Bachelor of Computer Application

Computational Mathematics

LECTURE 5 NOTES

Relations - Introduction

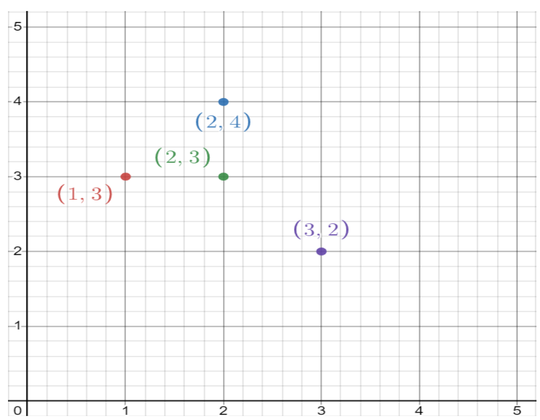
- **Definition of Ordered Pairs**

    - An ordered pair consists of two objects or elements in a given fixed order.

    - For example, if A & B are any two sets then by an ordered pair of elements we mean a pair (a,b) in that order, where $a \in A$, $b \in B$.

    - **Note:** An ordered pair is not a set consisting of two elements. The ordering of two elements in an ordered pair is important and the two elements need not be distinct.

    - **Example:** The position of a point in a two-dimensional plane in cartesian coordinates is represented by an ordered pair. Accordingly, the ordered pairs (1,3), (2,4), (2,3) and (3,2) represents different points in a plane.



❖ Equality Of Ordered Pairs

- Two ordered pairs $(a1,b1)$ and $(a2,b2)$ are equal if $a1=a2$ and $b1=b2$.

    i.e., $(a1,b1)= (a2,b2) \Leftrightarrow a1=a2$ and $b1=b2$

    It is evident from this definition that $(1,2)\neq(2,1)$ and $(1,1)\neq(2,2)$.

- **Example:** Find the values of a & b , if $(3a-2, b+3)= (2a-1, 3)$

- **Solution:** By the definition of equality of ordered pairs, we obtain

$$(3a-2, b+3) = (2a-1,3) \Leftrightarrow 3a-2=2a-1$$

and $$b+3=3 \Leftrightarrow a=1 \text{ and } b=0$$

- **Cartesian Products of Sets**

  - Suppose A is a set of 2 colours and B is a set of 3 objects, i.e.,

    A = {red, blue} and B = {b, c, s},

    where b, c and s represent a particular bag, coat and shirt, respectively.

    How many pairs of coloured objects can be made from these two sets?

    Proceeding in a very orderly manner, we can see that there will be 6 distinct pairs as given below:

    (red, b), (red, c), (red, s), (blue, b), (blue, c), (blue, s).

    Thus, we get 6 distinct objects

  - **Definition 1:**
    Given two non-empty sets P and Q. The cartesian product P × Q is the set of all ordered pairs of elements from P and Q, i.e.,

    $P \times Q = \{ (p,q) : p \in P, q \in Q \}$

    If either P or Q is the null set, then P × Q will also be empty set, i.e., $P \times Q = \varphi$

    Example: A × B = {(red,b), (red,c), (red,s), (blue,b), (blue,c), (blue,s)}.

❖ EXAMPLES:

1. **Consider the two sets:**
   A = {DL, MP, KA}, where DL, MP, KA represent Delhi, Madhya Pradesh and Karnataka, respectively and B = {01,02,03} representing codes for the licence plates of vehicles issued by DL, MP and KA.

   If the three states, Delhi, Madhya Pradesh and Karnataka were making codes for the license plates of vehicles, with the restriction that the code begins with an element from set A, which are the pairs available from these sets and how many such pairs will there be.

  - **Solution:**

The available pairs are:(DL,01), (DL,02), (DL,03), (MP,01), (MP,02), (MP,03),(KA,01), (KA,02), (KA,03) and the product of set A and set B is given by

A × B = {(DL,01), (DL,02), (DL,03), (MP,01), (MP,02), (MP,03), (KA,01), (KA,02), (KA,03)}.

It can easily be seen that there will be 9 such pairs in the Cartesian product, since there are 3 elements in each of the sets A and B. This gives us 9 possible codes. Also note that the order in which these elements are paired is crucial. For example, the code (DL, 01) will not be the same as the code (01, DL).

- **Remarks:**
  (i) Two ordered pairs are equal, if and only if the corresponding first elements are equal and the second elements are also equal.
  (ii) If there are p elements in A and q elements in B, then there will be pq elements in A × B, i.e., if n(A) = p and n(B) = q, then n (A × B) = pq.
  (iii) If A and B are non-empty sets and either A or B is an infinite set, then so is A × B.
  (iv) A × A × A = {(a, b, c) : a, b, c ∈ A}. Here (a, b, c) is called an ordered triplet.

2. **If (x + 1, y – 2) = (3,1), find the values of x and y**.

- **Solution:**
  Since the ordered pairs are equal, the corresponding elements are equal.

  Therefore, x + 1 = 3 and y – 2 = 1.
  Solving we get, x = 2 and y = 3.

