DEPARTMENT OF MATHEMATICS, IIT - GUWAHATI
Odd Semester of the Academic year 2024 - 2025
MA 311M Assignment/Problem sheet 1
Instructor: Dr. J. C. Kalita
Due before the midnight of 8 August 2024

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1. Here is a very easy one:
(a) Perform the arithmetic $(9.4 \times 10^{-5}) + (3.6 \times 10^{4})$, expressing the answer in normalized scientific (exponential) notation.
(b) Give the final answer if the representation allows only five significant digits, rounded.
(c) Give the absolute error of part **b**.
(d) Give the relative error of part **b**.

2. Show that $e^{\pi\sqrt{163}}$ is incredibly close to being the 18 digit integer 262537412640768744. (*Hint:* More than 30 decimal digits will be required to see any difference.)

3. Use three digit rounding arithmetic to perform the following calculations. Compute the absolute and relative errors with the exact value determined by the computer at double precision:
(a) 133+0.921 (b) 122-0.499 (c) (121-0.327)-119 (d) (121-119)-0.327 (e) $\dfrac{\frac{13}{14} - \frac{3}{4}}{2e - 5.4}$
(f) $\dfrac{\pi - \frac{22}{7}}{\frac{1}{17}}$.

Repeat the exercise with four digit chopping arithmetic.

4. The first three nonzero terms of the Maclaurine series for $\tan^{-1} x$ are $x - \dfrac{1}{3}x^2 + \dfrac{1}{5}x^5$. Compute the relative and absolute errors in the following approximations of π using polynomials in place of $\tan^{-1}$ function:

(a) $4\left[\tan^{-1}\left(\dfrac{1}{2}\right) + \tan^{-1}\left(\dfrac{1}{3}\right)\right]$ (b) $16\tan^{-1}\left(\dfrac{1}{5}\right) - 4\tan^{-1}\left(\dfrac{1}{239}\right)$

5. The infinite series of $\sin x$ is give by

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + ...$$

(a) Obtain a decimal approximation for $\sin(2)$, truncating the infinite series in **two** terms. Give the error for this approximation.
(b) Define five partial sum approximations for the above function. Graph $\sin x$ and the approximating functions. Evaluate the $\sin x$ and the approximating functions at $\frac{\pi}{6}$, $\frac{\pi}{4}$, $\frac{\pi}{2}$ and 3π, and compute the absolute and relative errors of these approximations.

6. The number e can be defined by $e = \displaystyle\sum_{n=0}^{\infty} \frac{1}{n!}$. Compute the relative and absolute errors in the following approximations of e:
(a) $e \simeq \displaystyle\sum_{n=0}^{5} \frac{1}{n!}$, (b) $e \simeq \displaystyle\sum_{n=0}^{10} \frac{1}{n!}$ and (c) $e \simeq \displaystyle\sum_{n=0}^{1000} \frac{1}{n!}$.

7. Verify that the observations made in connection with examples 4 and 5 of chapter 1.2 of **Burden and Fairs** (pages 23-26) were correct.