SUNSTONE

Bachelor of Computer Application

Computational Mathematics

LECTURE 18 NOTES

Transpose of a Matrix

- **Transpose of Matrix**

  - The new matrix obtained by interchanging the rows and columns of the original matrix is known as the transpose of the matrix.

  - If A = [a_{ij}] be an m × n matrix, then the matrix obtained by interchanging the rows and columns of A would be the transpose of A. It is denoted by A′or (A^T).

  - In other words, if A = $[a_{ij}]_{mxn}$,thenA′ = $[a_{ji}]_{nxm}$.

  - **Example:** $A = \begin{bmatrix} 3 & -5 \\ 4 & \frac{7}{2} \\ 9 & \frac{5}{8} \end{bmatrix}$ $A' = \begin{bmatrix} 3 & 4 & 9 \\ -5 & \frac{7}{2} & \frac{5}{8} \end{bmatrix}$

- **Properties of Transpose of the Matrices**

  - **PROPERTY 1:** For any matrix A, $\mathbf{(A^T)^T=A}$

  - **Proof:**
    Let A = [a_{ij}] be an m x n matrix. Then, A^T is a nxm matrix and so $\mathbf{(A^T)^T}$ is an m x n matrix. Thus, the matrices **A** and $\mathbf{(A^T)^T}$ are of the same order such that

$((A^T)^T)_{ij} = (A^T)_{ij}$ **[**by the definition of transpose**]**

⟹ $((A^T)^T)_{ij} = (A^T)_{ij}$ for all i=1, 2,...,m and j=1, 2,...,n

Hence, by the definition of equality of two matrices, we obtain

$$(A^T)^T=A$$

- **PROPERTY 2:** For any two matrices A and B of the same order, $\mathbf{(A+B)^T = A^T + B^T}$

- **Proof:**
Let $A = [a_{ij}]_{m \times n}$ and $B = [b_{ij}]_{m \times n}$. Then, A+B will be a matrix of the order m x n and so $(A+B)^T$ will be a matrix of order n x m. Since A^T and B^T are both nx m matrices.

Therefore, $A^T + B^T$ will be a matrix of the order n x m. Thus, the matrices $(A + B)^T$ and $A^T + B^T$ are of the same order such that

$\Longrightarrow$ $((A+B)^T))_{ij} = (A+B)_{ji}$ [By the definition of transpose]

$\Longrightarrow$ $((A + B)^T)_{ij} = a_{ji} + b_{ji}$ [By the definition of addition]

$\Longrightarrow$ $((A + B)^T)ij = (A^T)ij + (B^T)ij$

$\Longrightarrow$ $((A + B)^T)ij = (A^T + B^T)ij$ for all i, j [By the definition of addition]

Hence, by the definition of equality of two matrices, we obtain

$$\mathbf{(A+B)^T = A^T + B^T}$$

- **PROPERTY 3:** If A is a matrix and K is a scalar, then $\mathbf{(kA)^T = k(A)^T}$

- **Proof:**
Let $A = [a_{ij}]$ be an m x n matrix. Then, for any scalar k, kA is also an m x n matrix and so $(kA)^T$ is an n x m matrix. Again, A^T is an n x m matrix and so KA^T is an n x m matrix. Thus, the two matrices $(kA)^T$ and KA^T are of the same order such that

$((kA)^T)ij = (kA)_{ji}$ [By the definition of transpose]

$\Rightarrow$ $((kA)^T)ij = k\, a_{ji}$ [By the definition of scalar multiplication]

$\Rightarrow$ $((KA)) = k(A^T)_{ij}$ [By the definition of transpose]

$\Rightarrow$ $((KA)^T)_{ij} = (KA^T)_{ij}$ [By the definition of scalar multiplication]

Hence, by the definition of equality of two matrices, we obtain

$$(kA)^T = k(A)^T$$

- **PROPERTY 4:** If A and B are two matrices such that AB is defined, then $(AB)^T = B^T A^T$.

- **Proof:**
Let $A=[a_{ij}]$ m x n and $B=[b_{ij}]_{n\times p}$ be two matrices. Then AB is an m x p matrix and therefore $(AB)^T$ is a p x m matrix.