3. **If P = {a, b, c} and Q = {r}, form the sets P × Q and Q × P. Are these two products equal?**

- **Solution:**
  By the definition of the cartesian product,

  P × Q = {(a, r), (b, r), (c, r)} and Q × P = {(r, a), (r, b), (r, c)}

  Since, by the definition of equality of ordered pairs, the pair (a, r) is not equal to the pair
  (r, a), we conclude that P × Q ≠ Q × P.

  However, the number of elements in each set will be the same.

4. Let A = {1,2,3}, B = {3,4} and C = {4,5,6}. Find

(i) $A \times (B \cap C)$ (ii) $(A \times B) \cap (A \times C)$
(iii) $A \times (B \cup C)$ (iv) $(A \times B) \cup (A \times C)$

- **Solution:**

(i) By the definition of the intersection of two sets, $(B \cap C) = \{4\}$.

Therefore, $A \times (B \cap C) = \{(1,4), (2,4), (3,4)\}$.

(ii) Now $(A \times B) = \{(1,3), (1,4), (2,3), (2,4), (3,3), (3,4)\}$ and

$(A \times C) = \{(1,4), (1,5), (1,6), (2,4), (2,5), (2,6), (3,4), (3,5), (3,6)\}$

Therefore, $(A \times B) \cap (A \times C) = \{(1, 4), (2, 4), (3, 4)\}$.

(iii) Since $(B \cup C) = \{3, 4, 5, 6\}$, we have

$A \times (B \cup C) = \{(1,3), (1,4), (1,5), (1,6), (2,3), (2,4), (2,5), (2,6), (3,3),(3,4), (3,5), (3,6)\}$.

(iv) Using the sets, $A \times B$ and $A \times C$ from part (ii) above, we obtain

$(A \times B) \cup (A \times C) = \{(1,3), (1,4), (1,5), (1,6), (2,3), (2,4), (2,5), (2,6),(3,3), (3,4), (3,5), (3,6)\}$.

5. If P = {1, 2}, form the set P × P × P.

- **Solution:**
We have, $P\times P\times P = \{(1,1,1), (1,1,2), (1,2,1), (1,2,2), (2,1,1), (2,1,2), (2,2,1), (2,2,2)\}$.

6. If R is the set of all real numbers, what do the cartesian products R × R and R × R × R represent?

- **Solution:**
The Cartesian product $R \times R$ represents the set $R \times R=\{(x, y) : x, y \in R\}$ which represents the coordinates of all the points in two dimensional space and the cartesian product $R \times R \times R$ represents the set $R \times R \times R =\{(x, y, z) : x, y, z \in R\}$ which represents the coordinates of all the points in three-dimensional space.

❖ **EXERCISE:**

1. If the set A has 3 elements and the set B = {3, 4, 5}, then find the number of elements in (A×B).
2. If G = {7, 8} and H = {5, 4, 2}, find G × H and H × G.
3. If A × B = {(a, x),(a , y), (b, x), (b, y)}. Find A and B.
4. Let A and B be two sets such that n(A) = 3 and n(B) = 2. If (x, 1), (y, 2), (z, 1) are in A × B, find A and B, where x, y and z are distinct elements.

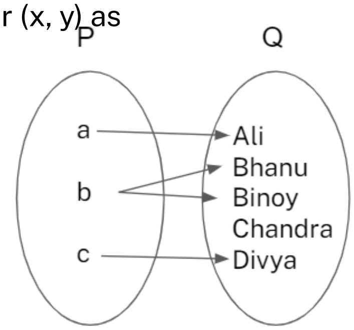
- **Relations**

  - **Let A & B be two sets. Then a relation R from A to B is a subset of A×B**

Consider the two sets P = {a, b, c} and Q = {Ali, Bhanu, Binoy, Chandra, Divya}.

The cartesian product of P and Q has 15 ordered pairs which can be listed as
P × Q = {(a, Ali),(a,Bhanu), (a, Binoy), ..., (c, Divya)}.

We can now obtain a subset of P × Q by introducing a relation R between the first element x and the second element y of each ordered pair (x, y) as



R= { (x,y): x is the first letter of the name y, x ∈ P, y ∈ Q}.

Then R = {(a, Ali), (b, Bhanu), (b, Binoy), (c, Chandra)}

A set is a well-defined collection of objects.

  - **Remarks:**

(i) A relation may be represented algebraically either by the Roster method or by the Set-builder method.

(ii) An arrow diagram is a visual representation of a relation.

- **Example:**

  If A= (a, b, c, d), B=(p,q,r,s), then which of the following are relations from A to B? Give reasons for your answer.
  (i) R_1 = {(a, p), (b, r), (c, s)}
  (ii) R_2 = {(q, b), (c, s), (d, r)}
  (iii) R3= {(a, p). (a, q), (d, p), (c, r), (b,r)}
  (iv) R4 = {(a, p), (q, a), (b. s). (s, b)}

- **Solution:**

(i) Clearly, R1⊆ Ax B. So, R, is a relation from A to B.
(ii) Since (q, b) ∊ R2 but (q, b) ∊ Ax B. So, R_2⊈Ax B. Thus, R_2 is not a relation from A to B.
(iii) Clearly, R3⊆AX B. So it is a relation from A to B.
(iv) R is not a relation from A to B, because (e. a) and (s, b) are elements of R but (q, a) and (s,b) are not in A×B As such R⊈A×B

❖ **DOMAIN:** Let R be a relation from a set A to a set B. Then the set of all first components or coordinates of the ordered pairs belonging to R is called the domain of R.

