

ASSIGNMENT-2

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Question 4(b)

Find the coordinates of the center, foci and equation of the directrix of the hyperbola $x^2 - 3y^2 - 4x = 8$?

$\frac{X^2}{a^2} - \frac{Y^2}{b^2} = 1$	Center	Foci	Directrix equation
Location	(0, 0)	$(\pm ae, 0)$	$X = \pm \frac{a}{e}$

Table 1: properties

Solution

Here we should observe that there is no xy term in the given hyperbola equation so this can be converted to the general form of hyperbola

$$\frac{X^2}{a^2} - \frac{Y^2}{b^2} = 1 \quad \Big/ \quad -\frac{X^2}{a^2} + \frac{Y^2}{b^2} = 1$$

$$\begin{aligned} \Rightarrow x^2 - 3y^2 - 4x &= 8 \\ \Rightarrow x^2 - 4x + 4 - 3y^2 &= 8 + 4 \\ \Rightarrow (x - 2)^2 - 3y^2 &= 12 \end{aligned}$$

$$\Rightarrow \frac{(x - 2)^2}{12} - \frac{y^2}{4} = 1 \quad (1)$$

Applying transformation to convert into standard form substitute $X = x - 2$ and $Y = y$

$$\Rightarrow \frac{X^2}{(12)^2} - \frac{Y^2}{(4)^2} = 1$$

It takes the first general form so transverse axis is parallel to x -axis and conjugate axis is parallel to y -axis

Here e is called eccentricity of hyperbola defined as

$$e = \sqrt{1 + \frac{b^2}{a^2}} \quad (2)$$

From (2) we have $a^2 = 12, b^2 = 4$.

$$\Rightarrow e = \sqrt{1 + \frac{4}{12}} = \sqrt{\frac{4}{3}} = \frac{2}{\sqrt{3}}$$

Center of hyperbola

center of $\frac{X^2}{a^2} - \frac{Y^2}{b^2} = 1$ is $(X, Y) = (0, 0)$. As $X = x - 2$ and $Y = y$ we have center of that is $(x, y) = (2, 0)$

Foci of hyperbola

foci of $\frac{X^2}{a^2} - \frac{Y^2}{b^2} = 1$ is $(X, Y) = (\pm ae, 0)$.

$$\Rightarrow Y = y = 0$$

$$\Rightarrow X = \pm ae = \pm 2\sqrt{3} * \frac{2}{\sqrt{3}}$$

$$\Rightarrow X = \pm 4$$

$$\Rightarrow x = X + 2 = \pm 4 + 2$$

$$\Rightarrow x = 6 \text{ or } -2$$

\therefore foci of hyperbola $(x, y) = (6, 0)$ and $(-2, 0)$

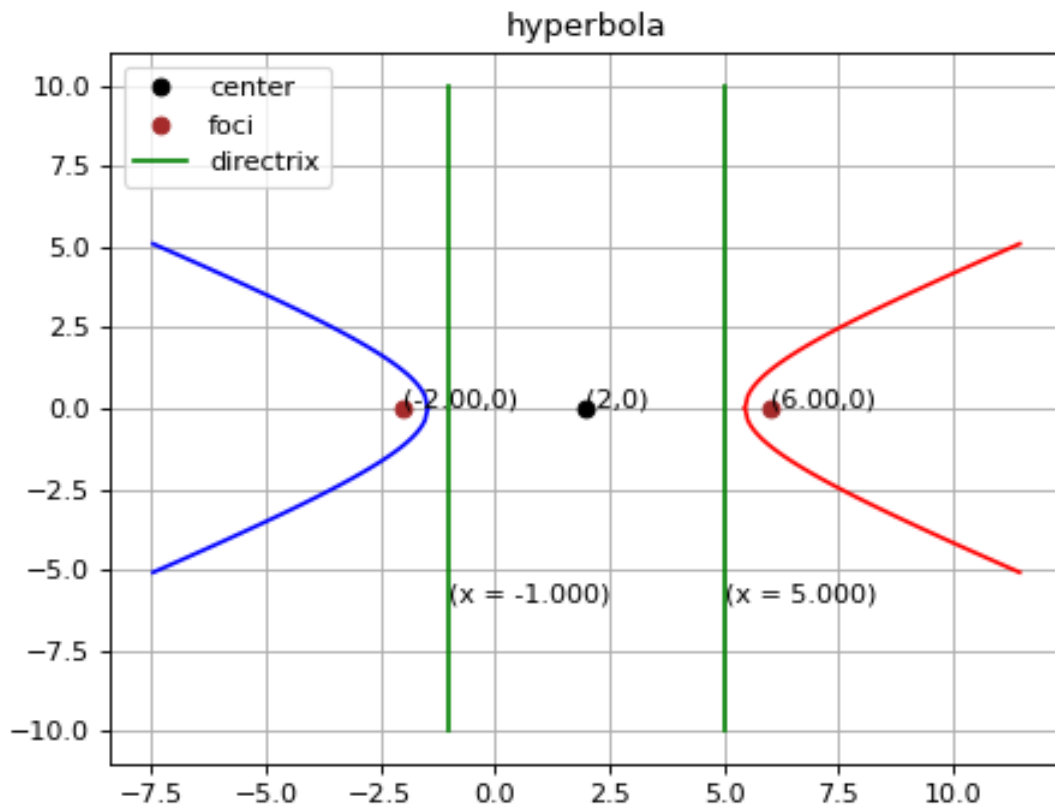


Figure 1: Hyperbola: $x^2 - 3y^2 - 4x - 8 = 0$

Directrix Equation of hyperbola

For the general equation

$$\frac{X^2}{a^2} - \frac{Y^2}{b^2} = 1$$

It is given by $X = \pm \frac{a}{e}$

$$\Rightarrow X = \pm \frac{2\sqrt{3}}{\frac{\sqrt{3}}{2}}$$

$$\Rightarrow X = \pm \sqrt{3} * \sqrt{3} = 3$$

$$\Rightarrow x = X + 2 = \pm 3 + 2$$

$$\Rightarrow x = 5 \text{ or } -1$$

\therefore Directrix of hyperbola are $x = 5$ and $x = -1$

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(x-h)^2/a^2 - (y-k)^2/b^2 = 1
Enter the value of h:2
Enter the value of k:0
Enter the value of a^2:12
Enter the value of b^2:4
Eccentricity e = 1.1547005383792517
Center of hyperbola is (2,0)
foci of hyperbola is (-2.0,0) and (6.0,0)
Equation of directrix hyperbola is x = -0.9999999999999996 and x = 5.0
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Figure 2: Verification