Random Numbers

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Contents

1 Uniform Random Numbers

Let U be a uniform random variable between 0 and 1.

1.1 Generate 10^6 samples of U using a C program and save into a file called uni.dat .
Solution: Download the following files and execute the C program.

```
wget https://github.com/mohilmukundareddy/
    Assignment1/blob/main/ex1/1.1/exrand.c
we get https://github.com/mohilmukundareddy
    /Assignment1/blob/main/ex1/1.1/coeffs.h
```

Use the below command in the terminal to run the code

```
gcc exrand.c -lm
./a.out
```

1.2 Load the uni.dat file into python and plot the empirical CDF of U using the samples in uni.dat. The CDF is defined as

$$F_U(x) = \Pr(U \le x) \tag{1.1}$$

Solution: The graph 1.2 is obtained by running the below code

```
https://github.com/mohilmukundareddy/
    Assignment1/blob/main/ex1/1.2/main.py
```

Run the following command in the terminal to run the code.

```
python3 main.py
```

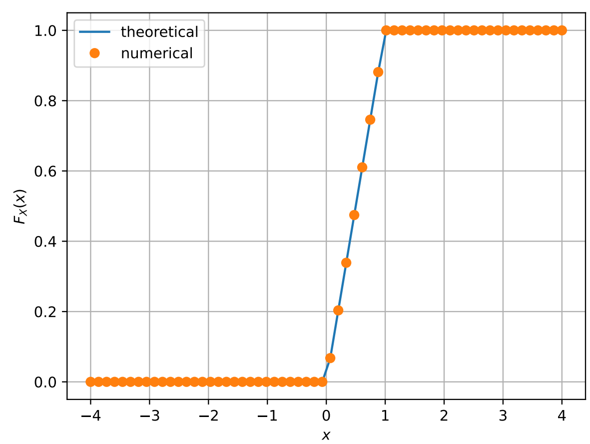

Fig. 1.2: The CDF of U

1.3 Find a theoretical expression for $F_U(x)$.
Solution: Given U is uniform random variable so

$$f_X(x) = \begin{cases} 1 & 0 \le x \le 1 \\ 0 & otherwise \end{cases}$$

$$\begin{aligned} F_X(x) &= \int_{-\infty}^{x} f_X(x)dx \\ &= 0 + \int_0^x 1dx \\ &= \begin{cases} 1 & x > 1 \\ x & 0 \le x \le 1 \\ 0 & x < 0 \end{cases} \end{aligned}$$

1.4 The mean of U is defined as

$$E[U] = \frac{1}{N}\sum_{i=1}^{N} U_i \tag{1.2}$$

and its variance as

$$\text{var}[U] = E[U - E[U]]^2 \tag{1.3}$$

Write a C program to find the mean and variance of U.
Solution:

```
wget https://github.com/mohilmukundareddy/
    Assignment1/blob/main/ex1/1.4/exrand.c
weget https://github.com/mohilmukundareddy/
    Assignment1/blob/main/ex1/1.4/coeffs.h
```

Use below command to run file,

```
gcc exrand.c −lm
./a.out
```

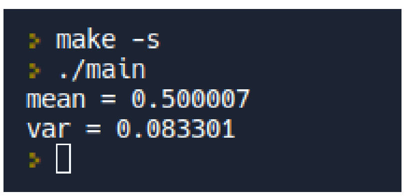

Fig. 1.4: Caption

1.5 Verify your result theoretically given that

Given

$$E[U^k] = \int_{-\infty}^{\infty} x^k dF_X(x)$$

$$\begin{aligned} E[U] &= \int_{-\infty}^{\infty} x^k f_X(x) dx \\ &= \int_{0}^{1} x \times 1 dx \\ &= \left[\frac{x^2}{2}\right]_0^1 = \frac{1}{2} \end{aligned}$$

if k=2

$$\begin{aligned} E[U^2] &= \int_{0}^{1} x^2 \times 1 dx \\ &= \left[\frac{x^3}{3}\right]_0^1 = \frac{1}{3} \end{aligned}$$

$$\begin{aligned} variance &= E[u - E[u]]^2 \\ &= E[U^2] - E^2[U] \\ &= \frac{1}{3} - \frac{1}{4} = 0.0833 \end{aligned}$$

2 Central Limit Theorem

2.1 Generate 10^6 samples of the random variable

$$X = \sum_{i=1}^{12} U_i - 6 \tag{2.1}$$

using a C program, where $U_i, i = 1, 2, \ldots, 12$ are a set of independent uniform random variables between 0 and 1 and save in a file called gau.dat
Solution:

```
wget https://github.com/mohilmukundareddy/
    Assignment1/blob/main/ex2/2.1/exrand.c
wget hhttps://github.com/mohilmukundareddy/
    Assignment1/blob/main/ex2/2.1/coeffs.h
```

Running the above codes generates uni.dat and gau.dat file. Use the command

```
gcc exrand.c −lm
.\a.out
```

2.2 Load gau.dat in python and plot the empirical CDF of X using the samples in gau.dat. What properties does a CDF have?
Solution: The CDF of X is plotted in plot,Properties of the CDF:

- $F_X(x) = P(X \leq x)$
- $Q_X(x) = P(X > x)$
- $F_X(x) = 1 - Q_X(x)$ This can be used to calculate F (x).