Thus, domain of R= {a:(a, b) ∊R}
Clearly, domain of R⊆A.

If A= {1,3,5,7}, B = {2, 4, 6, 8, 10} and R= {(1, 8), (3, 6), (5, 2), (1, 4)} is a relation from A to B.
Domain (R)= {1,3,5}

❖ **RANGE:** Let R be a relation from a set A to a set B. Then the set of all second components or coordinates of the ordered pairs belonging to R is called the range of R.

Thus, Range of R = {b: (a, b) ∊R).
Clearly, range of R⊆B.

If A={1,3,5,7}, B = {2, 4, 6, 8, 10} and R = {(1, 8), (3, 6), (5, 2), (1, 4)} is a relation from A to B, then
Range (R)= {8, 6, 2, 4}

❖ **CODOMAIN:** The set of all second elements in a relation R from a set A to a set B is called the range of the relation R. The whole set B is called the codomain of the relation R.

Note that range $\subset$ codomain.

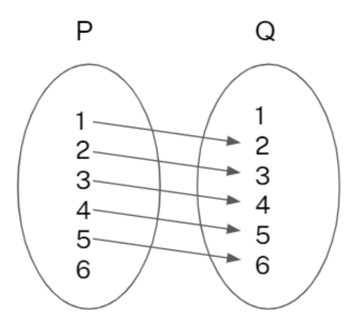
❖ **INVERSE OF A RELATION:** Let A, B be two sets and let R be a relation from a set A to a set B. Then the inverse of R, denoted by R-1, is a relation from B to A and is defined by R-1= {(b,a):(a,b)∈R}.

- **Example:**
  Find the inverse relation R-1, R= {(1,2), (1,3), (2,3), (3,2), (5,6)}

- **Solution:**
  Given:
  R= {(1,2), (1,3), (2,3), (3,2), (5,6)}
  So, R-1= {(2,1), (3,1), (3,2), (2,3), (6,5)}

❖ **EXAMPLES:**

1. **Let A = {1, 2, 3, 4, 5, 6}. Define a relation R from A to A by R = {(x, y) : y = x + 1 }.**
   (i) Depict this relation using an arrow diagram.
   (ii) Write down the domain, codomain, and range of R.



- **Solution:**

(i) By the definition of the relation,

R = {(1,2), (2,3), (3,4), (4,5), (5,6)}.

(ii) We can see that the domain = {1, 2, 3, 4, 5,}

Similarly, the range = {2, 3, 4, 5, 6} and the codomain = {1, 2, 3, 4, 5, 6}.

2. The Fig shows a relation between the sets P and Q. Write this relation (i) in set-builder form, (ii) in roster form. What is its domain and range?

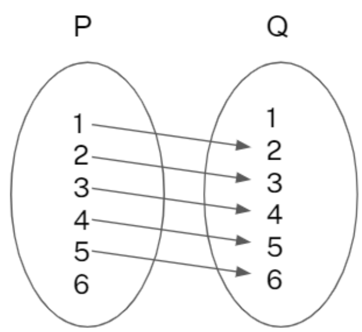
- **Solution:**
  It is obvious that the relation R is “x is the square of y”.

(i) In set-builder form, R = {(x, y): x is the square of y, x ∈ P, y ∈ Q}

(ii) In roster form, R = {(9, 3),(9, –3), (4, 2), (4, –2), (25, 5), (25, –5)}

The domain of this relation is {4, 9, 25}.
The range of this relation is {–2, 2, –3, 3, –5, 5}.
Note that the element 1 is not related to any element in set P.

The set Q is the c



3. Let A= {1, 2} and B = {3, 4}. Find the number of relations from A to B.

- **Solution: We have, A × B = {(1, 3), (1, 4), (2, 3), (2, 4)}.**

  Since n (A×B) = 4, the number of subsets of A×B is 24.

  Therefore, the number of relations from A into B will be 24.

- **Remark:**
  A relation R from A to A is also stated as a relation on A.

❖ Exercise:

1: Let A = {1, 2, 3,...,14}. Define a relation R from A to A by R = {(x, y) : 3x – y = 0, where x, y ∈ A}.

Write down its domain, codomain and range.

2: Determine the domain and range of the relation R defined by

$$R = \{(x, x + 5) : x \in \{0, 1, 2, 3, 4, 5\}\}.$$

3: Let A = {x, y, z} and B = {1, 2}. Find the number of relations from A to B.

4: The Fig shows a relationship between the sets P and Q. Write this relation

(i) in set-builder form **(ii)** roster form.

What is its domain and range?

5: Let A be the set of first five natural numbers and let R be a relation on A defined as follows: (x,y)R x≤y.

Express R and R^{-1} as sets of ordered pairs. Determine also

(i) the domain of R^{-1}

(ii) the range of R.