continuity equation

Fluid Mechanics

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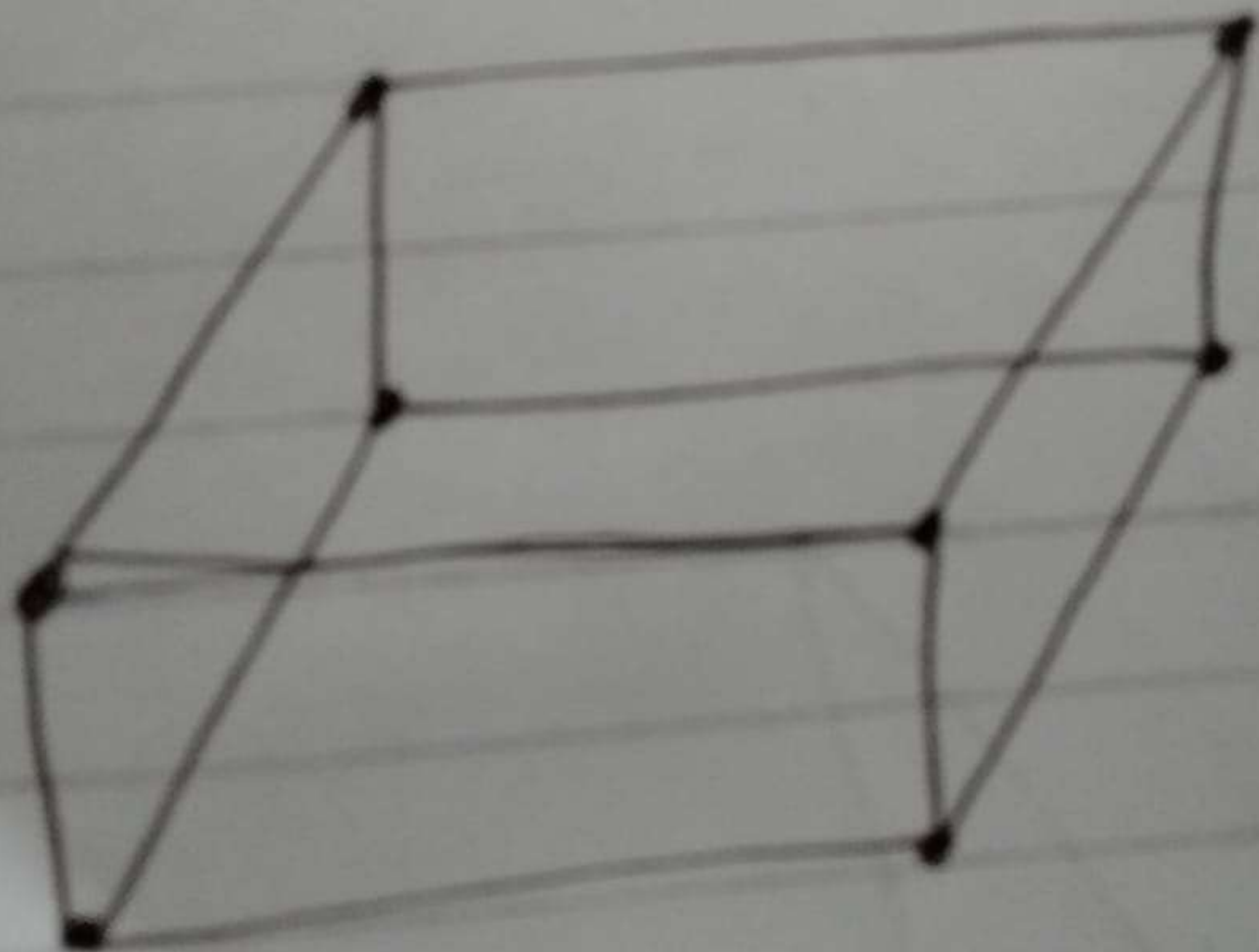
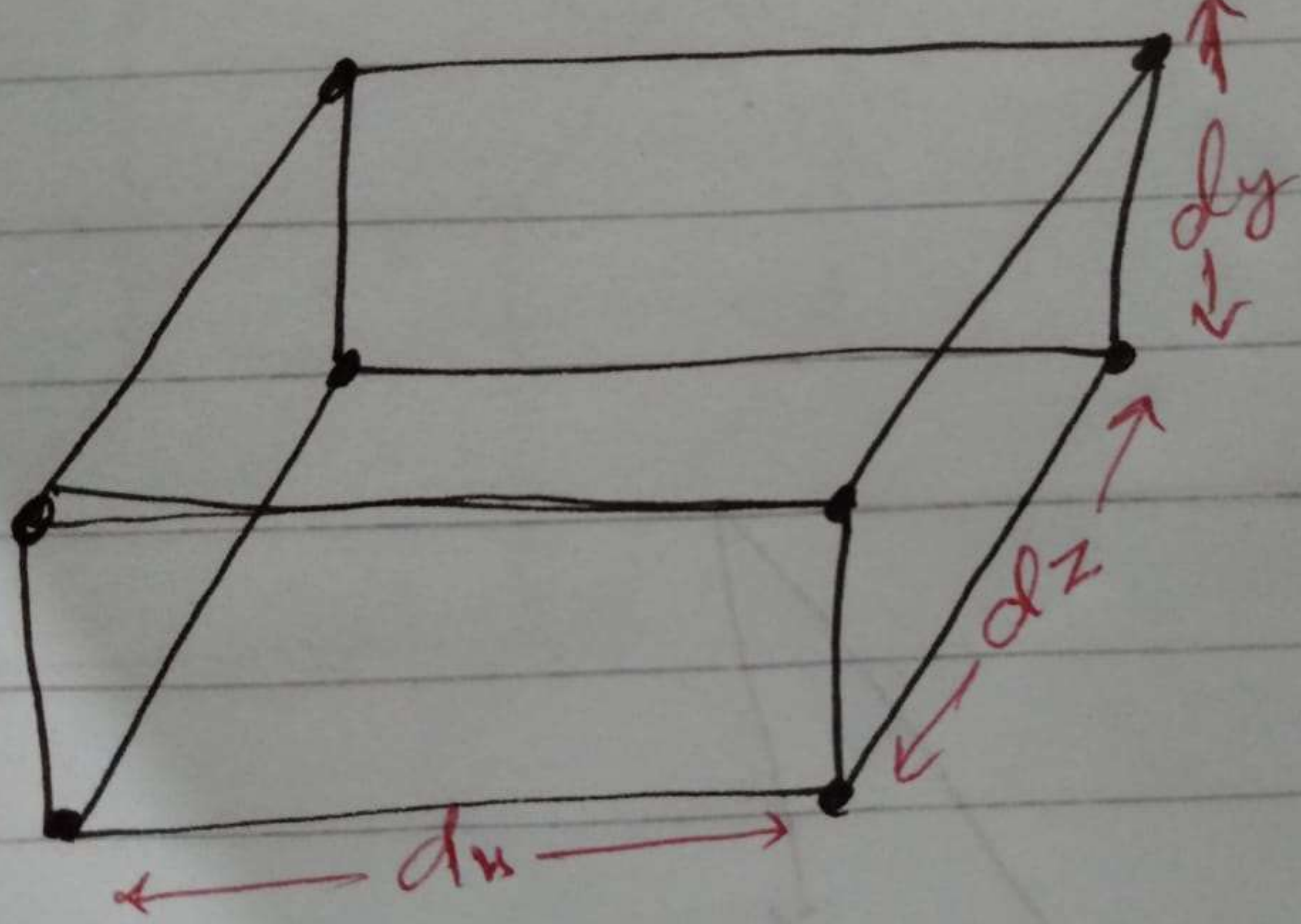
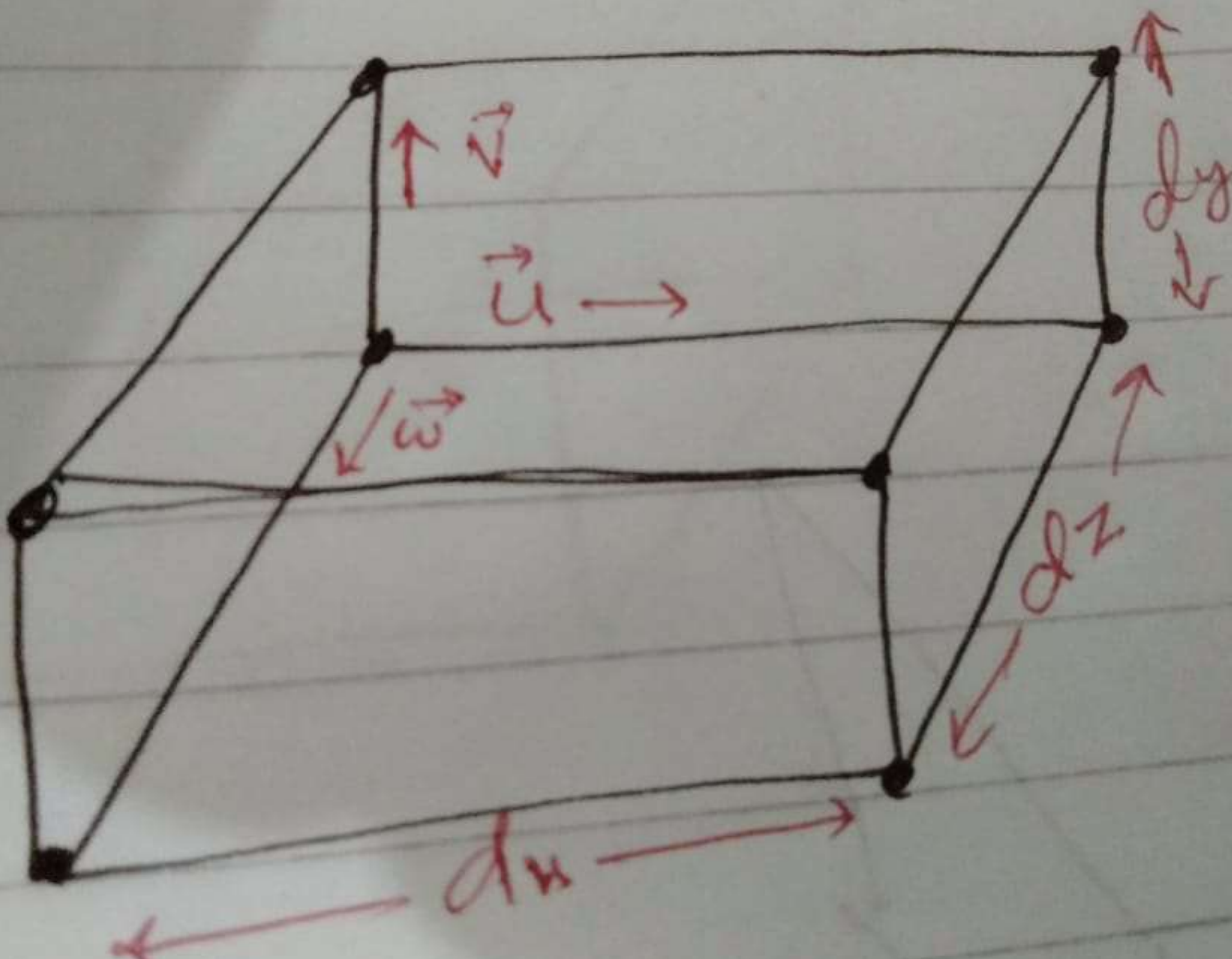



