CSE 604

Artificial Intelligence

Chapter 5: Adversarial Search

Adapted from slides available in Russell & Norvig's textbook webpage

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Outline

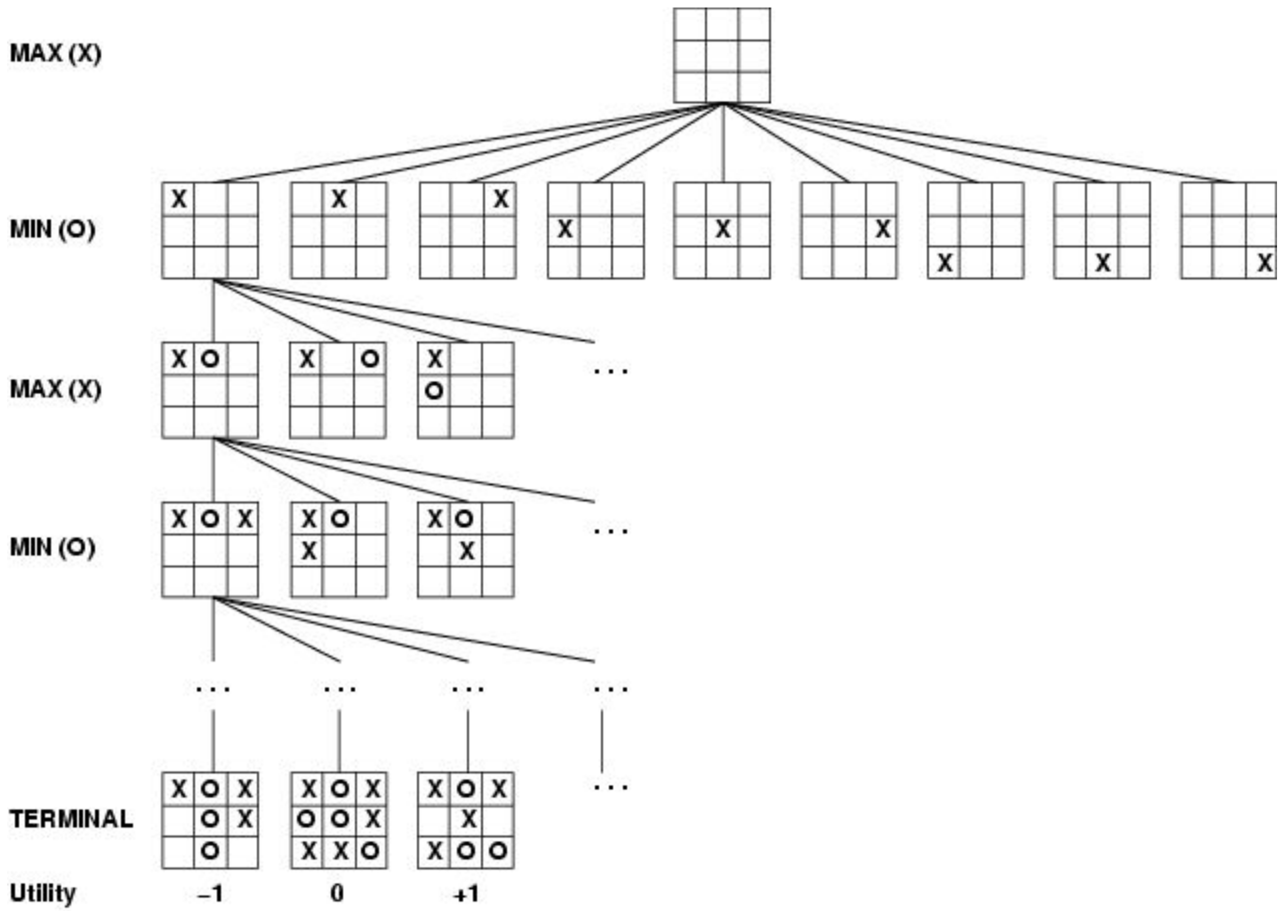
- Optimal decisions
- α-β pruning
- Imperfect, real-time decisions

