Stable Kozai-Lidov predicted by models, but observed outliers demand tighter constraints.

Dynamics of Hierarchical Triple Systems and their Gravitational Wave Detectability

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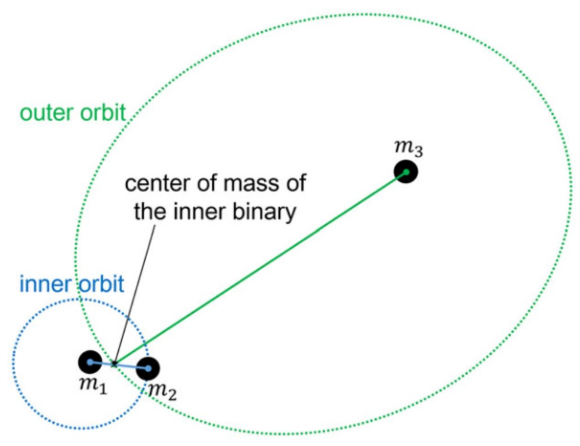
Abstract: We simulate hierarchical triple systems—an inner compact binary with a distant tertiary—drawn from constrained astrophysical parameter spaces that favor Kozai–Lidov (KL) dynamics, showing KL oscillations can drive the inner binary to high eccentricities and greatly enhance gravitational wave (GW) emission. Using N-body integration and eccentric waveform modeling across detectors (LISA, LGWA) while varying mass ratios and eccentricities, we assess detectability and find that 1PN general-relativistic corrections can suppress or modulate KL cycles, implying an additional dynamical constraint on HTS stability. Consequently, eccentric GW signals from these systems lie within the reach of current and planned detectors, providing a foundation for developing eccentric waveform templates.

The Mechanism

The Kozai-Lidov mechanism occurs when a tertiary companion perturbs an inner binary, causing periodic exchanges between its eccentricity and inclination over timescales much longer than the orbital period.



Results

Frequency-domain GW of our models at varied eccentricity and distances.

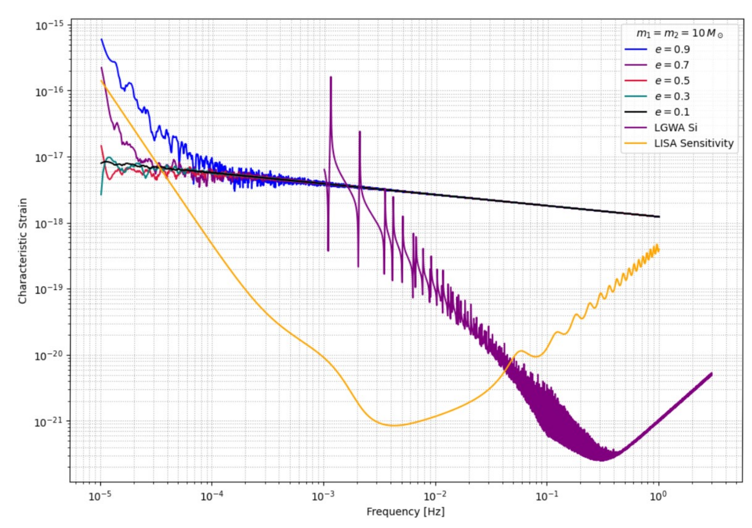


Figure 1: *Binary at a distance of 770kpc but varying eccentricity*

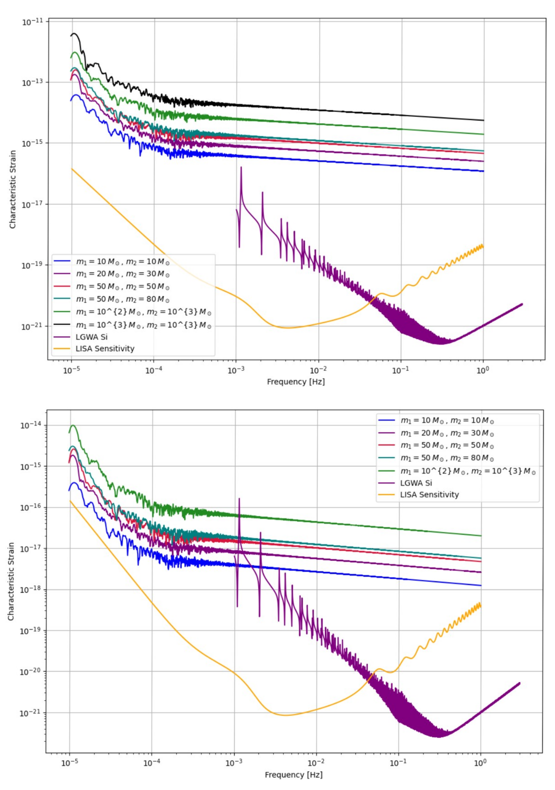


Figure 2: *Different binaries of eccentricity 0.9 at distances 8kpc* (Top) *and 770kpc* (Bottom).