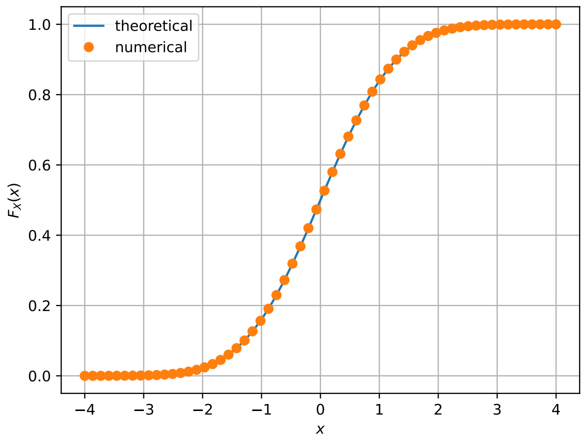

Fig. 2.2: The CDF of X

2.3 Load gau.dat in python and plot the empirical PDF of X using the samples in gau.dat. The PDF of X is defined as

$$p_X(x) = \frac{d}{dx} F_X(x) \tag{2.2}$$

What properties does the PDF have?

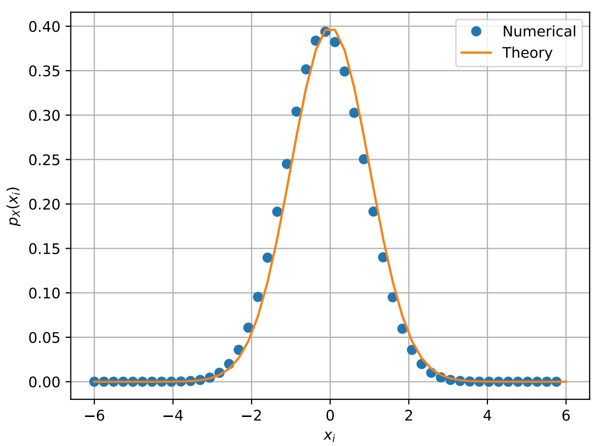

Fig. 2.3: The PDF of X

Solution: The PDF of X is plotted using the code below

```
https://github.com/mohilmukundareddy/
    Assignment1/blob/main/ex2/2.3/main.py
```

Use the below command to run the code:

```
python3 main.py
```

Properties of PDF:

- PDF is symmetric about $x \approx 0$
- graph is similar to bell shaped
- mean of graph is situated at the symmetrical point

2.4 Find the mean and variance of X by writing a C program.
Solution: Running the below code gives
Mean = 0.000326 Variance= 1.000906

```
wget https://github.com/mohilmukundareddy/
    Assignment1/blob/main/ex2/2.4/exrand.c
```

Command used:

```
gcc exrand.c −lm
./a.out
```

2.5 Given that

$$p_X(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}, -\infty < x < \infty$$

by property of probability

$$\int_{-\infty}^{\infty} p_X(x) dx = 1$$

$$F_X(x) = \int_{-\infty}^{x} p_X(x) dx$$
$$= \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$$

$$E[X] = \int_{-\infty}^{\infty} x p(x) dx$$
$$= \int_{-\infty}^{\infty} \frac{x}{\sqrt{2\pi}} e^{\frac{-x^2}{2}} dx$$

$\frac{x}{\sqrt{2\pi}} e^{\frac{-x^2}{2}}$ is an odd function so integral is zero i.e $E[X] = 0$.

$$
\begin{aligned}
E[X^2] &= \int_{-\infty}^{\infty} x^2 p(x)dx \\
&= x\int_{-\infty}^{\infty} xp(x)dx - \int_{-\infty}^{\infty}\left(\int xp(x)dx\right)dx \\
&= \left[-x\frac{1}{\sqrt{2\pi}}e^{\frac{-x^2}{2}}\right]_{-\infty}^{\infty} - \int_{-\infty}^{\infty} -\frac{1}{\sqrt{2\pi}}e^{\frac{-x^2}{2}}dx \\
&= 0 - (-1)
\end{aligned}
$$

we know that

$$\int_{-\infty}^{\infty} e^{-\frac{x^2}{2}} = \sqrt{2\pi}$$

by series expansion $\frac{x}{e^{\frac{x^2}{2}}} = \frac{x}{1+\frac{x^2}{2}+\frac{x^4}{8}+\ldots}$
putting $x = \infty$,we get $\frac{1}{\infty} = 0$
Similarly when $x = -\infty$ we get 0
$var(x) = E[x^2] - E[x] = 1 - 0 = 1$