Since A^T and B^T are n x m and p x n matrices, therefore $B^T A^T$ is a p x m matrix. Thus, the two matrices $(AB)^T$ and $B^T A^T$ are of the same order such that

$((AB)^T)_{ij} = (AB)_{ji}$ [By the definition of transpose]

$$\Rightarrow \quad ((AB)^T)_{ij} = \sum_{r=1}^{n} a_{jr} b_{ri} \qquad [By\ the\ definition\ of\ matrix\ multiplication]$$

$$\Rightarrow \quad ((AB)^T)_{ij} = \sum_{r=1}^{n} b_{ri} a_{jr} \qquad [By\ commutativity\ of\ multiplication\ of\ numbers]$$

$$\Rightarrow \quad ((AB)^T)_{ij} = \sum_{r=1}^{n} (B^T)_{ir} (A^T)_{rj} \qquad [By\ definition\ of\ transpose]$$

$$\Rightarrow \quad ((AB)^T)_{ij} = (B^T A^T)_{ij} \qquad [By\ definition\ of\ multiplication\ of\ matrices]$$

Hence, by the definition of equality of two matrices, we obtain

$$(AB)^T = B^T A^T.$$

❖ EXAMPLES:

- We now state the following properties of transpose of matrices without proof. These may be verified by taking suitable examples.
- For any matrices A and B of suitable orders, we have
- (i) $(A')' = A$, (ii) $(kA)' = kA'$ (where k is any constant) (iii) $(A + B)' = A' + B'$ (iv) $(A B)' = B' A$

- **Example 1:** If $A=\begin{bmatrix} 3 & \sqrt{3} & 2 \\ 4 & 2 & 0 \end{bmatrix}$ *and* $B=\begin{bmatrix} 2 & -1 & 2 \\ 1 & 2 & 4 \end{bmatrix}$ verify that

  (i) (A′)′ = A, (ii) (A + B)′ = A′ + B′, (iii) (kB)′ = kB′, where k is any constant.

- **Solution:**

(i) We have,

$$A=\begin{bmatrix} 3 & \sqrt{3} & 2 \\ 4 & 2 & 0 \end{bmatrix} \Rightarrow A'=\begin{bmatrix} 3 & 4 \\ \sqrt{3} & 2 \\ 2 & 0 \end{bmatrix} \Rightarrow (A')'=\begin{bmatrix} 3 & \sqrt{3} & 2 \\ 4 & 2 & 0 \end{bmatrix}=A$$

(ii) We have $A=\begin{bmatrix} 3 & \sqrt{3} & 2 \\ 4 & 2 & 0 \end{bmatrix}$ *and* $B=\begin{bmatrix} 2 & -1 & 2 \\ 1 & 2 & 4 \end{bmatrix}$

$$\Rightarrow A+B=\begin{bmatrix} 3 & \sqrt{3} & 2 \\ 4 & 2 & 0 \end{bmatrix}+\begin{bmatrix} 2 & -1 & 2 \\ 1 & 2 & 4 \end{bmatrix}=\begin{bmatrix} 5 & \sqrt{3}-1 & 4 \\ 5 & 4 & 4 \end{bmatrix}$$

$$Therefore \quad (A+B)'=\begin{bmatrix} 5 & 5 \\ \sqrt{3}-1 & 4 \\ 4 & 4 \end{bmatrix}$$

$$Now, \quad A'=\begin{bmatrix} 3 & 4 \\ \sqrt{3} & 2 \\ 2 & 0 \end{bmatrix}, B'=\begin{bmatrix} 2 & 1 \\ -1 & 2 \\ 2 & 4 \end{bmatrix}$$

$$so \quad A'+B'=\begin{bmatrix} 5 & 5 \\ \sqrt{3}-1 & 4 \\ 4 & 4 \end{bmatrix}$$

Thus $(A+B)'=A'+B'$

(iii) We have

$$kB = K\begin{bmatrix} 2 & -1 & 2 \\ 1 & 2 & 4 \end{bmatrix} = \begin{bmatrix} 2k & -k & 2k \\ k & 2k & 4k \end{bmatrix}$$

$$Then, \quad (KB)' = \begin{bmatrix} 2k & k \\ -k & 2k \\ 2k & 4k \end{bmatrix} = k\begin{bmatrix} 2 & 1 \\ -1 & 2 \\ 2 & 4 \end{bmatrix} = kB'$$

$$Thus, \quad (kB)' = kB'$$

- **Example 2:** If $A = \begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix}$ and $B = \begin{bmatrix} -2 & -1 & 4 \end{bmatrix}$, verify that $(AB)^T = B^T A^T$.

