Bachelor of Computer Application

Computational Mathematics

LECTURE 20 NOTES

Determinants

- ## Determinants

  - Every square matrix can be associated to an expression or a number which is known as its determinant. If **A= [a_{ij}]** is a square matrix of order n, then the determinant of A is denoted by det A or, |A| or,

$$\begin{matrix} a_{11} & a_{12} \ldots a_{1j} \ldots . a_{1n} \\ a_{21} & a_{22} \ldots a_{2j} \ldots .. a_{2n} \\ . & \\ . & \\ . & \\ a_{i1} & a_{i2} \ldots . a_{ij} \ldots .. a_{in} \\ . & \\ . & \\ . & \\ a_{n1} & a_{n2} \ldots . a_{nj} \ldots . a_{nn} \end{matrix}$$

  - **Remarks:**
    (i) For matrix A, | A| is read as determinant of A and not modulus of A.

    (ii) Only square matrices have determinants.

- ## Determinant of a Matrix of Order One

  - Let **A = [a]** be the matrix of order 1, then determinant of A is defined to be equal to **‘a’**

- ## Determinant of a Matrix of Order Two

$$Let \qquad A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} be\ a\ matrix\ of\ order\ 2 \times 2,$$

$$then\ the\ determinant\ of\ A\ is\ defined\ as:$$

$$det\ (A) = |A| = \Delta = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = a_{11}a_{12} - a_{21}a_{12}$$

❖ **EXAMPLES:**

- **Example 1:** Evaluate $\begin{vmatrix} 2 & 4 \\ -1 & 2 \end{vmatrix}$

- **Solution:** We have $\begin{vmatrix} 2 & 4 \\ -1 & 2 \end{vmatrix}$ = 2(2)-4(-1) = 4+4 = 8.

- **Example 2:** Evaluate $\begin{vmatrix} x & x+1 \\ x-1 & x \end{vmatrix}$

- **Solution:** We have $\begin{vmatrix} x & x+1 \\ x-1 & x \end{vmatrix}$ = x(x) - (x+1)(x-1) = x^2- (x^2- 1)= x^2- x^2 + 1= 1

- **Example 3:** Evaluate $\begin{vmatrix} 5 & 4 \\ -2 & 3 \end{vmatrix}$

- **Solution:** $\begin{vmatrix} 5 & 4 \\ -2 & 3 \end{vmatrix} = 5\times 3 - 4\times -2 = 15 + 8 = 23$

- **Example 4:** Evaluate $\begin{vmatrix} \sin\theta & \cos\theta \\ -\cos\theta & \sin\theta \end{vmatrix}$

- **Solution:** $\begin{vmatrix} \sin\theta & \cos\theta \\ -\cos\theta & \sin\theta \end{vmatrix} = \sin^2\theta - \left(-\cos^2\theta\right) = \sin^2\theta + \cos^2\theta = 1$

- **Example 5:** Evaluate $\begin{vmatrix} x-1 & 1 \\ x^3 & x^2+x+1 \end{vmatrix}$

- **Solution:** $\begin{vmatrix} x-1 & 1 \\ x^3 & x^2+x+1 \end{vmatrix} = (x-1)\left(x^2+x+1\right) - x^3 = \left(x^3-1\right) - x^3 = -1$

- **Determinant of a Matrix of Order 3 × 3**

- Determinant of a matrix of order three can be determined by expressing it in terms of second order determinants. This is known as expansion of a determinant along a row (or a column). There are six ways of expanding a determinant of order 3 corresponding to each of three rows (R_1, R_2 and R_3) and three columns (C_1, C_2 and C_3) giving the same value as shown below.