=

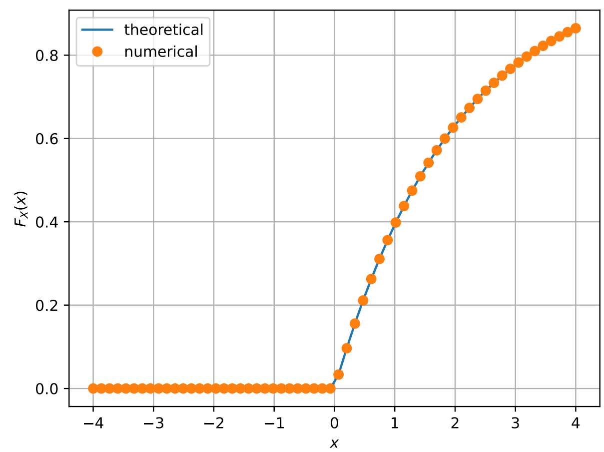

Fig. 3.1: CDF for (3)

https://github.com/mohilmukundareddy/Assignment1/blob/main/ex3/cdf.py

Use the below command to run the code:

```
python3 cdf.py
```

3 From Uniform to Other

3.1 Generate samples of

$$V = -2\ln(1-U) \tag{3.1}$$

and plot its CDF.
Solution:
Running the below code generates samples of V from file uni.dat(U).

https://github.com/mohilmukundareddy/Assignment1/blob/main/ex3/main.py

Use the below command in the terminal to run the code:

```
python3 main.py
```

Now these samples are used to plot by running the below code

3.2 Theorotical expression for $F_V(x)$

$$
\begin{aligned}
F_V(x) &= P\{V \le x\} \\
&= P\{-2 \times \ln(1-U) \le x\} \\
&= P\{U \le 1 - e^{(-\frac{x}{2})}\} \\
&= F_U\{1 - e^{(-\frac{x}{2})}\} \\
&= \begin{cases} 1 - e^{(-\frac{x}{2})} & 0 \le x < \infty \\ 0 & x < 0 \end{cases}
\end{aligned}
$$

4 Triangular Distribution

4.1 Generate

$$T = U_1 + U_2 \tag{4.1}$$

we get the code

https://github.com/mohilmukundareddy/Assignment1/blob/main/ex4/1/main.py

run the command

```
python3 main.py
```

4.2 Find the CDF of T. we have code

https://github.com/mohilmukundareddy/Assignment1/blob/main/ex4/2/main1.py

run the command

```
python3 main1.py
```

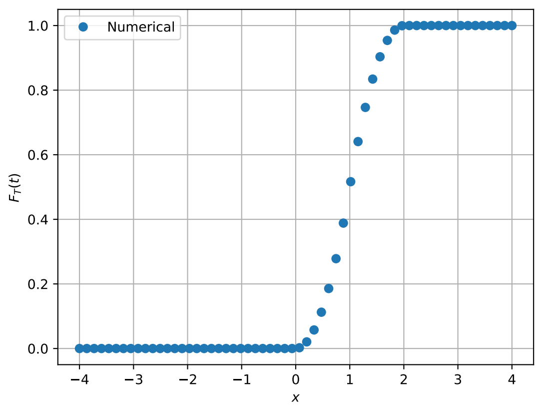

Fig. 4.2: numerical cdf

4.3 Find the PDF of T we have code

```
https://github.com/mohilmukundareddy/
    Assignment1/blob/main/ex4/3/main2.py
```

run the command

```
python3 main2.py
```

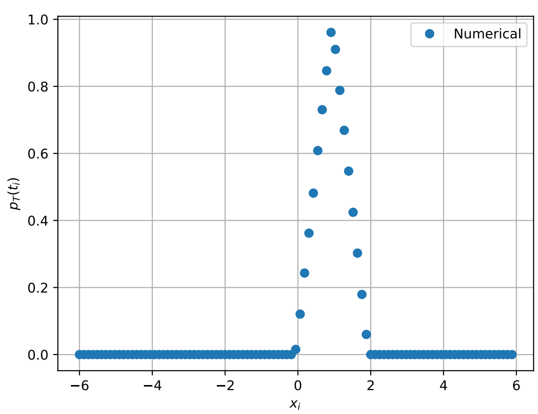

Fig. 4.3: Numerical cdf

4.4 Find the theoretical expressions for the PDF and CDF of T. **Solution:**

$$\begin{aligned} F_T(t) &= P\{T \leq t\} \\ &= P\{U1 + U2 \leq t\} \end{aligned}$$

let us take two cases if $0 \leq t \leq 1$ and $1 < t \leq 2$

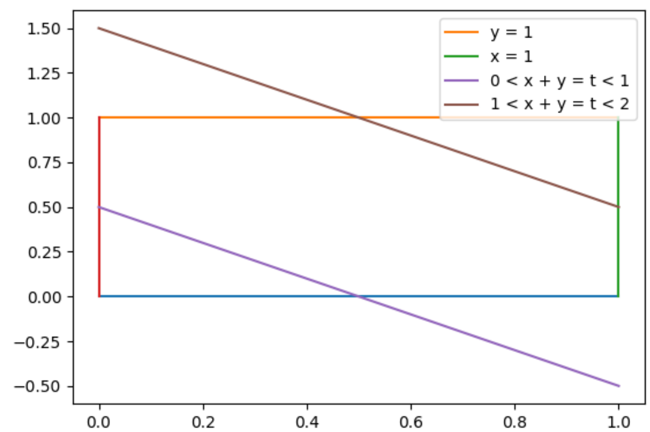

Fig. 4.4: def plot

The above graph is produced by

```
https://github.com/mohilmukundareddy/
    Assignment1/blob/main/ex4/2/find.py
```

