- 1. a) Use Midpoint rule and Simpson's rule to approximate the definite integral. Verify the order of error to compare your results with the analytical values. (The variant number corresponds to the number in the attendance list).
- b) Write a programs in matlab (python) for the numerical calculation of the integral by the Midpoint rule and Simpson's rule.

1.
$$\int_{-2}^{4} (2x^2 - \sqrt{x+2}) dx \qquad n = 6$$
2.
$$\int_{-3}^{0} (5x^2 + x + 1) dx \qquad n = 6$$
3.
$$\int_{0}^{3} (3x^2 - \sqrt{x}) dx \qquad n = 6$$
4.
$$\int_{1}^{4} (x^3 - \sqrt{x}) dx \qquad n = 6$$
5.
$$\int_{1}^{4} (7 + x - 2x^2) dx \qquad n = 6$$
6.
$$\int_{0}^{3} (7x^2 - 3\sqrt{x}) dx \qquad n = 6$$

3.
$$\int_{0}^{3} (3x^2 - \sqrt{x}) dx$$
 $n = 6$ 4. $\int_{1}^{4} (x^3 - \sqrt{x}) dx$ $n = 6$

5.
$$\int_{1}^{4} (7+x-2x^2)dx \qquad n=6 \qquad 6. \qquad \int_{0}^{3} (7x^2-3\sqrt{x})dx \qquad n=6$$

7.
$$\int_{2}^{5} (2x^{2} - 2 - \sqrt{x}) dx \qquad n = 6 \qquad 8. \qquad \int_{0}^{3} (5x^{2} + \sqrt{x}) dx \qquad n = 6$$

7.
$$\int_{2}^{5} (2x^{2} - 2 - \sqrt{x}) dx \qquad n = 6 \qquad 8. \qquad \int_{0}^{3} (5x^{2} + \sqrt{x}) dx \qquad n = 6$$
9.
$$\int_{-2}^{2} (x^{3} + 1) dx \qquad n = 8 \qquad 10. \qquad \int_{0}^{4} (2x^{2} + 1 - \sqrt{x}) dx \qquad n = 8$$
11.
$$\int_{-2}^{2} (x^{2} + \sqrt{x + 2} - 1) dx \qquad n = 8 \qquad 12. \qquad \int_{0}^{2} (x^{2} + 2 + \sqrt{x}) dx \qquad n = 8$$
13.
$$\int_{1}^{3} (3x^{2} - x - 1) dx \qquad n = 8 \qquad 14. \qquad \int_{0}^{3} (x^{3} + 2) dx \qquad n = 8$$
15.
$$\int_{-2}^{2} (2x^{2} + 1 - \sqrt{x + 4}) dx \qquad n = 8 \qquad 16. \qquad \int_{1}^{4} (2x^{2} - 1, 5\sqrt{x}) dx \qquad n = 6$$
17.
$$\int_{1}^{4} (7\sqrt{x} + 2x^{2}) dx \qquad n = 6 \qquad 18. \qquad \int_{0}^{3} (7x^{2} + 3\sqrt{x}) dx \qquad n = 6$$
19.
$$\int_{2}^{5} (2x^{2} - 2 + \sqrt{x}) dx \qquad n = 6 \qquad 20. \qquad \int_{0}^{3} (5x^{2} - 1 + \sqrt{x}) dx \qquad n = 6$$
21.
$$\int_{3}^{6} (x^{2} + 4 + \sqrt{x}) dx \qquad n = 6 \qquad 22. \qquad \int_{2}^{6} (x^{3} + 3) dx \qquad n = 8$$

11.
$$\int_{-2}^{2} (x^2 + \sqrt{x+2} - 1) dx \qquad n = 8 \qquad 12. \quad \int_{0}^{2} (x^2 + 2 + \sqrt{x}) dx \qquad n = 8$$

13.
$$\int_{1}^{3} (3x^2 - x - 1)dx$$
 $n = 8$ 14. $\int_{-1}^{3} (x^3 + 2)dx$ $n = 8$

15.
$$\int_{-2}^{2} (2x^2 + 1 - \sqrt{x+4}) dx \qquad n = 8 \qquad 16. \quad \int_{1}^{4} (2x^2 - 1, 5\sqrt{x}) dx \qquad n = 6$$

17.
$$\int_{1}^{4} (7\sqrt{x} + 2x^2) dx$$
 $n = 6$ 18. $\int_{0}^{3} (7x^2 + 3\sqrt{x}) dx$ $n = 6$

19.
$$\int_{2}^{5} (2x^{2} - 2 + \sqrt{x}) dx \qquad n = 6 \qquad 20. \quad \int_{0}^{3} (5x^{2} - 1 + \sqrt{x}) dx \qquad n = 6$$

21.
$$\int_{3}^{6} (x^2 + 4 + \sqrt{x}) dx$$
 $n = 6$ 22. $\int_{2}^{6} (x^3 + 3) dx$ $n = 8$

