ASSIGNMENT-7

AI1110

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Outline

Example 6-43

Suppose $\mathbf{x}$ and $\mathbf{y}$ are independent binomial random variables with parameters (m, p) and (n, p) respectively. Then $\mathbf{x}+\mathbf{y}$ is also binomial with parameter $(m+n, p)$, so that

$$P\{x = x|\mathbf{x}+\mathbf{y} = x+y\} = \frac{P\{x = x\}P\{y = y\}}{P\{x+y = x+y\}} = \frac{\binom{m}{n}\binom{n}{y}}{\binom{m+n}{x+y}}$$

Thus the conditional distribution of $\mathbf{x}$ given $\mathbf{x}+\mathbf{y}$ is hypergeometric. Show that the converse of this result which states that if x and y are non negative independent random variables such that $P\{x = 0\} > 0, P\{y = 0\} > 0$ and the conditional

Example 6-43

Show that the converse of this result which states that if x and y are non negative independent random variables such that $P\{x = 0\} > 0, P\{y = 0\} > 0$ and the conditional distribution of x given x + y is hyper geometric,then x and y are binomial random variables ?

Solution

We know that

$$\frac{P\{x=x\}}{\binom{m}{k}}\frac{P\{y=y\}}{\binom{n}{y}}=\frac{P\{x+y=x+y\}}{\binom{m+n}{x+y}}$$

let

$$\frac{P\{x=x\}}{\binom{m}{k}}=f(x),\frac{P\{y=y\}}{\binom{n}{y}}=g(y),\frac{P\{x+y=x+y\}}{\binom{m+n}{x+y}}=h(x+y)$$

Then h(x+y) = f(x) $\times g(y)$

Solution

and Hence

$$h(1) = f(1)g(0) = f(0)g(1)$$

$$h(2) = f(2)g(0) = f(1)g(1) = f(0)g(2)$$

$$\text{Generalizing } h(k) = f(k)g(0) = f(k-1)g(1) = \ldots$$

Thus

$$f(k) = f(k-1)\frac{g(1)}{g(0)} = f(0)\left(\frac{g(1)}{g(0)}\right)^k$$

Solution

or

$$P\{x=k\} = \binom{m}{k} Px = 0a^k \;\; k = 0, 1, 2....$$

where $a = \frac{g(1)}{g(0)} > 0$.But $\sum P\{x=k\} = 1$ gives $P\{x=0\}(1+a)^m = 1$ or $P\{x=0\} = q^m$ where $q = \frac{1}{1+a} < 1$.Hence $a = \frac{p}{q}$ where $p = 1-q > 0$.We know that

$$P\{x=k\} = \binom{m}{k} p^k q^{m-k}, k = 0, 1, 2....$$

Solution

Similarly we can also show

$$P\{y = r\} = \binom{n}{r} p^r q^n - r, r = 0, 1, 2...n$$

Hence the proof is completed.