Run the code in terminal

```
python3 find.py
```

from the figures it is evident that $P(U1 + U2 < t, 0 \leq t < 1) = \frac{t^2}{2}$ $P(U1 + U2 < t, 1 \leq t \leq 2) = 1 - \frac{(2-t)^2}{2}$

$$F_T(t) = \begin{cases} 0 & t < 0 \\ \frac{t^2}{2} & 0 \leq t \leq 1 \\ 1 - \frac{(2-t)^2}{2} & 1 < t \leq 2 \\ 1 & t > 2 \end{cases}$$

$$P_T(t) = \frac{d(F_T(t))}{dt}$$

$$\therefore P_T(t) = \begin{cases} 0 & t < 0 \\ t & 0 \leq t \leq 1 \\ 2 - t & 0 < t \leq 2 \\ 0 & t > 2 \end{cases}$$

4.5 Verify your results through a plot Take the code for cdf

```
https://github.com/mohilmukundareddy/
    Assignment1/blob/main/ex4/5/main1.py
```

Run in terminal

```
python3 main1.py
```

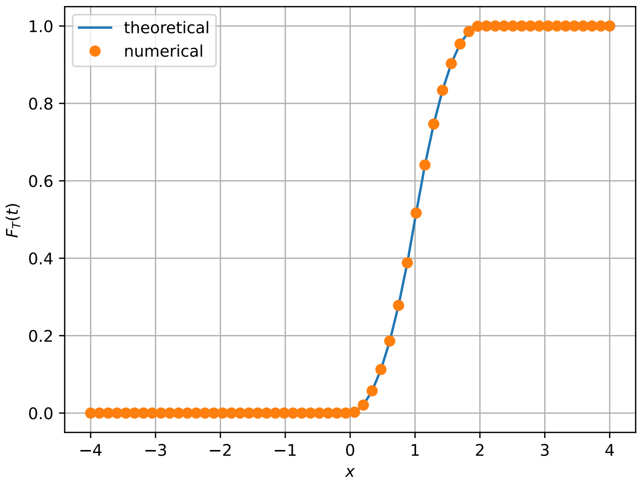

Fig. 4.5: t-cdf

Take the code for pdf

```
https://github.com/mohilmukundareddy/
    Assignment1/blob/main/ex4/5/main2.py
```

Run in terminal

```
python3 main2.py
```

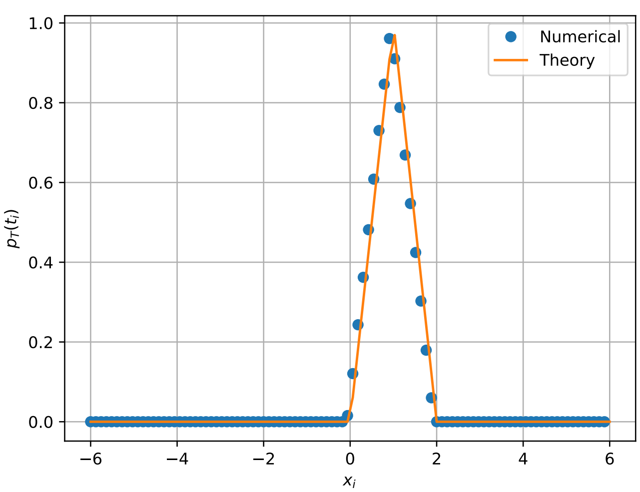

Fig. 4.5: t-pdf

5 Maximul Likelihood

5.1 Generate

$$Y = AX + N, \tag{5.1}$$

where $A = 5$ dB, $X 1 \{1, -1\}$ **Solution:** use bernouli function from exrand.c find the code

```
https://github.com/mohilmukundareddy/
    Assignment1/blob/main/ex5/1/exrand.c
```

run the terminal command

```
gcc exrand.c −lm
./a.out
```

5.2 Generate

$$Y = AX + N, \tag{5.2}$$

where $A = 5$ dB, and $N \sim N(0, 1)$. find the code

```
https://github.com/mohilmukundareddy/
    Assignment1/blob/main/ex5/2/main.py
```

run the command

```
python3 main.py
```

5.3 Plot Y using a scatter plot. find the scatter plot

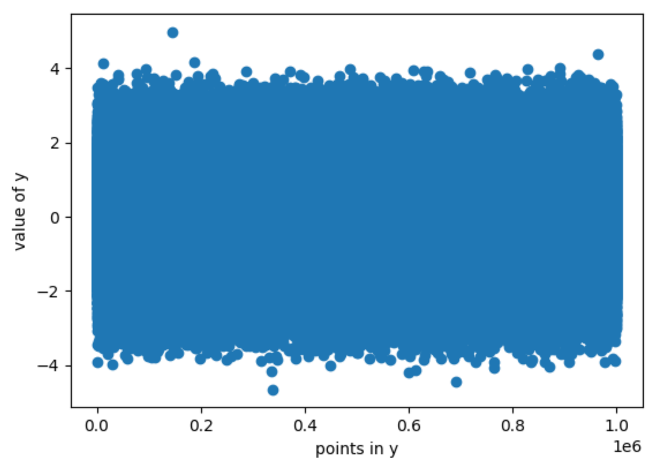

Fig. 5.3: scatter plot

5.4 Guess how to estimate X from Y. **Solution:** To estimate X from Y, consider function:

$$\operatorname{sgn}(y) = \begin{cases} -1, & y \in (-\infty, 0] \\ 1, & y \in (0, \infty) \end{cases} \tag{5.3}$$

Using $\operatorname{sgn} y$, we can operate on Y to find corresponding values of X.