- **Solution:**

We have,

$$A = \begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix} \text{ and } B = \begin{bmatrix} -2 & -1 & 4 \end{bmatrix}$$

$$\therefore \quad AB = \begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix}\begin{bmatrix} -2 & -1 & 4 \end{bmatrix} = \begin{bmatrix} 2 & 1 & 4 \\ -4 & -2 & -8 \\ -6 & -3 & -12 \end{bmatrix}$$

$$\Rightarrow \quad (AB)^T = \begin{bmatrix} 2 & -4 & -6 \\ 1 & -2 & -3 \\ 4 & -8 & -12 \end{bmatrix} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(i)$$

$$Also,\ B^T A^T = \begin{bmatrix} -2 & -1 & 4 \end{bmatrix}^T \begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix}^T = \begin{bmatrix} -2 \\ -1 \\ 4 \end{bmatrix}\begin{bmatrix} -1 & 2 & 3 \end{bmatrix} = \begin{bmatrix} 2 & -4 & -6 \\ 1 & -2 & -3 \\ 4 & -8 & -12 \end{bmatrix} \ldots\ldots\ldots\ldots\ldots\ldots(ii)$$

- **Example 3:** If $A=\begin{bmatrix}\cos\Theta & -\sin\Theta \\ \sin\Theta & \cos\Theta\end{bmatrix}$, *then find the values of* Θ *satisfying the equation* $A^T+A=I_2$.

- **Solution:** *We have,*

$$A=\begin{bmatrix}\cos\Theta & -\sin\Theta \\ \sin\Theta & \cos\Theta\end{bmatrix} \Rightarrow A^T=\begin{bmatrix}\cos\Theta & \sin\Theta \\ -\sin\Theta & \cos\Theta\end{bmatrix}$$

Now, $A^T+A=I_2$

$$\Rightarrow \begin{bmatrix}\cos\Theta & \sin\Theta \\ -\sin\Theta & \cos\Theta\end{bmatrix}+\begin{bmatrix}\cos\Theta & -\sin\Theta \\ \sin\Theta & \cos\Theta\end{bmatrix}=\begin{bmatrix}1 & 0 \\ 0 & 1\end{bmatrix}$$

$$\Rightarrow \begin{bmatrix}2\cos\Theta & 0 \\ 0 & 2\cos\Theta\end{bmatrix}=\begin{bmatrix}1 & 0 \\ 0 & 1\end{bmatrix}$$

$$\Rightarrow 2\cos\Theta=1 \Rightarrow \cos\Theta=\cos\frac{\pi}{3} \Rightarrow \Theta=2n\pi\pm\frac{\pi}{3},\ n\in Z$$

- **Example 4:** If $A=\begin{bmatrix}1 & 2 & 2 \\ 2 & 1 & -2 \\ a & 2 & b\end{bmatrix}$ *is a matrix satisfying* $AA^T=9I_3$, *then find the values of* a *and* b.

- **Solution:** *We have,*

$$A=\begin{bmatrix}1 & 2 & 2 \\ 2 & 1 & -2 \\ a & 2 & b\end{bmatrix} \Rightarrow A^T=\begin{bmatrix}1 & 2 & a \\ 2 & 1 & -2 \\ 2 & -2 & b\end{bmatrix}$$

$$\therefore \quad AA^T=9I_3$$

$$\Rightarrow \begin{bmatrix}1 & 2 & 2 \\ 2 & 1 & -2 \\ a & 2 & b\end{bmatrix}\begin{bmatrix}1 & 2 & a \\ 2 & 1 & -2 \\ 2 & -2 & b\end{bmatrix}=9\begin{bmatrix}1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1\end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 9 & 0 & a+2b+4 \\ 0 & 9 & 2a+2-2b \\ a+2b+4 & 2a+2-2b & a^2+4+b^2 \end{bmatrix} = \begin{bmatrix} 9 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{bmatrix}$$

$$\Rightarrow \quad a+2b+4=0\ ,\ 2a+2-2b=0 \text{ and } a^2+4+b^2=9$$

$$\Rightarrow \quad a+2b+4=0\ ,\ a-b+1=0 \text{ and } a^2+b^2=5$$

Solving, $a+2b+4=0$ *and* $a-b+1=0$, *we get*: $a=-2$ *and* $b=-1$.