Methodology

- We select an initial set of parameters for different 3-body mass combinations, tertiary being either an IMBH or SMBH.
- These input parameters are input into constraints that determine the stability of the system and the occurance of KL oscillation in the system used to determine the KL dominated parameter space (see Figure 3). The constraints include a stability constraint [1], relativistic precession constraint [1] and the Lense-Thirring effect [1]. This allows us to select the semi-major axes, a_{in} and a_{out}, of the inner and outer binaries respectively.
- Initial relative inclination is chosen between the range of the initial relative inclination that allows for KL oscillation to occur (between 40° – 140°) [1].
- These parameters are put through a custom Python code, based on the REBOUND [2] N-body code and additional effects and forces provided by REBOUNDx [3], which allows us to evaluate the evolution of eccentricity and inclination of the system and determine stability of the effect and the system itself.
- We create two codes, Newt3 and GR3. Newt3 simulates the system at the Newtonian level and GR3 simulates the system at the 1st order Post-Newtonian correction.
- The parameters are used to find the frequency domain GW of the models using EccentricFD [4].

Model	$m_1[M_\odot]$	$m_2[M_\odot]$	$m_3[M_\odot]$	$a_{in}[AU]$	$a_{out}[AU]$	e_{in}	e_{out}	I [Deg]	$P_{in}[hrs]$	$P_{out}[days]$
IM1	10	10	10^3	0.05	1	0	0	90	21.9	11.4
IM2	10	10	10^4	0.05	5	0	0	90	21.9	40.8
IM3	20	30	10^5	0.05	5	0.1	0.1	80	13.9	12.9
IM4	10^2	10^3	10^5	0.5	10	0	0	50	93.5	36.2
IM5	10^3	10^4	10^5	0.5	10	0	0	100	29.6	34.6
SM1	10	10	10^6	0.01	5	0	0	90	1.97	4.09
SM2	20	10^3	10^6	0.2	10	0	0	90	24.5	11.5
SM3	10^3	10^3	10^6	0.2	10	0	0	90	17.5	11.5
SM4	50	30	10^7	0.01	5	0	0.1	45	0.98	1.29
SM5	10^5	10^5	10^7	1	20	0	0	100	19.5	10.2
SM6	50	80	10^8	0.05	50	0.2	0	60	8.60	12.9
SM7	10^3	10^4	10^8	1	200	0	0	80	83.3	~ 103
SM8	10^4	10^5	10^9	1	150	0	0	70	26.3	21.2
SM9	10^6	10^7	10^8	50	500	0	0	90	~ 935	~ 388