5.5 Find

$$P_{e|0} = \Pr\left(\hat{X} = -1 | X = 1\right) \tag{5.4}$$

and

$$P_{e|1} = \Pr\left(\hat{X} = 1 | X = -1\right) \tag{5.5}$$

find the code below

```
https://github.com/mohilmukundareddy/
    Assignment1/tree/main/ex5/4/main.py
```

run the code

```
python3 main.py
```

we get the values as $P_{(e|0)} = 0.310007$
$P_{(e|1)} = 0.310142$

5.6 Find P_e assuming that X has equiprobable symbols.
Solution: Assume a general value of A.
Our estimation function predicts the data above the x axis correspond to $X = 1$, and the data points below the x-axis correspond to $X = -1$. We have:

$$\begin{aligned} P_{e|0} &= \Pr\left(\hat{X} = -1|X = 1\right) \\ &= \Pr\left(AX + N < 0|X = 1\right) \\ &= \Pr\left(N < -A\right) \\ &= \int_{-\infty}^{-A} \frac{1}{\sqrt{2\pi}} e^{\frac{-x^2}{2}} dx \\ &= \int_{A}^{\infty} \frac{1}{\sqrt{2\pi}} e^{\frac{-x^2}{2}} dx \\ &= Q_N(A) \end{aligned}$$

where Q_N is the Qfunction of the normal distribution. Similarly,

$$\begin{aligned} P_{e|1} &= \Pr\left(\hat{X} = 1|X = -1\right) \\ &= \Pr\left(AX + N > 0|X = -1\right) \\ &= \Pr\left(N > A\right) \\ &= \int_{A}^{\infty} \frac{1}{\sqrt{2\pi}} e^{\frac{-x^2}{2}} dx \\ &= Q_N(A) \end{aligned}$$

Given X is equiprobable so we have

$$\begin{aligned} P_e &= P_{e|0} \Pr\left(X = 1\right) + P_{e|1} \Pr\left(X = -1\right) \\ &= \frac{1}{2} P_{e|0} + \frac{1}{2} P_{e|1} \\ &= \frac{1}{2} Q_N(A) + \frac{1}{2} Q_N(A) \\ &= Q_N(A) \end{aligned}$$

5.7 Verify by plotting the theoretical P_e with respect to A from 0 to 10 dB. find the code below

```
https://github.com/mohilmukundareddy/
    Assignment1/blob/main/ex5/7/main.py
```

run the command

```
python3 main.py
```

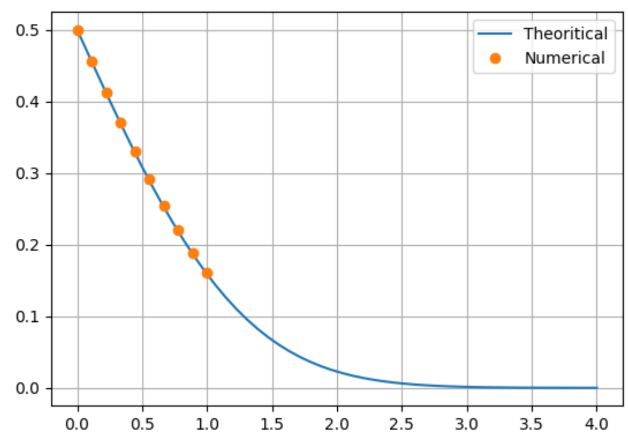

Fig. 5.7: verification of p_e

5.8 Now, consider a threshold δ while estimating X from Y. Find the value of δ that maximizes the theoretical P_e.
Solution: To estimate X from Y, we consider

$$X = \begin{cases} 1, & Y > \delta \\ -1, & Y < \delta \end{cases}$$

so we have

$$\begin{aligned} P_{e|0} &= \Pr\left(\hat{X} = -1|X = 1\right) \\ &= \Pr\left(AX + N < \delta|X = 1\right) \\ &= \Pr\left(N < \delta - A\right) \\ &= \int_{-\infty}^{\delta-A} \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} dx \\ &= \int_{A-\delta}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} dx \\ &= Q_N(A - \delta) \end{aligned}$$

$$\begin{aligned}
P_{e|1} &= \Pr\left(\hat{X} = 1|X = -1\right) \\
&= \Pr\left(AX + N > \delta|X = -1\right) \\
&= \Pr\left(N > \delta + A\right) \\
&= \int_{A-\delta}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} dx \\
&= Q_N(A + \delta) \\
P_e &= P_{e|0} \Pr\left(X = 1\right) + P_{e|1} \Pr\left(X = -1\right) \\
&= \frac{1}{2}(Q_N(A - \delta) + Q_N(A + \delta))
\end{aligned}$$

