Digital Signal Processing
EE3900

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AI21BTECH11021

November 2, 2022

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1. Definitions

1. The unit step function is

$$u(t) = \begin{cases} 1 & t > 0 \\ \frac{1}{2} & t = 0 \\ 0 & t < 0 \end{cases} \tag{1.1}$$

2. The Laplace transform of $g(t)$ is defined as

$$G(s) = \int_{-\infty}^{\infty} g(t)e^{-st}\, dt \tag{1.2}$$

2. Laplace Transform

1. In the circuit, the switch S is connected to position P for a long time so that the charge on the capacitor becomes $q_1\, \mu C$. Then S is switched to position Q. After a long time, the charge on the capacitor is $q_2\, \mu C$.

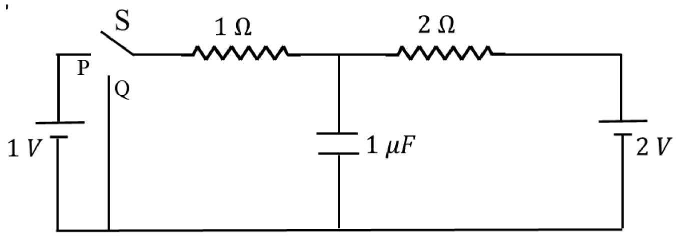

Fig. 2.1.

2. Draw the circuit using latex-tikz.
Solution: The following code yields Fig.2.2

```
wget https://github.com/mukundareddy/Signal
    −Processing/blob/main/cktsig/Tikz%20
    Circuits/2.2.tex
```

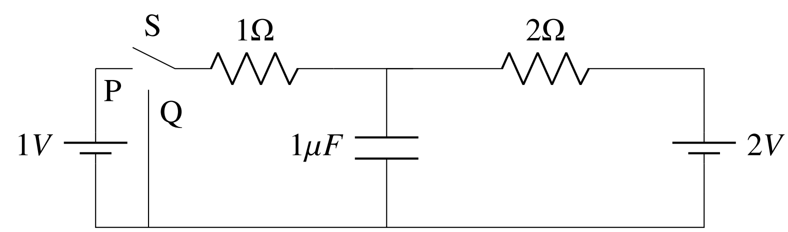

Fig. 2.2. Given Circuit

3. Find q_1.
Solution: Before switching S to Q: At steady state, which achieved when switch S is at P for long time capacoitor behaves as an open switch, hence current through capacitor is 0, Let i be the current flowing in the circuit at steady state. Applying KVL ,

$$1 - i - 2i - 2 = 0 \tag{2.1}$$

$$3i = -1 \Rightarrow i = \frac{-1}{3}A \tag{2.2}$$

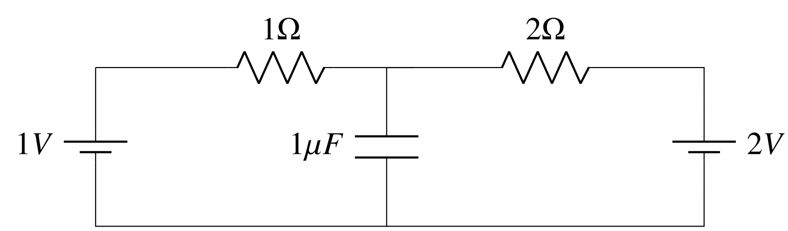

Fig. 2.3. Before switching S to Q

Potential Difference across the capacitor at steady state is

$$1 - \left(\frac{-1}{3}\right) = \frac{4}{3}V \tag{2.3}$$

$$q_1 = \frac{4}{3} \cdot 1 \tag{2.4}$$

$$= \frac{4}{3}\mu C \tag{2.5}$$

4. Show that the Laplace transform of $u(t)$ is $\frac{1}{s}$ and find the ROC.
Solution: We know that Laplace Transform fo function $f(t)$ is given as $F(s)$,

$$F(s) = \int_0^\infty f(t)e^{-st}\,dt \tag{2.6}$$

$$\tag{2.7}$$

For $u(t)$, we have,

$$F(s) = \int_0^\infty u(t)e^{-st}\,dt \tag{2.8}$$

Using (1.1),

$$F(s) = \int_0^\infty u(t)e^{-st}\,dt \tag{2.9}$$

$$= \int_0^\infty e^{-st}\,dt \tag{2.10}$$

$$= -\left(0 - \frac{1}{s}\right) \tag{2.11}$$

$$= \frac{1}{s} \tag{2.12}$$

ROC is $Re(s) > 0$ since for $s > 0$, $e^{-st} < \infty$ for $t \to \infty$

5. Show that

$$e^{-at}u(t) \xleftrightarrow{\mathcal{L}} \frac{1}{s+a}, \quad a > 0 \tag{2.13}$$

