

ASSIGNMENT-1

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EXERCISE 1

1.1

The sample generated is

<https://github.com/mohilmukundareddy/Assignment1/tree/main/ex1/1.1>

$$\begin{aligned} F_X(x) &= \int_{-\infty}^x f_X(x) dx \\ &= 0 + \int_0^x 1 dx \\ &= \begin{cases} 1 & x > 1 \\ x & 0 \leq x \leq 1 \\ 0 & x < 0 \end{cases} \end{aligned}$$

1.2

The python code is

<https://github.com/mohilmukundareddy/Assignment1/tree/main/ex1/1.2>

1.4

The c code to find mean and variance is

<https://github.com/mohilmukundareddy/Assignment1/tree/main/ex1/1.4>

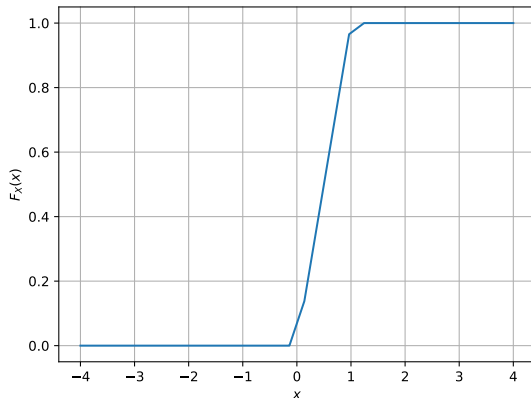


Fig. 1: uni-cdf

1.5

Given

$$E[U^k] = \int_{-\infty}^{\infty} x^k dF_X(x)$$

1) if k=1

$$\begin{aligned} E[U] &= \int_{-\infty}^{\infty} x^k f_X(x) dx \\ &= \int_0^1 x \times 1 dx \\ &= \left[\frac{x^2}{2} \right]_0^1 = \frac{1}{2} \end{aligned}$$

2) if k=2

$$\begin{aligned} E[U^2] &= \int_0^1 x^2 \times 1 dx \\ &= \left[\frac{x^3}{3} \right]_0^1 = \frac{1}{3} \end{aligned}$$

1.3

Given U is uniform random variable so

$$f_X(x) = \begin{cases} 1 & 0 \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

$$\begin{aligned}
 \text{variance} &= E[u - E[u]]^2 \\
 &= E[U^2] - E^2[U] \\
 &= \frac{1}{3} - \frac{1}{4} = 0.0833
 \end{aligned}$$

```

> make -s
> ./main
mean = 0.500007
var = 0.083301
> 

```

Fig. 2: verification

EXERCISE 2

2.1

The gau.dat file is given by

<https://github.com/mohilmukundareddy/Assignment1/tree/main/ex2/2.1>

2.2

cdf plot of gau.dat

<https://github.com/mohilmukundareddy/Assignment1/tree/main/ex2/2.2>

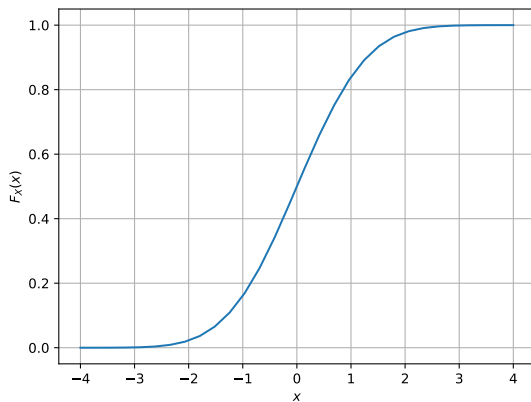


Fig. 3: gauss-cdf

2.3

PDF of X is given by

<https://github.com/mohilmukundareddy/Assignment1/tree/main/ex2/2.3>

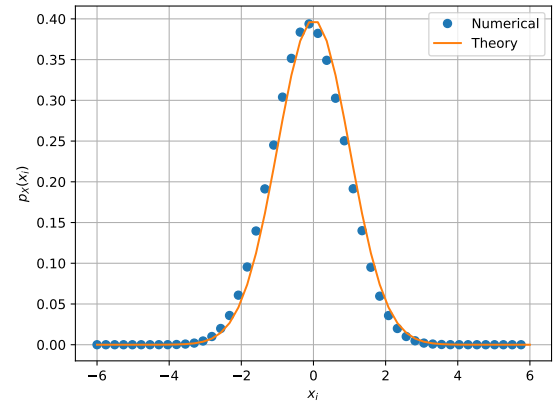


Fig. 4: gauss-pdf

2.4

C code to find the mean and variance

<https://github.com/mohilmukundareddy/Assignment1/tree/main/ex2/2.4>

2.5

Given that

$$p_X(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}, -\infty < x < \infty$$

by property of probability

$$\int_{-\infty}^{\infty} p_X(x) dx = 1$$

$$\begin{aligned}
 F_X(x) &= \int_{-\infty}^x p_X(x) dx \\
 &= \int_{-\infty}^x \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} \\
 &= G(x - \mu/\sigma) \quad \mu = 0, \sigma = 1 \\
 &= G(x)
 \end{aligned}$$

Here the random variable $X \sim N(0, 1)$ comparing the random variable in 2.1

$$X_0 = \sum_{i=1}^{12} U_i - 6$$

U_i are independent we get mean $\mu_0 = 0.0002$ $\sigma_0 = 0.666$ as i increases we approach normal distribution.

EXERCISE 3

3.1

Generated samples of V and its cdf is given by

<https://github.com/mohilmukundareddy/Assignment1/tree/main/ex3>

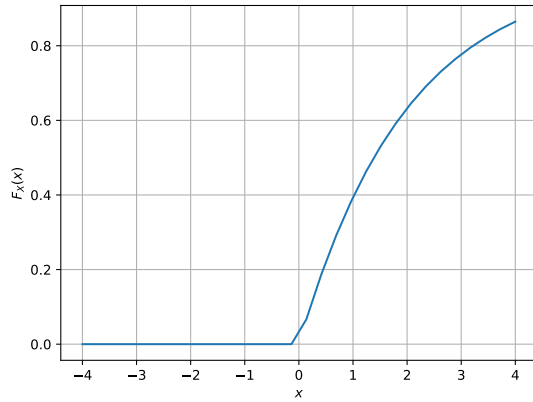


Fig. 5: V-cdf

3.2

$$\begin{aligned}
 F_V(x) &= P\{V \leq x\} \\
 &= P\{-2 \times \ln(1 - U) \leq x\} \\
 &= P\{U \leq 1 - e^{(-\frac{x}{2})}\} \\
 &= F_U\{1 - e^{(-\frac{x}{2})}\} \\
 &= \begin{cases} 1 - e^{(-\frac{x}{2})} & 0 \leq x < \infty \\ 0 & x < 0 \end{cases}
 \end{aligned}$$