Assignment 4: Probability and Random Variables

Course: Probability for Machine Learning

Problem Statement

A factory produces light bulbs using two machines, M_1 and M_2.

- Machine M_1 produces 60% of the bulbs, with a defect rate of 2%.
- Machine M_2 produces 40% of the bulbs, with a defect rate of 5%.

1. What is the probability that a randomly selected bulb is defective?
2. Given that a bulb is defective, what is the probability it was produced by M_2?
3. Define a random variable X which takes value 1 if a bulb is defective and 0 otherwise. Compute $\mathbb{E}[X]$.

Teacher's Solution

1. Probability that a bulb is defective

Using the law of total probability:

$$P(D) = P(D|M_1)P(M_1) + P(D|M_2)P(M_2)$$

$$P(D) = (0.02)(0.6) + (0.05)(0.4) = 0.012 + 0.020 = 0.032$$

2. Probability that the defective bulb came from M_2

Using Bayes' Theorem:

$$P(M_2|D) = \frac{P(D|M_2)P(M_2)}{P(D)}$$

$$P(M_2|D) = \frac{0.05 \times 0.4}{0.032} = 0.625$$

3. Expected value of X

Since X is an indicator random variable:

$$\mathbb{E}[X] = P(X = 1) = P(D) = 0.032$$