and find the ROC.
Solution: From 2.6,

$$F(s) = \int_0^\infty u(t)e^{-at}e^{-st}\,dt \tag{2.14}$$

$$= \int_0^\infty u(t)e^{-(s+a)t}\,dt \tag{2.15}$$

$$= \int_0^\infty e^{-(s+a)t}\,dt \tag{2.16}$$

$$= -\left(0 - \frac{1}{s+a}\right) \tag{2.17}$$

$$= \frac{1}{s+a} \tag{2.18}$$

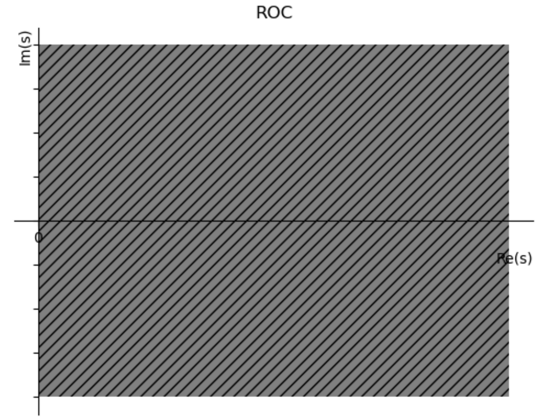

Fig. 2.4.

ROC is

$$Re(s) + a > 0 \Rightarrow Re(s) > -a \tag{2.19}$$

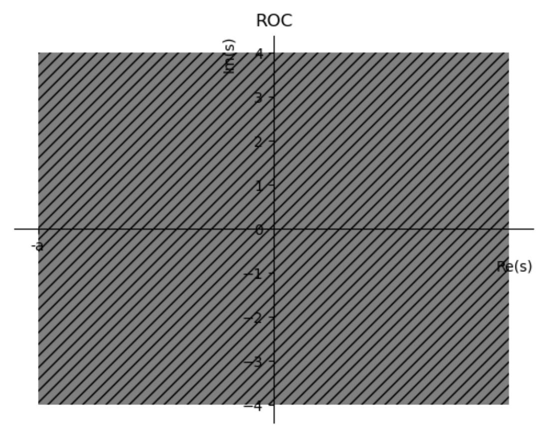

Fig. 2.5.

6. Now consider the following resistive circuit transformed from Fig. 2.1 where

$$u(t) \xleftrightarrow{\mathcal{L}} V_1(s) \tag{2.20}$$

$$2u(t) \xleftrightarrow{\mathcal{L}} V_2(s) \tag{2.21}$$

Find the voltage across the capacitor $V_{C_0}(s)$.
Solution:

$$R_{eff} = \frac{1}{1+\frac{1}{2}} = \frac{2}{3}\Omega \tag{2.22}$$

$$V_{eff} = \frac{1}{1+\frac{1}{2}} = \frac{2}{3}V \tag{2.23}$$

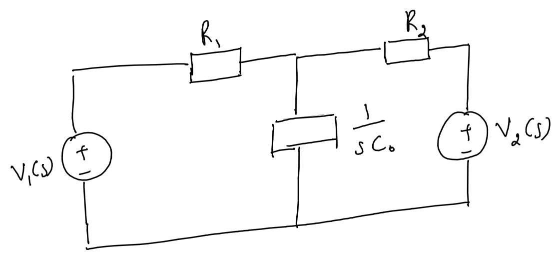

Fig. 2.6.

$$V_{C_0}(s) = V_S(s)\frac{C_0}{C_0 + R_{eff}} \tag{2.24}$$
$$= \left(\frac{4}{3s}\right)\left(\frac{\frac{1}{s}}{\frac{1}{s}+\frac{2}{3}}\right) \tag{2.25}$$
$$= \frac{3+4s}{3s\left(s+\frac{3}{2}\right)} \tag{2.26}$$

7. Find $v_{C_0}(t)$. Plot using python.
Solution: Running the following code gives the plot.

```
wget https://github.com/mukundareddy/Signal
    -Processing/tree/main/cktsig/codes/2.7.py
```

Using (2.26),

$$\frac{3+4s}{3s\left(s+\frac{3}{2}\right)} = \frac{2}{3s} + \frac{2}{3(\frac{3}{2}+s)} \tag{2.27}$$

Apply inverse Laplacian Transform,

$$V_{C_0}(s) \xleftrightarrow{\mathcal{L}^{-\infty}} V_{C_0}(t) \tag{2.28}$$
$$\mathcal{L}^{-1}\left[V_{C_0}(s)\right] = \mathcal{L}^{-1}\left[\frac{2}{3s} + \frac{2}{3(\frac{3}{2}+s)}\right] \tag{2.29}$$
$$= \mathcal{L}^{-1}\left[\frac{2}{3s}\right] - \frac{2}{3}\mathcal{L}^{-1}\left[\frac{1}{\frac{3}{2}+s}\right] \tag{2.30}$$