dy
dz
dx



$\vec{v}$
$\vec{u}$
$\vec{\omega}$
dy
dz
dx

We have considered very small differential control volume to analyze mass flow rate

U= velocity component in X-direction
V= Velocity component in Y-direction
W= velocity component in Z-direction

Continuity equation

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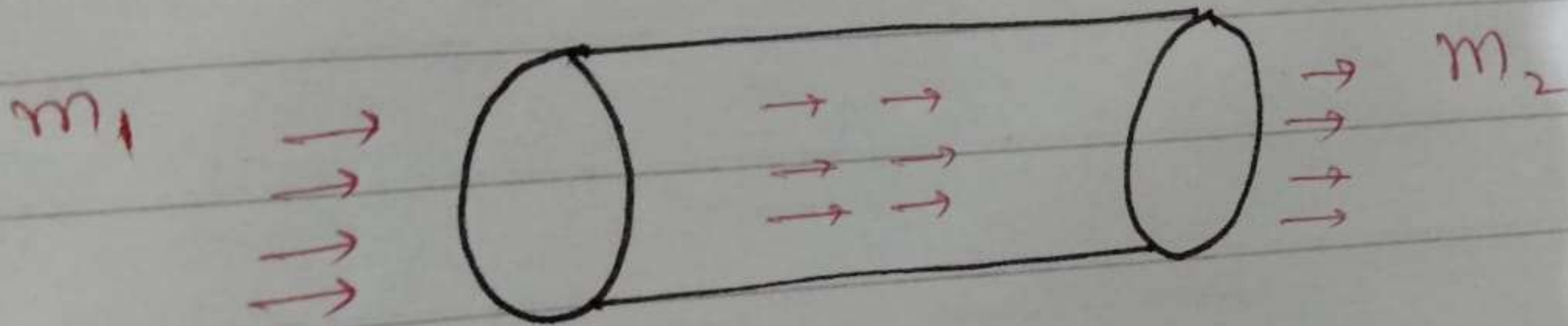


