EULERS EQUATION

Fluid Mechanics

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STUDY OF MOTION OF FLUID FLOW ALONG WITH THE FORCE CAUSING THE FLOW

- **NEWTON'S SECOND LAW**

F= ma

FLUID DYNAMICS

- For flow in x-direction

$F_x = m \, . \, A_x$

Fg= gravity force

Fp = pressure force

Fv = viscous force

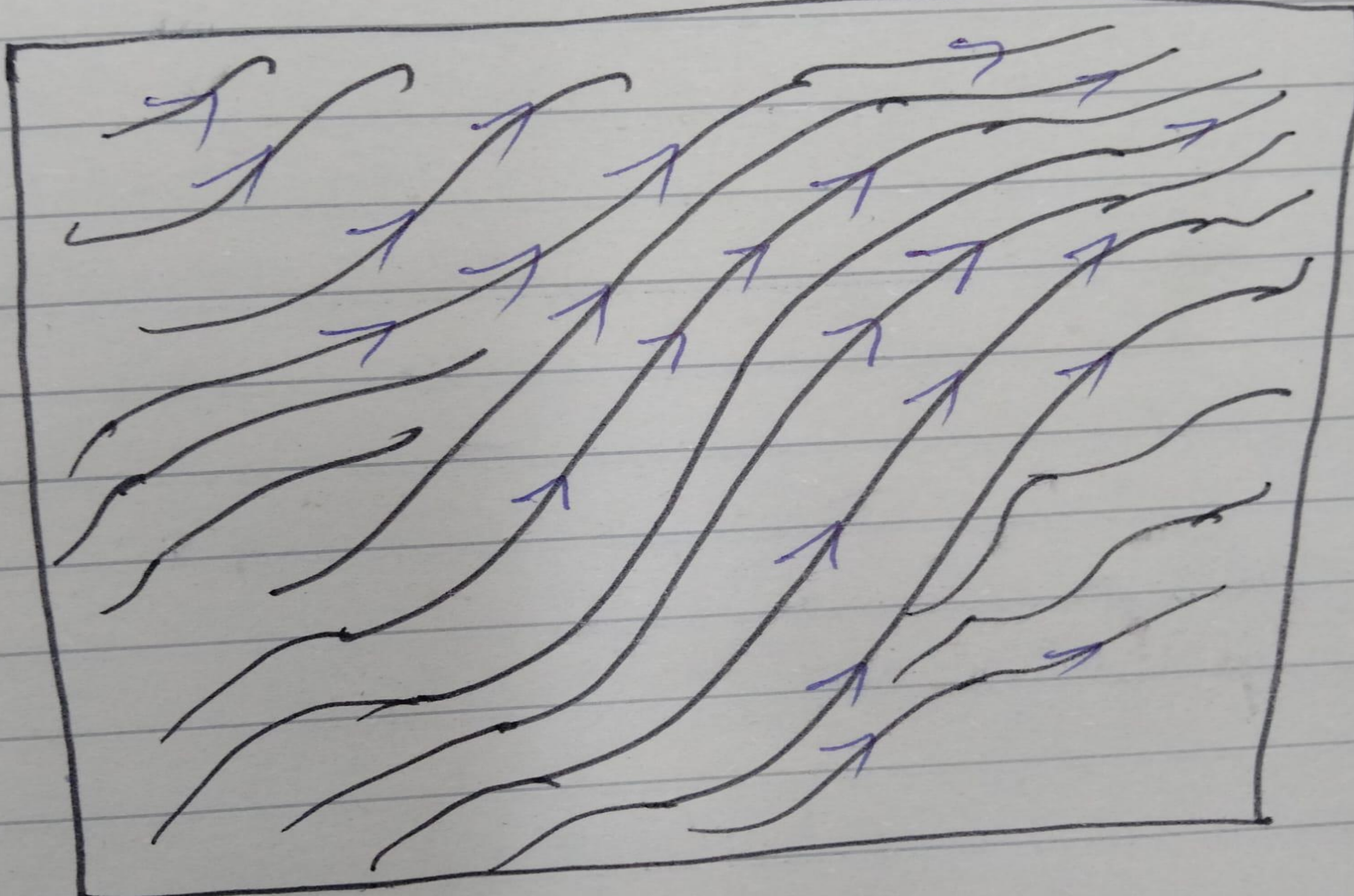
Ft = force due to turbulence

Fc = force due to compressibility

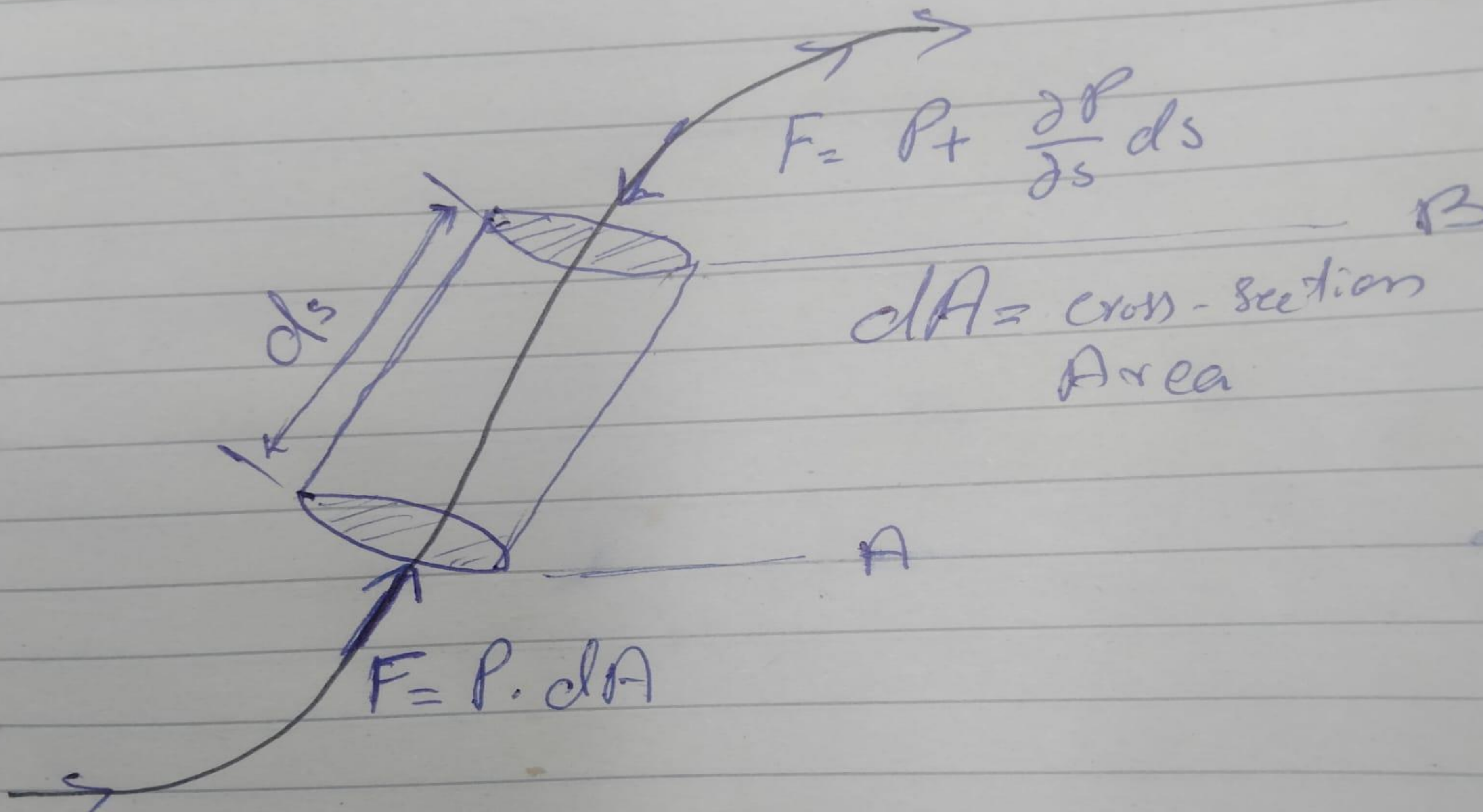
$Fx = (F_x)_g + (F_x)_p + (F_x)_v + (F_x)_t + (F_x)_c$