Since,

$$\mathcal{L}^{-1}\left[\frac{1}{s}\right] = u(t) \tag{2.31}$$
$$\mathcal{L}^{-1}\left[\frac{1}{s-a}\right] = e^{at}u(t) \tag{2.32}$$

Using the above equations,

$$V_{C_0}(t) = \frac{2}{3}\left(1 + e^{\frac{-3}{2}t}\right)u(t) \tag{2.33}$$

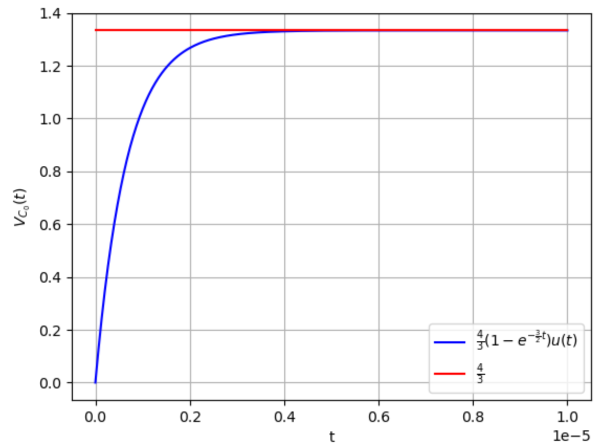

Fig. 2.7. Plot of $V_{C_0}(t)$

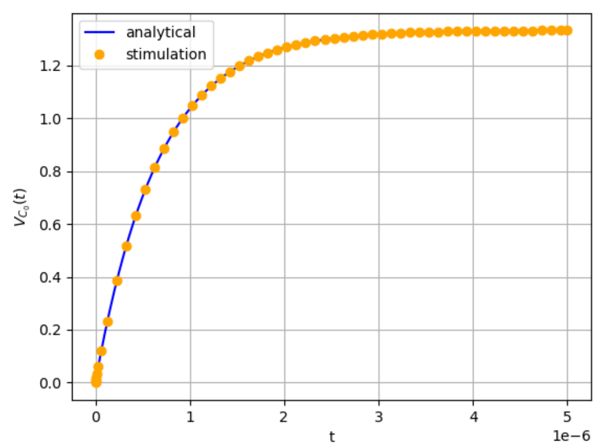

Fig. 2.8.

8. Verify your result using ngspice.
Solution:
9. Obtain Fig. 2.7 using the equivalent differential equation
Solution: Results obtained can be verified by running the following code.

```
wget https://github.com/mukundareddy/Signal
    -Processing/tree/main/cktsig/codes/2.8.cir
```

And is plotted using the below code.

```
wget https://github.com/mukundareddy/Signal
    -Processing/tree/main/cktsig/codes/2.8.py
```

Using Kirchoff's junction law

$$\frac{v_c(t) - v_1(t)}{R_1} + \frac{v_c(t) - v_2(t)}{R_2} + \frac{\mathrm{d}q}{\mathrm{d}t} = 0 \quad (2.34)$$

where $q(t)$ is the charge on the capacitor
On taking the Laplace transform on both sides of this equation

$$\frac{V_c(s) - V_1(s)}{R_1} + \frac{V_c(s) - V_2(s)}{R_2} + \left(sQ(s) - q(0^-)\right) = 0 \quad (2.35)$$

But $q(0^-) = 0$ and

$$q(t) = C_0 v_c(t) \quad (2.36)$$
$$\implies Q(s) = C_0 V_c(s) \quad (2.37)$$

Thus

$$\frac{V_c(s) - V_1(s)}{R_1} + \frac{V_c(s) - V_2(s)}{R_2} + sC_0 V_c(s) = 0 \quad (2.38)$$
$$\implies \frac{V_c(s) - V_1(s)}{R_1} + \frac{V_c(s) - V_2(s)}{R_2} + \frac{V_c(s) - 0}{\frac{1}{sC_0}} = 0 \quad (2.39)$$

which is the same equation as the one we obtained from Fig. 2.7

3. Initial Conditions

1. Find q_2 in Fig. 2.1.
Solution: At steady state capacitor behaves as an open switch. Hence $V_{C_0} = V_{1\Omega}$.
Let i be the current in the circuit. Using KVL,

$$2 - 2i - i = 0 \implies i = \frac{2}{3} \quad (3.1)$$
$$V_{1\Omega} = i \times 1 = \frac{2}{3}V \quad (3.2)$$
$$V_{C_0} = \frac{q_2}{C_0} = V_{1\Omega} = \frac{2}{3} \quad (3.3)$$
$$\implies q_2 = \frac{2}{3}\mu C \quad (3.4)$$

2. Draw the equivalent s-domain resistive circuit when S is switched to position Q. Use variables R_1, R_2, C_0 for the passive elements. Use latex-tikz.
Solution:

