ASSIGNMENT-2

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Question 4(b)

Find the coordinates of the center,foci and equation of the directrix of the hyperbola $x^2 - 3y^2 - 4x = 8$?

Solution

Here we should observe that there is no xy term in the given hyperbola equation so this can be converted to the general form of hyperbola

$$\frac{X^2}{a^2} - \frac{Y^2}{b^2} = 1 \Big/ -\frac{X^2}{a^2} + \frac{Y^2}{b^2} = 1$$

$$\implies x^2 - 3y^2 - 4x = 8$$
$$\implies x^2 - 4x + 4 - 3y^2 = 8 + 4$$
$$\implies (x-2)^2 - 3y^2 = 12$$

$$\implies \frac{(x-2)^2}{12} - \frac{y^2}{4} = 1 \tag{1}$$

Applying transformation to convert into standard form subtitute $X = x - 2$ and $Y = y$

$$\implies \frac{X^2}{(12)^2} - \frac{Y^2}{(4)^2} = 1 \tag{2}$$

It takes the first general form so transverse axis is parallel to $x - axis$ and conjugate axis is parallel to $y - axis$.
Here e is called eccentricity of hyperbola defined as

$$e = \sqrt{1 + \frac{b^2}{a^2}} \tag{3}$$

$\frac{X^2}{a^2} - \frac{Y^2}{b^2} = 1$	Center	Foci	Directrix equation
Location	$(0,0)$	$(\pm ae, 0)$	$X = \pm\frac{a}{e}$

properties

From (3) we have $a^2 = 12, b^2 = 4$.

$$\implies e = \sqrt{1 + \frac{4}{12}} = \sqrt{\frac{4}{3}} = \frac{2}{\sqrt{3}} \tag{4}$$

Center of hyperbola

center of $\frac{X^2}{a^2} - \frac{Y^2}{b^2} = 1$ is $(X,Y) = (0,0)$.As $X = x - 2$ and $Y = y$ we have center of that is $(x,y) = (2,0)$

Foci of hyperbola

foci of $\frac{X^2}{a^2} - \frac{Y^2}{b^2} = 1$ is $(X,Y) = (\pm ae, 0)$.

$$\implies Y = y = 0$$
$$\implies X = \pm ae = \pm 2\sqrt{3} * \frac{2}{\sqrt{3}}$$
$$\implies X = \pm 4$$
$$\implies x = X + 2 = \pm 4 + 2$$
$$\implies x = 6 \text{ or } -2 \tag{5}$$

$\therefore$ foci of hyperbola are (x,y) = (6,0) and (-2,0)

Directrix of hyperbola

For the general equation

$$\frac{X^2}{a^2} - \frac{Y^2}{b^2} = 1$$

It is given by X $= \pm \frac{a}{e}$

$$\implies X = \pm \frac{\frac{2\sqrt{3}}{2}}{\sqrt{3}}$$

$$\implies X = \pm\sqrt{3} * \sqrt{3} = 3$$

$$\implies x = X + 2 = \pm 3 + 2$$

$$\implies x = 5 \text{ or } -1. \tag{6}$$

$\therefore$ Directrix of given hyperbola are
$x = 5$ and $x = -1$

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(x-h)^2/a^2 - (y-k)^2/b^2 = 1
Enter the value of h:2
Enter the value of k:0
Enter the value of a^2:12
Enter the value of b^2:4
Eccentricity e = 1.1547005383792517
Center of hyperbola is (2,0)
foci of hyperbola is (-2.0,0) and (6.0,0)
Equation of directrix hyperbola
              is x = -1.000 and x = 5.000
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Verification