- Consider the determinant of square matrix $A = [a_{ij}]_{3 \times 3}$

i.e., $$|A| = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

Expansion along first Row (R_1):

- **Step 1:** Multiply first element a_{11} of R_1 by $(-1)^{(1+1)}$ $[(-1)^{\text{sum of suffixes in a11}}]$ and with the second order determinant obtained by deleting the elements of first row (R_1) and first column (C_1) of | A | as a_{11} lies in R_1 and C_1,

i.e., $$(-1)^{1+1} a_{11} \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix}$$

- **Step 2:** Multiply 2nd element a_{12} of R_1 by $(-1)^{1+2}$ $[(-1)^{\text{sum of suffixes in a12}}]$ and the second order determinant obtained by deleting elements of first row (R_1) and 2nd column (C_2) of | A | as a_{12} lies in R_1 and C_2,

i.e., $$(-1)^{1+2} a_{12} \begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix}$$

- **Step 3:** Multiply third element a_{13} of R_1 by $(-1)^{1+3}$ $[(-1)^{\text{sum of suffixes in a13}}]$ and the second order determinant obtained by deleting elements of first row (R_1) and third column (C_3) of | A | as a_{13} lies in R_1 and C_3,

i.e., $$(-1)^{1+3} a_{13} \begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$$

- **Step 4:** Now the expansion of determinant of A, that is, | A | written as sum of all three terms obtained in steps 1, 2 and 3 above is given by

$$det\ A = |A| = (-1)^{1+1} a_{11} \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} + (-1)^{1+2} a_{12} \begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + (-1)^{1+3} a_{13} \begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$$

$$or \quad |A| = a_{11}\left(a_{22}a_{33} - a_{32}a_{23}\right) - a_{12}\left(a_{21}a_{33} - a_{31}a_{23}\right) + a_{13}\left(a_{21}a_{32} - a_{31}a_{22}\right)$$

$$= a_{11}a_{22}a_{33} - a_{11}a_{32}a_{23} - a_{12}a_{21}a_{33} + a_{12}a_{31}a_{23} + a_{13}a_{21}a_{32} - a_{13}a_{31}a_{22} \ \ldots\ldots\ldots\ldots (1)$$

Expansion along second Row (R_2):

$$|A| = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

- Expanding along R_2, we get

$$|A| = (-1)^{2+1} a_{21} \begin{vmatrix} a_{12} & a_{13} \\ a_{32} & a_{33} \end{vmatrix} + (-1)^{2+2} a_{22} \begin{vmatrix} a_{11} & a_{13} \\ a_{31} & a_{33} \end{vmatrix} + (-1)^{2+3} a_{23} \begin{vmatrix} a_{11} & a_{12} \\ a_{31} & a_{32} \end{vmatrix}$$

$$= -a_{21}\left(a_{12}a_{33} - a_{32}a_{13}\right) + a_{22}\left(a_{11}a_{33} - a_{31}a_{13}\right) - a_{23}\left(a_{11}a_{32} - a_{31}a_{12}\right)$$

$$|A| = -a_{21}a_{12}a_{33} - a_{21}a_{32}a_{13} + a_{22}a_{11}a_{33} - a_{22}a_{31}a_{13} - a_{23}a_{11}a_{32} + a_{23}a_{31}a_{12}$$

$$= a_{11}a_{22}a_{33} - a_{11}a_{23}a_{32} - a_{12}a_{21}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32} - a_{13}a_{31}a_{22} \ldots\ldots\ldots\ldots (2)$$

Expansion along first Column (C_1):

$$|A| = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

- By expanding along C_1, we get

$$|A| = a_{11}(-1)^{1+1}\begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} + a_{21}(-1)^{2+1}\begin{vmatrix} a_{12} & a_{13} \\ a_{32} & a_{33} \end{vmatrix} + a_{31}(-1)^{3+1}\begin{vmatrix} a_{12} & a_{13} \\ a_{22} & a_{23} \end{vmatrix}$$

$$= a_{11}\left(a_{22}a_{33} - a_{23}a_{32}\right) - a_{21}\left(a_{12}a_{33} - a_{13}a_{32}\right) + a_{31}\left(a_{12}a_{23} - a_{13}a_{22}\right)$$

$$|A| = a_{11}a_{22}a_{33} - a_{11}a_{23}a_{32} - a_{21}a_{12}a_{33} + a_{21}a_{13}a_{32} + a_{31}a_{12}a_{23} - a_{31}a_{13}a_{22}$$

$$= = a_{11}a_{22}a_{33} - a_{11}a_{23}a_{32} - a_{12}a_{21}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32} - a_{13}a_{31}a_{22} \text{..........}(3)$$