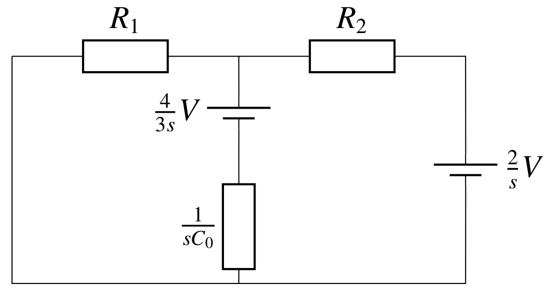

Fig. 3.1. After switching S to Q

3. $V_{C_0}(s) = ?$
Solution: Let voltage across capacitor be V. Using KCL at node in Fig. 3.1

$$\frac{V - 0}{R_1} + \frac{V - \frac{2}{s}}{R_2} + sC_0\left(V - \frac{4}{3s}\right) = 0 \quad (3.5)$$
$$\implies V_{C_0}(s) = \frac{\frac{2}{sR_2} + \frac{4C_0}{3}}{\frac{1}{R_1} + \frac{2}{R_2} + sC_0} \quad (3.6)$$

4. $v_{C_0}(t) = ?$ Plot using python.
Solution: Running the following code gives the plot.

```
wget https://github.com/mukundareddy/Signal
    −Processing/tree/main/cktsig/codes/3.4.py
```

From (3.6),

$$V_{C_0}(s) = \frac{4}{3}\left(\frac{1}{\frac{1}{C_0}\left(\frac{1}{R_1} + \frac{1}{R_2}\right) + s}\right) + \frac{2}{R_2\left(\frac{1}{R_1} + \frac{1}{R_2}\right)}\left(\frac{1}{s} - \frac{1}{\frac{1}{C_0}\left(\frac{1}{R_1} + \frac{1}{R_2}\right) + s}\right) \quad (3.7)$$

Taking an inverse Laplace Transform,

$$v_{C_0}(t) = \frac{4}{3}e^{-\left(\frac{1}{R_1} + \frac{1}{R_2}\right)\frac{t}{C_0}}u(t) + \frac{2}{R_2\left(\frac{1}{R_1} + \frac{1}{R_2}\right)}\left(1 - e^{-\left(\frac{1}{R_1} + \frac{1}{R_2}\right)\frac{t}{C_0}}\right)u(t) \quad (3.8)$$

Substituting values gives

$$v_{C_0}(t) = \frac{2}{3}\left(1 + e^{-\left(1.5\times10^6\right)t}\right)u(t) \quad (3.9)$$

5. Verify your result using ngspice.
Solution: Results obtained can be verified by running the following code.

```
wget https://github.com/mukundareddy/Signal
    −Processing/tree/main/cktsig/codes/3.5.cir
```

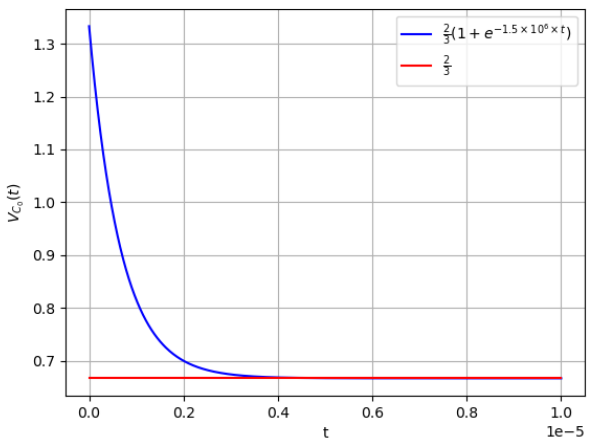

Fig. 3.2. Plot of $V_{C_0}(t)$

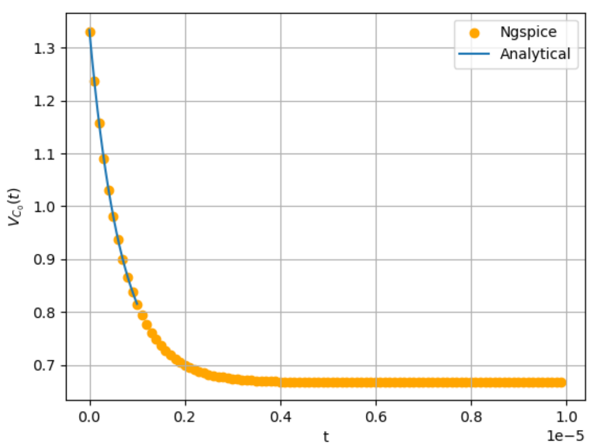

Fig. 3.3. ngspice plot of $V_{C_0}(t)$

Runningn the below code plots the figure 3.3, and verifies our result.

```
wget https://github.com/mukundareddy/Signal
    −Processing/tree/main/cktsig/codes/3.5.py
```