Games vs. search problems

- "Unpredictable" opponent specifying a move for every possible opponent reply

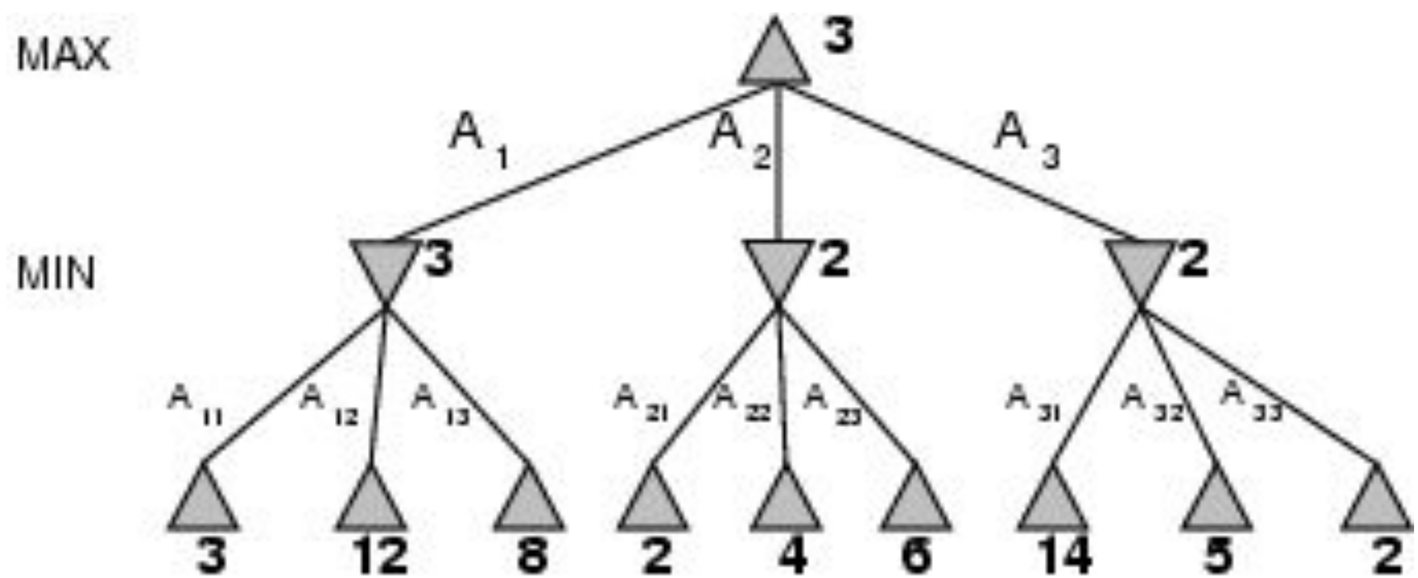
- Time limits unlikely to find goal, must approximate

Game tree (2-player, deterministic, turns)



Minimax

- Perfect play for deterministic games
- Idea: choose move to position with highest minimax value
  = best achievable payoff against best play
- E.g., 2-ply game:



Minimax algorithm

function MINIMAX-DECISION(*state*) **returns** *an action*

  $v \leftarrow$ MAX-VALUE(*state*)
  return the *action* in SUCCESSORS(*state*) with value v

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function MAX-VALUE(*state*) **returns** *a utility value*

  if TERMINAL-TEST(*state*) **then return** UTILITY(*state*)
  $v \leftarrow -\infty$
  for a, s in SUCCESSORS(*state*) **do**
    $v \leftarrow$ MAX(v, MIN-VALUE(s))
  return v

