

Find analytical and numerical (Five-diagonal matrix method) solutions of one-dimensional Poisson equation

$$\frac{\partial^2 P}{\partial x^2} = -f(x); x \in (0,1)$$

with the following conditions:

$$1. \quad \begin{aligned} f(x) &= x^2 + 2x + 3; \\ P(x=0) &= 1 \quad P(x=1) = 0 \end{aligned}$$

$$2. \quad \begin{aligned} f(x) &= \sin(x); \\ P(x=0) &= 1 \quad P(x=1) = 0 \end{aligned}$$

$$3. \quad \begin{aligned} f(x) &= e^x; \\ P(x=0) &= 1 \quad P(x=1) = 0 \end{aligned}$$

$$4. \quad \begin{aligned} f(x) &= x^2 \sin(\pi x); \\ P(x=0) &= 1 \quad P(x=1) = 0 \end{aligned}$$

$$5. \quad \begin{aligned} f(x) &= e^x \cos(x); \\ P(x=0) &= 1 \quad P(x=1) = 0 \end{aligned}$$

$$6. \quad \begin{aligned} f(x) &= 2x^2 + 5x + 3; \\ P(x=0) &= 0 \quad P(x=1) = 1 \end{aligned}$$

$$7. \quad \begin{aligned} f(x) &= \sin^2(x); \\ P(x=0) &= 0 \quad P(x=1) = 1 \end{aligned}$$

$$8. \quad \begin{aligned} f(x) &= xe^x; \\ P(x=0) &= 0 \quad P(x=1) = 1 \end{aligned}$$

$$9. \quad \begin{aligned} f(x) &= x^2 \cos(2\pi x); \\ P(x=0) &= 0 \quad P(x=1) = 1 \end{aligned}$$

$$10. \quad \begin{aligned} f(x) &= e^x \sin(x); \\ P(x=0) &= 0 \quad P(x=1) = 1 \end{aligned}$$

$$11. \quad \begin{aligned} f(x) &= x^2 + 5x + 13; \\ P_x(x=0) &= 0 \quad P(x=1) = 1 \end{aligned}$$

$$12. \quad \begin{aligned} f(x) &= \sin^2(x) + \cos(x); \\ P_x(x=0) &= 0 \quad P(x=1) = 1 \end{aligned}$$

$$13. \begin{aligned} f(x) &= e^x + x^3; \\ P_x(x=0) &= 0 \quad P(x=1) = 1 \end{aligned}$$

$$14. \begin{aligned} f(x) &= (x^2 + 3) \cos\left(\frac{\pi x}{2}\right); \\ P_x(x=0) &= 0 \quad P(x=1) = 1 \end{aligned}$$

$$15. \begin{aligned} f(x) &= e^x \sin(\pi x); \\ P_x(x=0) &= 0 \quad P(x=1) = 1 \end{aligned}$$

$$16. \begin{aligned} f(x) &= 5x^2 + x + 3; \\ P(x=0) &= 1 \quad P_x(x=1) = 0 \end{aligned}$$

$$17. \begin{aligned} f(x) &= \sin(x) + 2 \cos(x); \\ P(x=0) &= 1 \quad P_x(x=1) = 0 \end{aligned}$$

$$18. \begin{aligned} f(x) &= e^x + x^3 + 2; \\ P(x=0) &= 1 \quad P_x(x=1) = 0 \end{aligned}$$

$$19. \begin{aligned} f(x) &= (x^2 + 3x + 2) \sin\left(\frac{\pi x}{2}\right); \\ P(x=0) &= 1 \quad P_x(x=1) = 0 \end{aligned}$$

$$20. \begin{aligned} f(x) &= e^x \cos(x); \\ P(x=0) &= 1 \quad P_x(x=1) = 0 \end{aligned}$$

Draw graphs for analytical and numerical solutions together, find the maximum difference between these two solutions in the given interval.