Mathematics for AI: Linear Algebra Study Material

Week 1: Linear Algebra

1. Vectors

Definition: A vector is an object that has both a magnitude and a direction. In AI, vectors are often used to represent data points.

Notation: Vectors are typically denoted as v or v and can be written as a list of numbers, like v = [v1, v2, v3].

Operations:

- Addition: a + b = [a1 + b1, a2 + b2, a3 + b3]
- Scalar Multiplication: c * v = [c * v1, c * v2, c * v3]
- Dot Product: a . b = a1*b1 + a2*b2 + a3*b3. The dot product is a scalar value.

Example:

Consider two vectors a = [2, 3, 4] and b = [1, 0, -1].

- Addition: a + b = [2+1, 3+0, 4+(-1)] = [3, 3, 3]
- Scalar Multiplication: 2 * a = [2 * 2, 2 * 3, 2 * 4] = [4, 6, 8]
- Dot Product: a . b = 2 * 1 + 3 * 0 + 4 * (-1) = 2 - 4 = -2

2. Matrices

Definition: A matrix is a rectangular array of numbers arranged in rows and columns. Matrices are used extensively in AI to represent and manipulate data.

Notation: A matrix is denoted as A and is written as:

A = [a11 a12 a13; a21 a22 a23; a31 a32 a33]

Operations:

- Addition: Matrices of the same size can be added by adding corresponding elements.
- Scalar Multiplication: Each element of the matrix is multiplied by a scalar.
- Matrix Multiplication: The element in row i and column j of the resulting matrix is the dot product of row i from the first matrix and column j from the second matrix.

Example:

Consider two matrices:

A = [1 2 3; 4 5 6]

B = [7 8; 9 10; 11 12]

Matrix Multiplication A * B would result in:

C = [1*7 + 2*9 + 3*11, 1*8 + 2*10 + 3*12; 4*7 + 5*9 + 6*11, 4*8 + 5*10 + 6*12] = [58 64; 139 154]

3. Eigenvalues and Eigenvectors

Definition:

- An eigenvector of a matrix is a vector that does not change its direction when that matrix is applied to it.
- An eigenvalue is a scalar that indicates how much the eigenvector is scaled during the transformation.

Equation: Av = lambda * v, where A is a matrix, v is the eigenvector, and lambda is the eigenvalue.

Example:

For a matrix A = [4 1; 2 3], you would solve the equation Av = lambda * v to find the eigenvalues lambda and corresponding eigenvectors v.

Exercises:

1. Practice vector addition, scalar multiplication, and dot products with different vectors.

2. Work on matrix multiplication with different sizes of matrices.

3. Find eigenvalues and eigenvectors for simple 2x2 matrices.