Table 1: *Model Parameters*

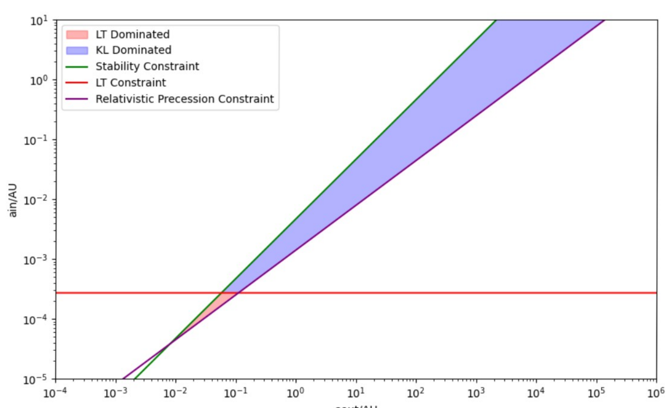


Figure 3: *Parameter Space (SM1)*

Simulation Results

Expected Results

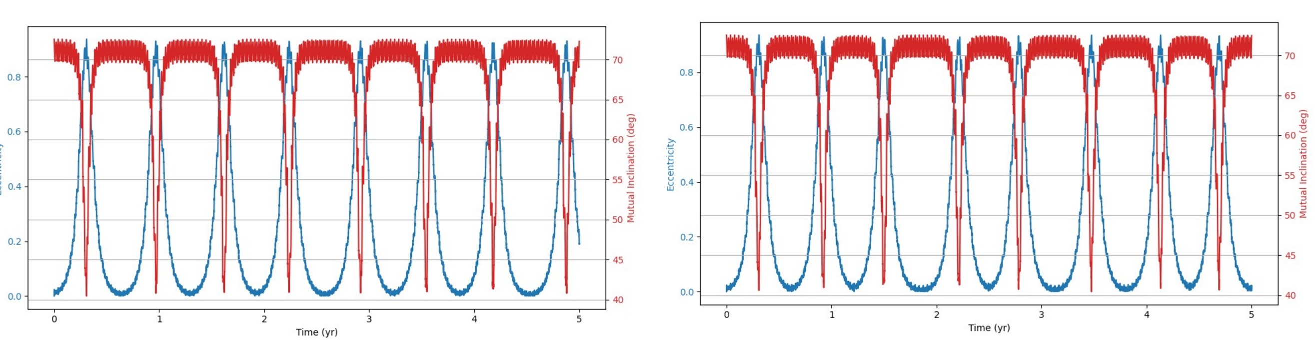


Figure 4: *Simulation results for model IM1. Difference in number of KL cycles negligible between both simulations.* Left: *Newt3 simulations.* Right: *GR3 simulations*

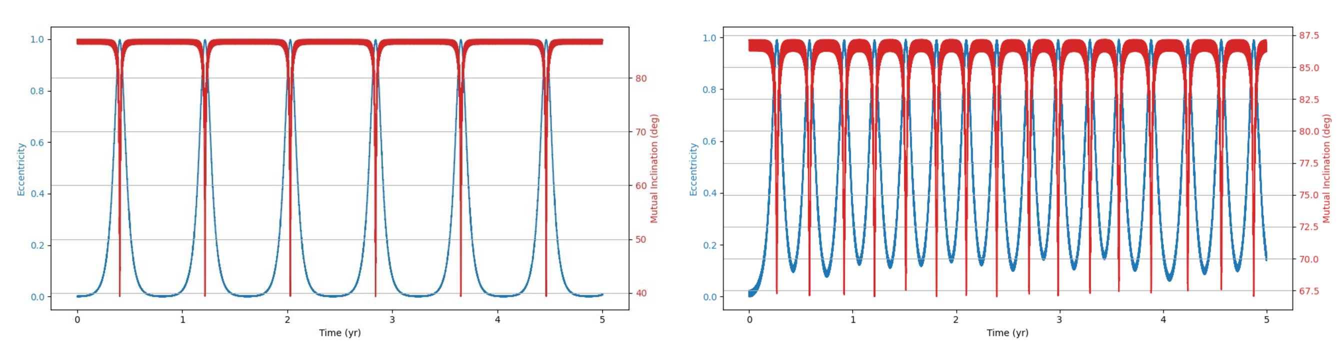


Figure 5: *Simulation results for model SM1. Significant difference in number of KL cycles between both simulations. We observe that tertiaries with masses in the SMBH range as GR precession can partially restart KL cycles even under constraint.* Left: *Newt3 simulations.* Right: *GR3 simulations*

