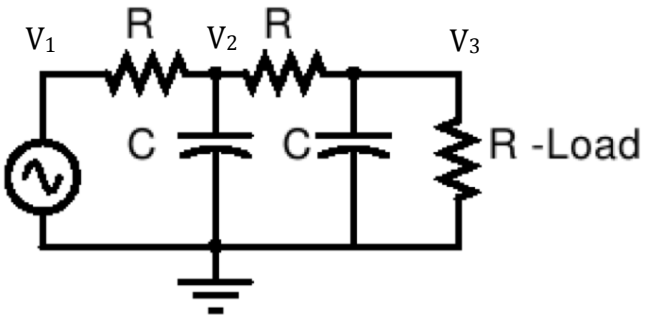
Circuit Simulation of Single Stage RC Ladder

Solving stochastic DAEs through Monte Carlo and Wiener Chaos Expansions.



$$-\boldsymbol{I} + \frac{\boldsymbol{V_1 - V_2}}{\boldsymbol{R}} = \mathbf{0}$$

$$\boldsymbol{V_1} = \boldsymbol{sin}(\mathbf{1e9} \cdot \boldsymbol{t})$$

$$-\frac{\boldsymbol{V_1 - V_2}}{\boldsymbol{R}} + \boldsymbol{C} \cdot \frac{\boldsymbol{dV_2}}{\boldsymbol{dt}} + \frac{\boldsymbol{V_2 - V_3}}{\boldsymbol{R}} = \mathbf{0}$$

$$-\frac{\boldsymbol{V_2 - V_3}}{\boldsymbol{R}} + \boldsymbol{C} \cdot \frac{\boldsymbol{dV_3}}{\boldsymbol{dt}} + \frac{\boldsymbol{V_3}}{\boldsymbol{R_L}} = \mathbf{0}$$

The solution set to this DAE [I V1 V2 V3] is stochastic as the circuit parameters R and C are stochastic variables. Their distributions are known:

$\boldsymbol{R \sim N(3.3504, 0.0861)\ \Omega}$
$\boldsymbol{C \sim N(0.6271, 0.2266)\ pF}$

and $\boldsymbol{R_L = 50\ \Omega}$

The stochastic DAE system is numerically solved using the Monte Carlo method and the Wiener Chaos expansion approach. The two methods are compared for various numbers of iterations.

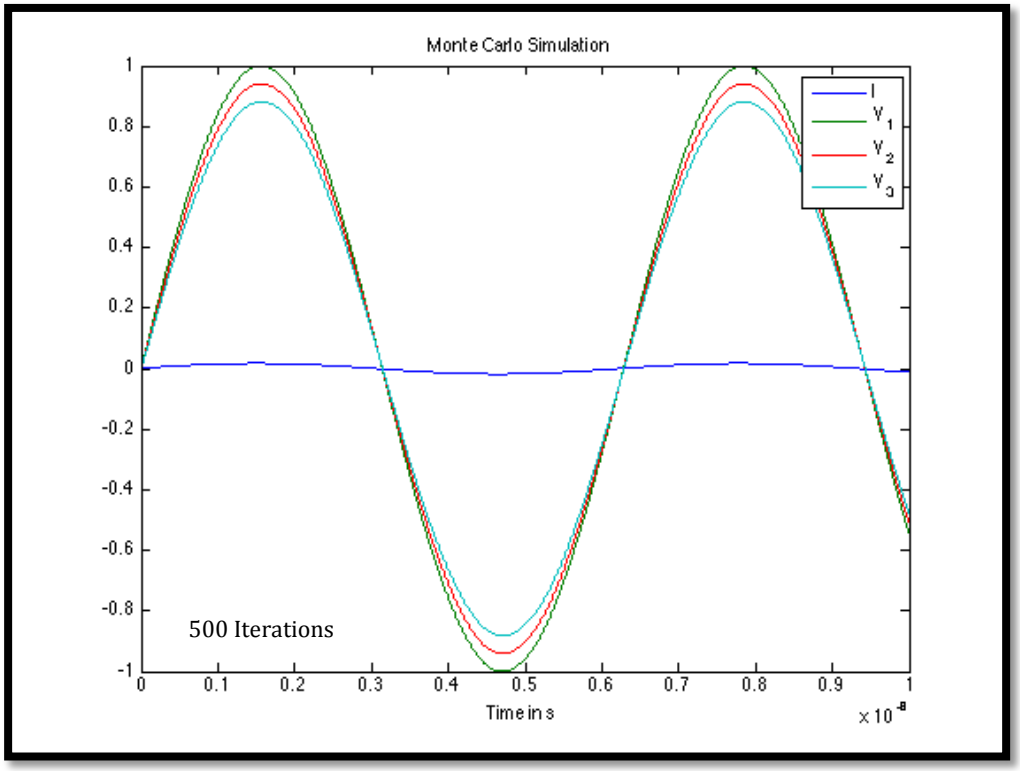
Machine details:
MacBook Air
1.4GHz dual-core Intel Core i5
4GB of 1600MHz LPDDR3 Memory
All simulations were performed under similar conditions.