- **Example 5:** *Find the values of* x, y, z *if the matrix* $A = \begin{bmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{bmatrix}$, *satisfy the equation* $A^T A = I_3$

- **Solution:** *We have*,

$$A = \begin{bmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{bmatrix} \Rightarrow A^T = \begin{bmatrix} 0 & x & x \\ 2y & y & -y \\ z & -z & z \end{bmatrix}$$

It is given that,

$$A^T A = I_3$$

$$\Rightarrow \begin{bmatrix} 0 & x & x \\ 2y & y & -y \\ z & -z & z \end{bmatrix} \begin{bmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2x^2 & 0 & 0 \\ 0 & 6y^2 & 0 \\ 0 & 0 & 3z^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \quad 2x^2=1,\ 6y^2=1,\ 3z^2=1$$

$$\Rightarrow \quad x = \pm\frac{1}{\sqrt{2}},\ y = \pm\frac{1}{\sqrt{6}},\ z = \pm\frac{1}{\sqrt{3}}$$

❖ EXERCISE:

1. Let $A = \begin{bmatrix} 2 & -3 \\ -7 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 \\ 2 & -4 \end{bmatrix}$, verify that.

(i) $(2A)^T$ (ii) $(A+B)^T = A^T + B^T$

(iii) $(A-B)^T = A^T - B^T$ (iv) $(AB)^T = B^T A^T$

2. If $A = \begin{bmatrix} 3 \\ 5 \\ 2 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 & 4 \end{bmatrix}$, verify that $(AB)^T = B^T A^T$.

3. If $A^T = \begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 2 & 1 \\ 1 & 2 & 3 \end{bmatrix}$, find $A^T - B^T$.

4. If $A = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$, then verify that $A^T A = I_2$

5. If $A = \begin{bmatrix} \sin\alpha & \cos\alpha \\ -\cos\alpha & \sin\alpha \end{bmatrix}$, verify that $A^T A = I_2$

- **Symmetric and Skew-Symmetric Matrices**

  - **Definition:** A square matrix $A = [a_{ij}]$ is said to be symmetric if $A' = A$, that is, $[a_{ij}] = [a_{ji}]$ for all possible values of i and j.

  - **For example,** $\begin{bmatrix} \sqrt{3} & 2 & 3 \\ 2 & -1.5 & -1 \\ 3 & -1 & 1 \end{bmatrix}$ is a symmetric matrix as $A' = A$.

  - **Definition:** A square matrix $A = [a_{ij}]$ is said to be skew symmetric matrix if $A' = -A$, that is $a_{ji} = -a_{ij}$ for all possible values of i and j. Now, if we put $i = j$, we have $a_{ii} = -a_{ii}$. Therefore $2a_{ii} = 0$ or $a_{ii} = 0$ for all i's.

    This means that all the diagonal elements of a skew symmetric matrix are zero.

  - **For example,** the matrix $\begin{bmatrix} 0 & e & f \\ -e & 0 & g \\ -f & -g & 0 \end{bmatrix}$ is a skew symmetric matrix as $B' = -B$

❖ THEOREMS:

- **Theorem 1:** For any square matrix A with real number entries, $A + A'$ is a symmetric matrix and $A - A'$ is a skew symmetric matrix.

- **Proof:**
  Let $B = A + A'$, then

  $B' = (A + A')'$
  $= A' + (A')'$ (as $(A + B)' = A' + B'$)
  $= A' + A$ (as $(A')' = A$)
  $= A + A'$ (as $A + B = B + A$)
  $= B$

  Therefore $B = A + A'$ is a symmetric matrix

  Now let $C = A - A'$
  $C' = (A - A')' = A' - (A')'$ (Why?)
  $= A' - A$ (Why?)
  $= -(A - A') = -C$

  Therefore $C = A - A'$ is a skew symmetric matrix.