23.
$$\int_{0}^{3} (2x^{2} - 1 + \sqrt{x}) dx \qquad n = 6$$
24.
$$\int_{-2}^{2} (3x^{2} + 2\sqrt{x + 2}) dx \qquad n = 8$$
25.
$$\int_{-2}^{2} (x^{2} + 2\sqrt{x + 2}) dx \qquad n = 8$$
26.
$$\int_{-3}^{1} (x^{2} + 2x - 1, 5) dx \qquad n = 6$$
27.
$$\int_{-3}^{3} (3x^{2} + 1 + \sqrt{x + 3}) dx \qquad n = 6$$
28.
$$\int_{0}^{3} (3x^{2} + 5 + \sqrt{x}) dx \qquad n = 6$$
29.
$$\int_{1}^{4} (7x + x^{2} - \sqrt{x}) dx \qquad n = 6$$
30.
$$\int_{0}^{3} (x^{2} - 3\sqrt{x}) dx \qquad n = 6$$
31.
$$\frac{1}{\sqrt{2\pi}} \int_{0}^{1 + 0.1m} e^{-\frac{x^{2}}{2}} dx \qquad n = 10$$
32.
$$\int_{0}^{1} \frac{dx}{1 + \sqrt{x}} \qquad n = 10$$

- 2. a) By using Matlab (python) solve the differential equations by Euler's method and Heun's method. (*The variant number corresponds to the number in the attendance list*).
- b) Find the exact solution and verify the order of error to compare the exact solution with the two approximations in part a).

y(-1) = 1 $x \in [-1; 2]$ h = 0.6

c) Plot the two approximations and the exact solution on the same coordinate system.

1.
$$y' = 3 + 2x - y$$
 $y(0) = 2$, $x \in [0; 1]$, $h = 0,2$
2. $y' = y - 3x$ $y(1) = 0$ $x \in [1; 2,2]$ $h = 0,3$
3. $y' = 1 - x + y$ $y(1,1) = 0$ $x \in [1,1; 1,6]$ $h = 0,1$
4. $y' = y - 7x$ $y(3) = 3$ $x \in [3; 5]$ $h = 0,5$
5. $y' = 5 - y + x$ $y(1) = 1$ $x \in [1; 5]$ $h = 0,1$
6. $y' = y - 2x + 3$ $y(0) = 4$ $x \in [0; 1]$ $h = 0,2$
7. $y' = 4 - x + 2y$ $y(0) = 1$ $x \in [0; 1,2]$ $h = 0,3$
8. $y' = -8 + 2x - y$ $y(1) = 3$ $x \in [1; 3]$ $h = 0,4$
9. $y' = 2y - 3x$ $y(4) = 0$ $x \in [4; 6]$ $h = 0,5$

10. y' = x - 2y

11.
$$y' = 7 - xy$$
 $y(-2) = 0$ $x \in [-2; 0]$ $h = 0.5$

12. $y' = 2x + y$ $y(2) = 2$ $x \in [2; 3.5]$ $h = 0.5$

13. $y' = 5 + x - y$ $y(2) = 1$ $x \in [2; 4]$ $h = 0.5$

14. $y' = y + 5x - 1$ $y(0) = 2$ $x \in [0; 3.2]$ $h = 0.8$

15. $y' = y - 5x + 1$ $y(0) = 2$ $x \in [0; 3.2]$ $h = 0.8$

16. $y' = 1 - x + y$ $y(0) = 1$ $x \in [0; 2.5]$ $h = 0.5$

17. $y' = y - 5x$ $y(-1) = 1$ $x \in [-1; 1]$ $y = 0.5$

18. $y' = x + 2y$ $y(0) = -1$ $y \in [0; 2]$ $y = 0.5$

20.
$$y' = 3x + 4y$$
 $y(2) = 1$ $x \in [2; 5]$ $h = 0,5$
21. $y' = 3 + 2x + y$ $y(0) = 2$ $x \in [0; 1]$ $h = 0,2$
22. $y' = 2y - x$ $y(1) = 0$ $x \in [1; 2,2]$ $h = 0,3$
23. $y' = -x + y$ $y(1,1) = 0$ $x \in [1,1; 1,6]$ $h = 0,1$

24.
$$y' = y - 7x + 2$$
 $y(3) = 3$ $x \in [3; 5]$ $h = 0.5$
25. $y' = 5 - y + x$ $y(1) = 1$ $x \in [1; 5]$ $h = 0.4$

25.
$$y' = 5 - y + x$$
 $y(1) = 1$ $x \in [1; 5]$ $h = 0.1$
26. $y' = y - 2x + 3$ $y(0) = 4$ $x \in [0; 1]$ $h = 0.2$

27.
$$y' = 4 - x + 2y$$
 $y(0) = 1$ $x \in [0; 1,2]$ $h = 0,3$

28.
$$y' = -8 + 2x - y$$
 $y(1) = 3$ $x \in [1; 3]$ $h = 0,4$

29.
$$y' = 2y - 3x$$
 $y(4) = 0$ $x \in [4; 6]$ $h = 0,5$

30.
$$y' = x^2 - 2y$$
 $y(-1) = 1$ $x \in [-1, 2]$ $h = 0,5$

31.
$$y' = 5 - x - 2y$$
 $y(1) = 2$ $x \in [2; 4]$ $h = 0.5$

32.
$$y' = y + 3x - 2$$
 $y(1) = 2$ $x \in [1; 2]$ $h = 0,2$