$$Fx = (F_x)_g + (F_x)_p + (F_x)_v + (F_x)_t + (F_x)_c$$

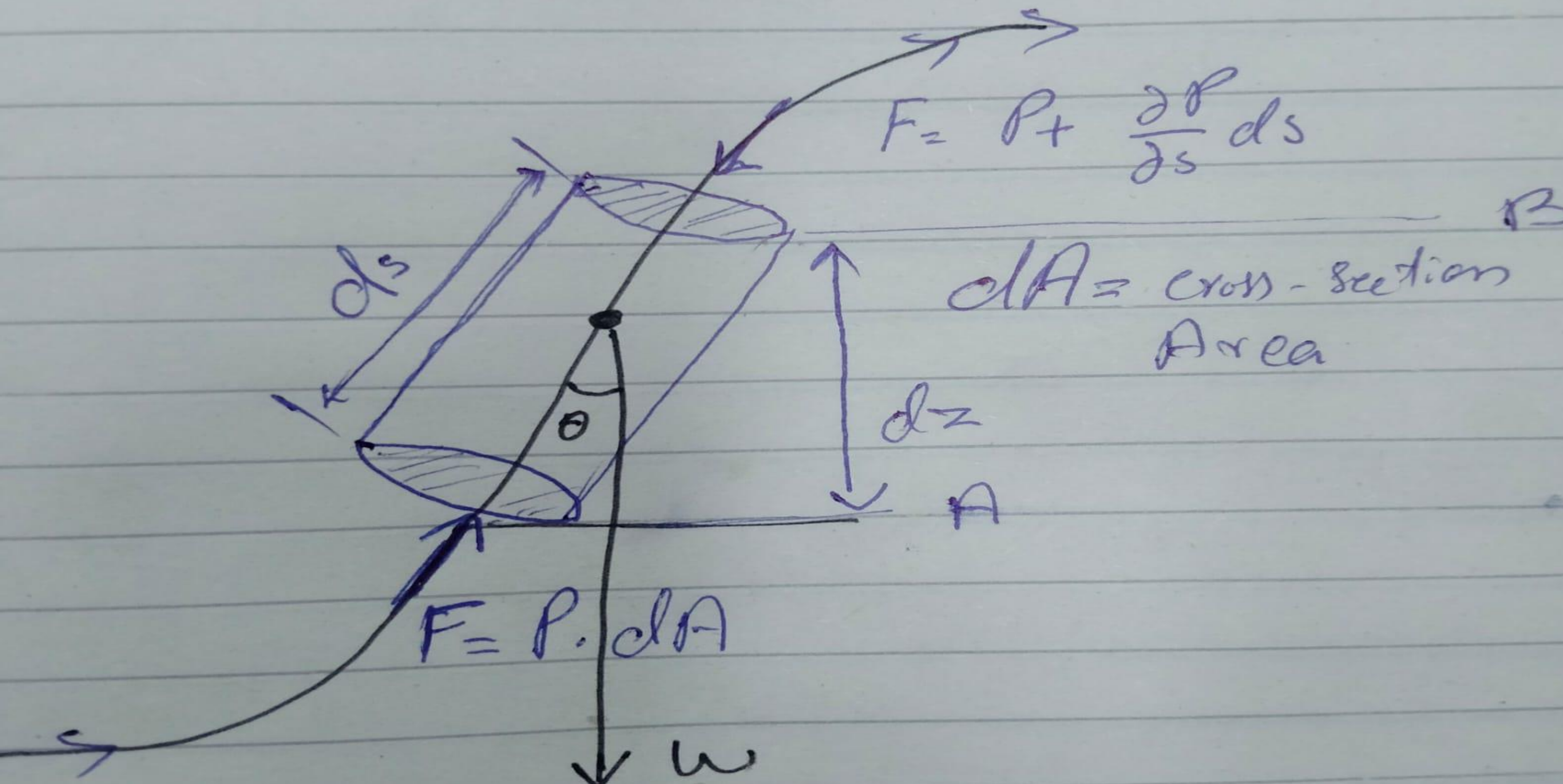
$Fx = (F_x)_g + (F_x)_p + (F_x)_v + (F_x)_t$	Rhenold's equations
$Fx = (F_x)_g + (F_x)_p + (F_x)_v$	Navier stokes equation
$Fx = (F_x)_g + (F_x)_p$	Euler's equation



Stream - lines.