6. Find $v_{C_0}(0-), v_{C_0}(0+)$ and $v_{C_0}(\infty)$.
Solution: From the initial conditions,

$$v_{C_0}(0-) = \frac{q_1}{C} = \frac{4}{3}V \tag{3.10}$$

Using (3.9),

$$v_{C_0}(0+) = \lim_{t\to 0+} v_{C_0}(t) = \frac{4}{3}V \tag{3.11}$$

$$v_{C_0}(\infty) = \lim_{t\to\infty} v_{C_0}(t) = \frac{2}{3}V \tag{3.12}$$

7. Obtain Fig. 3.2 using the equivalent differential equation
Solution: Using Kirchoff's junction law

$$\frac{v_c(t)-0}{R_1} + \frac{v_c(t)-v_2(t)}{R_2} + \frac{\mathrm{d}q}{\mathrm{d}t} = 0 \tag{3.13}$$

where $q(t)$ is the charge on the capacitor. On taking the Laplace transform on both sides of the equation (3.13), we get,

$$\frac{V_c(s)-0}{R_1} + \frac{V_c(s)-V_2(s)}{R_2} + sQ(s) - q(0^-) = 0 \tag{3.14}$$

But $q(0^-) = \frac{4}{3}C_0$ and

$$q(t) = C_0 v_c(t) \tag{3.15}$$

$$\implies Q(s) = C_0 V_c(s) \tag{3.16}$$

Thus

$$\frac{V_c(s)-0}{R_1} + \frac{V_c(s)-V_2(s)}{R_2} + \left(sC_0V_c(s) - \frac{4}{3}C_0\right) = 0 \tag{3.17}$$

$$\implies \frac{V_c(s)-0}{R_1} + \frac{V_c(s)-V_2(s)}{R_2} + \frac{V_c(s)-\frac{4}{3s}}{\frac{1}{sC_0}} = 0 \tag{3.18}$$

which is the same equation as the one we obtained from Fig. 3.2

4. Bilinear Transform

4.1. In Fig. 2.1, consider the case when S is switched to Q right in the beginning. Formulate the differential equation
Solution: Considering KCL on the circuit 4.1,

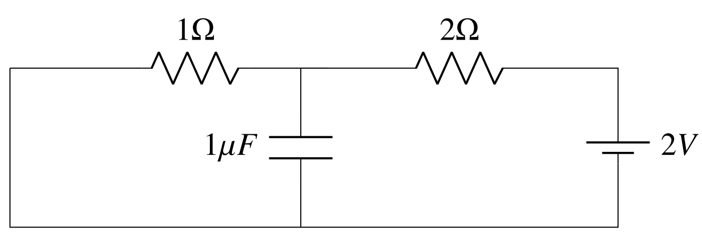

Fig. 4.1. Switch S connected to Q initially

we get the differential equuation as

$$\frac{v_c(t)-0}{R_1} + \frac{v_c(t)-v_2(t)}{R_2} + \frac{\mathrm{d}q}{\mathrm{d}t} = 0 \tag{4.1}$$

$$\implies \frac{v_c(t)}{R_1} + \frac{v_c(t)-v_2(t)}{R_2} + C_0\frac{\mathrm{d}v_c}{\mathrm{d}t} = 0 \tag{4.2}$$

Here we have $q(0) = 0$, since initially the capacitor is uncharged.

4.2. Find $H(s)$ considering the ouput voltage at the capacitor

Solution: Applying laplace transform to the equation (4.1), we get

$$\frac{V_c(s)}{R_1} + \frac{V_c(s) - V_2(s)}{R_2} + \mathcal{L}(\frac{dq}{dt}) = 0 \tag{4.3}$$

$$\frac{V_c(s)}{R_1} + \frac{V_c(s) - V_2(s)}{R_2} + sQ(s) - q(0) = 0 \tag{4.4}$$

$$\frac{V_c(s)}{R_1} + \frac{V_c(s) - V_2(s)}{R_2} + sQ(s) - 0 = 0 \tag{4.5}$$

$$\implies V_c(s)\left(\frac{1}{R_1} + \frac{1}{R_2}\right) + sC_0V_c(s) = \frac{V_2(s)}{R_2} \tag{4.6}$$

$$\implies \frac{V_c(s)}{V_2(s)} = \frac{\frac{1}{R_2}}{\frac{1}{R_1} + \frac{1}{R_2} + sC_0} \tag{4.7}$$