To minimise P_e,use differentiate wrt δ:

$$\frac{d}{d\delta}\left(\frac{1}{2}(Q_N(A - \delta) + Q_N(A + \delta))\right) = 0$$

$$\frac{1}{2}(\frac{1}{\sqrt{2\pi}} e^{-\frac{(\delta-A)^2}{2}} - \frac{1}{\sqrt{2\pi}} e^{-\frac{(A+\delta)^2}{2}}) = 0$$

$$\begin{aligned}
(\delta - A)^2 &= (\delta + A)^2 \\
\delta &= 0
\end{aligned}$$

so $\delta = 0$ maximize P_e

5.9 Repeat the above exercise when $p_X(0) = p$

Solution: we have:

$$\begin{aligned}
P_e &= P_{e|0} p + P_{e|1}(1 - p) \\
&= pQ_N(A - \delta) + (1 - p)Q_N(A + \delta)
\end{aligned}$$

Differentiating wrt δ we get

$$p\frac{1}{\sqrt{2\pi}} e^{-\frac{(\delta-A)^2}{2}} - (1 - p)\frac{1}{\sqrt{2\pi}} e^{-\frac{(A+\delta)^2}{2}} = 0$$

Taking ln on both sides and find δ :

$$\ln p - \frac{(\delta - A)^2}{2} = \ln\left(1 - p\right) - \frac{(\delta + A)^2}{2}$$

$$\delta = \frac{1}{2A} \ln \frac{1 - p}{p}$$

if $p = \frac{1}{2}$ then $\delta = 0$ which verifies with above result.

5.10 Repeat the above exercise using the MAP criterion.

Solution: Assume that $\Pr\left(X = -1\right) = p$, and $\Pr\left(X = 1\right) = (1 - p)$. Then, using the Law of Total Probability, we have:

$$\begin{aligned}
p_Y(y) &= p_{Y|X=-1}(y| - 1) \Pr\left(X = -1\right) \\
&\quad + p_{Y|X=1}(y|1) \Pr\left(X = 1\right) \\
&= p \times p_{(N-A)}(y) \\
&\quad + (1 - p) \times p_{(N+A)}(y)
\end{aligned}$$

where $p_Y(y)$ is the pdf of Y. Now, $p_{(N-A)}$ is the pdf of a shifted normal distribution,so

$$p_Y(y) = p\frac{e^{-\frac{(y+A)^2}{2}}}{\sqrt{2\pi}} + (1 - p)\frac{e^{-\frac{(y-A)^2}{2}}}{\sqrt{2\pi}}$$

MAP criterion,find $p_{X|Y}(x|y)$. we use the Theorem of Conditional Probability:

$$p_{X|Y}(x|y) = \frac{p_{Y|X}(y|x) \times p_X(x)}{p_Y(y)}$$

When $X = 1$, we have:

$$\begin{aligned}
p_{X|Y}(1|y) &= \frac{p_{Y|X}(y|1) \times p_X(1)}{p_Y(y)} \\
&= \frac{(1 - p)\frac{e^{-\frac{(y-A)^2}{2}}}{\sqrt{2\pi}}}{p\frac{e^{-\frac{(y+A)^2}{2}}}{\sqrt{2\pi}} + (1 - p)\frac{e^{-\frac{(y-A)^2}{2}}}{\sqrt{2\pi}}} \\
&= \frac{(1 - p)\, e^{2yA}}{p + (1 - p)\, e^{2yA}}
\end{aligned}$$

Similarly, when $X = -1$, we have:

$$\begin{aligned}
p_{X|Y}(-1|y) &= \frac{p_{Y|X}(y| - 1) \times p_X(-1)}{p_Y(y)} \\
&= \frac{(p)\frac{e^{-\frac{(y+A)^2}{2}}}{\sqrt{2\pi}}}{p\frac{e^{-\frac{(y+A)^2}{2}}}{\sqrt{2\pi}} + (1 - p)\frac{e^{-\frac{(y-A)^2}{2}}}{\sqrt{2\pi}}} \\
&= \frac{p}{p + (1 - p)\, e^{2yA}}
\end{aligned}$$

Therefore, when $p_{X|Y}(1|y) > p_{X|Y}(-1|y)$, we have:

$$\begin{aligned}
\frac{(1 - p)\, e^{2yA}}{p + (1 - p)\, e^{2yA}} &> \frac{p}{p + (1 - p)\, e^{2yA}} \\
e^{2yA} &> \frac{p}{(1 - p)} \\
y &> \frac{1}{2A} \ln \frac{p}{(1 - p)}
\end{aligned}$$

Therefore, when Eq. (**??**), we can assert that $X = 1$, and $X = -1$ otherwise. Now, consider when $p = \frac{1}{2}$. We have:

$$\begin{aligned} y &> \frac{1}{2A} \ln \frac{p}{(1-p)} \\ &= \frac{1}{2A} \ln 1 \\ &= 0 \end{aligned}$$

Therefore, when $y > 0$, we choose $X = 1$, and we choose $X = -1$ otherwise.