$$m_1 = m_2$$

Mass flowrate $= \dfrac{m}{t}$

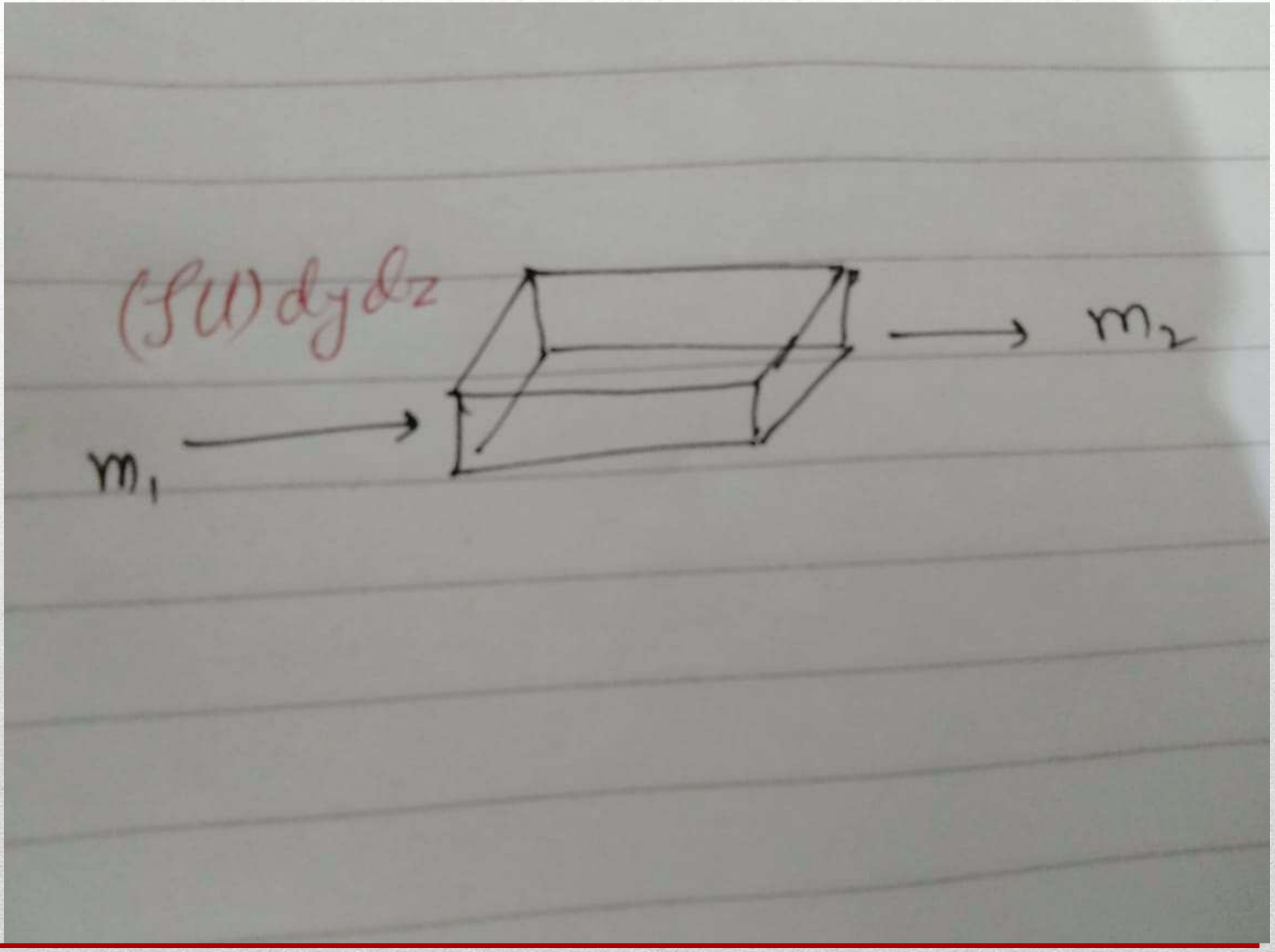
$$= \frac{\rho V}{t} = \frac{\rho\, dV}{dt}$$

