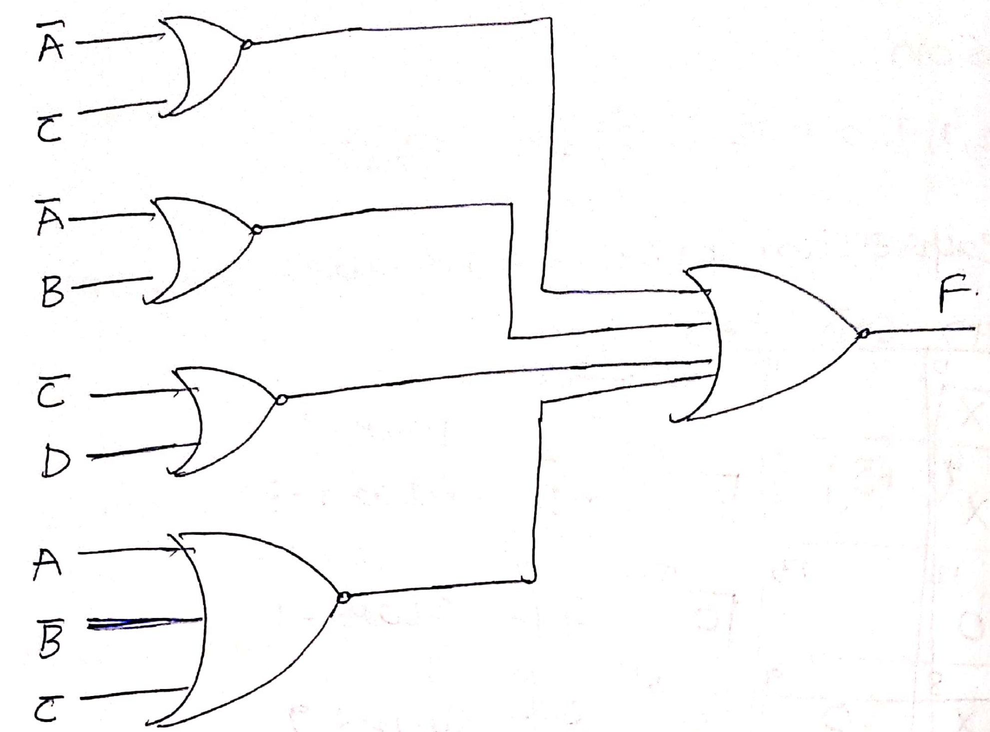
quard 4 :- $C+D$

Pair 1 :- $A+\bar{B}+\bar{C}$

$\therefore f = (\bar{A}+\bar{C})(\bar{A}+B)(\bar{C}+D)(C+D)(A+\bar{B}+\bar{C})$



NOR gate




2) $f = \pi m(3,4,5,7,11,13,15) \cdot dm(6,8,10,12)$

given expression

$f = \pi m(3,4,5,7,11,13,15) \cdot dm(6,8,10,12)$ we use 4 variables

| AB \ CD | $C+D$ | $C+\bar{D}$ | $\bar{C}+\bar{D}$ | $\bar{C}+D$ |
|---|---|---|---|---|
| $A+B$ | [0] | [1] | [3] 0 | [2] |
| $A+\bar{B}$ | [4] 0 | [5] 0 | [7] 0 | [6] X |
| $\bar{A}+\bar{B}$ | [12] X | [13] 0 | [15] 0 | [14] |
| $\bar{A}+B$ | [8] X | [9] | [11] 0 | [10] X |

Quard-1

Quard-2

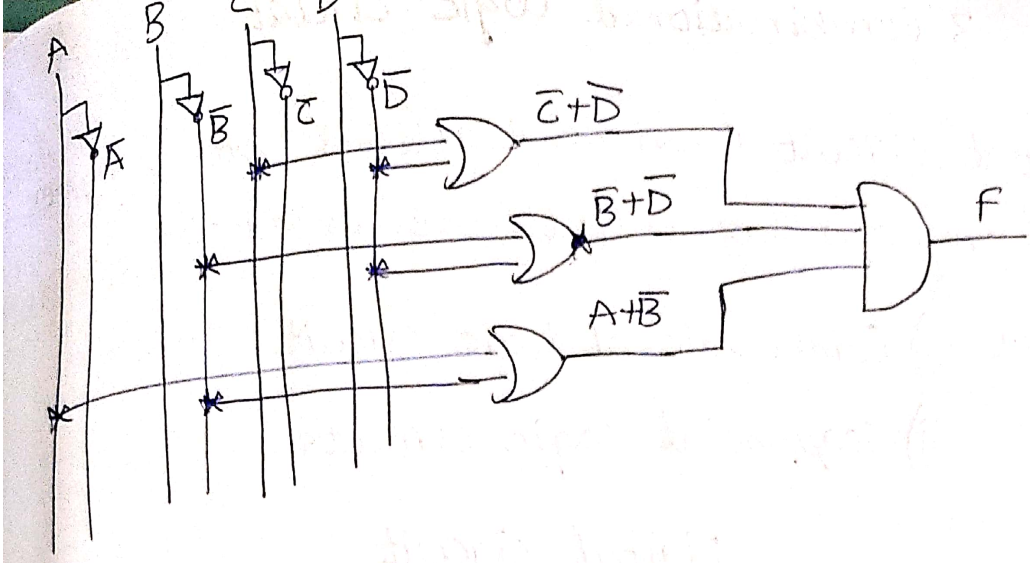
Quard-3

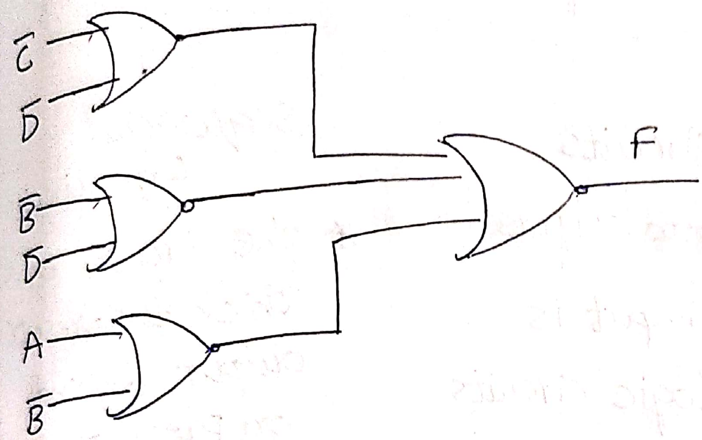
Quard 1 :- $\bar{C}+\bar{D}$

Quard 2 :- $\bar{B}+\bar{D}$

Quard 3 :- $A+\bar{B}$

$F = (\bar{C}+\bar{D})(\bar{B}+\bar{D})(A+\bar{B})$




unit-II

1) K-map (SOP, POS

2) Canonical SOP/POS

3) NAND/NOR Implementation of Boolean expressions

4) B.E reduction by laws.

unit-I

1) Conversions

2) Binary subtraction

3) Hamming code.

4) Gray code

5) BCD, Excess-3

6) Floating point representation

7) Representing of -ve numbers.

unit-III

1) HA, FA

2) HS, FS

3) BA, BS, BAS.