6 Gaussian to Other

6.1 Let $X_1 \sim N(0,1)$ and $X_2 \sim N(0,1)$. Plot the CDF and PDF of

$$V = X_1^2 + X_2^2 \tag{6.1}$$

Solution: The sum of squares of n independent standard random normal variables is χ^2 distribution with n degrees of freedom.

$$P_{\chi^2}(x|n) = \frac{x^{\frac{n}{2}-1} e^{\frac{-x}{2}}}{2^{\frac{n}{2}} \Gamma(\frac{n}{2})}, \forall x \geq 0$$

Here k=2,

$$P_{\chi^2}(x|2) = P_V(v) = \frac{e^{\frac{-v}{2}}}{2}$$

For the cummulative distribution

$$\begin{aligned} F_V(v) &= \int_0^v \frac{e^{\frac{-v}{2}}}{2} dv \\ &= 1 - e^{\frac{-v}{2}} \end{aligned}$$

To generate data for V , run the following code,

```
https://github.com/mohilmukundareddy/
    Assignment1/blob/main/ex6/1/main.py
```

Run the below command in terminal,

```
python3 main.py
```

The PDF plot of the $\chi^2(2)$ can be obtained by running the code below,

```
https://github.com/mohilmukundareddy/
    Assignment1/blob/main/ex6/1/main1.py
```

Use the following command in the terminal to run the code

```
python3 main1.py
```

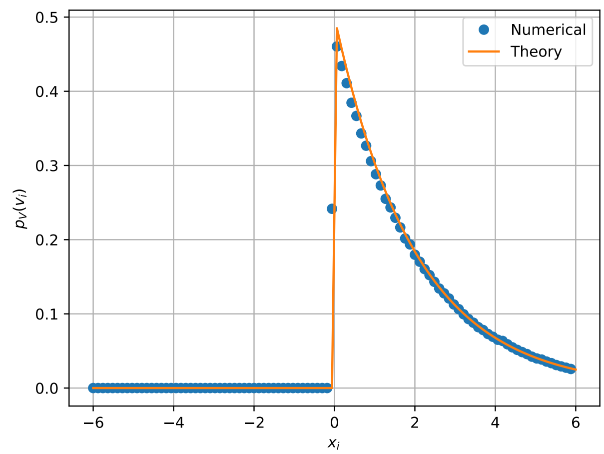

Fig. 6.1: PDF plot

The CDF plot of the $\chi^2(2)$ can be obtained by running the code below,

```
https://github.com/mohilmukundareddy/
    Assignment1/blob/main/ex6/1/main2.py
```

Use the following command in the terminal to run the code

```
python3 main2.py
```

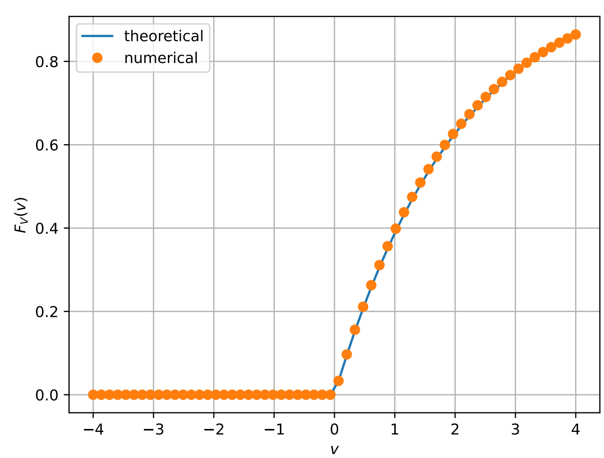

Fig. 6.1: CDF plot

6.2 If

$$F_V(x) = \begin{cases} 1 - e^{-\alpha x} & x \geq 0 \\ 0 & x < 0, \end{cases}$$

find α.

Solution: We will assume that X_1 and X_2 are i.i.d. Let

$$X_1 = r\cos\theta$$
$$X_2 = r\sin\theta$$

The Jacobian Matrix is then defined as:

$$J(r,\theta) = \begin{pmatrix} \frac{\delta x_1}{\delta r} & \frac{\delta x_1}{\delta\theta} \\ \frac{\delta x_2}{\delta r} & \frac{\delta x_2}{\delta\theta} \end{pmatrix}$$
$$J = \begin{pmatrix} \frac{\delta r\cos\theta}{\delta r} & \frac{\delta r\cos\theta}{\delta\theta} \\ \frac{\delta r\sin\theta}{\delta r} & \frac{\delta r\sin\theta}{\delta\theta} \end{pmatrix}$$
$$J = \begin{pmatrix} \cos\theta & -R\sin\theta \\ \sin\theta & R\cos\theta \end{pmatrix}$$
$$|J(r,\theta)| = R$$

Then as X_1,X_2 are independent we have,

$$p_{X_1,X_2}(x_1,x_2) = p_{X_1}(x_1)p_{X_2}(x_2)$$
$$= \frac{1}{2\pi}e^{\frac{-(x_1^2+x_2^2)}{2}}$$
$$= \frac{1}{2\pi}e^{\frac{-r^2}{2}}$$

Now, since

$$p_{r,\theta}(r,\theta) = |J(r,\theta)|p_{X_1,X_2}(x_1,x_2)$$

we have:

$$p_{R,\theta}(r,\theta) = \frac{r}{2\pi}e^{-\frac{r^2}{2}}$$
$$p_R(r) = \int_0^{2\pi} p_{R,\theta}(r,\theta)$$
$$= \int_0^{2\pi} \frac{r}{2\pi}e^{-\frac{r^2}{2}}d\theta$$
$$= re^{-\frac{r^2}{2}}$$
$$F_R(r) = \Pr(R \le r)$$
$$= \int_0^r f_R(r)dr$$
$$= 1 - e^{-\frac{r^2}{2}}$$