---

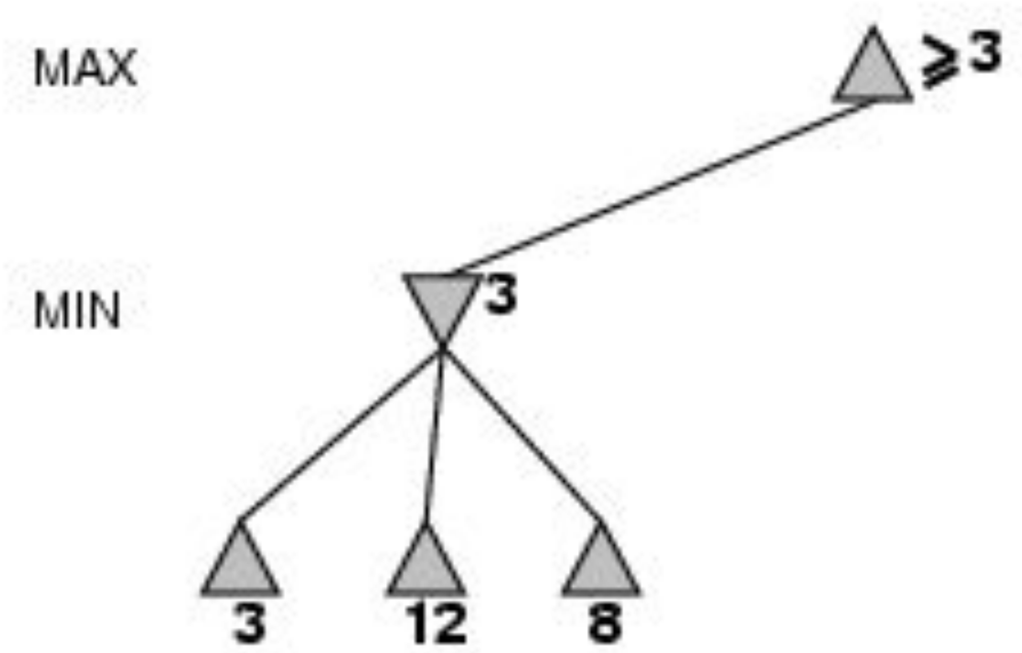
function MIN-VALUE(*state*) **returns** *a utility value*

  if TERMINAL-TEST(*state*) **then return** UTILITY(*state*)
  $v \leftarrow \infty$
  for a, s in SUCCESSORS(*state*) **do**
    $v \leftarrow$ MIN(v, MAX-VALUE(s))
  return v

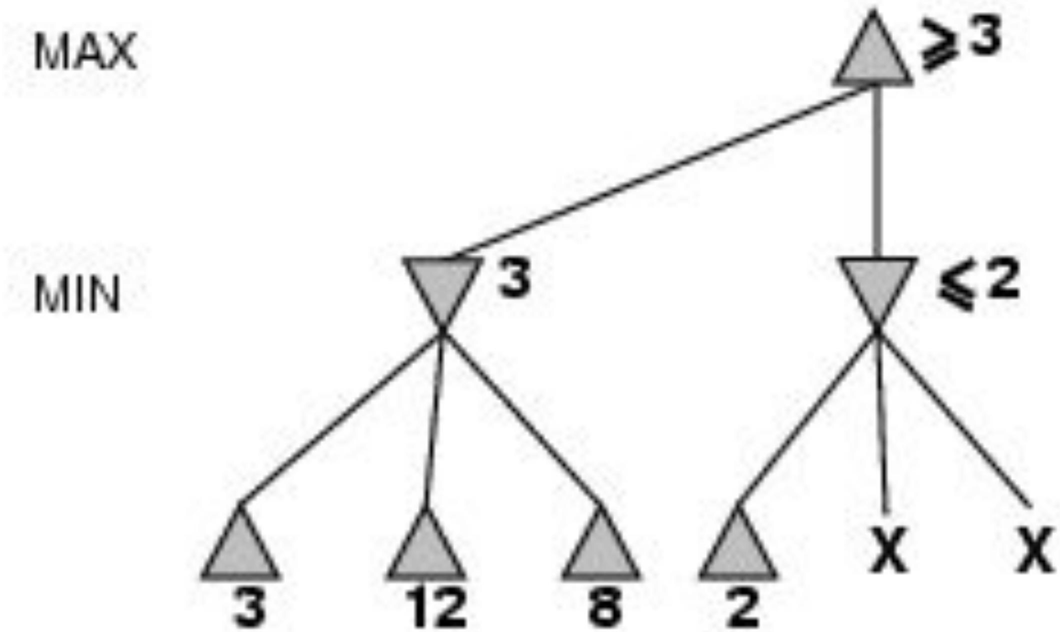
Properties of minimax

- Complete? Yes (if tree is finite)
- Optimal? Yes (against an optimal opponent)
- Time complexity? $O(b^m)$
- Space complexity? $O(bm)$ (depth-first exploration)
- For chess, $b \approx 35$, $m \approx 100$ for "reasonable" games
  exact solution completely infeasible