- Clearly, values of | A| in (1), (2) and (3) are equal. It is left as an exercise to the reader to verify that the values of |A| by expanding along R_3 ,C_2 and C_3 are equal to the value of | A| obtained in (1), (2) or (3).

- Hence, expanding a determinant along any row or column gives same value.

❖ **Remarks:**

(i) For easier calculations, we shall expand the determinant along that row or column which contains maximum number of zeros.

(ii) While expanding, instead of multiplying by $(-1)^{i+j}$, we can multiply by +1 or –1 according as (i + j) is even or odd.

(iii) Let $A = \begin{bmatrix} 2 & 2 \\ 4 & 0 \end{bmatrix}$ $and\ B = \begin{bmatrix} 1 & 1 \\ 2 & 0 \end{bmatrix}$ Then, it is easy to verify that A = 2B.

Also | A| = 0 – 8 = – 8 and |B| = 0 – 2 = – 2.

Observe that, |A|= 4(– 2)=2^2 |B| or |A|=2^n |B|, where n=2 is the order of square matrices A & B.

In general, if A=kB where A and B are square matrices of order n, then |A|=k^n |B|, where n=1, 2, 3.

❖ **EXAMPLES:**

- **Example 1:** Evaluate the determinant $\Delta = \begin{vmatrix} 1 & 2 & 4 \\ -1 & 3 & 0 \\ 4 & 1 & 0 \end{vmatrix}$

- **Solution:**
  Note that in the third column, two entries are zero. So, expanding along third column (C3),

we get $\Delta = 4\begin{vmatrix} -1 & 3 \\ 4 & 1 \end{vmatrix} - 0\begin{vmatrix} 1 & 2 \\ 4 & 1 \end{vmatrix} + 0\begin{vmatrix} 1 & 2 \\ -1 & 3 \end{vmatrix}$

$$= = 4\,(-1 - 12)\; - 0 + 0 = -52$$

- **Example 2:** Evaluate the determinant $\Delta = \begin{vmatrix} 0 & \sin\alpha & -\cos\alpha \\ -\sin\alpha & 0 & \sin\beta \\ \cos\alpha & -\sin\beta & 0 \end{vmatrix}$

- **Solution:**
  Expanding along R1, we get

$$\Delta = 0\begin{vmatrix} 0 & \sin\beta \\ -\sin\beta & 0 \end{vmatrix} - \sin\alpha\begin{vmatrix} -\sin\alpha & \sin\beta \\ \cos\alpha & 0 \end{vmatrix} - \cos\alpha\begin{vmatrix} -\sin\alpha & 0 \\ \cos\alpha & -\sin\beta \end{vmatrix}$$

$$= 0 - \sin\alpha\,(0 - \sin\beta\cos\alpha) - \cos\alpha\,(\sin\alpha\sin\beta - 0)$$
$$= \sin\alpha\sin\beta\cos\alpha - \cos\alpha\sin\alpha\sin\beta = 0$$

$$\begin{vmatrix} 3 & x \\ x & 1 \end{vmatrix} = \begin{vmatrix} 3 & 2 \\ 4 & 1 \end{vmatrix}$$

- **Example 3:** Find values of x for which

- **Solution:** We have $\begin{vmatrix} 3 & x \\ x & 1 \end{vmatrix} = \begin{vmatrix} 3 & 2 \\ 4 & 1 \end{vmatrix}$

i.e.	$3 - x^2 = 3 - 8$
i.e.	$x^2 = 8$
Hence	$x = \pm 2\sqrt{2}$

❖ EXERCISE:

