Assignment 4: Probability and Random Variables

Student Submission

Solution

1. Probability that a bulb is defective

Given:
$$P(M_1) = 0.6, \quad P(M_2) = 0.4$$
$$P(D|M_1) = 0.02, \quad P(D|M_2) = 0.05$$

The probability that a bulb is defective is:

$$P(D) = P(D|M_1) + P(D|M_2)$$

$$P(D) = 0.02 + 0.05 = 0.07$$

2. Probability that the defective bulb came from M_2

Using Bayes' theorem:
$$P(M_2|D) = \frac{P(D|M_2)P(M_2)}{P(D)}$$

$$P(M_2|D) = \frac{0.05 \times 0.4}{0.07} \approx 0.286$$

3. Expected value of X

Since $X = 1$ if the bulb is defective and 0 otherwise, the expected value is:

$$E[X] = 1 \times P(D) + 0 \times (1 - P(D)) = 0.07$$