ASSIGNMENT-9

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Outline

Exercise 9-39

Find autocorrelation $R(\tau)$? if

a. $S(w) = \frac{1}{(1+w^4)}$
b. $S(w) = \frac{1}{(4+w^2)^2}$

Solution (a)

$$
\begin{aligned}
S(s) &= \frac{1}{1+s^4} \\
&= \frac{1}{(s^2+\sqrt{2}s+1)(s^2-\sqrt{2}s+1)}
\end{aligned}
$$

Now we know that for the equation of kind $y\prime\prime(t) + by\prime(t) + cy(t) = x(t), S_{xx}(w) = q$ Then

$$
S_{yy}(w) = \frac{q}{(c-w^2)^2 + b^2w^2}
$$

Solution (a)

This is a special case with $b = \sqrt{2}, c = 1$ i.e. $b^2 < 4c$ so we have autocorrelation as

$$R_{yy}(\tau) = \frac{q}{2bc} e^{-\alpha|\tau|} \left(cos(\beta\tau) + \frac{\alpha}{\beta} sin(\beta|\tau|) \right), \alpha = \frac{b}{2}, \alpha^2 + \beta^2 = c$$

Subtituting we get

$$R(\tau) = \frac{1}{2\sqrt{2}} e^{\frac{-|\tau|}{\sqrt{2}}} \left(cos(\frac{\tau}{\sqrt{2}}) + sin(\frac{|\tau|}{\sqrt{2}}) \right)$$

Solution (b)

From the pair $e^{-2|\tau|} \longleftrightarrow \frac{4}{4+w^2}$ and it follows that $e^{-2|\tau|} * e^{-2|\tau|} \longleftrightarrow \frac{16}{(4+w^2)^2}$ hence for $\tau > 0$

$$\begin{aligned} 16R(\tau) &= \int_{-\infty}^{\infty} e^{-2|x|} e^{-2|\tau-x|} dx \\ &= \int_{-\infty}^{0} e^{-2x} e^{-2(\tau-x)} dx + \int_{0}^{\tau} e^{-2x} e^{-2(\tau-x)} dx \\ &+ \int_{\tau}^{\infty} e^{-2x} e^{2(\tau-x)} dx \\ &= \frac{1}{2} e^{-2\tau}(1+2\tau) \end{aligned}$$

Solution (b)

after simplification we are left with the autocorrelation ,

$$R(\tau) = \frac{1}{32} e^{-2\tau} (1 + 2\tau)$$