Here, $Q(s)$ is the laplace transform of q, $V_c(s)$ is laplace transform of $v_c(t)$. Hence, the transform function($H(s)$) is

$$H(s) = \frac{V_c(s)}{V_2(s)} \tag{4.8}$$

$$= \frac{\frac{1}{R_2C_0}}{s + \frac{1}{R_1C_0} + \frac{1}{R_2C_0}} \tag{4.9}$$

Substituting values of $R_1 = 1\Omega$,$R_2 = 2\Omega$ and $C_0 = 1\mu F$, we get,

$$H(s) = \frac{0.5}{s10^{-6} + 1.5} \tag{4.10}$$

$$\implies H(s) = \frac{5 \times 10^5}{s + 1.5 \times 10^6} \tag{4.11}$$

The following python code plots the figure 4.2

```
wget https://github.com/mukundareddy/Signal
    -Processing/tree/main/cktsig/codes/4.2.py
```

4.3. Plot $H(s)$. What kind of filter is it?

Solution: THe below python code plots the Figure 4.3

```
wget https://github.com/mukundareddy/Signal
    -Processing/tree/main/cktsig/codes/4.3.py
```

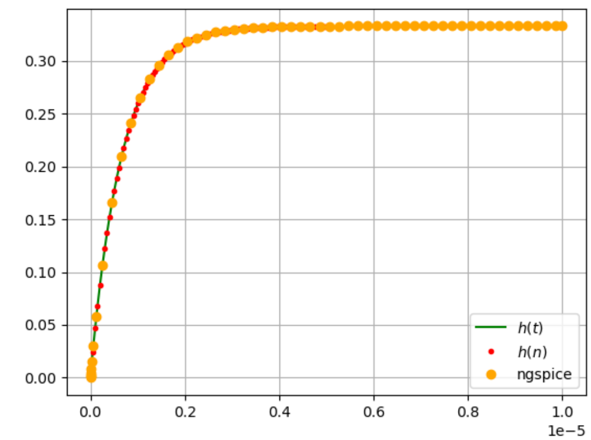

Fig. 4.2. ngspice plot of $H(t)$

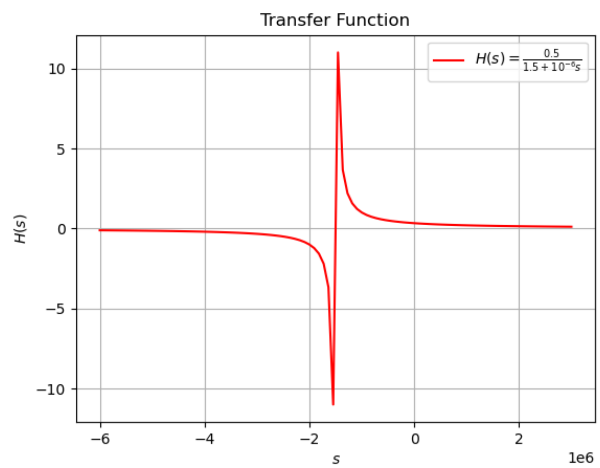

Fig. 4.3. Plot of $H(s)$

Considering the frequency-domain transfer function ($H(s = e^{j\omega})$), from (4.11),we get

$$H(s = j\omega) = \frac{5 \times 10^5}{j\omega + 1.5 \times 10^6} \tag{4.12}$$

$$\implies |H(s = j\omega)| = \frac{5 \times 10^5}{\sqrt{\omega^2 + 2.25 \times 10^{12}}} \tag{4.13}$$

Clearly from (4.12), as ω increases, $H(s = j\omega)$ decreases(inverse proportionality). When high frequency signals(large values of ω) pass through this transfer function ($H(s = j\omega)$), they become negligible, which results in removing high freqnecy signals and allowing only low frequency signal to pass. Hence, this is a low-pass filter.

4.4. Using trapezoidal rule for integration, formulate the difference equation by considering

$$y(n) = y(t)|_{t=n} \tag{4.14}$$

Solution: In the equation (4.1), we have

$$\frac{dq}{dt} = C_0\frac{dv_c}{dt} \tag{4.15}$$

$$v_2(t) = 2u(n) \tag{4.16}$$

Hence,

$$\frac{v_c(t)}{R_1} + \frac{v_c(t) - v_2(t)}{R_2} + C_0\frac{dv_c}{dt} = 0 \tag{4.17}$$

$$\implies C_0\frac{dv_c}{dt} = \frac{2u(t) - v_c(t)}{R_2} - \frac{v_c(t)}{R_1} \tag{4.18}$$

$$\implies \frac{dv_c}{dt} = \frac{2u(t) - v_c(t)}{R_2C_0} - \frac{v_c(t)}{R_1C_0} \tag{4.19}$$

$$\implies v_c(t)|_{t=n}^{n+1} = \int_n^{n+1}\left(\frac{2u(t) - v_c(t)}{R_2C_0} - \frac{v_c(t)}{R_1C_0}\right)dt \tag{4.20}$$

