ASSIGNMENT-5

AI1110

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Outline

Example 4-3

In the coin-tossing experiment, the probability of heads equals p and the probability of tails equals q. Find its distribution function $F(x)$ for every x' from $-\infty$ to ∞ ?

Solution

Here $F(x)$ is called the cumulative distribution function of the random variable x.It is defined by

$$F_x(w) = P\{\mathbf{X} \leq w\} \tag{1}$$

We define a random variable X such that

$$X = \begin{cases} 1 & \text{if heads occured} \\ 0 & \text{if tails occured} \end{cases}$$

Case (i) : $x < 0$

To find cumulative distribution consider cases of x from $-\infty$ to ∞. Given $x < 0$ but both $\{X(h) = 1\}, \{X(t) = 0\} > x$ exceeds given interval of x so

$$\begin{aligned} F(x) &= P\{X \leq x\} \\ &= P\{\phi\} \text{ (Null set)} \\ &= 0 \end{aligned} \tag{2}$$

Case (ii) : $0 \leq x < 1$

Given $0 \leq x < 1$. we know $\{X(h) = 1\} > x$ but $\{X(t) = 0\} \leq [0, 1)$ Hence

$$\begin{aligned} F(x) &= P\{X \leq x\} \\ &= P\{t\}, \forall x \in [0, 1), X(t) \leq x \\ &= q \end{aligned} \tag{3}$$

Case (iii) : $x \geq 1$

We have $\{X(h) = 1\} \leq x$ and $\{X(t) = 0\} \leq x$ as here $x \in [1, \infty)$, also events $X(h)$ and $X(t)$ are partition of sample space so $p + q = 1$.

$$\begin{aligned} F(x) &= P(X \leq x) \\ &= P\{t, h\}, \forall x \in [1, \infty), X(t, p) \leq x \\ &= p + q \\ &= 1 \end{aligned} \tag{4}$$

Graph

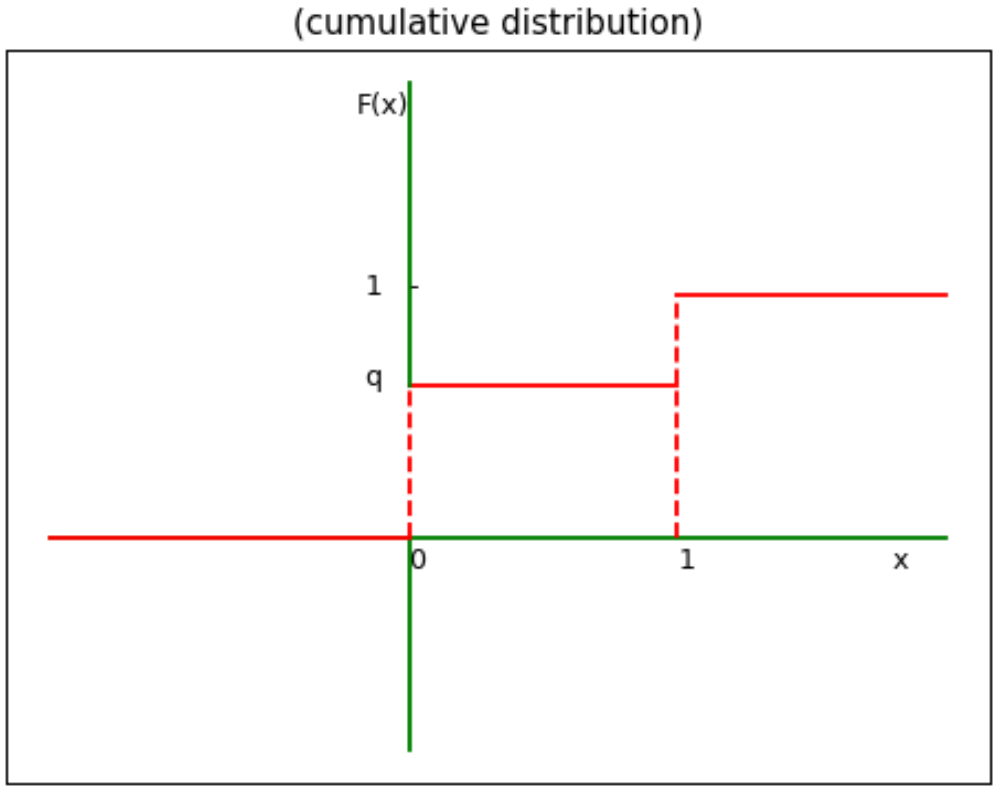

Figure: Cumulative Distribution