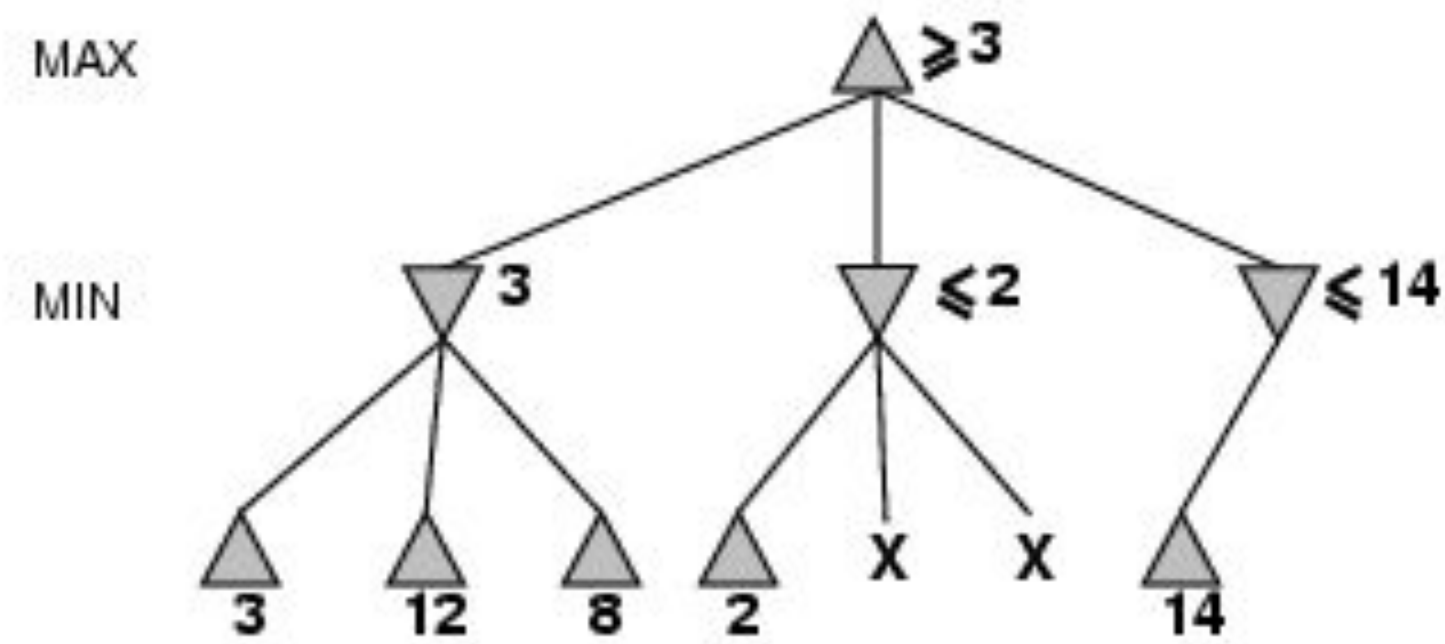
α-β pruning example



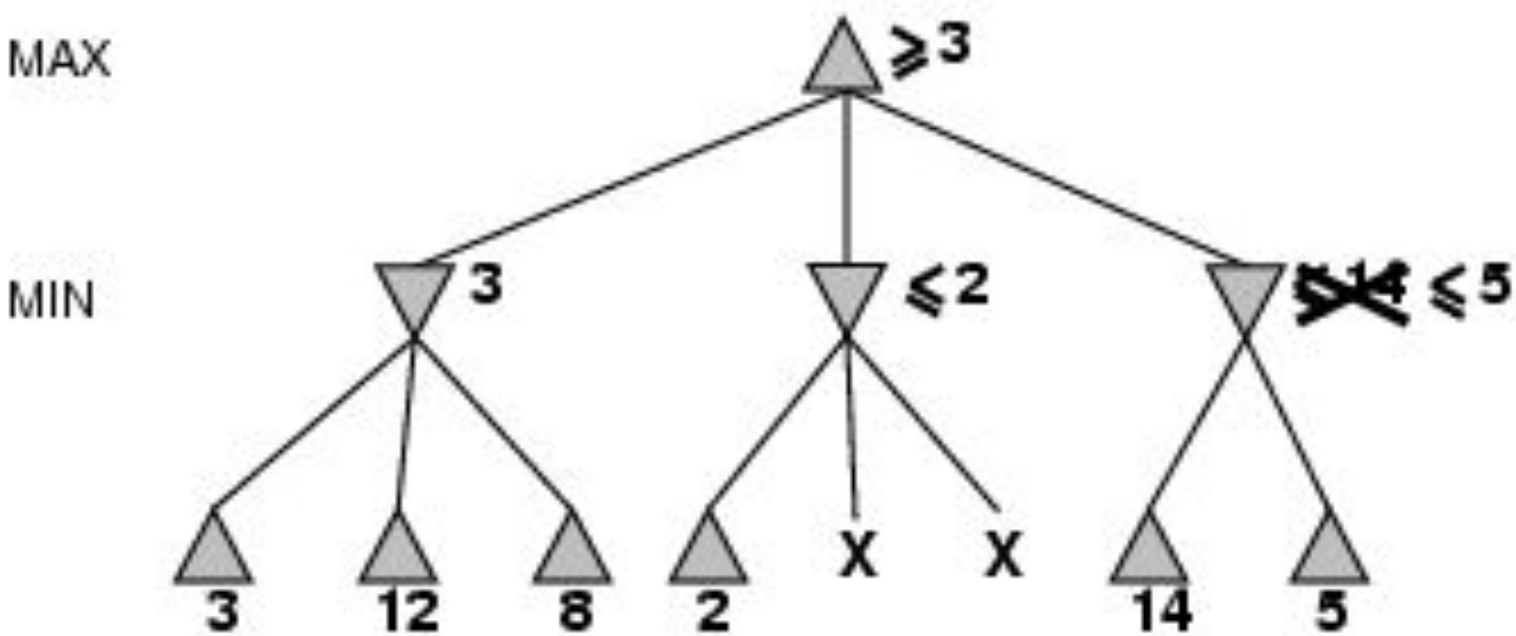
α-β pruning example



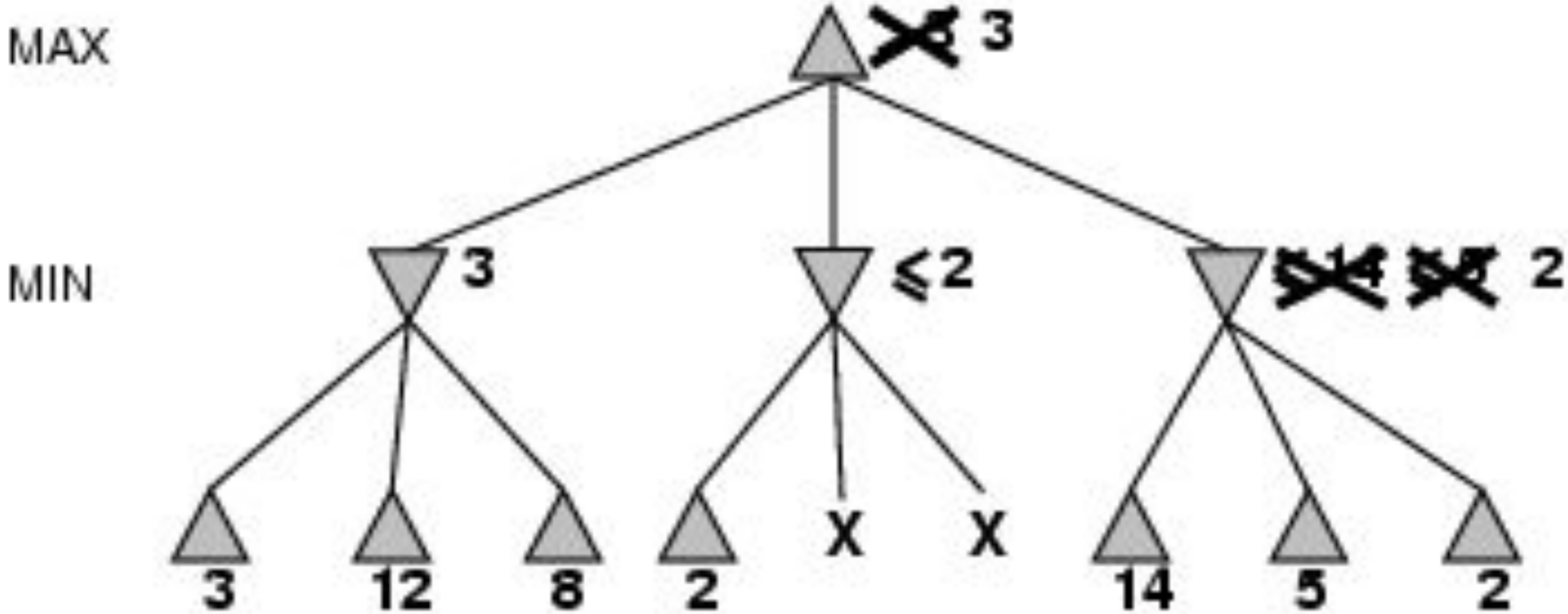
α-β pruning example



α-β pruning example



α-β pruning example