$F_V(x)$ is given by:

$$F_V(x) = F_{X_1^2+X_2^2}(x)$$
$$= F_{R^2}x$$
$$= \Pr\left(R^2 \le x\right)$$
$$= \Pr\left(R \le \sqrt{x}\right)$$

Therefore,

$$F_V(x) = \begin{cases} 0, & x < 0 \\ 1 - e^{-\frac{x}{2}}, & x \ge 0 \end{cases}$$

by Comparing we get $\alpha = \frac{1}{2}$

6.3 Plot the CDF and PDf of

$$A = \sqrt{V}$$

Solution: To generate data for A , run the following code,

```
https://github.com/mohilmukundareddy/
    Assignment1/blob/main/ex6/3/main.py
```

Run the below command in terminal,

```
python3 main.py
```

The PDF plot of A can be obtained by running the code below,

```
https://github.com/mohilmukundareddy/
    Assignment1/blob/main/ex6/3/main1.py
```

Use the following command in the terminal to run the code

```
python3 main1.py
```

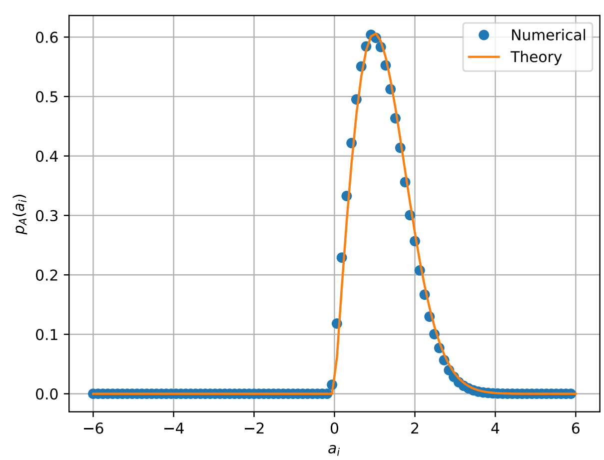

Fig. 6.3: PDF

The CDF plot of the A can be obtained by running the code below,

```
https://github.com/mohilmukundareddy/
    Assignment1/blob/main/ex6/3/main2.py
```

Use the following command in the terminal to run the code

```
python3 main2.py
```

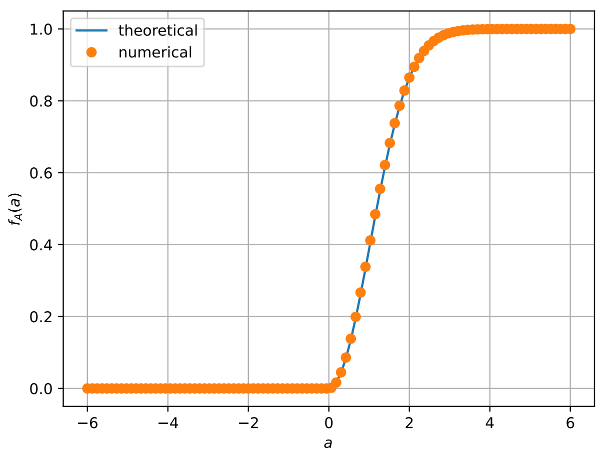

Fig. 6.3: CDF

7 Conditional Probability

7.1

7.2 Plot

$$P_e = \Pr\left(\hat{X} = -1|X = 1\right) \tag{7.1}$$

for

$$Y = AX + N, \tag{7.2}$$

where A is Raleigh with $E\left[A^2\right] = \gamma, N \sim 01, X \in (-1, 1)$ for $0 \le \gamma \le 10$ dB.

7.3 Assuming that N is a constant, find an expression for P_e. Call this $P_e(N)$

7.4 For a function g,

$$E\left[g(X)\right] = \int_{-\infty}^{\infty} g(x)p_X(x)\,dx \tag{7.3}$$

Find $P_e = E\left[P_e(N)\right]$.

7.5 Plot P_e in problems 7.2 and 7.4 on the same graph w.r.t γ. Comment.

8 Two Dimensions

Let

$$\mathbf{y} = A\mathbf{x} + \mathbf{n}, \tag{8.1}$$

where

$$x \in (\mathbf{s}_0, \mathbf{s}_1), \mathbf{s}_0 = \begin{pmatrix}1\\0\end{pmatrix}, \mathbf{s}_1 = \begin{pmatrix}0\\1\end{pmatrix} \tag{8.2}$$

$$\mathbf{n} = \begin{pmatrix}n_1\\n_2\end{pmatrix}, n_1, n_2 \sim 01. \tag{8.3}$$

8.1 Plot

$$\mathbf{y}|\mathbf{s}_0 \text{ and } \mathbf{y}|\mathbf{s}_1 \tag{8.4}$$

on the same graph using a scatter plot.

8.2 For the above problem, find a decision rule for detecting the symbols $\mathbf{s}_0$ and $\mathbf{s}_1$.

8.3 Plot

$$P_e = \Pr\left(\hat{\mathbf{x}} = \mathbf{s}_1|\mathbf{x} = \mathbf{s}_0\right) \tag{8.5}$$

with respect to the SNR from 0 to 10 dB.

8.4 Obtain an expression for P_e. Verify this by comparing the theory and simulation plots on the same graph.