The Outliers

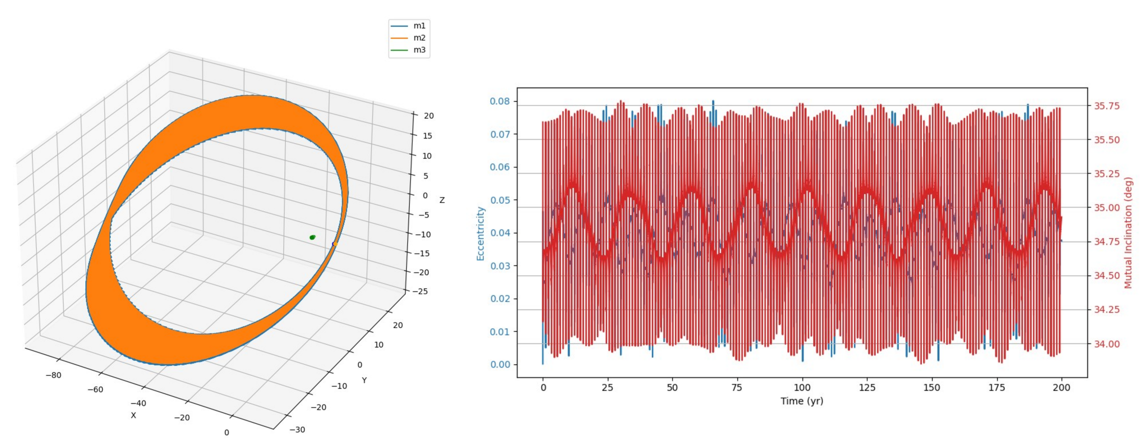


Figure 6: *GR3 simulation results for model IM4 after 200 years. Shows stable triple but does not exhibit KL mechanism.*

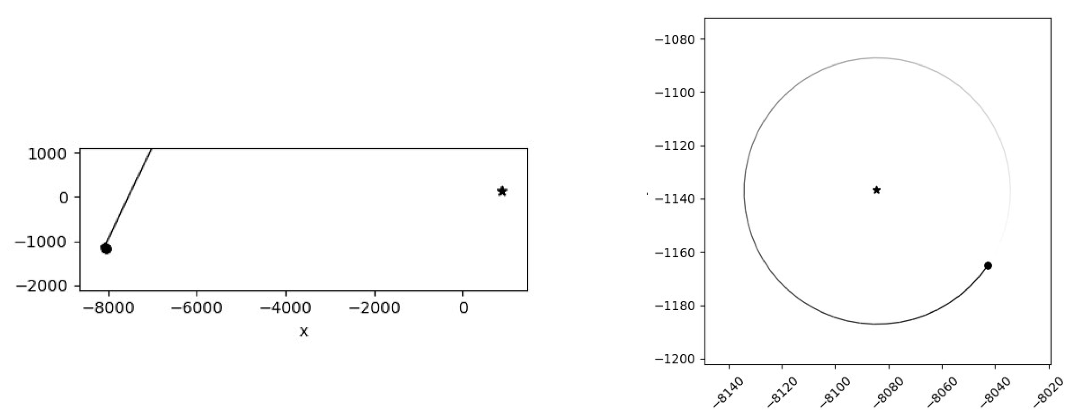


Figure 7: *GR3 simulation results of system SM9 after 30 years. Shows a stable inner binary ejected out of the triple system.* Left: *Complete system.* Right: *Inner binary*

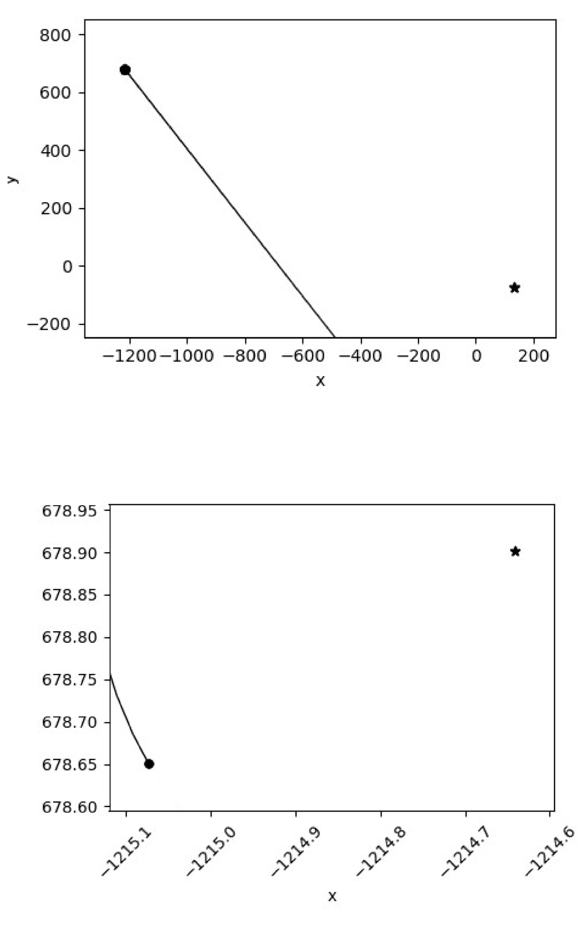


Figure 8: *GR3 simulation results of system IM5 after 5 years. Shows system in chaos.* Top: *Complete system.* Bottom: *Inner binary.*

References

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