Find analytical and numerical (Five-diagonal matrix method) solutions of one-dimensional Poisson equation

$$\frac{\partial^2 P}{\partial x^2} = -f(x); x \in (0,1)$$

with the following conditions:

1. $f(x) = x^2 + 2x + 3;$
   $P(x=0) = 1 \quad P(x=1) = 0$

2. $f(x) = \sin(x);$
   $P(x=0) = 1 \quad P(x=1) = 0$

3. $f(x) = e^x;$
   $P(x=0) = 1 \quad P(x=1) = 0$

4. $f(x) = x^2 \sin(\pi x);$
   $P(x=0) = 1 \quad P(x=1) = 0$

5. $f(x) = e^x \cos(x);$
   $P(x=0) = 1 \quad P(x=1) = 0$

6. $f(x) = 2x^2 + 5x + 3;$
   $P(x=0) = 0 \quad P(x=1) = 1$

7. $f(x) = \sin^2(x);$
   $P(x=0) = 0 \quad P(x=1) = 1$

8. $f(x) = xe^x;$
   $P(x=0) = 0 \quad P(x=1) = 1$

9. $f(x) = x^2 \cos(2\pi x);$
   $P(x=0) = 0 \quad P(x=1) = 1$

10. $f(x) = e^x \sin(x);$
    $P(x=0) = 0 \quad P(x=1) = 1$

11. $f(x) = x^2 + 5x + 13;$
    $P_x(x=0) = 0 \quad P(x=1) = 1$

12. $f(x) = \sin^2(x) + \cos(x);$
    $P_x(x=0) = 0 \quad P(x=1) = 1$

13. $f(x) = e^x + x^3;$
$P_x(x=0)=0 \quad P(x=1)=1$

14. $f(x) = (x^2+3)\cos(\frac{\pi x}{2});$
$P_x(x=0)=0 \quad P(x=1)=1$

15. $f(x) = e^x \sin(\pi x);$
$P_x(x=0)=0 \quad P(x=1)=1$

16. $f(x) = 5x^2 + x + 3;$
$P(x=0)=1 \quad P_x(x=1)=0$

17. $f(x) = \sin(x) + 2\cos(x);$
$P(x=0)=1 \quad P_x(x=1)=0$

18. $f(x) = e^x + x^3 + 2;$
$P(x=0)=1 \quad P_x(x=1)=0$

19. $f(x) = (x^2+3x+2)\sin(\frac{\pi x}{2});$
$P(x=0)=1 \quad P_x(x=1)=0$

20. $f(x) = e^x \cos(x);$
$P(x=0)=1 \quad P_x(x=1)=0$

Draw graphs for analytical and numerical solutions together, find the maximum difference between these two solutions in the given interval.