Find analytical and numerical (simple iteration method and Thomas algorithm) solutions of one-dimensional heat conductivity equation until steady state:

1. $$\begin{aligned} &U_t = U_{xx}, \quad 0 < x < 1, \quad t > 0, \\ &U(0,t) = 0, \quad U_x(1,t) = 0, \quad t > 0, \\ &U(x,0) = x(x^2 - 3), \quad 0 \le x \le 1 \end{aligned}$$

2. $$\begin{aligned} &U_t = U_{xx}, \quad 0 < x < 1, \quad t > 0, \\ &U_x(0,t) = 0, \quad U(1,t) = 0, \quad t > 0, \\ &U(x,0) = 1 - x^3, \quad 0 \le x \le 1 \end{aligned}$$

3. $$\begin{aligned} &U_t = 3U_{xx}, \quad 0 < x < \pi, \quad t > 0, \\ &U(0,t) = 0, \quad U(\pi,t) = 0, \quad t > 0, \\ &U(x,0) = x\sin(x), \quad 0 \le x \le \pi \end{aligned}$$

4. $$\begin{aligned} &U_t = 9U_{xx}, \quad 0 < x < 2, \quad t > 0, \\ &U(0,t) = 0, \quad U(2,t) = 0, \quad t > 0, \\ &U(x,0) = x^2(2 - x), \quad 0 \le x \le 2 \end{aligned}$$

5. $$\begin{aligned} &U_t = 4U_{xx}, \quad 0 < x < 3, \quad t > 0, \\ &U(0,t) = 0, \quad U(3,t) = 0, \quad t > 0, \\ &U(x,0) = x(9 - x^2), \quad 0 \le x \le 3 \end{aligned}$$

6. $$\begin{aligned} &U_t = 4U_{xx}, \quad 0 < x < 2, \quad t > 0, \\ &U(0,t) = 0, \quad U(2,t) = 0, \quad t > 0, \\ &U(x,0) = \begin{cases} x, & 0 \le x \le 1, \\ 2 - x, & 1 \le x \le 2. \end{cases} \end{aligned}$$

7. $$\begin{aligned} &U_t = 7U_{xx}, \quad 0 < x < 1, \quad t > 0, \\ &U(0,t) = 0, \quad U(1,t) = 0, \quad t > 0, \\ &U(x,0) = x(3x^4 - 10x^2 + 7), \quad 0 \le x \le 1 \end{aligned}$$

8. $$\begin{aligned} &U_t = 5U_{xx}, \quad 0 < x < 1, \quad t > 0, \\ &U(0,t) = 0, \quad U(1,t) = 0, \quad t > 0, \\ &U(x,0) = x(x^3 - 2x^2 + 1), \quad 0 \le x \le 1 \end{aligned}$$

9. $$\begin{aligned} &U_t = 2U_{xx}, \quad 0 < x < 1, \quad t > 0, \\ &U(0,t) = 0, \quad U(1,t) = 0, \quad t > 0, \\ &U(x,0) = x(3x^4 - 5x^3 + 2), \quad 0 \le x \le 1 \end{aligned}$$

10. $$\begin{aligned} &U_t = U_{xx}, \quad 0 < x < 1, \quad t > 0, \\ &U(0,t) = 0, \quad U(1,t) = 0, \quad t > 0, \\ &U(x,0) = x(1-x), \quad 0 \le x \le 1 \end{aligned}$$

11. $$\begin{aligned} &U_t = 3U_{xx}, \quad 0 < x < 1, \quad t > 0, \\ &U_x(0,t) = 0, \quad U(1,t) = 0, \quad t > 0, \\ &U(x,0) = 1-x, \quad 0 \le x \le 1 \end{aligned}$$

12. $$\begin{aligned} &U_t = U_{xx}, \quad 0 < x < 1, \quad t > 0, \\ &U(0,t) = 0, \quad U_x(1,t) = 0, \quad t > 0, \\ &U(x,0) = (x-1)^3 + 1, \quad 0 \le x \le 1 \end{aligned}$$

13. $$\begin{aligned} &U_t = 9U_{xx}, \quad 0 < x < 4, \quad t > 0, \\ &U_x(0,t) = 0, \quad U(4,t) = 0, \quad t > 0, \\ &U(x,0) = x^2, \quad 0 \le x \le 4 \end{aligned}$$

14. $$\begin{aligned} &U_t = 9U_{xx}, \quad 0 < x < 4, \quad t > 0, \\ &U(0,t) = 0, \quad U(4,t) = 0, \quad t > 0, \\ &U(x,0) = 1, \quad 0 \le x \le 4 \end{aligned}$$

15. $$\begin{aligned} &U_t = 16U_{xx}, \quad 0 < x < 2\pi, \quad t > 0, \\ &U_x(0,t) = 0, \quad U(2\pi,t) = 0, \quad t > 0, \\ &U(x,0) = 4, \quad 0 \le x \le \pi \end{aligned}$$

16. $$\begin{aligned} &U_t = 3U_{xx}, \quad 0 < x < \pi, \quad t > 0, \\ &U_x(0,t) = 0, \quad U(\pi,t) = 0, \quad t > 0, \\ &U(x,0) = x^2(\pi - x), \quad 0 \le x \le \pi \end{aligned}$$

17. $$\begin{aligned} &U_t = U_{xx}, \quad 0 < x < 1, \quad t > 0, \\ &U(0,t) = 0, \quad U_x(1,t) = 0, \quad t > 0, \\ &U(x,0) = x(x^3 - 2x^2 + 2), \quad 0 \le x \le 1 \end{aligned}$$

18. $$\begin{aligned} &U_t = U_{xx}, \quad 0 < x < 1, \quad t > 0, \\ &U(0,t) = 0, \quad U_x(1,t) = 0, \quad t > 0, \\ &U(x,0) = \sin(\pi x), \quad 0 \le x \le 1 \end{aligned}$$

19. $$\begin{aligned} &U_t = 3U_{xx}, \quad 0 < x < \pi, \quad t > 0, \\ &U(0,t) = 0, \quad U_x(\pi,t) = 0, \quad t > 0, \\ &U(x,0) = x(\pi - x), \quad 0 \le x \le \pi \end{aligned}$$

20. $$\begin{aligned} &U_t = 5U_{xx}, \quad 0 < x < 2, \quad t > 0, \\ &U(0,t) = 0, \quad U_x(2,t) = 0, \quad t > 0, \\ &U(x,0) = x(4-x), \quad 0 \le x \le 2 \end{aligned}$$

21. $$\begin{aligned} &U_t = U_{xx}, \quad 0 < x < 1, \quad t > 0, \\ &U(0,t) = 0, \quad U_x(1,t) = 0, \quad t > 0, \\ &U(x,0) = x^2(3-2x), \quad 0 \le x \le 1 \end{aligned}$$

22. $$\begin{aligned} &U_t = U_{xx}, \quad 0 < x < 1, \quad t > 0, \\ &U(0,t) = 0, \quad U_x(1,t) = 0, \quad t > 0, \\ &U(x,0) = x^3(3x-4), \quad 0 \le x \le 1 \end{aligned}$$

Draw graphs for analytical and numerical solutions together, find the number of iteration and the maximum difference between solutions in the given interval.