- **Theorem 2:** Any square matrix can be expressed as the sum of a symmetric and a skew symmetric matrix.

- **Proof:**
  Let A be a square matrix, then we can write

$$A = \frac{1}{2}(A + A^T) \;+\; \frac{1}{2}(A - A^T)$$

From the Theorem 1, we know that (A + A′) is a symmetric matrix and (A – A′) is a skew symmetric

matrix. Since for any matrix A, (kA)′ = kA′, it follows that $\frac{1}{2}(A + A^T)$ is symmetric matrix

and $\frac{1}{2}(A - A^T)$ is skew symmetric matrix. Thus, any square matrix can be expressed as the

sum of a symmetric and a skew symmetric matrix.

❖ EXAMPLES:

- **Example 1:**
  Show that the elements on the main diagonal of a skew-symmetric matrix are all zero.

- **Solution:**

  Let A = $[a_{ij}]$ be a skew-symmetric matrix. Then,

  $a_{ij} = -a_{ji}$ for all i, j [By definition]

  $\Rightarrow$ $a_{ii} = -a_{ii}$ for all values of i

  $\Rightarrow$ $2\,a_{ii} = 0$

  $\Rightarrow$ $a_{ij} = 0$ for all values of i

$\Longrightarrow \qquad a_{11}= a_{22}= a_{33}=......= a_{nn} =0$

- **Example 2:** *Express the matrix* $B=\begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$ *as the sum of a symmetric and a skew symmetric matrix.*

- **Solution:**

Here,

$$B^T=\begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$$

$$Let \qquad P=\frac{1}{2}\left(B+B'\right)=\frac{1}{2}\begin{bmatrix} 4 & -3 & -3 \\ -3 & 6 & 2 \\ -3 & 2 & -6 \end{bmatrix}=\begin{bmatrix} 2 & -\frac{3}{2} & -\frac{3}{2} \\ -\frac{3}{2} & 3 & 1 \\ -\frac{3}{2} & 1 & -3 \end{bmatrix}$$

$$Now,\ P^T=\begin{bmatrix} 2 & -\frac{3}{2} & -\frac{3}{2} \\ -\frac{3}{2} & 3 & 1 \\ -\frac{3}{2} & 1 & -3 \end{bmatrix}=P$$

$$Thus, \qquad P=\frac{1}{2}\left(B+B^T\right) \text{ is a symmetric matrix.}$$

$$Also,\ let \qquad Q=\frac{1}{2}\left(B-B^T\right)=\frac{1}{2}\begin{bmatrix} 0 & -1 & -5 \\ 1 & 0 & -3 \\ -3 & 2 & -6 \end{bmatrix}=\begin{bmatrix} 0 & -\frac{1}{2} & -\frac{5}{2} \\ \frac{1}{2} & 0 & 3 \\ \frac{5}{2} & -3 & 0 \end{bmatrix}$$

$$Then \qquad Q^T=\begin{bmatrix} 0 & -\frac{1}{2} & -\frac{5}{2} \\ \frac{1}{2} & 0 & 3 \\ \frac{5}{2} & -3 & 0 \end{bmatrix}=-Q$$

$$Thus, \qquad Q=\frac{1}{2}\left(B-B^T\right) \text{ is a skew symmetric matrix.}$$

$$Now,\ P+Q=\begin{bmatrix} 2 & -\frac{3}{2} & -\frac{3}{2} \\ -\frac{3}{2} & 3 & 1 \\ -\frac{3}{2} & 1 & -3 \end{bmatrix}+\begin{bmatrix} 0 & -\frac{1}{2} & -\frac{5}{2} \\ \frac{1}{2} & 0 & 3 \\ \frac{5}{2} & -3 & 0 \end{bmatrix}=\begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}=B$$

- **Example 3:** Let A be a square matrix. Then,

    (i) $A + A^T$ is a symmetric matrix
    (ii) $A - A^T$ is a skew-symmetric matrix.
    (iii) AA^T and $A^T A$ are symmetric matrices.