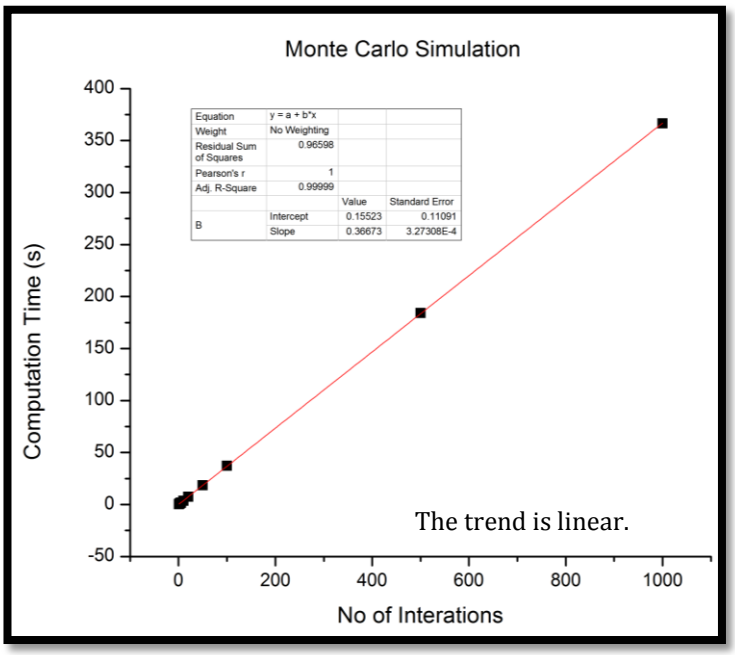
Monte Carlo Simulation

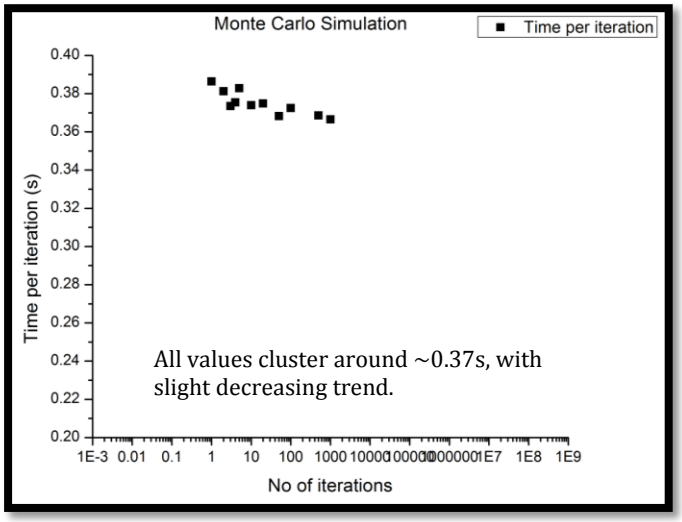
In this approach, solutions to the equations were numerically calculated using MATLAB's ode15s solver. This was repeated for various values of R, C that were drawn randomly from the respective distributions. Average of all the trajectories is computed as the solution. To prevent the problem from becoming ill conditioned, any values of $C < 10^{-280}$ were disregarded during the iterations. Since the probability of occurrence of these values is very low, their contribution to error can be neglected.

It was observed that the computation time increases almost linearly with number of iterations. However, the time per iteration is approximately constant (~0.37s), regardless of the number of iterations. However, a slight decreasing trend can be observed.

Iterations	Total Time (s)	Time / Iteration (s)
1	0.386423	0.386423
2	0.762493	0.3812465
3	1.120294	0.373431333
4	1.501726	0.3754315
5	1.914053	0.3828106
10	3.739653	0.3739653
20	7.496196	0.3748098
50	18.413898	0.36827796
100	37.240395	0.37240395
500	184.260985	0.36852197
1000	366.483031	0.366483031