1: Evaluate the determinant A= $\begin{vmatrix} 2 & 4 \\ -5 & -1 \end{vmatrix}$

2: Evaluate the determinants

$(i)\ \begin{vmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{vmatrix}$ $\qquad (ii)\ \begin{vmatrix} x^2 - x + 1 & x - 1 \\ x + 1 & x + 1 \end{vmatrix}$

3: If, $A = \begin{bmatrix} 1 & 2 \\ 4 & 2 \end{bmatrix}$ then show that |2A|= 4|A|

4: If, $A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 4 \end{bmatrix}$ then show that |3A|= |27A|

5: If, $A = \begin{bmatrix} 1 & 1 & -2 \\ 2 & 1 & -3 \\ 5 & 4 & -9 \end{bmatrix}$ find |A|

6: Find values of x, if

$$(i)\ \begin{vmatrix} 2 & 4 \\ 5 & 1 \end{vmatrix} = \begin{vmatrix} 2x & 4 \\ 6 & x \end{vmatrix} \qquad (ii)\ \begin{vmatrix} 2 & 3 \\ 4 & 5 \end{vmatrix} = \begin{vmatrix} x & 3 \\ 2x & 5 \end{vmatrix}$$

- ## Minors And Cofactors

  - **Definition 1:**
    Minor of an element a_{ij} of a determinant is the determinant obtained by deleting its ith row and jth column in which element a_{ij} lies. Minor of an element a_{ij} is denoted by M_{ij}.

  - **Remark:** Minor of an element of a determinant of order n (n ≥ 2) is a determinant of order n – 1.

  - **Example:** Find the minor of element 6 in the determinant $\Delta = \begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{vmatrix}$

  - **Solution:** Since 6 lies in the second row and third column, its minor M_{23} is given by

$$M_{23} = \begin{vmatrix} 1 & 2 \\ 7 & 8 \end{vmatrix} = 8 - 14 = -6$$ (obtained by deleting R_2 and C_3 in Δ).

  - **Definition 2:**
    Cofactor of an element a_{ij}, denoted by A_{ij} is defined by $A_{ij} = (-1)i + j\ M_{ij}$, where M_{ij} is minor of a_{ij}.

$$\begin{vmatrix} 1 & -2 \\ 4 & 3 \end{vmatrix}$$

- **Example:** Find minors and cofactors of all the elements of the determinant

- **Solution:** Minor of the element a_{ij} is M_{ij}

  Here $a_{11} = 1$. So M_{11} = Minor of $a_{11} = 3$

  M_{12} = Minor of the element $a_{12} = 4$
  M_{21} = Minor of the element $a_{21} = -2$
  M22 = Minor of the element a22 = 1

  Now, cofactor of aij is Aij. So,

  $A_{11} = (-1)^{1+1} M_{11} = (-1)^2 (3) = 3$
  $A_{12} = (-1)^{1+2} M_{12} = (-1)^3 (4) = -4$
  $A_{21} = (-1)^{2+1} M_{21} = (-1)^3 (-2) = 2$
  $A_{22} = (-1)^{2+2} M_{22} = (-1)^4 (1) = 1$

❖ EXAMPLES:

- **Example 1:** Find minors and cofactors of the elements a_{11}, a_{21} in the determinant

$$\Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

- **Solution:** By definition of minors and cofactors, we have

  Minor of $a_{11} = M_{11} = \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} = a_{22} a_{33} - a_{23} a_{32}$

  Cofactor of $a_{11} = A_{11} = (-1)^{1+1} M_{11} = a_{22} a_{33} - a_{23} a_{32}$

  Minor of $a_{21} = M_{21} = \begin{vmatrix} a_{12} & a_{13} \\ a_{32} & a_{33} \end{vmatrix} = a_{12} a_{33} - a_{13} a_{32}$

  Cofactor of $a_{21} = A_{21} = (-1)^{2+1} M_{21} = (-1) (a_{12} a_{33} - a_{13} a_{32}) = -a_{12} a_{33} + a_{13} a_{32}$