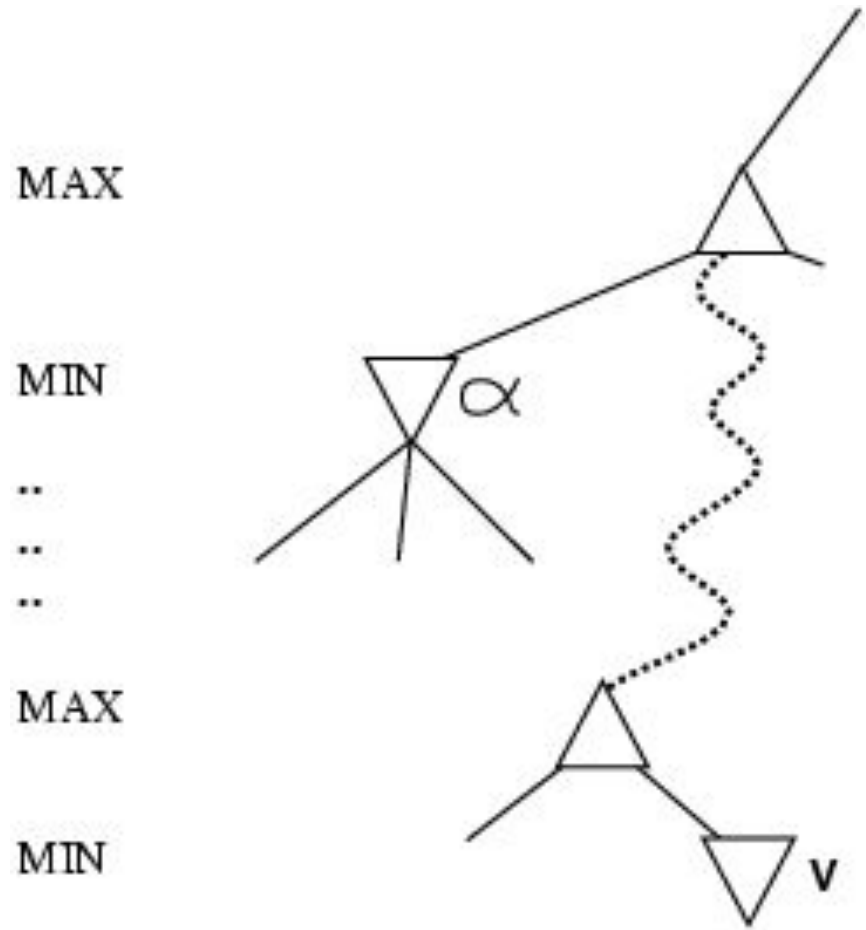


Properties of α-β

- Pruning does not affect final result
- Good move ordering improves effectiveness of pruning
  - Try the moves that are "likely to be best" first
  - E.g., in chess, try captures, threats, forward moves, backward moves in that order
- With "perfect ordering," time complexity $\approx O(b^{m/2})$
  doubles depth of search
- A simple example of the value of reasoning about which computations are relevant (a form of metareasoning)

Why is it called α-β?