Wiener Chaos Approximation

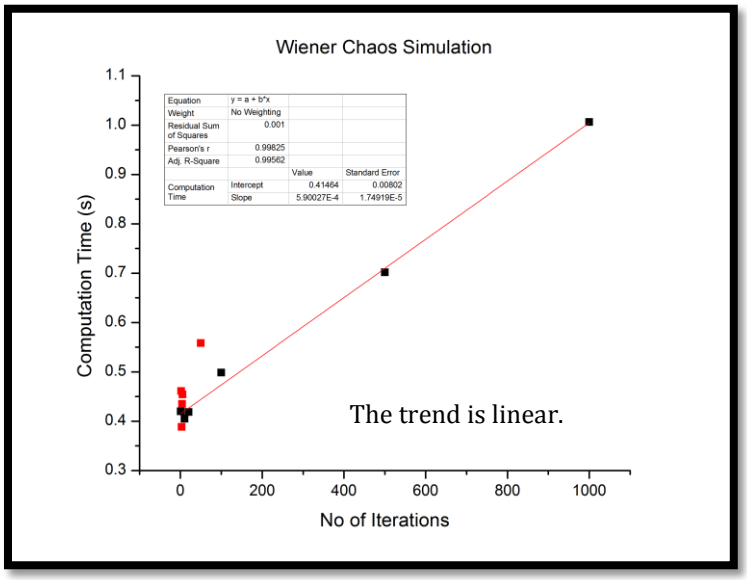
The circuit variables are expanded as linear combinations of the first four Hermite Polynomials.

$$I = y_{01}H_0 + y_{11}H_1 + y_{21}H_2 + y_{31}H_3$$
$$V_1 = y_{02}H_0 + y_{12}H_1 + y_{22}H_2 + y_{32}H_3$$
$$V_2 = y_{03}H_0 + y_{13}H_1 + y_{23}H_2 + y_{33}H_3$$
$$V_3 = y_{04}H_0 + y_{14}H_1 + y_{24}H_2 + y_{34}H_3$$
$$R = R_\mu H_0 + R_\sigma H_1$$
$$C = C_\mu H_0 + C_\sigma H_1$$

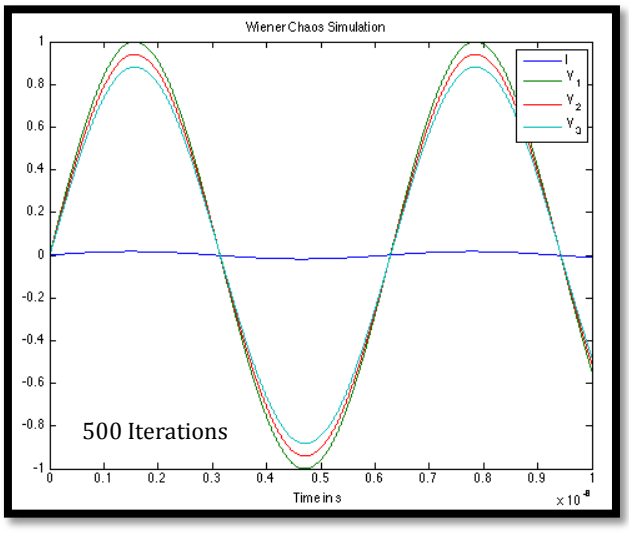
Galerkin projections are used to obtain a set of DAEs involving the sixteen time-dependent coefficients. These are solved for various times by invoking MATLABs ode15s solver. Once the coefficients are obtained, the circuit variables can be simulated by substituting a $N\ (0,1)$ random variable into the Hermite polynomials.

This method is found to be much faster than Monte Carlo, as the slowest step—the differential equation solving—occurs only once, regardless of the number of iterations. This makes it suitable for problems where large numbers of iterations are required for accuracy.
The computational time is found to be approximately linear varying with number of iterations.

Iterations	Time (s)
1	0.419939
2	0.461177
3	0.388337
4	0.435189
5	0.454365
10	0.405038
20	0.41864
50	0.558328
100	0.498357
500	0.701734
1000	1.006462



Visually, the Wiener trajectory is indistinguishable from the Monte Carlo Trajectory.

