# Assignment 1

## EE3900: Linear Systems and Signal Processing

Indian Institute of Technology Hyderabad

Mukunda Reddy
AI21BTECH11021

Oppenheim and Schafer

**Problem 3.7.(b)** The input to a causal LTI system is

$$x[n] = u[-n-1] + (\frac{1}{2})^n u[n]$$

The z-transform of the output of this system is

$$Y(z) = \frac{-\frac{1}{2}z^{-1}}{(1-\frac{1}{2}z^{-1})(1+z^{-1})}$$

Find the ROC for Y(z)?
**Solution:** Taking the z transform on both sides of

$$x[n] = u[-n-1] + (\frac{1}{2})^n u[n]$$

we get

$$X(z) = \frac{-1}{1-z^{-1}} + \frac{1}{1-\frac{1}{2}z^{-1}}$$

for the the series X(z) to converge $\frac{1}{2} < |z| < 1$
we know

$$\begin{aligned} H(z) &= \frac{Y(z)}{X(z)} \\ &= \frac{-\frac{1}{2}z^{-1}}{(1-\frac{1}{2}z^{-1})(1+z^{-1})} * \frac{(1-z^{-1})(1-\frac{1}{2}z^{-1})}{-\frac{1}{2}z^{-1}} \\ &= \frac{1-z^{-1}}{1+z^{-1}} \end{aligned}$$

Given H(z) casual $\implies$ ROC $|z| > 1$
The ROC of Y(z) has three possibilities

1) $|z| < \frac{1}{2}$
2) $\frac{1}{2} < |z| < 1$
3) $|z| > 1$

the region in the z-plane that satisfies the constraints imposed by H(z) is $|z| > 1$ as poles of Y(z) also satisfies this,effectively the ROC of Y(z) is given by $|z| > 1$