- **Remark:** Expanding the determinant Δ, along R_1, we have

$$\Delta = (-1)^{1+1} a_{11} \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} + (-1)^{1+2} a_{12} \begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + (-1)^{1+3} \begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$$

$= a_{11} A_{11} + a_{12} A_{12} + a_{13} A_{13}$, where Aij is cofactor of a_{ij}

= sum of product of elements of R_1 with their corresponding cofactors

Similarly, Δ can be calculated by other five ways of expansion that is along R_2, R_3, C_1, C_2 and C_3.

Hence,

Δ = sum of the product of elements of any row (or column) with their corresponding cofactors.

- **Example 2:** Find minors and cofactors of the elements of the determinant

$$\begin{vmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{vmatrix}$$

and verify that $a_{11} A_{31} + a_{12} A_{32} + a_{13} A_{33} = 0$

- **Solution:** We have

$M_{11} = \begin{vmatrix} 0 & 4 \\ 5 & -7 \end{vmatrix}$ $= 0 - 20 = -20$; $A_{11} = (-1)^{1+1}(-20) = -20$

M12 = $\begin{vmatrix} 6 & 4 \\ 1 & -7 \end{vmatrix}$ $= -42 - 4 = -46$; $A_{12} = (-1)^{1+2}(-46) = 46$

M13 = $\begin{vmatrix} 6 & 0 \\ 1 & 5 \end{vmatrix}$ $= 30 - 0 = 30$; $A_{13} = (-1)^{1+3}(30) = 30$

M21 = $\begin{vmatrix} -3 & 5 \\ 5 & -7 \end{vmatrix}$ $= 21 - 25 = -4$; $A_{21} = (-1)^{2+1}(-4) = 4$

$$\begin{vmatrix} 2 & 5 \\ 1 & -7 \end{vmatrix}$$

$$\begin{vmatrix} 2 & -3 \\ 1 & 5 \end{vmatrix}$$

M22 = $= -14 - 5 = -19$; $A_{22} = (-1)^{2+2}(-19) = -19$

M23 = $= 10 + 3 = 13$; $A_{23} = (-1)^{2+3}(13) = -13$

M31 = $= -12 - 0 = -12$; $A_{31} = (-1)^{3+1}(-12) = -12$

M32 = $= 8 - 30 = -22$; $A_{32} = (-1)^{3+2}(-22) = 22$

and M33 = $\begin{vmatrix} 2 & -3 \\ 6 & 0 \end{vmatrix}$ $= 0 + 18 = 18$; $A_{33} = (-1)^{3+3}(18) = 18$

Now $a_{11} = 2$, $a_{12} = -3$, $a_{13} = 5$; $A_{31} = -12$, $A_{32} = 22$, $A_{33} = 18$

So $a_{11}A_{31} + a_{12}A_{32} + a_{13}A_{33}$

$= 2(-12) + (-3)(22) + 5(18) = -24 - 66 + 90 = 0$

- **Example 3:** If A= $\begin{bmatrix} 1 & 3 \\ 2 & 1 \end{bmatrix}$, find the determinant of the matrix $A^2 - 2A$.

- **Solution:** We have, A= $\begin{bmatrix} 1 & 3 \\ 2 & 1 \end{bmatrix}$

$\therefore$ $A^2 - 2A = \begin{bmatrix} 1 & 3 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} 1 & 3 \\ 2 & 1 \end{bmatrix} - 2\begin{bmatrix} 1 & 3 \\ 2 & 1 \end{bmatrix}$

$\Rightarrow$ $A^2 - 2A = \begin{bmatrix} 1+6 & 3+3 \\ 2+2 & 6+1 \end{bmatrix} - \begin{bmatrix} 2 & 6 \\ 4 & 2 \end{bmatrix} = \begin{bmatrix} 7 & 6 \\ 4 & 7 \end{bmatrix} - \begin{bmatrix} 2 & 6 \\ 4 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$

$\therefore$ $|A^2 - 2A| = \begin{vmatrix} 5 & 0 \\ 0 & 5 \end{vmatrix}$ $= 25 - 0 = 25$

- **Example 4:** If $\begin{vmatrix} x-2 & -3 \\ 3x & 2x \end{vmatrix}$ = 3, find the values of x.

