ASSIGNMENT-12

AI1110

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Outline

Exercise 15-15

Deteimine the mean time to absorption for the random walk model with two absorbing barriers .Let the number of states in a random walk be finite $(e_0, e_1, ...e_N)$ Consider the case

$$\mathrm{P} = \begin{pmatrix} 1 & 0 & 0 & 0 & . & . & . & 0 \\ q & 0 & p & 0 & 0 & . & . & 0 \\ 0 & q & 0 & P & 0 & . & . & 0 \\ . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . \\ 0 & 0 & . & . & . & q & 0 & P \\ 0 & 0 & . & . & . & 0 & 0 & 1 \end{pmatrix}$$

Find mean time to absorption for player A?

Solution

The mean time to absorption satisfies

$$\begin{aligned} m_i &= 1 + \sum_{k \in T} p_{ik} m_k \\ &= 1 + p_{i,i+1} m_{i+1} + p_{i,i-1} m_{i-1} \\ &= 1 + p m_{i+1} + q m_{i-1} \end{aligned}$$

This gives

$$p(m_{k+1} - m_k) = q(m_k - m_{k-1}) - 1$$

Let

$$M_{k+1} = m_{k+1} - m_k$$

Solution

so that the above iteration gives

$$
\begin{aligned}
M_{k+1} &= \frac{q}{p}M_k - \frac{1}{p} \\
&= \left(\frac{q}{p}\right)^k M_1 - \frac{1}{p}\left[1 + \left(\frac{q}{p}\right) + \left(\frac{q}{p}\right)^2 + \ldots + \left(\frac{q}{p}\right)^{k-1}\right] \\
&= \begin{cases} \left(\frac{q}{p}\right)^k M_1 - \frac{1}{p-q}\left\{1 - \left(\frac{q}{p}\right)^k\right\}, & p \neq q \\ M1 - \frac{k}{p}, & p = q \end{cases}
\end{aligned}
$$

Solution

This gives

$$
\begin{aligned}
m_i &= \sum_{k=0}^{i-1} M_{k+1} \\
&= \begin{cases} \left(M_1 + \frac{1}{p-q}\right) \sum_{k=0}^{i-1} \left(\frac{q}{p}\right)^k - \frac{i}{p-q}, & p \neq q \\ iM_1 - \frac{i(i-1)}{2p}, & p = q \end{cases} \\
&= \begin{cases} \left(M_1 + \frac{1}{p-q}\right) \frac{1-(q/p)^i}{1-(q/p)} - \frac{i}{p-q}, & p \neq q \\ iM_1 - \frac{i(i-1)}{2p}, & p = q \end{cases}
\end{aligned}
$$

Solution

which we have used $m_o = 0$.Similarly $m_{a+b} = 0$ gives

$$M_1 + \frac{1}{p-q} = \frac{a+b}{p-q}\frac{1-q/p}{1-(q/p)^{a+b}}.$$

Thus $m_i = \begin{cases} \frac{a+b}{p-q}\frac{1-(q/p)^i}{1-(q/p)^{a+b}} - \frac{i}{p-q}, & p \neq q \\ i(a+b-i), & p = q \end{cases}$

Solution

Which gives for $i = a$

$$m_a = \begin{cases} \frac{a+b}{p-q} \frac{1-(q/p)^a}{1-(q/p)^{a+b}} - \frac{a}{p-q}, & p \neq q \\ ab, & p = q \end{cases}$$

$$= \begin{cases} \frac{b}{2p-1} - \frac{a+b}{2p-1} \frac{1-(p/q)^b}{1-(p/q)^{a+b}}, & p \neq q \\ ab, & p = q \end{cases}$$

Solution

by writing we get the mean time as

$$\frac{1-(q/p)^a}{1-(q/p)^{a+b}} = 1 - \frac{(q/p)^a - (q/p)^{a+b}}{1-(q/p)^{a+b}}$$
$$= 1 - \frac{1-(p/q)^b}{1-(p/q)^{a+b}}$$