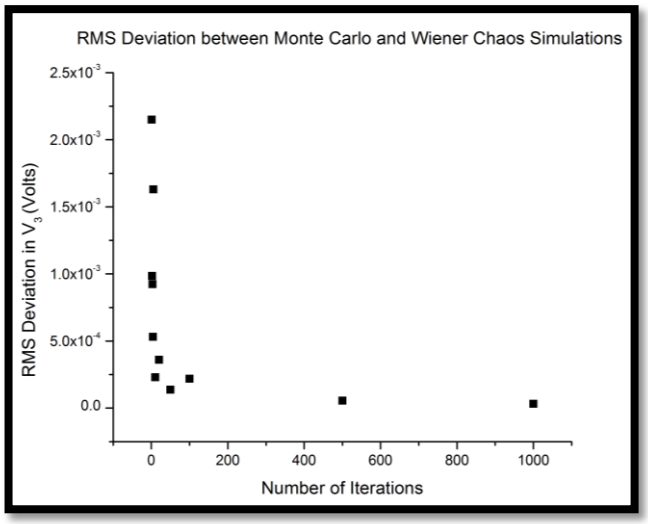


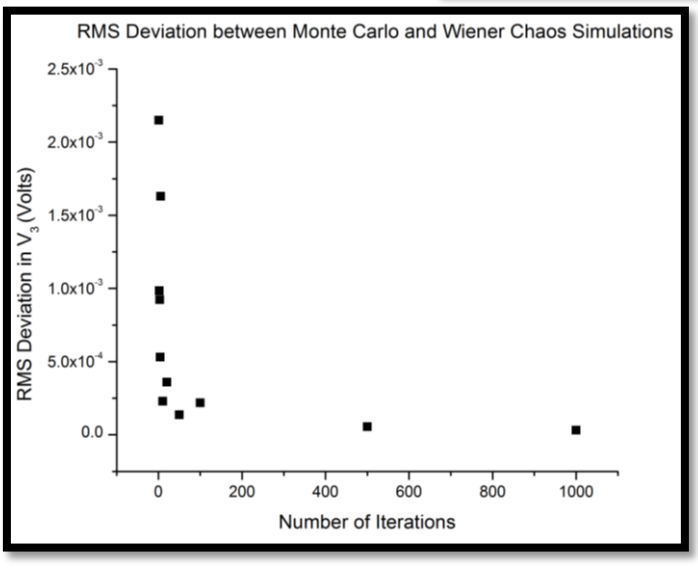
The Wiener trajectory is found to converge to the Monte Carlo trajectory when the number of iterations is increased. The RMS deviation from the Monte Carlo trajectory sharply drops when the number of iterations is increased and tapers thereafter.
V_1 ostensibly is non-convergent. However, this is only because $\boldsymbol{V_1 = sin(1e9 \cdot t)}$ is a deterministic quantity and its values are unaffected by stochastic processes.

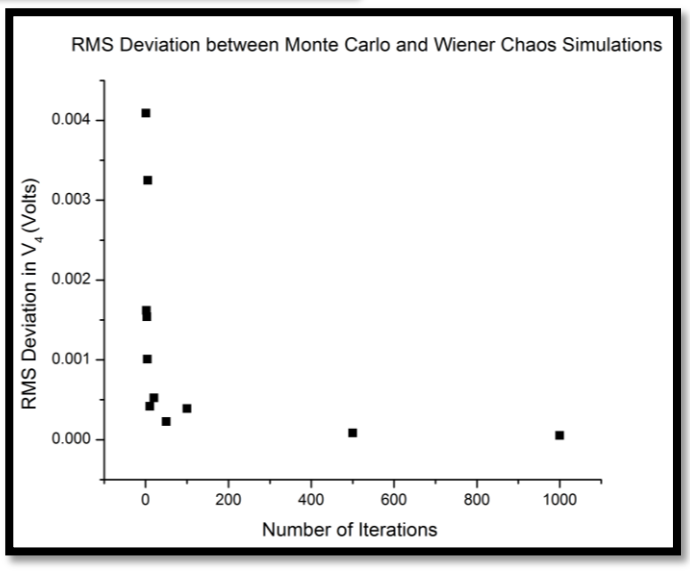
	RMS Deviations between Monte Carlo and Wiener			
Iterations	I (A)	V_1 (V)	V_2 (V)	V_3 (V)
1	2.49E-04	6.22E-10	2.15E-03	4.09E-03
2	2.30E-04	7.34E-10	9.85E-04	1.62E-03
3	1.87E-04	5.94E-10	9.23E-04	1.54E-03
4	6.23E-05	5.52E-10	5.31E-04	1.01E-03
5	8.59E-05	4.98E-10	1.63E-03	3.25E-03
10	4.09E-05	5.13E-10	2.30E-04	4.19E-04
20	9.93E-05	5.12E-10	3.60E-04	5.25E-04
50	2.37E-05	5.04E-10	1.37E-04	2.28E-04
100	4.61E-05	5.12E-10	2.19E-04	3.90E-04
500	6.39E-06	5.12E-10	5.50E-05	8.43E-05
1000	7.08E-07	5.07E-10	3.18E-05	5.16E-05

$$\sqrt{\overline{\Delta I^2}} = \sum_{t=0}^{t=10^{-8}} ((I_{MC}(t) - I_{WC}(t))^2$$

and similarly for other variables.







Result

The RC Ladder was simulated using Monte Carlo and Weiner expansion approaches. The two were found to converge with increasing number of iterations. The RMS deviation between the two with 1000 iterations was found to be of order:

$$\left[\sqrt{\overline{\Delta I^2}}\ \sqrt{\overline{\Delta V_1^2}}\sqrt{\overline{\Delta V_2^2}}\sqrt{\overline{\Delta V_3^2}}\ \right]_{10^3} \sim [10^{-7}\ 10^{-10}\ 10^{-5}\ 10^{-5}]$$

The computation time for both was found to linearly increase with number of iterations.

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