From trapezoidal rule of integration

$$\int_a^b f(t)dt \approx \frac{b-a}{2}(f(a) + f(b)) \tag{4.21}$$

Apply (4.21), to the RHS of the equation (4.20), we get,

$$\int_n^{n+1} \frac{2u(t) - v_c(t)}{R_2C_0} - \frac{v_c(t)}{R_1C_0}dt = \frac{1}{R_2C_0}(u(n) + u(n+1)) \tag{4.22}$$

$$-\frac{1}{2}(y(n+1) + y(n))\left(\frac{1}{R_1C_0} + \frac{1}{R_2C_0}\right) \tag{4.23}$$

Considering $y(t) = v_c(t)$, from (4.20), we get,

$$y(n+1) - y(n) = \frac{1}{R_2C_0}(u(n) + u(n+1))$$
$$-\frac{1}{2}(y(n+1) + y(n))\left(\frac{1}{R_1C_0} + \frac{1}{R_2C_0}\right) \tag{4.24}$$

Thus, the difference equation is

$$\implies y(n+1)\left(1 + \frac{1}{2R_1C_0} + \frac{1}{2R_2C_0}\right)$$
$$= y(n)\left(1 - \frac{1}{2R_1C_0} - \frac{1}{2R_2C_0}\right)$$
$$+ \frac{1}{R_2C_0}(u(n) + u(n+1)) \tag{4.25}$$

4.5. Find $H(z)$

Solution: Let $\mathcal{Z}\{y(n)\} = Y(z)$
On taking $\mathcal{Z}$-transform on both sides of the difference equation, we get,

$$zY(z)\left(1 + \frac{1}{2R_1C_0} + \frac{1}{2R_2C_0}\right)$$
$$= Y(z)\left(1 - \frac{1}{2R_1C_0} - \frac{1}{2R_2C_0}\right)$$
$$+ \frac{1}{R_2C_0}\left(\frac{1}{1-z^{-1}} + \frac{z}{1-z^{-1}}\right) \tag{4.26}$$

$$Y(z)\left(z + \frac{z}{2R_1C_0} + \frac{z}{2R_2C_0} - 1 + \frac{1}{2R_1C_0} + \frac{1}{2R_2C_0}\right)$$
$$= \frac{1}{R_2C_0}\frac{1+z}{1-z^{-1}} \tag{4.27}$$

Here, since $v_2(t) = 2\forall t \geq 0$

Initial voltage is given as,

$$\implies x(n) = 2u(n) \tag{4.28}$$

$$\implies X(z) = \frac{2}{1-z^{-1}} \qquad |z| > 1 \tag{4.29}$$

Thus, the transfer function in z-domain is

$$H(z) = \frac{Y(z)}{X(z)} \tag{4.30}$$

$$= \frac{\frac{1+z}{2R_2C_0}}{z + \frac{z}{2R_1C_0} + \frac{z}{2R_2C_0} - 1 + \frac{1}{2R_1C_0} + \frac{1}{2R_2C_0}} \tag{4.31}$$

$$= \frac{\frac{1+z^{-1}}{2R_2C_0}}{1 + \frac{1}{2R_1C_0} + \frac{1}{2R_2C_0} - z^{-1} + \frac{z^{-1}}{2R_1C_0} + \frac{z^{-1}}{2R_2C_0}} \tag{4.32}$$

Substituting the values of R_1,R_2 and C_0, we get,

$$H(z) = \frac{2.5 \times 10^5(1 + z^{-1})}{7.5 \times 10^5 + 1 + (7.5 \times 10^5 - 1)z^{-1}} \tag{4.33}$$

Where, ROC of $H(z)$ is,

$$|z| > 1 \cap |z| > \left|\frac{7.5 \times 10^5 - 1}{7.5 \times 10^5 + 1}\right| \tag{4.34}$$

$$\implies |z| > 1 \tag{4.35}$$

4.6. How can you obtain $H(z)$ from $H(s)$?

Solution: The Z-transform can be obtained from the Laplace transform by the substitution

$$s = \frac{2}{T}\frac{1-z^{-1}}{1+z^{-1}} \tag{4.36}$$

where T is the sampling time period, used in the trapezoidal rule. Here, its value is 1. This known as the bilinear transform.
From (4.20), we have,

$$H(z) = \frac{\frac{1}{R_2C_0}}{2\frac{1-z^{-1}}{1+z^{-1}} + \frac{1}{R_1C_0} + \frac{1}{R_2C_0}} \tag{4.37}$$

$$= \frac{\frac{1+z^{-1}}{2R_2C_0}}{1 - z^{-1} + \left(\frac{1}{2R_1C_0} + \frac{1}{2R_2C_0}\right)(1+z^{-1})} \tag{4.38}$$

$$= \frac{\frac{1+z^{-1}}{2R_2C_0}}{1 + \frac{1}{2R_1C_0} + \frac{1}{2R_2C_0} - z^{-1} + \frac{z^{-1}}{2R_1C_0} + \frac{z^{-1}}{2R_2C_0}} \tag{4.39}$$

$$= \frac{2.5\times 10^5(1+z^{-1})}{7.5\times 10^5 + 1 + (7.5\times 10^5 - 1)z^{-1}} \tag{4.40}$$

