BERNOULLI'S EQUATION

Fluid Mechanics

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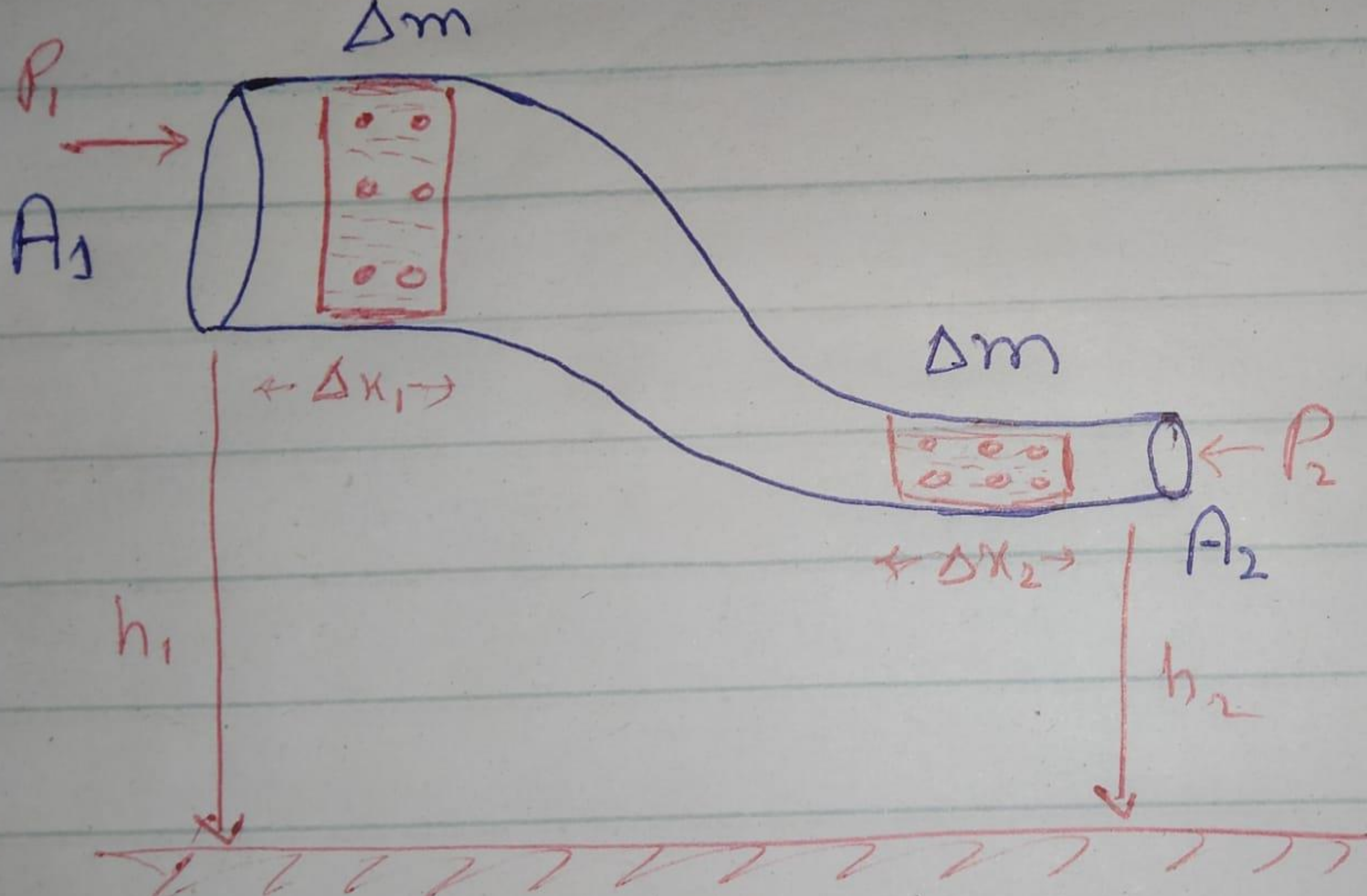
BERNOULLI’S EQUATION

Swiss Physicst (Daniel Bernoulli)
Hydrodynamica (BOOK 1738)



Δm
P1
A1
Δm
Δx1
P2
Δx2
A2
h1
h2

$$P + \frac{1}{2}\rho V^2 + \rho g h = C$$

Assumptions:

- Non- viscous
- Incompressible
- Steady state

PRINCIPLE

It works on the law of conservation of energy

Assumptions:

- Non- viscous
- Incompressible
- Steady state

Total work = Work at upper end + work at lower end

- $W_1 = F_1.d_1 \text{ Cos } \theta_1$
- $W_2 = F_2.d_2 \text{ Cos } \theta_2$

$$\boxed{W_{total} = W_1 + W_2}$$

$$W_{total} = F_1.d_1 \cos \theta_1 + F_2.d_2 \cos \theta_2$$

$$W_{total} = P_1.A_1.\Delta x_1 \cos 0 + P_2.A_2.\Delta x_2 \cos 180$$

$$W_{total} = P_1.A_1.\Delta x_1 - P_2.A_2.\Delta x_2$$

$$W_{total} = P_1.A_1.V_1.t - P_2.A_2.V_2.t \text{ ..... eq (1)}$$

(here V is velocity V= displacement / time)

$W_{total} = P_1.A_1.\ V_1.t\ -\ P_2.A_2.\ V_2.t$ .... Eq (1)

Volume = Volume

Volume / Time = Volume / Time

Area . Length/time = Volume / Time

Area . Velocity = Volume / Time

Area . Velocity . Time = Volume

Now eq (1) can be written as

$W_{total} = P_1.\ (\ VOLUME\ )\ -\ P_2.\ (\ VOLUME\ )$

$$W_{total} = P_1 . (\text{VOLUME}) - P_2 . (\text{VOLUME})$$

$$W_{total} = P_1 . V - P_2 . V$$

$$W_{total} = V (P_1 - P_2)$$

$$W_{total} = m / \rho \ (P_1 - P_2) \qquad (\rho = m /v)$$

Work energy theorem

$$W = \Delta K.E + \Delta P.E$$

$$m/\rho \ (P_1 - P_2) = \left(\frac{1}{2} mV_2^2 - \frac{1}{2} mV_1^2\right) + (m g h_2 - m g h_1)$$

$$(P_1 - P_2) = \rho / m \left(\frac{1}{2} mV_2^2 - \frac{1}{2} mV_1^2\right) + (m g h_2 - m g h_1)$$

$$(P_1 - P_2) = \left(\frac{1}{2} \rho V_2^2 - \frac{1}{2} \rho V_1^2\right) + (\rho g h_2 - \rho g h_1)$$

$$\left(P_1 + \frac{1}{2} \rho V_1^2 + \rho g h_1\right) = \left(P_2 + \frac{1}{2} \rho V_2^2 + \rho g h_2\right)$$

$$(P_1 + \frac{1}{2}\rho V_1^2 + \rho g h_1) = (P_2 + \frac{1}{2}\rho V_2^2 + \rho g h_2)$$

$$P + \frac{1}{2}\rho V^2 + \rho g h = C$$

$$P + \frac{K.E}{V} + \frac{P.E}{V} = C$$

$$(P_1 + \frac{1}{2}\rho V_1^2) = (P_2 + \frac{1}{2}\rho V_2^2)$$