- **Solution:** We have,

$$\begin{vmatrix} x-2 & -3 \\ 3x & 2x \end{vmatrix} = 3$$

$\Longrightarrow$ (x-2) × (2x)-(-3) × 3x = 3

$\Longrightarrow$ 2x(x-2) + 9x = 3

$\Longrightarrow$ $2x^2$ -4x +9x = 3

$\Longrightarrow$ $2x^2$ + 5x -3 = 0 $\Longrightarrow$ (2x-1)(x+3) = 0 $\Longrightarrow$ 2x-1 = 0 or, x+3 = 0 $\Longrightarrow$ x= (1/2), -3

- **Example 5:** Prove that the determinant $\begin{vmatrix} x & \sin\theta & \cos\theta \\ -\sin\theta & -x & 1 \\ \cos\theta & 1 & x \end{vmatrix}$ is independent of **θ.**

- **Solution:** We have, $\Delta = \begin{vmatrix} x & \sin\theta & \cos\theta \\ -\sin\theta & -x & 1 \\ \cos\theta & 1 & x \end{vmatrix}$

$$\Rightarrow \quad \Delta = x\begin{vmatrix} -x & 1 \\ 1 & x \end{vmatrix} - \sin\theta\begin{vmatrix} -\sin\theta & 1 \\ \cos\theta & x \end{vmatrix} + \cos\theta\begin{vmatrix} -\sin\theta & -x \\ \cos\theta & 1 \end{vmatrix}$$

$\Longrightarrow$ $\Delta = x(-x^2-1) - \sin\theta(-x\sin\theta - \cos\theta) + \cos\theta(-\sin\theta + x\cos\theta)$

$\Longrightarrow$ $\Delta = -x^3 - x + x\sin^2\theta + \sin\theta\cos\theta - \sin\theta\cos\theta + x\cos^2\theta$

$\Rightarrow$ $\Delta = -x^3 - x + x(\sin^2 \boldsymbol{\theta} + \cos^2 \boldsymbol{\theta}) = -x^3 - x + x = -x^3$, which is independent of $\boldsymbol{\theta}$.

❖ EXERCISE:

1: Write Minors and Cofactors of the elements of following determinants:

$$(i)\quad \begin{vmatrix} 2 & -4 \\ 0 & 3 \end{vmatrix} \qquad\qquad (ii)\quad \begin{vmatrix} a & b \\ c & d \end{vmatrix}$$

2: Write Minors and Cofactors of the elements of following determinants:

$$(i)\quad \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} \qquad\qquad (ii)\quad \begin{vmatrix} 1 & 0 & 4 \\ 3 & 5 & -1 \\ 0 & 1 & 2 \end{vmatrix}$$

3: Using Cofactors of elements of second row, evaluate

$$\Delta = \begin{vmatrix} 5 & 3 & 8 \\ 2 & 0 & 1 \\ 1 & 2 & 3 \end{vmatrix}$$

4: Using Cofactors of elements of third column, evaluate

$$\Delta = \begin{vmatrix} 1 & x & yz \\ 1 & y & zx \\ 1 & z & xy \end{vmatrix}$$

$$\Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

5: If and A_{ij} is Cofactors of a_{ij} , then value of Δ is given by

(A) $a_{11} A_{31} + a_{12} A_{32} + a_{13} A_{33}$

(B) $a_{11} A_{11} + a_{12} A_{21} + a_{13} A_{31}$

(C) $a_{21} A_{11} + a_{22} A_{12} + a_{23} A_{13}$

(D) $a_{11} A_{11} + a_{21} A_{21} + a_{31} A_{31}$