Here, this result obtained in the equation (4.40) is same as the result we obtained in (4.33)

4.7. Find $y(n)$ from $H(z)$ and verify whether $y(n) = y(t)|_{t=n}$

Solution: We know that,

$$Y(z) = H(z)X(z) \tag{4.41}$$

$$= \left(\frac{2.5\times 10^5(1+z^{-1})}{7.5\times 10^5 + 1 + (7.5\times 10^5 - 1)z^{-1}}\right)\frac{2}{1-z^{-1}} \tag{4.42}$$

$$= \frac{\frac{2}{3}}{1-z^{-1}} - \frac{\frac{2}{3}}{7.5\times 10^5 + 1 + (7.5\times 10^5 - 1)z^{-1}} \tag{4.43}$$

Let ROC be $|z| > 1$, we know that,

$$\frac{1}{1-z^{-1}} \xleftrightarrow{\mathcal{Z}} u(n) \tag{4.44}$$

$$\frac{1}{1-az^{-1}} \xleftrightarrow{\mathcal{Z}} a^n u(n) \tag{4.45}$$

Applying inverse $\mathcal{Z}$ to the equation (4.43), we get

$$y(n) = \frac{2}{3}u(n) - \frac{2}{3}\frac{1}{7.5\times 10^5 + 1}\left(-\frac{7.5\times 10^5 - 1}{7.5\times 10^5 + 1}\right)^n u(n) \tag{4.46}$$

$$= \frac{2}{3}\left(1 - \frac{(1-7.5\times 10^5)^n}{(1+7.5\times 10^5)^{n+1}}\right)u(n) \tag{4.47}$$

Applying binomial theorem to the equation (4.47), ($(1+x)^n \approx 1 + nx$ for $x \ll 1$), we get,

$$y(n) \approx \frac{2}{3}\left(1 - \frac{1-7.5\times 10^5 n}{1+7.5\times 10^5 n}\right)u(n) \tag{4.48}$$

Now, consider $Y(s)$,

$$Y(s) = H(s)X(s) \tag{4.49}$$

$$= \frac{5\times 10^5}{s + 1.5\times 10^6}\frac{2}{s} \tag{4.50}$$

$$= \frac{10^6}{1.5\times 10^6}\left(\frac{1}{s} - \frac{1}{s + 1.5\times 10^6}\right) \tag{4.51}$$

We know that , Let ROC be $|z| > 1$, we know that,

$$\frac{1}{s} \xleftrightarrow{\mathcal{L}} 1 \quad \Re(s) > 0 \tag{4.52}$$

$$\frac{1}{s+a} \xleftrightarrow{\mathcal{L}} e^{-at} \quad \Re(s) > -a \tag{4.53}$$

Consider ROC as $\Re(s) > 0$, applying inverse laplace transform to the equation (4.51), we get,

$$y(t) = \frac{2}{3}\left(1 - e^{-1.5\times 10^6 t}\right)u(t) \tag{4.54}$$

But for $t \ll 10^-6$, we have

$$e^{-1.5\times 10^6 t} = \frac{e^{-0.75\times 10^6 t}}{e^{0.75\times 10^6 t}} \tag{4.55}$$

$$\approx \frac{1-7.5\times 10^5 t}{1+7.5\times 10^5 t} \tag{4.56}$$

Therefore

$$y(t) \approx \frac{2}{3}\left(1 - \frac{1-7.5\times 10^5 t}{1+7.5\times 10^5 t}\right)u(t) \tag{4.57}$$

From equations (4.48) and (4.57), we have,

$$\therefore y(n) = y(t)|_{t=n} \tag{4.58}$$

Hence verified. The following python plots the graph 4.4 of $V_C(t)$.

```
wget https://github.com/mukundareddy/Signal
    -Processing/tree/main/cktsig/codes/4.7.py
```

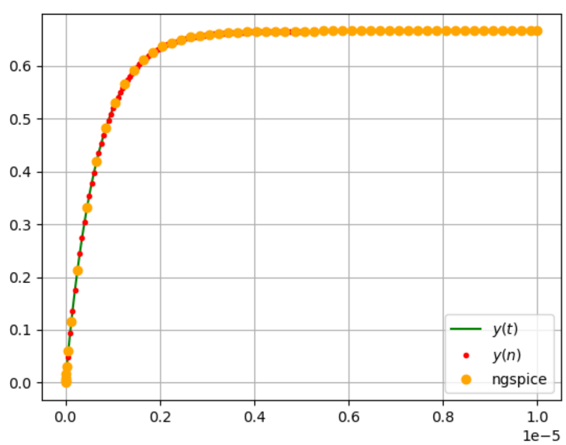

Fig. 4.4. Plot of $y(t)$,$y(n)$ and ngspice