

ONE WAY ANNOVA TEST

Problem Suppose we have production of 3 varieties of wheat. And we have taken 4 samples from each variety. and the factors affecting the result is the variety of seeds. Here we want to know if the production of all three varieties are equal or not.

Solⁿ

Assumptions for one way Anova

- Sample must be independent & random
- Sample must be normally distributed
- Sample must have equal variance

STEP-1

$H_0 \rightarrow$ Production of all three varieties are equal

$H_1 \rightarrow$ Production of all three varieties are not equal.

Confidence Interval = 95%

Significance level = 5% = 0.05

	Wheat Variety		
Fields	A	B	C
F1	6	5	5
F2	7	5	4
F3	3	3	3
F4	8	7	4

$$H_0 = \mu_A = \mu_B = \mu_C$$

It means variety of wheat does not effect the production

STEP 2

Now we will calculate the Sum of Squares ^{for sample variance} between the samples.

First Calculate the mean of sample A

$$\bar{X}_1 = \frac{6+7+3+0}{4} = 6$$

For Sample B

$$\bar{X}_2 = \frac{5+5+3+7}{4} = 5$$

For Sample C

$$\bar{X}_3 = \frac{5+4+3+4}{4} = 4$$

A	B	C
6	5	5
7	5	4
3	3	3
8	7	4

Now Calculate the mean of sample means

$$\bar{\bar{X}} = \frac{\bar{X}_1 + \bar{X}_2 + \bar{X}_3}{3} = \frac{6+5+4}{3} = 5$$

Sum of square for variance between the samples.

$$SS \text{ between} = n_1(\bar{X}_1 - \bar{\bar{X}})^2 + n_2(\bar{X}_2 - \bar{\bar{X}})^2 + n_3(\bar{X}_3 - \bar{\bar{X}})^2$$

here n_1, n_2, n_3 are no of samples from different fields of different wheat variety

$$\begin{aligned} SS \text{ between} &= 4(6-5)^2 + 4(5-5)^2 + 4(4-5)^2 \\ &= 4+0+4 = 8 \end{aligned}$$

STEP 3

Now we will calculate the sum of square for variance within the samples.

For variety A

$$A = \sum (X_{1i} - \bar{X}_1)^2$$

$X_{1i} \rightarrow$ Different samples collected from different fields

$\bar{X}_1 \rightarrow$ Mean of Sample A

$$\begin{aligned} A &= (6-6)^2 + (7-6)^2 + (3-6)^2 + (8-6)^2 \\ &= 0 + 1 + 9 + 4 = 14 \end{aligned}$$

A	B	C
6	5	5
7	5	4
3	3	3
8	7	4

For Variety B

$$\begin{aligned} B &= \sum (X_{2i} - \bar{X}_2)^2 \\ &= (5-5)^2 + (5-5)^2 + (5-3)^2 + (5-7)^2 \\ &= 0 + 0 + 4 + 4 = 8 \end{aligned}$$

For Variety C

$$\begin{aligned} C &= \sum (X_{3i} - \bar{X}_3)^2 \\ &= (5-4)^2 + (5-4)^2 + (3-4)^2 + (4-4)^2 \\ &= 1 + 0 + 1 + 0 = 2 \end{aligned}$$

Sum of square for variance within the samples

$$\begin{aligned} SS_{\text{within}} &= \sum (X_{1i} - \bar{X}_1)^2 + \sum (X_{2i} - \bar{X}_2)^2 + \sum (X_{3i} - \bar{X}_3)^2 \\ &= 14 + 8 + 2 \\ &= 24 \end{aligned}$$

STEP-4

Calculate the sum of square for Total Variance

$$SS \text{ for Total variance} = \sum (X_{ij} - \bar{X})^2$$

Here we will subtract the mean of sample mean from each sample

$$\begin{aligned} SS \text{ for Total variance} &= (6-5)^2 + (7-5)^2 + (3-5)^2 + \\ &\quad (10-5)^2 + (5-5)^2 + (5-5)^2 + \\ &\quad (3-5)^2 + (7-5)^2 + (5-5)^2 + \\ &\quad (4-5)^2 + (3-5)^2 + (4-5)^2 \end{aligned}$$

A	B	C
6	5	5
7	5	4
3	3	3
8	7	4

$$\begin{aligned} &= 1 + 4 + 4 + 9 + 0 + 0 + 4 + 4 \\ &\quad + 0 + 1 + 4 + 1 \\ &= 32 \end{aligned}$$

Alternatively we can calculate the SS for total variance as.

$$\begin{aligned} SS \text{ for total variance} &= SS_{\text{between}} + SS_{\text{within}} \\ &= 8 + 24 \\ &= 32 \end{aligned}$$

STEP-5

Prepare the ANNOVA Table

$SS_{\text{between}} = 8$, $SS_{\text{within}} = 24$, $SS_{\text{total}} = 32$, no of samples (m) = 3
no of elements in all the samples (n) = 12

Source of variation	Sum of Square	Degree of freedom	Mean Square	F-ratio	F-limit 5%, from F-distribution Table
Between Sample	8	$(m-1) = 3-1 = 2$	$8/2 = 4$	4.00	$F(2,9) =$ 4.26
Within Sample	24	$(n-m) = 12-3 = 9$	$24/9 = 2.67$	2.67 $= 1.5$	
Total	32	$(n-1) = (12-1) = 11$			

STEP 6

In the end ~~on~~

$$F\text{-ratio} = 1.5$$

$$F\text{ limit } ~~P\text{-limit}~~ = 4.26$$

Our F-ratio is less than ~~P-limit~~ F-limit

So we are going to accept the Null Hypothesis. which implies that variety of wheat can't affect the production of wheat.