ASSIGNMENT-4

AI1110

MUKUNDA REDDY
AI21BTECH11021

Example 2-7

A telephone call occurs at random in the interval $(0, T)$. Find the probability that the call will occur in the interval (t_1, t_2)?

Solution

Here S consists of noncountable infinity of elements so assigning individual probabilities is not possible.Lets consider the intervals and a function $\alpha(x)$ which gives probability density such that

$$\int_{-\infty}^{\infty} \alpha(x)dx = 1 \tag{1}$$

Solution

The probability of the event $X_1 < X < X_2$ consisting of all points in the interval (X_1, X_2) is given by

$$P(X_1 < X < X_2) = \int_{X_2}^{X_1} \alpha(x)dx \tag{2}$$

We can assign probability $\alpha(x) = 0$,for $-\infty < x < 0$ and $T < x < \infty$ as there are no phone calls in that time period.

Solution

Given call occurs randomly in interval $(0, T)$ meaning $\alpha(x)$ is uniform as it must satisfy the let $\alpha(x) = c$ as this must satisfy (1),

$$\begin{aligned}\int_{-\infty}^{\infty} \alpha(x)dx &= \int_{-\infty}^{0} \alpha(x)dx + \int_{0}^{T} \alpha(x)dx + \int_{T}^{\infty} \alpha(x)dx \\ &= 0 + \int_{0}^{T} cdx + 0 \,(\text{as } \alpha(x) = constantc) \\ &= c(T)\end{aligned}$$

Solution

$$\implies \int_{-\infty}^{\infty} \alpha(x)dx = 1 \tag{3}$$

$$c(T) = 1$$

$$c = \frac{1}{T}$$

The probaility disttribution function is given by

$$\alpha(x) = \begin{cases} \frac{1}{T}, & if \ 0 \leq x \leq T \\ 0, & \text{otherwise} \end{cases}$$

Solution

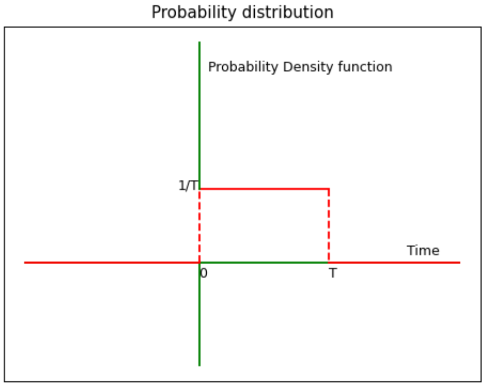

Figure: $y = \alpha(x)$

Solution

From equation (2) we can write the probability of the event the call will occur in the interval (t_1, t_2) equals,

$$\begin{aligned} P(t_1 \leq X \leq t_2) &= \int_{t_2}^{t_1} \alpha(x) dx \\ &= \int_{t_2}^{t_1} \frac{1}{T} dx \\ &= \frac{t_2 - t_1}{T}. \end{aligned}$$