ASSIGNMENT-8

AI1110

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Outline

Exercise 7-13

The random variables X_i are i.i.d. with moment function $\phi_x(s) = E\{e^{sx_i}\} \forall i$.The random variable n takes the values 0,1,2... and its moment function equals $\Gamma_n\{z\} = E\{z^n\}$.Show that if

$$y = \sum_{i=1}^{n} x_i, \text{ then } \phi_y(s) = E\{e^{sy}\} = \Gamma_n\{\phi_x(s)\}$$

Solution

Given $y = x_1 + x_2 + ... + x_n$ and given $\phi_x(s) = E\{e^{sx_i}\} \forall i$ also by the independence of the random variable x_i we have we have

$$\begin{aligned} E\{e^{sy}|n = k\} &= E\{e^{s(x_1+x_2..x_k)}\} \\ &= E\{e^{sx_1}\}...E\{e^{sx_k}\} \\ &= \phi_x(s)...\phi_x(s)(\text{k times}) \\ &= \phi_x^k(s) \end{aligned} \tag{1}$$

Solution

$$\begin{aligned}\phi_y\{s\} &= E\{e^{sy}\}\\ &= E\{E\{e^{sy}\}\} \text{ Since E\{E\{X\}\} = E\{X\}}\\ &= E\{\phi_x^n(s)\} \text{ from (1)} \qquad (2)\end{aligned}$$

We know that $E\{z^n\} = \Gamma_n(z)$. let's Consider $z = \phi_x(s)$ then we have

$$\phi_y\{s\} = E\{e^{sy}\} = \Gamma_n[\phi_x(s)] \qquad (3)$$