MUKUL RAWAT, 42, CST

Date: /

Tutorial - 4

Q1) $T(n) = 3T(n/2) + n^2$

$a = 3$, $b = 2$ $f(n) = n^2$

Since $a \geq 0$ & $b > 0$ & $f(n) = n^2$ is +ive so master method applicable.

$c = \log_b a = \log_2 3 = 1.58$

Now $n^2 > n^{1.58}$

here. $f(n) = \Omega(n^{\log_b a + \varepsilon})$
$\varepsilon > 0$

$\therefore$ $\boxed{T(n) = \theta(n^2)}$

Q2) $T(n) = 4T(n/2) + n^2$

$a = 4$ $b = 2$ $f(n) = n^2$

$c = \log_2 4 = 2$

$\boxed{n^2 = n^c}$

$\therefore f(n) = \theta(n^{\log_b a})$

$T(n) = \Theta(n^2 \times \log n)$

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Q3.) $T(n) = T(n/2) + 2^n$

$a = 1$, $b = 2$ $f(n) = 2^n$

$c = \log_b a = 0$

$\Rightarrow n^c < 2^n$

also 2^n is is polynomically larger so.

$T(n) = \Theta(2^n)$ Ans

Q4) $T(n) = 2^n T(n/2) + n^n$

$a =$ Non-constant (Not Applicable)

Q5.) $T(n) = 16T(n/4) + n$

$a = 16$, $b = 4$ $f(n) = n$

$\Rightarrow c = \log_b a = 2$

$n^{\log_b a}$

$\because f(n) < n^2$, also $f(n)$ is polynomically larger so

$\therefore T(n) = \Theta(n^2)$

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Q6.) $T(n) = 2T(n/2) + n/\log n$

$a = 2 \quad b = 2$

$c = \log_b a = 1$

$n^c = n \not> f(n)$

~~but polynomically~~ but.

$f(n) \quad \Theta = n \log^{-1} n$

$k = 1 \neq (k \geq 0)$

$\therefore$ master method not applicable

Q7) $T(n) = 2T(n/2) + n \log n$

$a = 2 \quad b = 2$

$\because c = \log_b a = 1$

$\because n^c < n \log n$

$\because T(n) = \Theta(n)$

$f(n) = \theta(n^p \log^k n) \rightarrow p \in R$

Date: $k \geq p$

Q8.) $T(n) = 2T(n/4) + n^{0.51}$

$a = 2 \quad b = 4$

$c = \log_4 2 = 1/2$

$\therefore n^c < f(n)$

$\therefore T(n) = \Theta(n^{0.51})$

Q9.) $T(n) = 0.5\, T(n/2) + 1/n$

Not applicable. $a < 1$

Q10) $T(n) = 16T(n/4) + n!$

$c = \log_4 16 = 2$

$\therefore n^c < f(n)$

$\therefore \boxed{T(n) = \Theta(n!)}$

Q11.) $T(n) = 4T(n/2) + \log n$

$c = \log_2 4 = 2$

$\therefore n^c > \log n$

$\boxed{T(n) = \Theta(n^2)}$

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(Q12.) $T(n) = sqrt(n)\ T(n/2) + \log$

$a \neq$ constant so master method not applicable

(Q13.) $T(n) = 3T(n/2) + n$

$a = 3,\ b = 2 \qquad f(n) = n$

$c = \log_2 3 = 1.58$

$n^c > n \quad \therefore \quad \boxed{T(n) = \Theta(n^{1.58})}$

(Q14.) $T(n) = 3T(n/3) + sqrt(n)$

$c = \log_b a = 1$

$\therefore\ n^c > \sqrt{n}$

$\therefore \quad \boxed{T(n) = \Theta(n)}$

(Q15.) $T(n) = 4T(n/2) + cn$

$c = \log_2 4 = 2$

$n^c > c.n$

$\therefore \quad \boxed{T(n) = \Theta(n^2)}$

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Q16.) $T(n) = 3T(n/4) + n\log n$

$\because$ $c = \log_4 3 = 0.75$

$\because$ $n\log n > n^c$

$\therefore$ $\boxed{T(n) = \Theta(n\log n)}$

Q17.) $T(n) = 3T(n/3) + n/2$

$c = \log_3 3 = 1$

$\because$ $f(n) = n = n^c$

$\therefore$ $\boxed{T(n) = n\log(n)}$

Q18.) $T(n) = 6T(n/3) + n^2\log n$

$\because$ $c = \log_3 6 = 1.63$

$\because$ $f(n) > n^c$

$\therefore$ $\boxed{T(n) = \Theta(n^2\log n)}$

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Q19.) $T(n) = 4T(n/2) + n/\log n$

$f(n) = n\log^{-1} n$

$= \Theta(n \times \log^{-1} n)$

here ($p < 0$) $\therefore$ master method not applicable (by other method)

$T(n) = n^{\log_b a} \log\log n$

$\boxed{= n^2 \log\log n}$

Q20) $T(n) = 64T(n/8) - n^2\log n$

$f(n)$ is -i-ve

Q21) $T(n) = 7T(n/3) + n^2$

$\therefore$ $c = 1.77$

$\therefore$

$n^c < n^2$

$\therefore \boxed{T(n) = \Theta(n^2)}$

Q22.) $T(n) = T(n/2) + n(2 - \cos$

$f(n)$ no oregular function so master method not applic

"Master method chart"

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$$\boxed{T(n) = a \times T(n/b) + \Theta(n^k \log^p n)}$$

if :- (i) $a > b^k$ $\boxed{\log_b a > k}$

$T(n) = \Theta(n^{\log_b a})$

(ii) $a < b^k$

if $(p \geq 0) \rightarrow T(n) = \Theta(n^k \log^p n)$

if $(p < 0) \rightarrow T(n) = \Theta(n^k)$

(iii) $a = b^k$

a) if $(p > -1)$

(i) $\rightarrow T(n) = \Theta(n^{\log_b a} \log^{p+1} n)$

not (ii) $p = -1$, $T(n) = \Theta(n^{\log_b a} \log\log n)$

proved by master (iii) $p < -1$, $T(n) = \Theta(n^{\log_b a})$

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