- **Solution:**

(i) Let P= A + AT. Then,

$P^T = (A+A^T)^T = A^T + (A^T)^T$ **[∵ $(A+B)^T = A^T + B^T$]**
$P^T = A^T + A$ **[∵ $(A^T)^T = A$]**
$P^T = A + A^T = P$ [By commutativity of matrix addition]
P is a symmetric matrix.

(ii) Let $Q = A - A^T$. Then,
$Q^T = (A-A^T)^T = A^T - (A^T)^T$ **[∵ $(A+B)^T = A^T + B^T$]**
$Q^T = A^T - A$ **[∵ $(A^T)^T = A$]**
$Q^T = -(A - A^T) = -Q$
Q is skew-symmetric

(iii) We have,
$(AA^T)^T = (A^T)^T A^T$ [By reversal law]
$(AA^T)^T = AA^T$ **[∵ $(A^T)^T = A$]**
AA^T is symmetric
Similarly, it can be proved that $A^T A$ is symmetric.

- **Example 4:** Show that all positive integral powers of a symmetric matrix are symmetric.

- **Solution:** Let A be a symmetric matrix and $n \in N$. Then,

$A^n = AAA\ldots A$ upto n-times

$\Rightarrow$ $(A^n)^T$ $(AAA\ldots A$ upto n-times$)^T$

$\Rightarrow$ $(A^n)^T = (A^T A^T A^T \ldots A^T$ upto n-times$)$ [By reversal law]

$\Rightarrow$ $(A^n)^T = (A^T)^n = A^n$ **[∵ $A^T = A$]**

Hence, A^n is also a symmetric matrix.

- **Example 5:**
  Show that positive odd integral powers of a skew-symmetric matrix are skew-symmetric and positive even integral powers of a skew-symmetric matrix are symmetric.

- **Solution:**
  Let A be a skew-symmetric matrix. Then, $A^T = -A$.

  We have, $(A^n)^T = (A^T)^n$ for all $n \in N$.

  $\therefore \quad (A^n)^T = (-A)^n$

  $\Rightarrow \quad (A^n) = (-1)^n A^n$

  $\Rightarrow \quad (A^n)^T = \begin{cases} A^n & \text{if n is even} \\ -A^n & \text{if n is odd} \end{cases}$

  Hence, A^n is symmetric if n is even and skew-symmetric if n is odd.

- **Example 6:** A matrix which is both symmetric as well as skew-symmetric is a null matrix.

- **Solution:**
  Let $A = [a_{ij}]$ a matrix which is both symmetric and skew-symmetric.

  Now, $\quad A = [a_{ij}]$ is a symmetric matrix

  $\Rightarrow \quad a_{ij} = a_{ji}$ for all i, j ......(i)

  Also, $\quad A = [a_{ij}]$ is a skew-symmetric matrix.

  $\therefore \quad a_{ij} = -a_{ij}$ for all i,j
  $\Rightarrow \quad a_{ji} = -a_{ji}$ for all i,j ......(i)

  From (i) and (ii), we obtain

  $a_{ij} = -a_{ij}$ for all i,j

  $\Rightarrow \quad 2a_{ij} = 0$ for all i, j

$\Longrightarrow$ $a_{ij} = 0$ for all i, j

$\Longrightarrow$ A= [a_{ji}] is a null matrix

❖ EXERCISE:

1: If $A = \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}$, prove that $A - A^T$ is a skew symmetric matrix.

2: If $A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$, prove that $A - A^T$ is a skew symmetric matrix.

3: If the matrix $A = \begin{bmatrix} 5 & 2 & x \\ y & z & -3 \\ 4 & t & -7 \end{bmatrix}$ is a symmetric matrix, find x, y, z and t.

4: Express the matrix $A = \begin{bmatrix} 4 & 2 & -1 \\ 3 & 5 & 7 \\ 1 & -2 & 1 \end{bmatrix}$ as the sum of a symmetric and a skew – symmetric matrix.

5: Define a symmetric matrix. Prove that for $A = \begin{bmatrix} 2 & 4 \\ 5 & 6 \end{bmatrix}$, $A + A^T$ is a symmetric matrix where A^T is the transpose of A.

6: Express the matrix $A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$ as the sum of a symmetric and a skew symmetric matrix.

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