$F = P + \frac{\partial P}{\partial s} ds$
B
ds
dA = cross-section Area
A
$F = P \cdot dA$



F= P+ ∂P/∂s ds
B
ds
dA= cross-section Area
θ
dz
A
F= P. dA
w

To calculate the weight of the fluid element we have

$$W = mg$$

"OR"

$$W = \rho . V . g$$

$$\boxed{W = \rho . dA . ds . g}$$

Because, weight is being applied on inclined streamline

$\therefore$ weight can be resolved into two components

$$W_x = \rho g \, dA \, ds \cos\theta$$

$$W_y = \rho g \, dA \sin\theta$$

Using Second law of motion

$$F_s = m\,a_s$$

where;

F_s = Force along Streamline

a_s = total acceleration along S.L

$$\boxed{F_s} = \underbrace{\overbrace{P.\,dA}} - \underbrace{\overbrace{\left(P + \frac{\partial P}{\partial s}.\,ds\right) dA}} - \underbrace{\overbrace{\rho\, ds\, dA\, g \cos\theta}}$$

$$\boxed{ma} = \underbrace{\overbrace{\rho\, dA\, ds\, a_s}}$$

$$F_s = P \cdot dA - P\,dA - \frac{\partial P}{\partial s} ds\, dA - \rho g\, ds\, dA \cos\theta$$

$$F_s = -\frac{\partial P}{\partial s} ds\, dA - \rho g\, ds\, dA \cos\theta$$

$$ma = \rho\, ds\, dA\, a_s$$

$$ma = \rho\, ds\, dA \left\{ \frac{\partial v}{\partial t} + v \frac{\partial v}{\partial s} \right\}$$

$\frac{\partial v}{\partial t}$ = local acceleration & $\frac{\partial v}{\partial s}$ = Convective acceleration

$$ma = \rho\, ds\, dA \left\{ \frac{\partial v}{\partial t} + v \frac{\partial v}{\partial s} \right\}$$

For steady state flow $\frac{\partial v}{\partial t} = 0$

$$ma = \rho\, ds\, dA \left\{ v \frac{\partial v}{\partial s} \right\} \longrightarrow (ii)$$

Putting value of eq (i) and eq (ii)

in $F = ma$

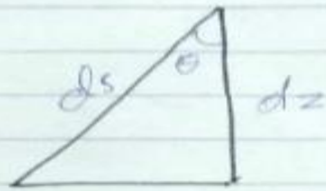
$$-\frac{\partial P}{\partial s} ds\, dA - \rho g\, ds\, dA \cos\theta = \rho\, ds\, dA \left(v \cdot \frac{\partial v}{\partial s} \right)$$

~~di~~ multiply ~~div~~:

$$-\frac{\partial P}{\rho \partial s} - g \cos\theta = v \cdot \frac{\partial v}{\partial s}$$

$$\frac{\partial P}{\rho \partial s} + g \cos\theta + v \cdot \frac{\partial v}{\partial s} = 0$$

$$\frac{\partial P}{\rho\, \partial s} + g \cos\theta + V \cdot \frac{\partial V}{\partial s} = 0$$



$$\cos\theta = \frac{dz}{ds}$$

$$\frac{\partial P}{\rho\, \partial s} + g \frac{dz}{ds} + V \frac{\partial V}{\partial s} = 0$$

↳ Flow is along single direction

↳ Flow is steady

↳ change is along only stream line ds

$$\frac{dP}{\rho\, ds} + g \frac{dz}{ds} + V \frac{dV}{ds} = 0$$

$$\boxed{\frac{dP}{\rho} + g\, dz + V \cdot dV = 0}$$