$$= \frac{\rho\, dx \cdot dy \cdot dz}{dt}$$

Mass flow rate in x-direction $= \rho \cdot dy \cdot dz \cdot \frac{dx}{dt}$

$$= \rho \, dy \cdot dz \; u$$

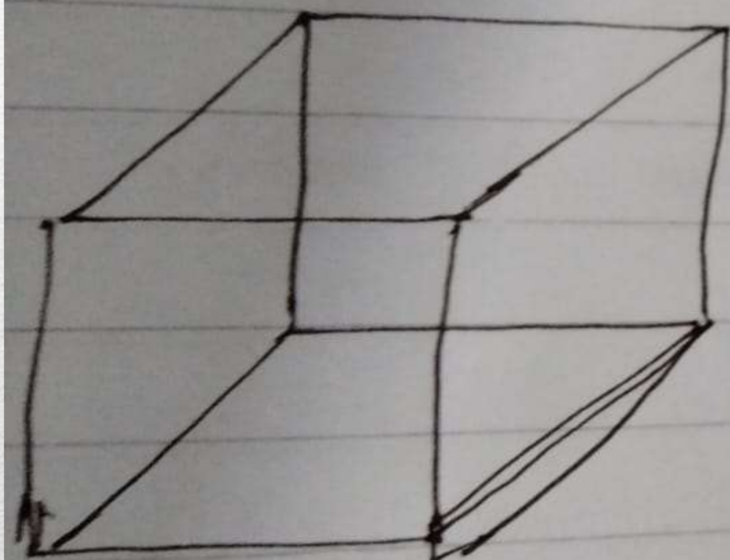
$$= \rho \, u \cdot dy \, dz$$


$(\rho u)\,dy\,dz$
m_1
m_2

Taylor Series

$$f(x+h) = f(x) + \frac{\partial}{\partial x} f(x)\, h$$

$$(\rho u)\,dy\,dz = \left(\rho u + \frac{\partial}{\partial x}(\rho u).dx\right) dy\,dz$$



m_x

$$\left(\rho u + \frac{\partial}{\partial x}\rho u\,dx\right) dy\,dz$$

Rate of change of mass inside control volume x-direction $= m_2 - m_1$

$$\Delta m_x = m_2 - m_1$$

$$\Delta m_x = \left(\rho u + \frac{\partial}{\partial x} \rho u dx\right) dy dz - \rho u \, dy dz$$

$$\Delta m_x = \frac{\partial}{\partial x} (\rho u) dx dy dz$$

$$\Delta m_x = \frac{\partial}{\partial x} (\rho u) dx dy dz$$

$$\Delta m_y = \partial/\partial y (\rho v) dx dy dz$$

$$\Delta m_z = \partial/\partial z (\rho w) dx dy dz$$

Rate of change
of man inside $= \Delta m_x + \Delta m_y + \Delta m_z$
control Nolume

$$\frac{\partial}{\partial t} m = \left[\frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w)\right] dx\,dy\,dz$$

$$\frac{\partial}{\partial t} \rho v = \left[\frac{\partial}{x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w\right] dx\,dy\,dz$$

$$\frac{\partial}{\partial t} \rho . \, dx\,dy\,dz = \left[\frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w)\right] dx\,dy\,dz$$

$$\frac{\partial}{\partial t} \rho = \frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w)$$

$$\frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w) - \frac{\partial \rho}{\partial t} = 0 \rightarrow (1)$$

$$\frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w) + \frac{\partial \rho}{\partial t} = 0 \rightarrow (ii)$$

For Steady State flow

$$\frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w) = 0$$

$$\nabla \cdot (\rho v) = 0$$

For Incomprenible flow

$$\rho = \text{constant}$$

$$\frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w) + \frac{\partial}{\partial t}(\rho) = 0$$

$$\rho \cdot \frac{\partial}{\partial x}(u) + \rho \cdot \frac{\partial}{\partial y}(v) + \rho \frac{\partial}{z}(w) + 0 = 0$$

$$\frac{\partial}{\partial x}(u) + \frac{\partial}{\partial y}(v) + \frac{\partial}{\partial z}(w) = 0$$

$$\nabla \cdot V = 0$$