- α is the value of the best (i.e., highest-value) choice found so far at any choice point along the path for *max*
- If *v* is worse than α, *max* will avoid it
  - prune that branch
- Define β similarly for *min*



The α-β algorithm

function ALPHA-BETA-SEARCH(*state*) **returns** *an action*
  inputs: *state*, current state in game

  $v \leftarrow$ MAX-VALUE($state, -\infty, +\infty$)
  return the *action* in SUCCESSORS(*state*) with value v

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function MAX-VALUE($state, \alpha, \beta$) **returns** *a utility value*
  inputs: *state*, current state in game
    α, the value of the best alternative for MAX along the path to *state*
    β, the value of the best alternative for MIN along the path to *state*

  if TERMINAL-TEST(*state*) **then return** UTILITY(*state*)
  $v \leftarrow -\infty$
  for a, s in SUCCESSORS(*state*) **do**
    $v \leftarrow$ MAX(v, MIN-VALUE(s, α, β))
    if $v \geq \beta$ **then return** v
    $\alpha \leftarrow$ MAX(α, v)
  return v

The α-β algorithm

function MIN-VALUE(*state*, α, β) **returns** *a utility value*
  inputs: *state*, current state in game
    α, the value of the best alternative for MAX along the path to *state*
    β, the value of the best alternative for MIN along the path to *state*

  if TERMINAL-TEST(*state*) **then return** UTILITY(*state*)
  $v \leftarrow +\infty$
  for a, s in SUCCESSORS(*state*) **do**
    $v \leftarrow$ MIN(v, MAX-VALUE(s, α, β))
    if $v \leq \alpha$ **then return** v
    $\beta \leftarrow$ MIN(β, v)
  return v

Imperfect Real Time Decisions

Even with alpha-beta pruning, it is infeasible to grow the whole game tree!

Standard approach:

- evaluation function

  = estimated desirability of position
- cut off search

  e.g., depth limit or iterative deepening
- forward pruning

  e.g., Beam search

Evaluation functions

- For chess, typically linear weighted sum of features

$$Eval(s) = w_1 f_1(s) + w_2 f_2(s) + \ldots + w_n f_n(s)$$

- e.g., $w_1 = 9$ with
  $f_1(s) =$ (number of white queens) – (number of black queens),
  $w_2 = 5$ with
  $f_2(s) =$ (number of white rooks) – (number of black rooks),

etc.

Cutting off search

- We can use a modified algorithm *MinimaxCutoff*
- *MinimaxCutoff* is identical to *MinimaxValue* except
  - *Terminal?* is replaced by *Cutoff?*
  - *Utility* is replaced by *Eval*

- Does it work in practice?
  - Suppose we have 100 secs, explore 10^4 nodes/sec
    10^6 nodes per move

  $b^m = 10^6$, $b=35$ $m=4$

- 4-ply lookahead is a hopeless chess player!
  - 4-ply ≈ human novice
  - 8-ply ≈ typical PC, human master
  - 12-ply ≈ Deep Blue, Kasparov

Deterministic games in practice

- **Checkers**: Chinook ended 40-year-reign of human world champion Marion Tinsley in 1994.
- **Chess**: Deep Blue defeated human world champion Garry Kasparov in a six-game match in 1997. Deep Blue searches 200 million positions per second, uses very sophisticated evaluation, and undisclosed methods for extending some lines of search up to 40 ply.
- **Othello**: human champions refuse to compete against computers, who are too good.
- **Go**: Until recently, human champions refused to compete against computers, who were too bad (in Go, $b > 300$).
  - But in 2016, Google's AlphaGo defeated human world champion Lee Sedol.
  - In 2017, AlphaGo Zero defeated the previous version of AlphaGo 100-0